

Riemann hypothesis

November 2, 2019

Yuji Masuda

(y_masuda0208@yahoo.co.jp)

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \frac{1}{7^s} + \cdots + \frac{1}{\infty^s}$$

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{1^s} + \frac{1}{2^s} + \cdots + \frac{1}{3^s}$$

$$\infty = 5m + 3 \quad \therefore m = \frac{\infty - 3}{5} = 0$$

$$\zeta(s) = \left(\frac{\infty - 3}{5}\right) \left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s}\right) + \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s}$$

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} = 0 \quad \therefore 2^s + 3^s = -1$$

$$2^s + 3^s = 27^s + 8^s = 3^{3s} + 2^{2s} = 3^{-2s} + 2^{-2s}$$

$$3^{-2s} + 2^{-2s} = \frac{1}{9^s} + \frac{1}{4^s} = \frac{1}{4^s} + \frac{1}{4^s} = \frac{2}{4^s} = -1$$

$$\therefore 4^s = -2$$

$$\therefore s = \frac{2i\pi n + i\pi + \log 2}{2\log 2}, \quad n \in \mathbb{Z} \quad \therefore s = \frac{2i\pi n + i\pi}{2\log 2} + \frac{1}{2}$$

That's all (proof end)