# On the fundamental mathematical constants $\pi$, $\phi$, ζ(2), ζ(6), ζ(8) and ζ(10): new interesting mathematical connections

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## Abstract

*In this research thesis, we have described the new possible mathematical connections between the following fundamental mathematical constants: π, ϕ, ζ(2), ζ(6), ζ(8) and ζ(10)*

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$$\frac{1}{(\sqrt{\phi\sqrt{5}})e^{2\pi/5}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \cfrac{e^{-10\pi}}{1 + \dots}}}}}$$

https://twitter.com/pickover/status/841670962034724865

**Mathematical connections between $\pi$, $\sqrt{n}$ and ζ(2), ζ(6), ζ(8), ζ(10)**

(((1 + 1/(-sqrt(2) + 1/(2 π))^2)))))

**Input:**

$$1 + \frac{1}{\left(-\sqrt{2} + \frac{1}{2\pi}\right)^2}$$

**Decimal approximation:**

1.634851249576022573393053114259819243054710568277478224956...

1.634851249…. that is a golden number

**Property:**

$1 + \frac{1}{\left(-\sqrt{2} + \frac{1}{2\pi}\right)^2}$ is a transcendental number

**Series representations:**

$$1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}=1+\frac{4\pi^2}{\left(1-2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

$$1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}=1+\frac{4\pi^2}{\left(1-2\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}=$$
$$1+\frac{1}{\left(\frac{1}{2\pi}-\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}$$

(((1.0061571663^2*1 + 1/(-sqrt(2) + 1/(2 π))^2)))))

**Input interpretation:**

$$1.0061571663^2\times 1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}$$

**Result:**

1.6472034929...

1.6472034929.... ≈ ζ(2) = $\frac{\pi^2}{6}=1.644934$ ...

**Series representations:**

$$1.00615716630000^2\times 1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}=$$
$$1.01235224329685+\frac{4\pi^2}{\left(1-2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

$$1.00615716630000^2\times 1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}=1.01235224329685+$$
$$\frac{4\pi^2}{\left(1-2\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$ for $(x\in\mathbb{R}$ and $x<0)$

$$1.00615716630000^2 \times 1 + \frac{1}{\left(-\sqrt{2} + \frac{1}{2\pi}\right)^2} = 1.01235224329685 + \frac{1}{\left(\frac{1}{2\pi} - \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2 (1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)^2}$$

2((((sqrt(((6*(((1.0061571663^2*1 + 1/(-sqrt(2) + 1/(2 π))^2)))))

**Input interpretation:**

$$2\sqrt{6\left(1.0061571663^2 \times 1 + \frac{1}{\left(-\sqrt{2} + \frac{1}{2\pi}\right)^2}\right)}$$

**Result:**

6.2875180977...

6.2875180977.... ≈ $2\pi$

**Series representations:**

$$2\sqrt{6\left(1.00615716630000^2 \times 1 + \frac{1}{\left(-\sqrt{2} + \frac{1}{2\pi}\right)^2}\right)} = 2\sqrt{5.0741134597811 + \frac{24\pi^2}{\left(1 - 2\pi\sqrt{2}\right)^2}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(5.0741134597811 + \frac{24\pi^2}{\left(1 - 2\pi\sqrt{2}\right)^2}\right)^{-k}$$

$$2\sqrt{6\left(1.00615716630000^2 \times 1 + \frac{1}{\left(-\sqrt{2} + \frac{1}{2\pi}\right)^2}\right)} = 2\sqrt{5.0741134597811 + \frac{24\pi^2}{\left(1 - 2\pi\sqrt{2}\right)^2}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(5.0741134597811 + \frac{24\pi^2}{\left(1-2\pi\sqrt{2}\right)^2}\right)^{-k}}{k!}$$

$$2\sqrt{6\left(1.00615716630000^2\times 1+\frac{1}{\left(-\sqrt{2}+\frac{1}{2\pi}\right)^2}\right)}=$$

$$2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(6.0741134597811+\frac{24\pi^2}{\left(1-2\pi\sqrt{2}\right)^2}-z_0\right)^k z_0^{-k}}{k!}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

(((Pi/(sqrt(2*3)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{2\times 3}}\right)^2$$

**Result:**

$$\frac{\pi^2}{6}$$

**Decimal approximation:**

1.644934066848226436472415166646025189218949901206798437735...

1.64493406.... = ζ(2)

**Integral representations:**

$$\left(\frac{\pi}{\sqrt{2\times 3}}\right)^2=\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\left(\frac{\pi}{\sqrt{2\times 3}}\right)^2=\frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2$$

$$\left(\frac{\pi}{\sqrt{2\times 3}}\right)^2=\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

(((Pi/(sqrt(6)))^2 * ((Pi/(sqrt(15)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\frac{\pi}{\sqrt{15}}\right)^2$$

**Result:**

$$\frac{\pi^4}{90}$$

**Decimal approximation:**

1.082323233711138191516003696541167902774750951918726907682...

1.082323…

**Property:**

$\frac{\pi^4}{90}$ is a transcendental number

**Series representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 = \sum_{k=1}^{\infty} \frac{1}{k^4}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 = \frac{16}{15} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^4}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 = \frac{128}{45} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^4$$

**Integral representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 = \frac{8}{45} \left(\int_0^{\infty} \frac{1}{1+t^2}\,dt\right)^4$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 = \frac{128}{45} \left(\int_0^{1} \sqrt{1-t^2}\,dt\right)^4$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 = \frac{8}{45} \left(\int_0^{1} \frac{1}{\sqrt{1-t^2}}\,dt\right)^4$$

(((Pi/(sqrt(6)))^2 * ((Pi/(sqrt(15)))^2 * ((Pi/(sqrt((10+11)/2))))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\right)$$

**Result:**

$\frac{\pi^6}{945}$

**Decimal approximation:**

1.017343061984449139714517929790920527901817490032853561842...

1.01734306…. the reciprocal of this result is 1/1.01734306… = 0.9829525922… that is very near to the dilaton value **0. 989117352243 = $\phi$**

**Property:**

$\frac{\pi^6}{945}$ is a transcendental number

**Series representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 = \sum_{k=1}^{\infty} \frac{1}{k^6}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 = \frac{64}{63} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^6}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 = \frac{4096}{945} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^6$$

**Integral representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 = \frac{64}{945} \left(\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)^6$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 = \frac{64}{945} \left(\int_0^{1} \frac{1}{\sqrt{1-t^2}}\, dt\right)^6$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\frac{\pi}{\sqrt{15}}\right)^2\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 = \frac{4096}{945}\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^6$$

(((Pi/(sqrt(6)))^2 * ((Pi/(sqrt(15)))^2 * ((Pi/(sqrt((10+11)/2))))^2 * ((Pi/(sqrt(10)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\left(\frac{\pi}{\sqrt{15}}\right)^2\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\left(\frac{\pi}{\sqrt{10}}\right)^2\right)$$

**Result:**

$$\frac{\pi^8}{9450}$$

**Decimal approximation:**

1.004077356197944339378685238508652465258960790649850020329...

1.004077356…. the reciprocal of this result is 1/1.004077356… = 0.9959392… that is very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

**Property:**

$\frac{\pi^8}{9450}$ is a transcendental number

**Series representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\frac{\pi}{\sqrt{15}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\left(\frac{\pi}{\sqrt{10}}\right)^2\right) = \frac{32\,768\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^8}{4725}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{10}}\right)^2\right) = \frac{\left(\sum_{k=0}^{\infty} -\frac{4(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}\right)^8}{9450}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{10}}\right)^2\right) = \frac{\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^8}{9450}$$

**Integral representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{10}}\right)^2\right) = \frac{128\left(\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)^8}{4725}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{10}}\right)^2\right) = \frac{32768\left(\int_0^{1} \sqrt{1-t^2}\, dt\right)^8}{4725}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{10}}\right)^2\right) = \frac{128\left(\int_0^{1} \frac{1}{\sqrt{1-t^2}}\, dt\right)^8}{4725}$$

(((Pi/(sqrt(6)))^2 * ((Pi/(sqrt(15)))^2 * ((Pi/(sqrt((10+11)/2))))^2 * ((Pi/(sqrt(10)))^2 * ((Pi/(sqrt(9+9/10)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\left(\frac{\pi}{\sqrt{10}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right)$$

**Result:**

$$\frac{\pi^{10}}{93555}$$

**Decimal approximation:**

1.000994575127818085337145958900319017006019531564477517257...

1.000994575…. the reciprocal of this result is 1/1.000994575… = 0.999006413… that is practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

**Property:**

$\frac{\pi^{10}}{93555}$ is a transcendental number

**Series representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\frac{\pi}{\sqrt{15}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\left(\left(\frac{\pi}{\sqrt{10}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1048576\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\frac{\pi}{\sqrt{15}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\left(\left(\frac{\pi}{\sqrt{10}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2\left(\frac{\pi}{\sqrt{15}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\left(\left(\frac{\pi}{\sqrt{10}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{10}}{93555}$$

**Integral representations:**

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\left(\frac{\pi}{\sqrt{10}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1024\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\left(\frac{\pi}{\sqrt{10}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1024\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\left(\frac{\pi}{\sqrt{10}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1048576\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^{10}}{93555}$$

Or:

(((Pi/(sqrt((10+2)/2)))^2 * ((Pi/(sqrt((10+20)/2)))^2 * ((Pi/(sqrt((10+11)/2))))^2 * ((Pi/(sqrt((10+10)/2)))^2 *  ((Pi/(sqrt(9+9/10)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{\frac{10+2}{2}}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+20}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{10+10}{2}}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right)$$

**Result:**

$$\frac{\pi^{10}}{93555}$$

**Decimal approximation:**

1.000994575127818085337145958900319017006019531564477517257...

**Property:**

$\frac{\pi^{10}}{93555}$ is a transcendental number

Or:

((Pi/(sqrt((1+3+8)/2)))^2 * ((Pi/(sqrt((1+8+21)/2)))^2 * ((Pi/(sqrt((3+5+13)/2)))^2 * ((Pi/(sqrt((2+5+13)/2)))^2 *  ((Pi/(sqrt(9+9/10)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right)\right)$$

**Result:**

$$\frac{\pi^{10}}{93555}$$

**Decimal approximation:**

1.000994575127818085337145958900319017006019531564477517257...

- **Property:**

$\frac{\pi^{10}}{93555}$ is a transcendental number

**Series representations:**

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1048576 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{\left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} \left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{10}}{93555}$$

## Integral representations:

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1024\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1024\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^{10}}{93555}$$

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right) = \frac{1048576\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^{10}}{93555}$$

((Pi/(sqrt(9+9/10)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2$$

**Result:**

$$\frac{10\pi^2}{99}$$

**Decimal approximation:**

0.996929737483773597862069797967287993466030243155635416809...

0.9969297…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

**Property:**

$\frac{10\pi^2}{99}$ is a transcendental number

**Series representations:**

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2=\frac{20}{33}\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2=-\frac{40}{33}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2 = \frac{80}{99}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2 = \frac{40}{99}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2 = \frac{160}{99}\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^2$$

$$\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2 = \frac{40}{99}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

sqrt(((((((( 9.9 * ((Pi/(sqrt(9+9/10)))^2)))))))

**Input:**

$$\sqrt{9.9\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2}$$

**Result:**

3.1415926535897932384626433832795028841971693993751058209 74...

3.14159265... = $\pi$

**Series representations:**

$$\sqrt{9.9\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2} = \sqrt{-1+\frac{9.9\,\pi^2}{\sqrt{\frac{99}{10}}^2}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+\frac{9.9\,\pi^2}{\sqrt{\frac{99}{10}}^2}\right)^{-k}$$

$$\sqrt{9.9\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2} = \sqrt{-1+\frac{9.9\,\pi^2}{\sqrt{\frac{99}{10}}^2}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+\frac{9.9\pi^2}{\sqrt{\frac{99}{10}}^2}\right)^{-k}}{k!}$$

•

$$\sqrt{9.9\left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2} = \sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{9.9\pi^2}{\sqrt{\frac{99}{10}}^2}-z_0\right)^k z_0^{-k}}{k!}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

**With regard the golden ratio and the mathematical connection with $\pi$, we have the following interesting expression:**

2+(((sqrt(5))+1)/2))*1/(sqrt(2))) + 2.21+(((sqrt(5))+1)/2))*1/(sqrt(3))) + 2.42+(((sqrt( 5))+1)/2))*1/(sqrt(5)))+ 2.53+(((sqrt(5))+1)/2))*1/(sqrt(7)))

**Input:**

$$2+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{2}}+2.21+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{3}}+2.42+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{5}}+2.53+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{7}}$$

**Result:**

12.5735…

12.5735… result very near to the black hole entropy 12.5664

**Input interpretation:**

$$\frac{12.5735}{4}$$

**Result:**

3.143375

$3.143375 \approx \pi$

2.57+(((sqrt(5))+1)/2))*1/(sqrt(8))) + 2.61+(((sqrt(5))+1)/2))*1/(sqrt(9)))+2.63+(((sqrt(5))+1)/2))*1/(sqrt(10)))+2.66+(((sqrt(5))+1)/2))*1/(sqrt(11)))

**Input:**

$$2.57+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{8}}+2.61+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{9}}+2.63+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{10}}+2.66+\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)\times\frac{1}{\sqrt{11}}$$

**Result:**

12.5809…

12.5809…

**Input interpretation:**

$$\frac{12.5809}{4}$$

**Result:**

3.145225

3.145225

(2.57+2.61+2.63+2.66)/4

**Input:**

$$\frac{1}{4}(2.57+2.61+2.63+2.66)$$

**Result:**

2.6175

2.6175

sqrt((((2.57+2.61+2.63+2.66)/4)))

**Input:**

$$\sqrt{\frac{1}{4}(2.57+2.61+2.63+2.66)}$$

**Result:**

1.617868968736343616999828034807702227323822820048926479819…

1.61786896873634……

This result is a very good approximation to the value of the golden ratio 1,618033988749...

From:

**Exact Renormalization Group Equations.**
**An Introductory Review.**

C. Bagnuls*and C. Bervillier†
C. E. Saclay, F91191 Gif-sur-Yvette Cedex, France
February 1, 2008

#### 4.1.2 Reparametrization invariance linearly realized and preserved

With a view to control the preservation of the reparametrization invariance, one may impose it evidently, i.e. linearly, via a particular choice of cutoff function and try to keep this realization through the derivative expansion [30]. This is what has been done in [22] for the Legendre version of the ERGE (see below). For the smooth cutoff version of the ERGE, the only acceptable cutoff function is power-law like [68, 30] (otherwise the cutoff should be sharp [68, 48]). Unfortunately, for the Polchinski version, the symmetry is broken at finite order in the derivative expansion and the regulators do not regulate, at least not in a finite order in the derivative expansion [33, 30, 68, 38, 48, 104]. Now considering the Legendre version of the ERGE of section 2.6 is sufficient to overcome this difficulty [33, 68, 48, 104, 38].

**The smooth cutoff Legendre version and the derivative expansion** By choosing a power-law cutoff function $\tilde{C}(q^2) = q^{2k}$ in eq. (39), one is sure that the derivative expansion will preserve the reparametrization invariance [33, 22] and that the exponent $\eta$ will be unambiguously defined.

Let us expand the Legendre (effective) action $\Gamma[\Phi]$ as follows:

$$\Gamma[\Phi] = \int d^d x \left\{ U(\varphi, t) + \frac{1}{2} Z(\varphi, t)(\partial_\mu \Phi)^2 \right\}$$

in which $\varphi$ is independent on $x$.

For $d = 3$ and $k = 1$, the first order of the derivative expansion yields (after a long but straightforward computation) the following two coupled equations for $U$ and $Z$ [22]:

$$\begin{aligned}\dot{U} &= \frac{1-\eta/4}{\sqrt{Z}\sqrt{U''+2\sqrt{Z}}} + 3U - \frac{1}{2}(1+\eta)\varphi U' \\ \dot{Z} &= -\frac{1}{2}(1+\eta)\varphi Z' - \eta Z + \left(1-\frac{\eta}{4}\right)\left\{\frac{1}{48}\frac{24ZZ''-19(Z')^2}{Z^{3/2}(U''+2\sqrt{Z})^{3/2}}\right. \\ &\left. -\frac{1}{48}\frac{58U'''Z'\sqrt{Z}+57(Z')^2+(Z''')^2 Z}{Z(U''+2\sqrt{Z})^{5/2}} + \frac{5}{12}\frac{(U''')^2 Z + 2U'''Z'\sqrt{Z}+(Z')^2}{\sqrt{Z}(U''+2\sqrt{Z})^{7/2}}\right\} \qquad (87)\end{aligned}$$

As expected, the search for a non trivial fixed point solution for these equations (a solution which is nonsingular up to $\varphi \to \infty$) produces a unique solution with an unambiguously defined $\eta$ [22]:

$$\eta = 0.05393 \qquad (88)$$

The linearization about this fixed point yields the eigenvalues:

$$\nu = 0.6181 \qquad (89)$$

$$\omega = 0.8975 \qquad (90)$$

and also a zero eigenvalue $\lambda = 0$ [22] which corresponds to the redundant operator $\mathcal{O}_1$ [eq. (24)] responsible for the moving along the line of equivalent fixed points. This is, of course, an expected confirmation of the preservation of the reparametrization invariance.

We note that:

1/ν = 1/ 0.6181 = 1,617861187510……

And, from the previous formula:

$$\sqrt{\frac{1}{4}(2.57+2.61+2.63+2.66)}$$

$= 1.6178689687363436169998280348077022273238228200489264798 19...$

1.617868968...

A very good mathematical connection!

## Mathematical connections with ζ(12) and ζ(14)

Now, we have that from:

((Pi/(sqrt((1+3+8)/2)))^2 * ((Pi/(sqrt((1+8+21)/2)))^2 * ((Pi/(sqrt((3+5+13)/2)))^2 * ((Pi/(sqrt((2+5+13)/2)))^2 * ((Pi/(sqrt(9+9/10)))^2

**Input:**

$$\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2 \left(\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2 \left(\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2 \left(\left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(2+5+13)}}\right)^2 \left(\frac{\pi}{\sqrt{9+\frac{9}{10}}}\right)^2\right)\right)\right)\right)\right)\right)$$

**Result:**

$$\frac{\pi^{10}}{93555}$$

we obtain:

π^10/93555 ((((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2

**Input:**

$$\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2$$

**Result:**

$$\frac{691\,\pi^{12}}{638512875}$$

**Decimal approximation:**

1.000246086553308048298637998047739670960416088458003404533...

1.0002460865…. the reciprocal of this result is 1/1.0002460865…. = 0,99975397399 result practically equal to the value of the following Rogers-Ramanujan mock theta function:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

**Property:**

$\frac{691\,\pi^{12}}{638512875}$ is a transcendental number

((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [((( sqrt(2*35)*Pi)^2 / (24^2+23*5)))]

**Input:**

$$\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\times\frac{(\sqrt{2\times 35}\,\pi)^2}{24^2+23\times 5}$$

**Result:**

$$\frac{2\,\pi^{14}}{18243225}$$

**Decimal approximation:**

1.000061248135058704829258545105135333747481696169154549482...

1.000061248135….

**Property:**

$\frac{2\,\pi^{14}}{18243225}$ is a transcendental number

- **Series representations:**

$$\frac{\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2(\sqrt{2\times 35}\,\pi)^2\right)\pi^{10}}{(24^2+23\times 5)\,93555} = \frac{536870912\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{14}}{18243225}$$

$$\frac{\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\sqrt{2\times 35}\,\pi\right)^2\right)\pi^{10}}{(24^2+23\times 5)\,93\,555} = \frac{2\left(\sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{14}}{18\,243\,225}$$

•

$$\frac{\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\sqrt{2\times 35}\,\pi\right)^2\right)\pi^{10}}{(24^2+23\times 5)\,93\,555} = \frac{2\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{14}}{18\,243\,225}$$

**Integral representations:**

$$\frac{\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\sqrt{2\times 35}\,\pi\right)^2\right)\pi^{10}}{(24^2+23\times 5)\,93\,555} = \frac{32\,768\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{14}}{18\,243\,225}$$

$$\frac{\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\sqrt{2\times 35}\,\pi\right)^2\right)\pi^{10}}{(24^2+23\times 5)\,93\,555} = \frac{32\,768\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^{14}}{18\,243\,225}$$

•

$$\frac{\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\sqrt{2\times 35}\,\pi\right)^2\right)\pi^{10}}{(24^2+23\times 5)\,93\,555} = \frac{536\,870\,912\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^{14}}{18\,243\,225}$$

Or:

((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]

**Input:**

$$\frac{\pi^{10}}{93\,555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}\,(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2\times\frac{2}{1382}\right)$$

**Result:**

$$\frac{2\pi^{14}}{18243225}$$

**Decimal approximation:**

1.000061248135058704829258545105135333747481696169154549482...

1.000061248135…

**Property:**

$\frac{2\pi^{14}}{18243225}$ is a transcendental number

Now, we have that:

691*2 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]

**Input:**

$$691\times 2\times\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2\times\frac{2}{1382}\right)$$

**Result:**

$$\frac{2764\,\pi^{14}}{18243225}$$

**Decimal approximation:**

1382.084644922651130074035309335297031239019704105771587385...

1382.0846 result very near to the rest mass of Sigma baryon 1382.8

**Property:**

$\frac{2764\,\pi^{14}}{18243225}$ is a transcendental number

or, equivalently:

1382 * ((((π^10/93555)))) * ((((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]

**Input:**

$$1382 \times \frac{\pi^{10}}{93\,555} \left(\sqrt{24^2 + 23 \times 5} \times \frac{\pi}{\sqrt{\frac{1}{2}(21 \times 13 \times 25 \times 2)}}\right)^2 \left(\left(\sqrt{2 \times 35}\ \pi\right)^2 \times \frac{2}{1382}\right)$$

**Result:**

$$\frac{2764\,\pi^{14}}{18\,243\,225}$$

**Decimal approximation:**

1382.084644922651130074035309335297031239019704105771587385...

1382.0846449…. as above

**Property:**

$\frac{2764\,\pi^{14}}{18\,243\,225}$ is a transcendental number

**Series representations:**

$$\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\right)\left(\sqrt{2\times 35}\ \pi\right)^2\ 2}{93\,555\times 1382} = \frac{741\,955\,600\,384\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{14}}{18\,243\,225}$$

$$\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\right)\left(\sqrt{2\times 35}\ \pi\right)^2\ 2}{93\,555\times 1382} = \frac{2764\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{14}}{18\,243\,225}$$

•

$$\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times5}\;\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\right)\left(\sqrt{2\times35}\;\pi\right)^2\,2}{93555\times1382} = \frac{2764\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{14}}{18243225}$$

**Integral representations:**

$$\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times5}\;\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\right)\left(\sqrt{2\times35}\;\pi\right)^2\,2}{93555\times1382} = \frac{45285376\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{14}}{18243225}$$

$$\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times5}\;\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\right)\left(\sqrt{2\times35}\;\pi\right)^2\,2}{93555\times1382} = \frac{45285376\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^{14}}{18243225}$$

- $$\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times5}\;\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\right)\left(\sqrt{2\times35}\;\pi\right)^2\,2}{93555\times1382} = \frac{741955600384\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^{14}}{18243225}$$

2*1382 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]

**Input:**

$$2\times1382\times\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times5}\times\frac{\pi}{\sqrt{\frac{1}{2}\left(21\times13\times25\times2\right)}}\right)^2\left(\left(\sqrt{2\times35}\;\pi\right)^2\times\frac{2}{1382}\right)$$

**Result:**

$$\frac{5528\,\pi^{14}}{18243225}$$

**Decimal approximation:**

2764.169289845302260148070618670594062478039408211543174770...

2764.1692… result very near to the rest mass of charmed Omega baryon 2765.9

**Property:**

$\frac{5528\,\pi^{14}}{18\,243\,225}$ is a transcendental number

**Series representations:**

$$\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93\,555\times1382}=\frac{1\,483\,911\,200\,768\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{14}}{18\,243\,225}$$

$$\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93\,555\times1382}=\frac{5528\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{1+2k}\right)^{14}}{18\,243\,225}$$

- $$\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93\,555\times1382}=\frac{5528\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{14}}{18\,243\,225}$$

**Integral representations:**

$$\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93\,555\times1382}=\frac{90\,570\,752\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{14}}{18\,243\,225}$$

$$\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93\,555\times1382}=\frac{90\,570\,752\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^{14}}{18\,243\,225}$$

•

$$\frac{(2\pi^{10}\ 1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left((\sqrt{2\times 35}\ \pi)^2\ 2\right)}{93555\times 1382} = \frac{1483911200768\left(\int_0^1\sqrt{1-t^2}\ dt\right)^{14}}{18243225}$$

We have that:

-(32^2-5)+((((((((2*1382 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))])))))))))

**Input:**

$$-(32^2-5)+ 2\times 1382\times\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\ \pi\right)^2\times\frac{2}{1382}\right)$$

**Result:**

$$\frac{5528\,\pi^{14}}{18243225}-1019$$

**Decimal approximation:**

1745.169289845302260148070618670594062478039408211543174770...

1745.169289… This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

**Property:**

$-1019+\frac{5528\,\pi^{14}}{18243225}$ is a transcendental number

Or:

-1019+((((((((2*1382 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))])))))))))

where $1019 = 32^2 - 5$

**Input:**

$$-1019 + 2\times 1382\times \frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times \frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\ \pi\right)^2\times \frac{2}{1382}\right)$$

**Result:**

$$\frac{5528\,\pi^{14}}{18\,243\,225} - 1019$$

**Decimal approximation:**

1745.1692898453022601480706186705940624780394082115431747 70...

1745.169289…. as above

**Property:**

$-1019 + \frac{5528\,\pi^{14}}{18\,243\,225}$ is a transcendental number

**Series representations:**

$$-1019 + \frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\ \pi\right)^2\ 2\right)}{93555\times 1382} = -1019 + \frac{1\,483\,911\,200\,768\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{14}}{18\,243\,225}$$

$$-1019 + \frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\ \pi\right)^2\ 2\right)}{93555\times 1382} = -1019 + \frac{5528\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{14}}{18\,243\,225}$$

•

$$-1019 + \frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\ \pi\right)^2\ 2\right)}{93555\times 1382} = -1019 + \frac{5528\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{14}}{18\,243\,225}$$

**Integral representations:**

$$-1019+\frac{(2\pi^{10}\ 1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left((\sqrt{2\times 35}\ \pi)^2\ 2\right)}{93555\times 1382}=$$
$$-1019+\frac{90570752\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{14}}{18243225}$$

$$-1019+\frac{(2\pi^{10}\ 1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left((\sqrt{2\times 35}\ \pi)^2\ 2\right)}{93555\times 1382}=$$
$$-1019+\frac{90570752\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^{14}}{18243225}$$

- $$-1019+\frac{(2\pi^{10}\ 1382)\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left((\sqrt{2\times 35}\ \pi)^2\ 2\right)}{93555\times 1382}=$$
$$-1019+\frac{1483911200768\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^{14}}{18243225}$$

-1745+((((((((2*1382 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]))))))))

**Input:**

$$-1745+2\times 1382\times \frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times \frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\ \pi\right)^2\times \frac{2}{1382}\right)$$

**Result:**

$$\frac{5528\pi^{14}}{18243225}-1745$$

**Decimal approximation:**

1019.169289845302260148070618670594062478039408211543174770...

1019.1692... result very near to the rest mass of Phi meson 1019.445

**Property:**

$-1745+\frac{5528\,\pi^{14}}{18\,243\,225}$ is a transcendental number

**Series representations:**

$$-1745+\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)}{93\,555\times 1382}=-1745+\frac{1\,483\,911\,200\,768\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}\right)^{14}}{18\,243\,225}$$

$$-1745+\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)}{93\,555\times 1382}=-1745+\frac{5528\left(\sum_{k=0}^{\infty}-\frac{4\,(-1)^k\,1195^{-1-2\,k}\left(5^{1+2\,k}-4\times 239^{1+2\,k}\right)}{1+2\,k}\right)^{14}}{18\,243\,225}$$

•

$$-1745+\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)}{93\,555\times 1382}=-1745+\frac{5528\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2\,k}+\frac{2}{1+4\,k}+\frac{1}{3+4\,k}\right)\right)^{14}}{18\,243\,225}$$

**Integral representations:**

$$-1745+\frac{(2\,\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)}{93\,555\times 1382}=-1745+\frac{90\,570\,752\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{14}}{18\,243\,225}$$

$$-1745+\frac{(2\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93555\times1382}=$$
$$-1745+\frac{90570752\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^{14}}{18243225}$$

- $$-1745+\frac{(2\pi^{10}\,1382)\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{93555\times1382}=$$
$$-1745+\frac{1483911200768\left(\int_0^1\sqrt{1-t^2}\,dt\right)^{14}}{18243225}$$

1/9.9*((((((((1382 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]))))))))

**Input:**

$$\frac{1}{9.9}\left(1382\times\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times5}\times\frac{\pi}{\sqrt{\frac{1}{2}\,(21\times13\times25\times2)}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\times\frac{2}{1382}\right)\right)$$

**Result:**

139.605...

139.605... result very near to the rest mass of Pion meson 139.57

**Series representations:**

$$\frac{1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)}{(93555\times1382)\,9.9}=\frac{0.00149213\,\pi^{14}\,\sqrt{69}^2\left(\sum_{k=0}^{\infty}69^{-k}\binom{\frac{1}{2}}{k}\right)^2}{\sqrt{6824}^2\left(\sum_{k=0}^{\infty}6824^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2 2\right)}{(93555\times 1382)\,9.9} = \frac{0.00149213\,\pi^{14}\,\sqrt{69}^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{69}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}{\sqrt{6824}^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6824}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

- $$\frac{1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2 2\right)}{(93555\times 1382)\,9.9} = \frac{0.00149213\,\pi^{14}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (70-z_0)^k z_0^{-k}}{k!}\right)^2}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (6825-z_0)^k z_0^{-k}}{k!}\right)^2} \quad \text{for not } ((z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

-144-5-0.99+((((((((1382 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2))))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]))))))))

**Input:**

$$-144-5-0.99+1382\times\frac{\pi^{10}}{93555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2\times\frac{2}{1382}\right)$$

**Result:**

1232.09464...

1232.09464... result very near to the rest mass of Delta baryon 1232

**Series representations:**

$$-144-5-0.99+\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\right)\left(\sqrt{2\times 35}\;\pi\right)^2\,2}{93\,555\times 1382}=$$
$$\frac{0.0147721\left(\pi^{14}\sqrt{69}^2\left(\sum_{k=0}^{\infty}69^{-k}\binom{\frac{1}{2}}{k}\right)^2-10\,153.6\sqrt{6824}^2\left(\sum_{k=0}^{\infty}6824^{-k}\binom{\frac{1}{2}}{k}\right)^2\right)}{\sqrt{6824}^2\left(\sum_{k=0}^{\infty}6824^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$-144-5-0.99+\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\right)\left(\sqrt{2\times 35}\;\pi\right)^2\,2}{93\,555\times 1382}=$$
$$\frac{0.0147721\left(\pi^{14}\sqrt{69}^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{69}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2-10\,153.6\sqrt{6824}^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6824}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2\right)}{\sqrt{6824}^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6824}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

- $$-144-5-0.99+\frac{\left(1382\,\pi^{10}\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\right)\left(\sqrt{2\times 35}\;\pi\right)^2\,2}{93\,555\times 1382}=$$

$$\left(0.0147721\left(\pi^{14}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(70-z_0)^k z_0^{-k}}{k!}\right)^2-10\,153.6\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6825-z_0)^k z_0^{-k}}{k!}\right)^2\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6825-z_0)^k z_0^{-k}}{k!}\right)^2\quad\text{for not }((z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

We note that:

$$\frac{2\,\pi^{14}}{18\,243\,225}$$

$$=1.000061248135058704829258545105135333747481696169154549482\ldots$$

And:

ln(((1.0000612481350587*(2 π^14)))) + (9.9/100)

**Input interpretation:**

$$\log(1.0000612481350587\,(2\,\pi^{14})) + \frac{9.9}{100}$$

log(x) is the natural logarithm

**Result:**

16.818427...

16.818427…. result very near to the black hole entropy 16.8741

**Series representations:**

$$\log(1.00006124813505870000 \times 2\,\pi^{14}) + \frac{9.9}{100} = 0.099 + \log(-1 + 2.00012249627011740000\,\pi^{14}) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(-1 + 2.00012249627011740000\,\pi^{14})^{-k}}{k}$$

$$\log(1.00006124813505870000 \times 2\,\pi^{14}) + \frac{9.9}{100} = 0.099 + 2\,i\,\pi \left\lfloor \frac{\arg(2.00012249627011740000\,\pi^{14} - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(2.00012249627011740000\,\pi^{14} - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

•

$$\log(1.00006124813505870000 \times 2\,\pi^{14}) + \frac{9.9}{100} = 0.099 + \left\lfloor \frac{\arg(2.00012249627011740000\,\pi^{14} - z_0)}{2\,\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(2.00012249627011740000\,\pi^{14} - z_0)}{2\,\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(2.00012249627011740000\,\pi^{14} - z_0)^k\,z_0^{-k}}{k}$$

**Integral representations:**

$$\log(1.00006124813505870000 \times 2\,\pi^{14}) + \frac{9.9}{100} = 0.099 + \int_1^{2.00012249627011740000\,\pi^{14}} \frac{1}{t}\,dt$$

$$\log(1.00006124813505870000 \times 2\pi^{14}) + \frac{9.9}{100} =$$

$$0.099 + \frac{1}{2 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1 + 2.00012249627011740000\,\pi^{14})^{-s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for} \quad -1 < \gamma < 0$$

From:

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\left(\frac{\pi}{\sqrt{15}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{10+11}{2}}}\right)^2\right)$$

$$\frac{\pi^6}{945}$$

1.0173430619844491397145179297909205279018174900328535618420...

We obtain:

(-34-5-1)+1.017343061984449139*945*((Pi/(sqrt((1+3+8)/2)))^2 * ((Pi/(sqrt((1+8+21)/2)))^2 * ((Pi/(sqrt((3+5+13)/2)))^2

**Input interpretation:**

$$(-34 - 5 - 1) + 1.017343061984449139 \times 945 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+3+8)}}\right)^2 \left(\left(\frac{\pi}{\sqrt{\frac{1}{2}(1+8+21)}}\right)^2 \left(\frac{\pi}{\sqrt{\frac{1}{2}(3+5+13)}}\right)^2\right)\right)$$

**Result:**

938.0626259506605138...

938.0626… result very near to the rest mass of proton 938.272

From:

$$\left(\frac{\pi}{\sqrt{6}}\right)^2 \left(\frac{\pi}{\sqrt{15}}\right)^2$$

$$\frac{\pi^4}{90}$$

1.082323233711138191516003696541167902774750951918726907682...

$\frac{\pi^4}{90}$ is a transcendental number

We obtain:

(((pi^4/90)))^6 - (1-1.0061571663)

**Input interpretation:**

$$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.0061571663)$$

**Result:**

1.6136235077...

1.6136235... that is a golden number

**Series representations:**

$$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.00615716630000) =$$
$$0.00615716630000 + 529.6448273856 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.00615716630000) =$$
$$0.00615716630000 + 1.881676423159 \times 10^{-12} \left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^{24}$$

- $$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.00615716630000) =$$
$$0.00615716630000 + 0.00003156929179344 \sqrt{3}^{24} \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k}{1+2k}\right)^{24}$$

**Integral representations:**

$$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.00615716630000) = 0.00615716630000 + 0.00003156929179344 \left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.00615716630000) = 0.00615716630000 + 529.6448273856 \left(\int_0^1 \sqrt{1-t^2}\,dt\right)^{24}$$

- $$\left(\frac{\pi^4}{90}\right)^6 - (1 - 1.00615716630000) = 0.00615716630000 + 0.00003156929179344 \left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^{24}$$

(((pi^4/90)))^6 *1.0061571663

**Input interpretation:**

$$\left(\frac{\pi^4}{90}\right)^6 \times 1.0061571663$$

**Result:**

1.6173637789...

1.6173637789... result that is very near to the 1 / 0.6181 = 1.617861187510...... and to the value of the golden ratio 1.618033988749...

**Series representations:**

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000 = 532.905938667796 \left(\sum_{k=0}^\infty \frac{(-1)^k}{1+2k}\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000 = 0.0000317636691729901 \left(-1.00000000000000 + \sum_{k=1}^\infty \frac{2^k}{\binom{2k}{k}}\right)^{24}$$

- 

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000 = 1.89326221781910\times 10^{-12}\left(\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^{24}$$

**Integral representations:**

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000 = 0.0000317636691729901\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000 = 532.905938667796\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^{24}$$

- 

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000 = 0.0000317636691729901\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^{24}$$

(((pi^4/90)))^6 *1.0061571663^5

**Input interpretation:**

$$\left(\frac{\pi^4}{90}\right)^6\times 1.0061571663^5$$

**Result:**

1.657566695...

1.65756... is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

**Series representations:**

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^5 = 546.15241596204\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^5 =$$
$$0.000032553220746639\left(-1.00000000000000+\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}\right)^{24}$$

- 

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^5 = 1.94032315889828 \times 10^{-12} \left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50 k)}{\binom{3k}{k}}\right)^{24}$$

**Integral representations:**

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^5 = 0.000032553220746639 \left(\int_0^{\infty} \frac{1}{1+t^2} \, dt\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^5 = 546.15241596204 \left(\int_0^1 \sqrt{1-t^2} \, dt\right)^{24}$$

•

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^5 = 0.000032553220746639 \left(\int_0^{\infty} \frac{\sin(t)}{t} \, dt\right)^{24}$$

(((pi^4/90)))^6 *1.0061571663^(2Pi)

**Input interpretation:**

$$\left(\frac{\pi^4}{90}\right)^6 \times 1.0061571663^{2\pi}$$

**Result:**

1.670674174...

1.670674174...

We note that 1.670674174... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

**Series representations:**

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^{2\pi} = 0.00003080360460665\, e^{0.02455315360608 \times \sum_{k=1}^{\infty} 2^k / \binom{2k}{k}} \left(-1.0000000000000 + 1.0000000000000 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^{24}$$

$$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^{2\pi} = \frac{4\,398\,046\,511\,104 \times 1.00615716630000^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{24}}{8\,303\,765\,625}$$

- $$\left(\frac{\pi^4}{90}\right)^6 1.00615716630000^{2\pi} = \frac{1.00615716630000^{2 \sum_{k=0}^{\infty} \left(2^{-k} (-6+50k)\right) / \binom{3k}{k}} \left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}\right)^{24}}{531\,441\,000\,000}$$

Now, from ζ(14):

$$\zeta(14) = 1 + \frac{1}{2^{14}} + \frac{1}{3^{14}} + \cdots = \frac{2\pi^{14}}{18243225} = 1.0000612\ldots$$

((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(( (sqrt(2*35)*Pi)^2 * 2/(1382)))]

**Input:**

$$\frac{\pi^{10}}{93555} \left(\sqrt{24^2 + 23 \times 5} \times \frac{\pi}{\sqrt{\frac{1}{2} (21 \times 13 \times 25 \times 2)}}\right)^2 \left(\left(\sqrt{2 \times 35}\, \pi\right)^2 \times \frac{2}{1382}\right)$$

**Result:**

$$\frac{2\pi^{14}}{18\,243\,225}$$

**Decimal approximation:**

1.000061248135058704829258545105135333747481696169154549482...

We can to obtain:

1/2 * 18243225 * ((((π^10/93555)))) * ((((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]

**Input:**

$$\frac{1}{2} \times 18\,243\,225 \times \frac{\pi^{10}}{93\,555} \left(\sqrt{24^2 + 23 \times 5} \times \frac{\pi}{\sqrt{\frac{1}{2}(21 \times 13 \times 25 \times 2)}}\right)^2 \left(\left(\sqrt{2 \times 35}\ \pi\right)^2 \times \frac{2}{1382}\right)$$

**Result:**

$$\pi^{14}$$

**Decimal approximation:**

$$9.122171181754353170204375110762816274502700883297762 25\ldots \times 10^6$$

$9.122171181\ldots * 10^6$

**Property:**

$\pi^{14}$ is a transcendental number

**Series representations:**

$$\frac{\left(18\,243\,225 \left(\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\left(\sqrt{2\times 35}\ \pi\right)^2\ 2\right)\right)\right)\pi^{10}}{(2\times 1382)\ 93\,555} = 268\,435\,456 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{14}$$

$$\frac{\left(18\,243\,225 \left(\left(\frac{\sqrt{24^2+23\times 5}\ \pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2 \left(\left(\sqrt{2\times 35}\ \pi\right)^2\ 2\right)\right)\right)\pi^{10}}{(2\times 1382)\ 93\,555} = \left(\sum_{k=0}^{\infty} -\frac{4(-1)^k\ 1195^{-1-2k}\left(5^{1+2k} - 4\times 239^{1+2k}\right)}{1+2k}\right)^{14}$$

•

$$\frac{\left(18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2\;2\right)\right)\pi^{10}\right)}{(2\times 1382)\;93\,555} = \left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{14}$$

**Integral representations:**

$$\frac{\left(18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2\;2\right)\right)\pi^{10}\right)}{(2\times 1382)\;93\,555} = 16\,384\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{14}$$

$$\frac{\left(18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2\;2\right)\right)\pi^{10}\right)}{(2\times 1382)\;93\,555} = 268\,435\,456\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^{14}$$

- $$\frac{\left(18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2\;2\right)\right)\pi^{10}\right)}{(2\times 1382)\;93\,555} = 16\,384\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^{14}$$

((((((((((1/2 * 18243225 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))])))))))))^1/14

**Input:**

$$\left(\frac{1}{2}\times 18\,243\,225\times\frac{\pi^{10}}{93\,555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}\,(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2\times\frac{2}{1382}\right)\right)\text{^}(1/14)$$

**Exact result:**

$\pi$

**Decimal approximation:**

- More digits

3.1415926535897932384626433832795028841971693993751058209749...

3.14159265… = $\pi$

**Property:**

$\pi$ is a transcendental number

- 

**Series representations:**

$$\sqrt[14]{\frac{18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)\right)\pi^{10}}{(2\times1382)\,93\,555}}=4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt[14]{\frac{18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)\right)\pi^{10}}{(2\times1382)\,93\,555}}=\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times239^{1+2k}\right)}{1+2k}$$

- 

$$\sqrt[14]{\frac{18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)\right)\pi^{10}}{(2\times1382)\,93\,555}}=\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

**Integral representations:**

$$\sqrt[14]{\frac{18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times5\,\pi}}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2\,2\right)\right)\pi^{10}}{(2\times1382)\,93\,555}}=4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt[14]{\frac{\left(18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)\right)\pi^{10}}{(2\times 1382)\,93\,555}} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

- $$\sqrt[14]{\frac{\left(18\,243\,225\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)\right)\pi^{10}}{(2\times 1382)\,93\,555}} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

1/6*((((((((((((((((((((((1/2 * 18243225 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))]))))))))))^1/14)))))))))))^2

**Input:**

$$\frac{1}{6}\left(\frac{1}{2}\times 18\,243\,225\times\frac{\pi^{10}}{93\,555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}\,(21\times 13\times 25\times 2)}}\right)^2 \left(\left(\sqrt{2\times 35}\;\pi\right)^2\times\frac{2}{1382}\right)\right)^\wedge (1/14)^2$$

**Exact result:**

$$\frac{\pi^2}{6}$$

**Decimal approximation:**

1.644934066848226436472415166646025189218949901206798437735...

1.644934… = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... Thence from $\frac{2\pi^{14}}{18\,243\,225}$ to $\frac{\pi^2}{6}$

**Property:**

$\frac{\pi^2}{6}$ is a transcendental number

**Series representations:**

$$\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2 2\right)\right)}{2\times93\,555\times1382}}^{\,2} = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2 2\right)\right)}{2\times93\,555\times1382}}^{\,2} = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

- $$\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2 2\right)\right)}{2\times93\,555\times1382}}^{\,2} = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

## Integral representations:

$$\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2 2\right)\right)}{2\times93\,555\times1382}}^{\,2} = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2 2\right)\right)}{2\times93\,555\times1382}}^{\,2} = \frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

- $$\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times5}\,\pi}{\sqrt{\frac{21\times13\times25\times2}{2}}}\right)^2\left(\left(\sqrt{2\times35}\,\pi\right)^2 2\right)\right)}{2\times93\,555\times1382}}^{\,2} = \frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

-(27/10^3)+1/6*((((((((((((((((((((((1/2 * 18243225 * ((((π^10/93555)))) * (((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2))) * [(((sqrt(2*35)*Pi)^2 * 2/(1382)))])))))))))))^1/14))))))))))))^2

**Input:**

$$-\frac{27}{10^3}+\frac{1}{6}\left(\frac{1}{2}\times 18\,243\,225\times\frac{\pi^{10}}{93\,555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}\,(21\times 13\times 25\times 2)}}\right)^2 \left(\left(\sqrt{2\times 35}\;\pi\right)^2\times\frac{2}{1382}\right)\right)\hat{}\,(1/14)^2$$

**Exact result:**

$$\frac{\pi^2}{6}-\frac{27}{1000}$$

**Decimal approximation:**

1.617934066848226436472415166646025189218949901206798437735...

1.61793406…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Property:**

$-\frac{27}{1000}+\frac{\pi^2}{6}$ is a transcendental number

**Series representations:**

$$-\frac{27}{10^3}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2\,2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}+\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$-\frac{27}{10^3}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2} = -\frac{27}{1000}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

- $$-\frac{27}{10^3}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2} = -\frac{27}{1000}+\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

**Integral representations:**

$$-\frac{27}{10^3}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2} = -\frac{27}{1000}+\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\;dt\right)^2$$

$$-\frac{27}{10^3}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2} = -\frac{27}{1000}+\frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2$$

$$-\frac{27}{10^3}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2} = -\frac{27}{1000}+\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

Or:

-1728/64000+1/6*((((((((((((((((((((((1/2 * 18243225 * ((((π^10/93555)))) *(((((((24^2+23*5)^1/2))Pi/(((sqrt((21*13*25*2)/2)))))^2)))* [((((sqrt(2*35)*Pi)^2 * 2/(1382)))])))))))))))^1/14)))))))))))))^2

**Input:**

$$-\frac{1728}{64000}+\frac{1}{6}\left(\frac{1}{2}\times 18\,243\,225\times\frac{\pi^{10}}{93\,555}\left(\sqrt{24^2+23\times 5}\times\frac{\pi}{\sqrt{\frac{1}{2}(21\times 13\times 25\times 2)}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2\times\frac{2}{1382}\right)\right)^{\wedge}(1/14)^2$$

**Exact result:**

$$\frac{\pi^2}{6}-\frac{27}{1000}$$

**Decimal approximation:**

1.617934066848226436472415166646025189218949901206798437735...

1.61793406...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Series representations:**

$$-\frac{1728}{64000}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\,\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\,\pi\right)^2\,2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}+\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$-\frac{1728}{64\,000}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

- $$-\frac{1728}{64\,000}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}+\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

**Integral representations:**

$$-\frac{1728}{64\,000}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}+\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$-\frac{1728}{64\,000}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}+\frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

- $$-\frac{1728}{64\,000}+\frac{1}{6}\sqrt[14]{\frac{18\,243\,225\,\pi^{10}\left(\left(\frac{\sqrt{24^2+23\times 5}\;\pi}{\sqrt{\frac{21\times 13\times 25\times 2}{2}}}\right)^2\left(\left(\sqrt{2\times 35}\;\pi\right)^2 2\right)\right)}{2\times 93\,555\times 1382}}^{\,2}=-\frac{27}{1000}+\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

From Wikipedia:

# Positive integers

## Even positive integers

For the even positive integers, one has the relationship to the Bernoulli numbers:

$$\zeta(2n) = (-1)^{n+1} \frac{B_{2n}(2\pi)^{2n}}{2(2n)!}$$

for $n \in \mathbb{N}$. The first few values are given by:

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6} = 1.6449\ldots$$ (OEIS: A013661)

(the demonstration of this equality is known as the Basel problem)

$$\zeta(4) = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90} = 1.0823\ldots$$ (OEIS: A013662)

(the Stefan–Boltzmann law and Wien approximation in physics)

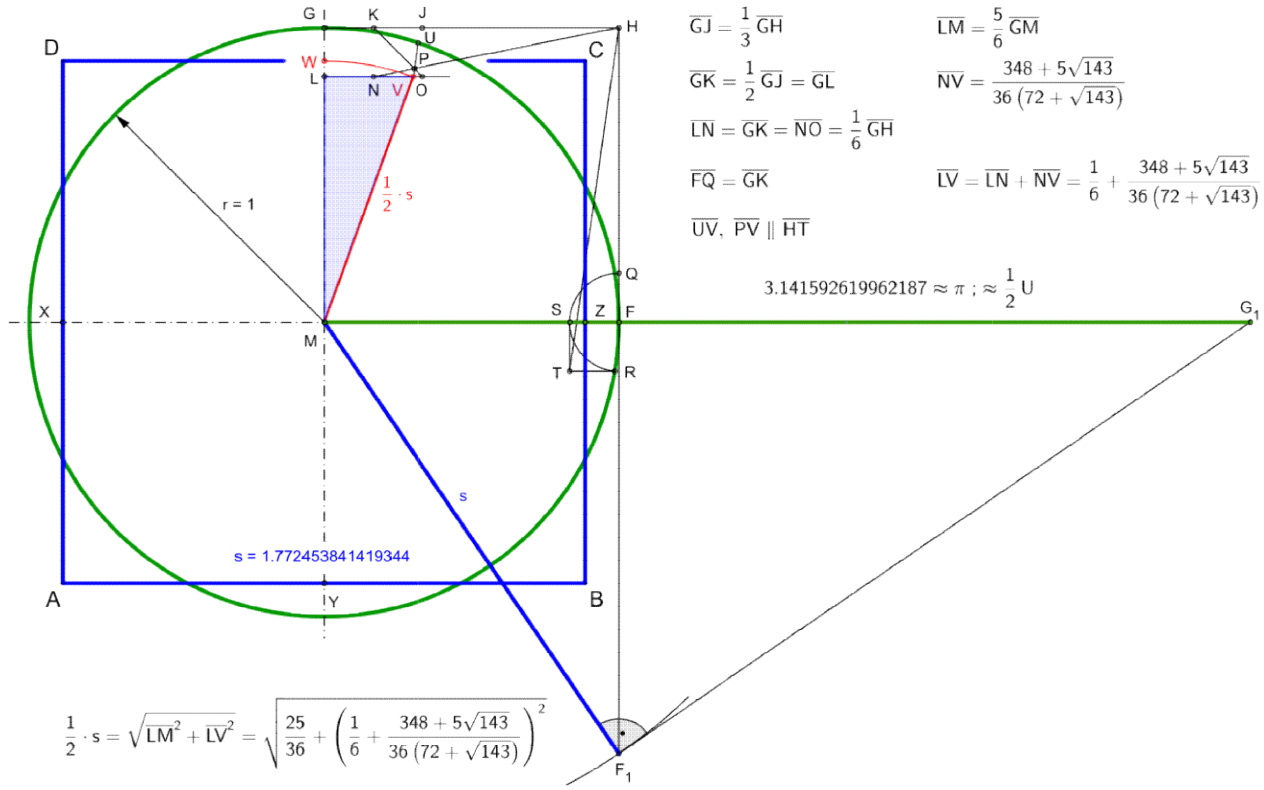
$$\zeta(6) = 1 + \frac{1}{2^6} + \frac{1}{3^6} + \cdots = \frac{\pi^6}{945} = 1.0173\ldots\ldots$$ (OEIS: A013664)

$$\zeta(8) = 1 + \frac{1}{2^8} + \frac{1}{3^8} + \cdots = \frac{\pi^8}{9450} = 1.00407\ldots\ldots$$ (OEIS: A013666)

$$\zeta(10) = 1 + \frac{1}{2^{10}} + \frac{1}{3^{10}} + \cdots = \frac{\pi^{10}}{93555} = 1.000994\ldots\ldots$$ (OEIS: A013668)

$$\zeta(12) = 1 + \frac{1}{2^{12}} + \frac{1}{3^{12}} + \cdots = \frac{691\pi^{12}}{638512875} = 1.000246\ldots$$ (OEIS: A013670)

$$\zeta(14) = 1 + \frac{1}{2^{14}} + \frac{1}{3^{14}} + \cdots = \frac{2\pi^{14}}{18243225} = 1.0000612\ldots$$ (OEIS: A013672).




Squaring the circle and from this the approximated half circumference (π)

$$\mathbf{2 \cdot \sqrt{\frac{25}{36} + \left(\frac{1}{6} + \frac{348 + 5\sqrt{143}}{36\,(72 + \sqrt{143})}\right)^2} = 1.772453841419343 76\ldots}$$

$$\mathbf{1.77245384141934376\ldots^2 = 3.141592619962188\ldots}$$

Now, we have that:

2*sqrt((((((((25/36+(((((1/6+(((348+5sqrt(143))/((36(72+sqrt(143)))))))^2))))))))

**Input:**

$$2\sqrt{\frac{25}{36} + \left(\frac{1}{6} + \frac{348 + 5\sqrt{143}}{36\,(72 + \sqrt{143})}\right)^2}$$

**Decimal approximation:**

1.772453841419343762029388284914552505633630521299223913874...

1.7724538…

10^3 * 2*sqrt((((((((25/36+(((((1/6+(((348+5sqrt(143))/((36(72+sqrt(143)))))))^2))))))))

**Input:**

$$10^3 \times 2 \sqrt{\frac{25}{36} + \left(\frac{1}{6} + \frac{348 + 5\sqrt{143}}{36(72 + \sqrt{143})}\right)^2}$$

**Result:**

$$2000 \sqrt{\frac{25}{36} + \left(\frac{1}{6} + \frac{348 + 5\sqrt{143}}{36(72 + \sqrt{143})}\right)^2}$$

**Decimal approximation:**

1772.4538414193437620293882849145525056336305212992239138 74...

1772.4538… result in the range of the mass of candidate "glueball" $f_0(1710)$ and the hypothetical mass of Gluino (**$1760 \pm 15$ MeV- 1785.16 GeV**).

((((((((((10^3 * 2*sqrt((((((((25/36+(((((1/6+(((348+5sqrt(143))/((36(72+sqrt(143)))))))^2)))))))))))))))) ^1/15

**Input:**

$$\sqrt[15]{10^3 \times 2 \sqrt{\frac{25}{36} + \left(\frac{1}{6} + \frac{348 + 5\sqrt{143}}{36(72 + \sqrt{143})}\right)^2}}$$

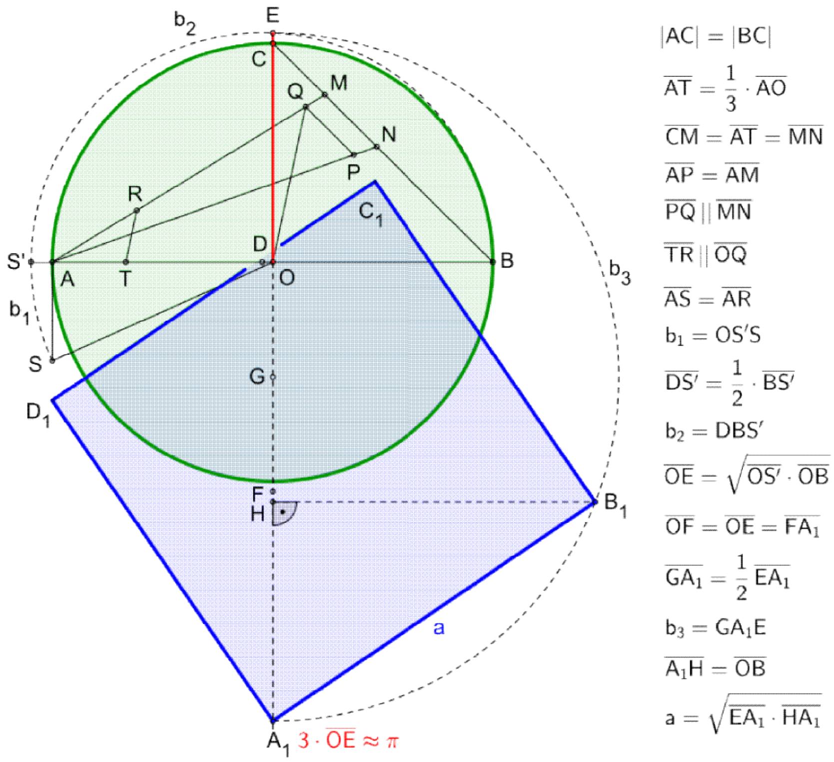
**Exact result:**

$$2^{4/15} \sqrt[5]{5} \sqrt[30]{\frac{25}{36} + \left(\frac{1}{6} + \frac{348 + 5\sqrt{143}}{36(72 + \sqrt{143})}\right)^2}$$

**Decimal approximation:**

1.6465376365706759478928531411560240050510741675587517426 07...

1.646537... ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...




Squaring the circle, approximate construction according to Ramanujan of 1914, with continuation of the construction (dashed lines, mean proportional red line).

$$\left(9^2 + \frac{19^2}{22}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{2143}{22}} = 3.14159265258264612 52\ldots$$

Now, we have that:

(((9^2+(19^2)/22))^1/4

**Input:**

$$\sqrt[4]{9^2 + \frac{19^2}{22}}$$

**Result:**

$$\sqrt[4]{\frac{2143}{22}}$$

**Decimal approximation:**

3.1415926525826461252060371796440223715578779831601261496 95...

3.14159265… = $\pi$

1 / (((9^2+(19^2)/22))^1/4

**Input:**

$$\frac{1}{\sqrt[4]{9^2 + \frac{19^2}{22}}}$$

**Result:**

$$\sqrt[4]{\frac{22}{2143}}$$

**Decimal approximation:**

0.318309886285836009187604674738153993933575850843891161903...

0.31830988...

-89+10^4 * 1 / (((9^2+(19^2)/22))^1/4

**Input:**

$$-89 + 10^4 \times \frac{1}{\sqrt[4]{9^2 + \frac{19^2}{22}}}$$

**Result:**

$$10\,000 \sqrt[4]{\frac{22}{2143}} - 89$$

**Decimal approximation:**

3094.098862858360091876046747381539939335758508438911619035...

3094.0988... result very near to the rest mass of J/Psi meson 3096.916

And:

((((((1/3(((-89+10^4 * 1 / (((9^2+(19^2)/22))^1/4))))))))^1/14

**Input:**

$$\sqrt[14]{\frac{1}{3}\left(-89 + 10^4 \times \frac{1}{\sqrt[4]{9^2 + \frac{19^2}{22}}}\right)}$$

**Exact result:**

$$\sqrt[14]{\frac{1}{3}\left(10\,000\sqrt[4]{\frac{22}{2143}}-89\right)}$$

**Decimal approximation:**

1.641510937539780637063173324640870068271457448212422139089...

$1.64151... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

**Computations and observations of Dr. Mariano Del Gaudio**

| NP | RQ | DIF.Prec. | DIF.PIGRECO/2 |
|---|---|---|---|
| 2 | 1,414213562 | | 0,156582764 |
| 3 | 1,732050808 | 0,317837 | -0,161254481 |
| 5 | 2,236067977 | 0,504017 | -0,665271651 |
| 7 | 2,645751311 | 0,409683 | -1,074954984 |
| 3,141592654 | 1,772453851 | | |
| 3,141592654 | 1,570796327 | CIRC/4 | |
| 3,141592654 | 0,785398163 | AREA/4 | |

$\sqrt{2}$ = 1,4142135623730950488016887242097

$\sqrt{3}$ = 1,7320508075688772935274463415059

$\sqrt{5}$ = 2,2360679774997896964091736687313

$\sqrt{7}$ = 2,6457513110645905905016157536393

$\sqrt{\pi}$ = 1,7724538509055160272981674833411

$\pi/2$ = 1,5707963267948966192313216916398

$\pi/4$ = 0,78539816339744830961566084581988

1,5707963267948966192313216916398 -1,4142135623730950488016887242097 =

= 0,1565827644218015704296329674301;

$(0,1565827644218015704296329674301)^{1/4}$ =

0,62905119112207220252548038624754

1,5707963267948966192313216916398 -1,7320508075688772935274463415059 =

= -0,1612544807739806742961246498661;

$-(0,1612544807739806742961246498661)^{1/4}$ =

-0,63369159634835856910289482534756

1,5707963267948966192313216916398 - 2,2360679774997896964091736687313 =

= -0,6652716507048930771778519770915

1,5707963267948966192313216916398 - 2,6457513110645905905016157536393 =

= -1,0749549842696939712702940619995;

$1 / -(1,0749549842696939712702940619995)^7$ =

1 / -1,6585628912291236591732875130487 =

-0,60293161343970652150412808559299

-27/10^3+1.0061571663^2+[1/4((((0.62905119112207220252548038624754)-(-0.63369159634835856910289482534756)-(-0.6652716507048930771778519770915)-(-0.60293161343970652150412808559299))))]

**Input interpretation:**

$$-\frac{27}{10^3}+1.0061571663^2+\frac{1}{4}(0.62905119112207220252548038624754 - -0.63369159634835856910289482534756 - -0.66527165070489307717785197709 15 - -0.60293161343970652150412808559299)$$

**Result:**

1.6180887562006034482675888185698975

1.61808875...

Or, with corrector 9.01826/1000, we obtain:

-(3*9.01826/10^3)+1.0061571663^2+[1/4((((0.629051191122072202525480386247 54)-(-0.63369159634835856910289482534756)-(-0.66527165070489307717785197709 15)-(-0.60293161343970652150412808559299))))]

**Input interpretation:**

$$-\left(3\times\frac{9.01826}{10^3}\right)+1.0061571663^2+\frac{1}{4}(0.62905119112207220252548038624754 - -0.63369159634835856910289482534756 - -0.66527165070489307717785197709 15 - -0.60293161343970652150412808559299)$$

**Result:**

1.6180339762006034482675888185698975

1.618033976… This result is practically the value of the golden ratio 1,618033988749...

With regard $\pi$ and $e$, we have the following beautiful Ramanujan formula:

https://ibmathsresources.com/2013/03/19/ramanujans-beauty-in-mathematics/

$$\sqrt{\frac{\pi e}{2}} = \cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\cfrac{5}{1+\cfrac{6}{1+\cfrac{7}{1+\cfrac{8}{\ddots}}}}}}}}} + \left\{1+\frac{1}{1\cdot 3}+\frac{1}{1\cdot 3\cdot 5}+\frac{1}{1\cdot 3\cdot 5\cdot 7}+\frac{1}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\cdots\right\},$$


**Acknowledgements**

I would like to thank Dr. **Mariano Del Gaudio** for the useful Table and fundamental suggestions that he has give me for the drafting of this paper and his kind availability with regard we

## References

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Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

**S. Ramanujan to G.H. Hardy 12 January 1920** - University of Madras
https://www.imsc.res.in/~rao/ramanujan/newnow/lastletter.pdf