

On some new mathematical connections between various equations of the Bouncing Cosmology, the Cosmological Constraints concerning the Dilaton Inflation and some sectors of Number Theory, principally the Rogers-Ramanujan continued fractions and the Ramanujan's mock theta functions

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Abstract

In this research thesis, we have described the new possible mathematical connections between some equations of various topics concerning the Bouncing Cosmology, the Cosmological Constraints regarding the Dilaton Inflation and some sectors of Number Theory, principally the Rogers-Ramanujan continued fractions and the Ramanujan's mock theta functions

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<https://www.pinterest.it/pin/444237950734694507/?lp=true>

If we consider a D-function, i.e. the transformed form Eulerian, e.g.

(A) $1 + \frac{v^2}{(1-v)^2} + \frac{v^4}{(1-v)^2(1-v^2)^2} + \frac{v^6}{(1-v)^2(1-v^2)^2(1-v^4)^2}$

(B) $1 + \frac{v^2}{1-v} + \frac{v^4}{(1-v)(1-v^2)} + \frac{v^6}{(1-v)(1-v^2)(1-v^3)}$

and consider determine the nature of the singularities at the points $v=1$, $v=1$, $v^2=1$, $v^4=1$, $v^6=1$, ... we know how beautifully the asymptotic nature form of this function can be expressed in a very neat and closed form in a exponential form. For instance when $v = e^{-t}$ and $t \rightarrow 0$

(A) $= \sqrt{\frac{t}{2\pi}} e^{\frac{t^2}{80} - \frac{5}{24}t^4} + o(1)$

(B) $= \frac{e^{\frac{t^2}{80}} - \frac{5}{80}t^4}{\sqrt{2\pi t^3}} + o(1)$

and similar results at other singularities. It is not necessary that there should be only one term and this there may be many terms but the number of terms must be finite. Also $o(1)$ may turn out to be $O(1)$. That is all. For instance when $v \rightarrow 1$ the function

$\frac{1}{(1-v)(v)(1-v)} - \frac{1}{v^{120}}$

is equivalent to the sum of five terms like $(*)$ together with $O(1)$ instead of $o(1)$.

If we take a number of functions, like (A) and (B) it is only in a limited number of cases the terms close as above; but in the majority of cases they never close as above. For instance, when $v = e^{-t}$ and $t \rightarrow 0$

(C) $1 + \frac{v^2}{(1-v)^2} + \frac{v^4}{(1-v)^2(1-v^2)^2} + \frac{v^6}{(1-v)^2(1-v^2)(1-v^4)^2} + \dots$

$$= \sqrt{\frac{t}{2\pi}} e^{\frac{t^2}{80} - a_1 t^4 + a_2 t^6 + \dots + O(a_n t^q)}$$

where $a_1 = \frac{1}{8\sqrt{5}}$, and so on.

<https://www.billtoole.net/wordpress/all/ramanujans-mock-theta-functions-letter-to-hardy-1920/>

Ramanujan mathematics applied to Cosmology

From the following vacuum equations concerning the brane supersymmetry breaking, we can to obtain, putting

$$4096 e^{-\pi \sqrt{18}}$$

instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a news possible mathematical solutions that are very near to the originals.

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

$$\begin{aligned} T e^{\gamma_E \phi} &= - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)}\phi} \\ 16 k' e^{-2C} &= \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)} \\ (A')^2 &= k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi} \end{aligned}$$

The results of these three equations are:

$$\textcolor{red}{7.61802....*10^{-7}; \quad 1.066522... * 10^{-5}; \quad 0.13533537...}$$

With respect to the exponential of the vacuum equation, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2 , while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

Dilaton value

$$0.989117352243 = \phi$$

If we put this value of dilaton in the previous three vacuum equations, we obtain the following results that are equals to the previous:

$$7.61802... * 10^{-7}; \quad 1.066522... * 10^{-5}; \quad 0.1353353...$$

The dilaton value obtained $0.989117352243 = \phi$ we note that is very near to the result of the following mean of these Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$\frac{e^{-\frac{2\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{(\varphi-1)^5 \sqrt[4]{5^3}} - 1}} - \varphi} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}} \approx 1.0000007913$$

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5} - \varphi + 1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Indeed:

$$1/4 * (1.0018674362 + 1.0000007913 + 0.9568666373 + 0.9991104684) =$$

= **0.9894613333** result very near to the dilaton value

From Wikipedia:

In physics, the **Planck mass**, denoted by m_p , is the unit of mass in the system of natural units known as Planck units. It is approximately 0.02 milligrams.

$$1m_p \approx 1.220910 \times 10^{19} \text{ GeV}/c^2 = 2.176435(24) \times 10^{-8} \text{ kg} = 21.76470 \mu\text{g}$$

$$= 1.3107 \times 10^{19} \text{ u}$$

Particle physicists and cosmologists often use an alternative normalization with the **reduced Planck mass**, which is

$$M_p \approx 4.341 \times 10^{-9} \text{ kg} = 2.435 \times 10^{18} \text{ GeV}/c^2.$$

The reduced Planck mass $M_{pl} = 2.435 \times 10^{18} \text{ GeV}$

We have that $4.341 \times 10^{-9} \text{ kg}$ is equal to

Input interpretation:

convert $4.341 \times 10^{-9} \text{ kg}$ (kilograms) to milligrams

Result:

0.004341 mg (milligrams)

Thence, we consider:

$$\sqrt{\frac{\hbar c}{8\pi G}} \approx 4,340 \mu\text{g}.$$

That is $4.341 \times 10^{-9} \text{ kg}$

From:

Towards a Nonsingular Bouncing Cosmology

Yi-Fu Cai, Damien A. Easson and Robert Brandenberger - arXiv:1206.2382v2 [hep-th] 22 Jun 2012

To illustrate that a nonsingular bounce can be achieved in our model, we numerically evolve the Einstein acceleration equation coupled to the field equation for ϕ , imposing the Hamiltonian constraint equation to set the initial conditions. In the numerical computations we work in units of the Planck mass M_p for all parameters. Specifically, these parameters are chosen to be:

$$V_0 = 10^{-7}, \quad g_0 = 1.1, \quad \beta = 5, \quad \gamma = 10^{-3}, \\ b_V = 5, \quad b_g = 0.5, \quad p = 0.01, \quad q = 0.1. \quad (24)$$

This Lagrangian admits the Ekpyrotic attractor solution for ϕ in a contracting universe:

$$\phi_c \simeq -\sqrt{\frac{q}{2}} \ln \left[\frac{2V_0 t^2}{q(1-3q)M_p^2} \right], \quad (17)$$

which yields an effective equation of state

$$w_c \simeq -1 + \frac{2}{3q}, \quad (18)$$

$$-\text{sqrt}(0.1/2) \ln(((2*9*10^{-7})/((0.1(1-3*0.1)*(4.341e-9)^2))))$$

Input interpretation:

$$-\sqrt{\frac{0.1}{2}} \log \left(\frac{2 \times 9 \times 10^{-7}}{0.1(1-3 \times 0.1)(4.341 \times 10^{-9})^2} \right)$$

$\log(x)$ is the natural logarithm

Result:

$$-6.24798946773771778755078997999103726598237940698998304291\dots$$

$$-6.24798946\dots$$

$$-1+2/(3*0.1)$$

Input:

$$-1 + \frac{2}{3 \times 0.1}$$

Result:

$$5.66\dots$$

$$5.6666\dots$$

$$\dot{\phi}_B^2 \simeq \frac{(g_0 - 1)M_p^2}{3\beta} \left[1 + \sqrt{1 + \frac{12\beta V_0}{(g_0 - 1)^2 M_p^4}} \right]$$

$$(1.1-1)*(4.341e-9)^2 / (3*5) * (1+sqrt(((1+(12*5*(10^-7))/((1.1-1)^2*(4.341e-9)^4)))$$

Input interpretation:

$$(1.1 - 1) \times \frac{(4.341 \times 10^{-9})^2}{3 \times 5} \left(1 + \sqrt{1 + \frac{\frac{12 \times 5}{10^7}}{(1.1 - 1)^2 (4.341 \times 10^{-9})^4}} \right)$$

Result:

$$0.000163299316185545332175025604980441083396275671591232127\dots$$

$$0.000163299316\dots$$

From the inverse of the above result, we obtain:

$$1/(1+0.000163299)$$

Input interpretation:

$$\frac{1}{1 + 0.000163299}$$

Result:

$$0.999836727662209488852679846233789868348288592821080910308\dots$$

0.999836727... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}\approx 0.9991104684$$

Next we consider the determination of the sound speed square in the neighborhood of the bounce. From the expression (26), we find that, in addition to the Hubble parameter, one needs to know the form of $\dot{\phi}^2$ around the bounce. We have evaluated this in our semi-analytical study of the background solution and the result is given in Eq. (22). Combining Eqs. (22), (26) and (36), we find that the sound speed parameter takes the approximate form

$$c_{sb}^2 \simeq \frac{1}{3} - \frac{2}{3 \sqrt{1 + \frac{12\beta V_0}{M_p^4(g_0-1)^2}}} , \quad (37)$$

in the bouncing phase. The subscript “ b ” in Eq. (37) indicates the bouncing phase. From this result, we see

$$1/3-2/((((3*sqrt(((1+(12*5*(10^-7))/(((1.1-1)^2*(4.341e-9)^4))))))))))$$

Input interpretation:

$$\frac{1}{3} - \frac{2}{3 \sqrt{1 + \frac{\frac{12 \times 5}{10^7}}{(1.1-1)^2 (4.341 \times 10^{-9})^4}}}$$

Result:

$$0.3333333333332820456966448622998272497616215898330006476\dots$$

$$0.3333\dots \approx 1/3$$

From the square root of the result, we obtain:

$$\sqrt{1/3-2/((((3*sqrt(((1+(12*5*(10^-7))/(((1.1-1)^2*(4.341e-9)^4))))))))}))})$$

Input interpretation:

$$\sqrt{\frac{1}{3} - \frac{2}{3 \sqrt{1 + \frac{\frac{12 \times 5}{10^7}}{(1.1-1)^2 (4.341 \times 10^{-9})^4}}}}$$

Result:

$$0.57735026918962532035\dots$$

$$0.5773502691\dots \approx \text{Euler-Mascheroni constant } 0.57721566\dots$$

Now, we have that:

$$2 / (((((3 * \text{sqrt}(((1 + (12 * 5 * (10^7)) / (((1.1 - 1)^2 * (4.341 \times 10^{-9})^4)))))))$$

Input interpretation:

$$\frac{2}{3 \sqrt[3]{1 + \frac{\frac{12 \times 5}{10^7}}{(1.1 - 1)^2 (4.341 \times 10^{-9})^4}}}$$

Result:

$$5.1287636688471033506083571711743500332685685421710313... \times 10^{-16}$$

$$5.128763668... * 10^{-16}$$

We note that if perform the ratio between the reduced Planck constant and the above result and raise to the square, we obtain:

$$(6.582119569 \times 10^{-16} / 5.1287636688471 \times 10^{-16})^2$$

Input interpretation:

$$\left(\frac{6.582119569 \times 10^{-16}}{5.1287636688471 \times 10^{-16}} \right)^2$$

Result:

$$1.647047634908671647317023087266146615111910459151923216787...$$

$$1.647047634... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

From:

$$\dot{\phi}_B^2 \simeq \frac{2M_p^2(g_0 - 1)}{3\beta}, \quad (38)$$

$$z^2 \simeq a^2 \frac{3\beta M_p^4 \dot{\phi}^4}{(2M_p^2 H - \gamma \dot{\phi}^3)^2}, \quad (39)$$

$$\dot{\phi} \simeq \dot{\phi}_B e^{-\frac{t^2}{T^2}}, \quad (40)$$

Inserting the values of the parameters $\Upsilon = 2.7 \times 10^{-4}$, $T = 0.5$, and $t_{R+} = -t_{R-} = 1$ (obtained from the numer-

To illustrate that a nonsingular bounce can be achieved in our model, we numerically evolve the Einstein acceleration equation coupled to the field equation for ϕ , imposing the Hamiltonian constraint equation to set the initial conditions. In the numerical computations we work in units of the Planck mass M_p for all parameters. Specifically, these parameters are chosen to be:

$$V_0 = 10^{-7}, \quad g_0 = 1.1, \quad \beta = 5, \quad \gamma = 10^{-3}, \\ b_V = 5, \quad b_g = 0.5, \quad p = 0.01, \quad q = 0.1. \quad (24)$$

From (38), we have:

$$(((2*(4.341e-9)^2 * (1.1-1))))/15$$

Input interpretation:

$$\frac{1}{15} (2 (4.341 \times 10^{-9})^2 (1.1 - 1))$$

Result:

$$2.5125708 \times 10^{-19}$$

$$2.5125708 * 10^{-19}$$

From (40), we have:

$$\dot{\phi} \simeq \dot{\phi}_B e^{-\frac{t^2}{T^2}},$$

$$5.0125550 \times 10^{-10} \exp(-1/(1/4))$$

Input interpretation:

$$5.0125550 \times 10^{-10} \exp\left(-\frac{1}{\frac{1}{4}}\right)$$

Result:

$$9.1808147... \times 10^{-12}$$

$$9.1808147... * 10^{-12}$$

From:

$$(9.1808147 * 10^{-12}) * 1 / (2.5125708 * 10^{-19}) * 1/512 + 64^{1/2}/2 + 64 + 16$$

Input interpretation:

$$\frac{9.1808147}{10^{12}} \times \frac{1}{\frac{2.5125708}{10^{19}}} \times \frac{1}{512} + \frac{64^2}{2} + 64 + 16$$

Result:

$$73494.26243900271387377422359600772244905496792368995134385\dots$$

$$73494.262439\dots$$

Further, we have the following mathematical connections:

$$\left[\frac{9.1808147}{10^{12}} \times \frac{1}{\frac{2.5125708}{10^{19}}} \times \frac{1}{512} + \frac{64^2}{2} + 64 + 16 \right] = 73494.262439 \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS} + \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS}} \right) =$$

$$-3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525\dots \Rightarrow$$

(the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$)

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700\dots$$

$$= 73491.7883254\dots \Rightarrow$$

(the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane)

$$\left(\begin{aligned} I_{21} &\ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leqslant p^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ &\ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\} \end{aligned} \right)$$

$$\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} = 73493.30662\dots$$

(the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series)

From (39) and H (Hubble parameter) = 1.00000000000000021978021978022:

$$z^2 \simeq a^2 \frac{3\beta M_p^4 \dot{\phi}^4}{(2M_p^2 H - \gamma \dot{\phi}^3)^2},$$

We have:

$$\frac{((15 \cdot (4.341 \cdot 10^{-9})^4 \cdot (9.1808147 \cdot 10^{-12})^4)))}{((2 \cdot (4.341 \cdot 10^{-9})^2 \cdot 1.0000000000000002 \cdot 1978021978022 \cdot 10^{-3} \cdot 9.1808147 \cdot 10^{-12}))^2}$$

Input interpretation:

$$\frac{15(4.341 \times 10^{-9})^4 (9.1808147 \times 10^{-12})^4}{\left(2(4.341 \times 10^{-9})^2 \times 1.00000000000000021978021978022 - \frac{9.1808147 \times 10^{-12}}{10^3}\right)^2}$$

Result:

$$4.5267433010382812066027798074565642017055336800392557... \times 10^{-49}$$

$$4.5267433010\ldots \times 10^{-49}$$

Note that, the 4096th root of the formula, is equal to:

$$((((15*(4.341e-9)^4*(9.1808147e-12)^4))))/(((2*(4.341e-9)^2*1.00000000000000021978021978022-10^{-3}*9.1808147e-12)))^2$$

Input interpretation:

$$\sqrt[4096]{\frac{15(4.341 \times 10^{-9})^4 (9.1808147 \times 10^{-12})^4}{(2(4.341 \times 10^{-9})^2 \times 1.00000000000000021978021978022 - \frac{9.1808147 \times 10^{-12}}{10^3})^2}}$$

Result:

$$0.973189049098250659823876425354988746941554110443619763894\dots$$

0.97318904909... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

The result $4.5267433010 \dots * 10^{-49}$ indicates a phase in which the universe is both bouncing and in a condition of high symmetry. The result obtained, reversing the equation, represents the dilatonic phase, therefore of inflation-expansion. The surprising fact is that this value is very close to that of a fundamental Rogers-Ramanujan continued fraction, in which e , ϕ , and π are present.

With regard $4096 = 64^2$, we want describe the following fundamentals Ramanujan equations.

From:

Modular equations and approximations to π - Srinivasa Ramanujan
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= \quad \quad \quad 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

We obtain, with the following Ramanujan equations, the new mathematical connection:

$$\begin{aligned} &64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}. \\ &= 2508927.99839293; \quad \textcolor{blue}{64 * 39202} = 2508928; \quad 2508928 \div \textcolor{blue}{4096} = 612.53125; \end{aligned}$$

$$612.53125 \times \textcolor{blue}{64} = \textcolor{blue}{39202};$$

$$\textcolor{blue}{39202} \times 2 - \textcolor{blue}{4096} - 276 + \textcolor{blue}{64} + 8 - 612.53125 = \textcolor{blue}{73491.46875}$$

$$2508927.9983929391347126585602054 \div 64 = 39201.9999748$$

$$(2508928 \div 39202 = 64)$$

Now, dividing 73492 and 39202, we obtain:

$$73492/39202$$

$$\frac{73492}{39202}$$

$$1.874700270394367634304372225906841487679200040814244171215\dots$$

1.87470027... (note that, dividing this result by 2, we obtain 0.937350135197, that is very near both to the value of the dilaton and to the mass of proton in GeV)

Further we observe that **0.937350135197** is also practically equal to the value of mesonic Regge slope as in the following Table:

From:

Rotating strings confronting PDG mesons

Jacob Sonnenschein and Dorin Weissman - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014

| Traj. | N | m | α' | a |
|-------------------|-------|-----------------------|-----------------|--|
| π/π_2 | $4+3$ | $m_{u/d} = 0 - 250$ | $0.770 - 0.801$ | $a_0 = (-0.34) - 0, \quad a_2 = (-1.53) - (-1.20)$ |
| a_1 | 4 | $m_{u/d} = 0 - 380$ | $0.777 - 0.862$ | $(-0.89) - (-0.20)$ |
| h_1 | 4 | $m_{u/d} = 0 - 265$ | $0.827 - 0.876$ | $(-0.85) - (-0.71)$ |
| ω/ω_3 | $5+3$ | $m_{u/d} = 240 - 345$ | $0.937 - 1.000$ | $a_1 = (-0.23) - (-0.04), \quad a_3 = (-1.54) - (-1.28)$ |
| ϕ | 3 | $m_s = 505 - 520$ | $1.005 - 1.045$ | 0.00 |
| Ψ | 4 | $m_c = 1390 - 1465$ | $0.464 - 0.514$ | $(-0.27) - (-0.10)$ |
| Υ | 6 | $m_b = 4730 - 4740$ | $0.417 - 0.428$ | 0.00 |
| χ_b | 3 | $m_b = 4820$ | 0.468 | -0.08 |

Table 3. The results of the meson WKB fits, all in the (n, M^2) plane. The ranges listed are those where χ^2 is within 10% of its optimal value. N is the number of data points in the trajectory.

2019 Review of Particle Physics.

M. Tanabashi et al. (Particle Data Group). Phys. Rev. D 98, 030001 (2018) and 2019 update.

LIGHT UNFLAVORED MESONS

($S = C = B = 0$)

For $I = 1$ (π, b, ρ, a): $u\bar{d}, (u\bar{u} - d\bar{d})/\sqrt{2}, d\bar{u}$;
for $I = 0$ ($\eta, \eta', h, h', \omega, \phi, f, f'$): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

INSPIRE search

$$\omega(1650) \quad I^G(J^{PC}) = 0^-(1^{--})$$

$\omega(1650)$ MASS

1670 ± 30 MeV

$\omega(1650)$ WIDTH

315 ± 35 MeV

The light (unflavored) scalar mesons may be divided into three groups:

mesons having a mass below $1 \text{ GeV}/c^2$

mesons having a mass between $1 \text{ GeV}/c^2$ and $2 \text{ GeV}/c^2$

other radially-excited unflavored scalar mesons above $2 \text{ GeV}/c^2$

from:

<https://physics.stackexchange.com/questions/30470/what-is-the-physical-significance-of-the-dilaton-in-string-theory>

The dilaton is a fundamental scalar field in closed string theory. The effective gravity equations in string theory includes the gravi-dilaton part that looks very similar to Brans-Dicke scalar-tensor theory of gravity (This is valid only at tree level). The dilaton field, as mention before, controls the string coupling constant so the genus expansion in string theory is directly related to the dilaton field and to corrections to General Relativity.

In the paper: Cosmological Constraints on Higgs-Dilaton Inflation

Manuel Trashorras, Savvas Nesseris, and Juan Garcia-Bellido - arXiv:1604.06760v3 [astro-ph.CO] 13 Sep 2016

The fundamental constraints are indicated in the following Table:

| Parameter | $w_0 w_a$ CDM | HDM (pred.) | HDM (obs.) | $w_0 w_a$ CDM | HDM (pred.) | HDM (obs.) |
|--------------------|---------------------------|--------------------------------|--------------------------------|---------------------------------|---------------------------------|---------------------------------|
| | Confidence level 68.3% | | Confidence level 95.5% | | | |
| $\Omega_b h^2$ | 0.02237 ± 0.00025 | 0.02231 ± 0.00022 | 0.02233 ± 0.00022 | $0.02237^{+0.00051}_{-0.00049}$ | $0.02231^{+0.00043}_{-0.00043}$ | $0.02233^{+0.00044}_{-0.00043}$ |
| $\Omega_c h^2$ | 0.1177 ± 0.0018 | 0.1181 ± 0.0011 | 0.1177 ± 0.0013 | $0.1177^{+0.0035}_{-0.0035}$ | $0.1181^{+0.0024}_{-0.0021}$ | $0.1177^{+0.0025}_{-0.0025}$ |
| $100\theta_{MC}$ | 1.04111 ± 0.00045 | 1.04106 ± 0.00040 | 1.04110 ± 0.00042 | $1.04111^{+0.00088}_{-0.00088}$ | $1.04106^{+0.00080}_{-0.00081}$ | $1.04110^{+0.00083}_{-0.00082}$ |
| τ_{RE} | $0.069^{+0.017}_{-0.019}$ | 0.066 ± 0.013 | 0.070 ± 0.014 | $0.069^{+0.038}_{-0.035}$ | $0.066^{+0.028}_{-0.025}$ | $0.070^{+0.027}_{-0.026}$ |
| $\ln(10^{10} A_s)$ | $3.067^{+0.032}_{-0.036}$ | 3.063 ± 0.025 | 3.068 ± 0.026 | $3.067^{+0.039}_{-0.036}$ | $3.063^{+0.045}_{-0.049}$ | $3.068^{+0.050}_{-0.050}$ |
| w_0 | -0.93 ± 0.10 | $-0.99999^{+0.0025}_{-0.0026}$ | -1.0001 ± 0.0032 | $-0.93^{+0.21}_{-0.20}$ | $-0.99999^{+0.0056}_{-0.0059}$ | $-1.0001^{+0.0072}_{-0.0074}$ |
| w_a | $-0.21^{+0.41}_{-0.31}$ | $-0.015^{+0.071}_{-0.048}$ | $0.001^{+0.039}_{-0.034}$ | $-0.21^{+0.69}_{-0.74}$ | $-0.02^{+0.18}_{-0.22}$ | $0.00^{+0.15}_{-0.16}$ |
| n_s | 0.9694 ± 0.0056 | $0.9663^{+0.0032}_{-0.0022}$ | $0.9693^{+0.0046}_{-0.0042}$ | $0.969^{+0.011}_{-0.011}$ | $0.9665^{+0.0045}_{-0.0051}$ | $0.9693^{+0.0083}_{-0.0082}$ |
| α_s | -0.0047 ± 0.0078 | -0.0027 ± 0.0073 | -0.0014 ± 0.0066 | $-0.005^{+0.015}_{-0.015}$ | $-0.003^{+0.015}_{-0.014}$ | $-0.001^{+0.013}_{-0.014}$ |
| $r_{0.05}$ | $0.045^{+0.017}_{-0.038}$ | 0.0002 ± 0.0017 | $0.00255^{+0.00070}_{-0.0010}$ | < 0.0964 | $0.0002^{+0.0031}_{-0.0033}$ | $0.0025^{+0.0017}_{-0.0016}$ |
| N_{inf} | n/a | n/a | 70^{+9}_{-10} | n/a | n/a | 70^{+20}_{-20} |

Table II: Constraints on cosmological parameters $\Omega_b h^2$, $\Omega_c h^2$, $100\theta_{MC}$, τ_{RE} , w_{DE}^0 , w_{DE}^a , $\ln(10^{10} A_s)$, n_s , α_s , $r_{0.05}$ and N_{inf} and $BKP + len. + low\ell + MPK + ext.$ for the $w_0 w_a$ CDM and HDM (observed and predicted) COSMOMC chains at the 68.3% and 95.5% confidence levels.

We note as n_s and w_0 have the values:

[0.9694-0.9665-0.9693] [0.969-0.9665-0.9693], with a mean of **0.968333....** and [-0.93, -0.99999, -1.0001] [-0.93, -0.99999, -1.0001], with a mean of **-0.9766966667**

We observe that, from the following expression

$$0.9991104684 - (-1 + (3.146016528)^2/6) + 0.9568666373 + 0.5683000031$$

where 0.9991104684, 0.9568666373 and 0.5683000031 are the values of the Rogers-Ramanujan continued fractions, while 3.146016528 is a good approximation to π , we obtain:

$$0.9991104684 - \left(-1 + \frac{3.146016528^2}{6} \right) + 0.9568666373 + 0.5683000031$$

$$1.874707109725137536$$

$$1.87470710971097...$$

Indeed:

$$1.874707109725137536 * 39202$$

$$1.874707109725137536 \times 39202$$

$$73492.268115444841686272$$

73492.268115...

Again onetime, we see as the Rogers-Ramanujan continued fractions are fundamental for the results that we have obtained in a cosmological context. These are:

$$4 \int_0^{\infty} \frac{tdt}{e^{\sqrt{5}t} \cosh t} = \frac{1}{1 + \frac{1^2}{1 + \frac{1^2}{1 + \frac{2^2}{1 + \frac{2^2}{1 + \frac{3^2}{1 + \frac{3^2}{1 + \dots}}}}}} \approx 0.5683000031$$

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

From:

is independent of k . The amplification factor \mathcal{F} for the perturbation modes during the bounce phase is

$$\begin{aligned}\mathcal{F} &= e^{\int_{B_-}^{B_+} \omega d\tau} \\ &\simeq \exp \left[\sqrt{\Upsilon + \frac{2}{T^2}} t + \frac{2 + 3\Upsilon T^2 + \Upsilon^2 T^4}{3T^4 \sqrt{\Upsilon + \frac{2}{T^2}}} t^3 \right] \Big|_{B_-}^{B_+}. \end{aligned}\tag{47}$$

Inserting the values of the parameters $\Upsilon = 2.7 \times 10^{-4}$, $T = 0.5$, and $t_{B_+} = -t_{B_-} = 1$ (obtained from the numerical solution for the cosmological background) we obtain

$$\mathcal{F} \simeq e^{11} \sim O(10^5)\tag{48}$$

$$\exp(((\sqrt((((2.7e-4+(2/0.5^2)))))))) + (((((2+3*2.7e-4*0.5^2+(2.7e-4)^2*0.5^4))/(((3*0.5^4*\sqrt((2.7e-4+(2/0.5^2)))))))))))$$

Input interpretation:

$$\exp \left(\sqrt{2.7 \times 10^{-4} + \frac{2}{0.5^2}} + \frac{2 + 3 \times 2.7 \times 10^{-4} \times 0.5^2 + (2.7 \times 10^{-4})^2 \times 0.5^4}{3 \times 0.5^4 \sqrt{2.7 \times 10^{-4} + \frac{2}{0.5^2}}} \right)$$

Result:

735.1166706468927763076647539659887722848005904472674374011...

735.116670646...

For $B = 81.56$, we obtain:

$$735.116670646 * 81.56 - 81.56$$

Input interpretation:

735.116670646 * 81.56 - 81.56

Result:

59874.55565788776

$$59874.55565788776 \approx e^{11} = 59874.1417151978$$

Note that, from the previous Ramanujan equations, we obtain:

$$2508952 \div (21 * 2) = 59736.9523809;$$

$59736.9523809 + (276/2) = 59874.9523809$ result very near to the above values

From:

By requiring v_k and v'_k to be continuous at the space-like surfaces of τ_{B-} and τ_{B+} , one can track how each Fourier mode evolves through the bouncing phase and derive the detailed expressions of all the coefficients appearing in the above solutions of the perturbations equations. We leave the detailed calculation for the second part of the Appendix, and here just write down the final expression of the perturbation which is

$$v_k^e(\tau) \simeq \mathcal{F} \frac{\gamma_E \Gamma_{\nu_c} c_2(k) k^{-\nu_c}}{2^{1-\nu_c} \pi (\tau_{B-} - \tilde{\tau}_{B-})^{\nu_c - \frac{1}{2}}} \left(\frac{\tau - \tilde{\tau}_{B+}}{\tau_{B+} - \tilde{\tau}_{B+}} \right)^{\frac{1}{2}} \quad (52)$$

where $\gamma_E \simeq 0.58$ is the Euler-Mascheroni constant.

As a consequence, the scale invariance of primordial fluctuations which exited the Hubble radius during the matter-dominated phase of contraction is preserved through the Ekpyrotic phase and the nonsingular bouncing phase, and the final power spectrum will be scale-invariant in the expanding phase. To find the amplitude of the spectrum, we apply the definition of P_ζ to obtain

$$P_\zeta \simeq \mathcal{F}^2 \frac{\gamma_E^2 H_m^2}{192\pi^2 M_p^2}, \quad (81)$$

where the parameter H_m is the physical Hubble parameter at the end of matter-dominated period of contraction.

For

$$\mathcal{F} \simeq e^{11} \sim O(10^5) = 59874.5556$$

Hubble parameter = 1.00000000000000021978021978022

$$M_P^2 = (9.1808147 \times 10^{-12})^2$$

$\gamma_E = 0.5772156649$, we obtain:

$$P_\zeta \simeq \mathcal{F}^2 \frac{\gamma_E^2 H_m^2}{192\pi^2 M_p^2}$$

$$\frac{(((59874.5556^2 * 0.5772156649^2 * 1.000000000000000021978021978022^2))) / (((192\pi^2 * (9.1808147e-12)^2)))$$

Input interpretation:

$$\frac{59874.5556^2 \times 0.5772156649^2 \times 1.000000000000000021978021978022^2}{192\pi^2 (9.1808147 \times 10^{-12})^2}$$

Result:

$$7.4782056772743363026645464055926098151835874678419427... \times 10^{27}$$

And:

$$1/((((((59874.5556^2 * 0.5772156649^2 * 1.000000000000000021978021978022^2))) / (((192\pi^2 * (9.1808147e-12)^2))))))$$

Input interpretation:

$$\frac{1}{59874.5556^2 \times 0.5772156649^2 \times 1.000000000000000021978021978022^2}$$

$$\frac{1}{192\pi^2 (9.1808147 \times 10^{-12})^2}$$

Result:

$$1.3372191714904543453413712162995524627934078427778430... \times 10^{-28}$$

$$1.3372191714... \times 10^{-28}$$

We note that:

$$((((((59874.5556^2 * 0.5772156649^2 * 1.000000000000000021978021978022^2))) / (((192\pi^2 * (9.1808147e-12)^2))))))^1/(4096*8)$$

Input interpretation:

$$\sqrt[4096 \times 8]{\frac{59874.5556^2 \times 0.5772156649^2 \times 1.000000000000000021978021978022^2}{192\pi^2 (9.1808147 \times 10^{-12})^2}}$$

Result:

$$1.00196059226...$$

$$\textcolor{red}{1.00196059226...}$$

And:

$$1 / (((((((59874.5556^2 * 0.5772156649^2 * 1.00000000000000021978021978022^2))) / (((192\pi^2 * (9.1808147e-12)^2))))))^1/(4096*8)$$

Input interpretation:

$$\frac{1}{4096 \times 8 \sqrt{\frac{59874.5556^2 \times 0.5772156649^2 \times 1.00000000000000021978021978022^2}{192 \pi^2 (9.1808147 \times 10^{-12})^2}}}$$

Result:

0.998043244145...

0.998043244145 result very near to the dilaton value **0.989117352243 = ϕ**

Further, we have that:

$$\log_{1.00196059226} (((((((59874.5556^2 * 0.5772156649^2 * 1.00000000000000021978021978022^2))) / (((192\pi^2 * (9.1808147e-12)^2)))))))$$

Input interpretation:

$$\log_{1.00196059226} \left(\frac{59874.5556^2 \times 0.5772156649^2 \times 1.00000000000000021978021978022^2}{192 \pi^2 (9.1808147 \times 10^{-12})^2} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

32767.9999...

32767.999... \approx 32768

$$32768 = 64^2 \times 8$$

From:

CHAOTIC INFLATION

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The part of the universe inside a domain filled with a homogeneous field φ expands as de Sitter space with the scale factor $a(t) = a_0 \exp(Ht)$, where

$$H = (\frac{8}{3}\pi V(\varphi)/M_p^2)^{1/2} = (\frac{2}{3}\pi\lambda)^{1/2}\varphi^2/M_p. \quad (1)$$

The equation of motion of the field φ inside this domain is

$$\ddot{\varphi} + 3H\dot{\varphi} = -\lambda\varphi^3, \quad (2)$$

which implies that at $\varphi^2 \gg M_p^2/6\pi$

$$\varphi = \varphi_0 \exp\{-[\sqrt{\lambda}M_p/(6\pi)^{1/2}]t\}. \quad (3)$$

This means that at $\lambda \ll 1$ the typical time $\Delta t \sim (6\pi)^{1/2}/\sqrt{\lambda}M_p$, during which the field φ decreases considerably, is much greater than the Planck time $t_p \sim (6M_p)^{-1}$ (see below). During the main part of this period the universe expands exponentially, and the scale factor of the universe after expansion grows as follows:

$$a(\Delta t) \sim a_0 \exp(H\Delta t) \sim a_0 \exp(2\pi\varphi_0^2/M_p^2). \quad (4)$$

From eq. (4) it follows that inflation of the universe is sufficiently large ($\exp(H\Delta t) \gtrsim \exp(65)$ [8]) if

$$\varphi_0 \gtrsim 3M_p. \quad (5)$$

Such a value of φ_0 is quite possible if $\frac{1}{4}\lambda\varphi_0^4 \lesssim M_p^4$, which implies that

$$\lambda \lesssim 10^{-2}. \quad (6)$$

$$M_p = 1.2209 * 10^{19} \text{ GeV}$$

$$\varphi_0 = 3.6627 * 10^{19} \text{ GeV}; t = 5.391 * 10^{44} \text{ s}$$

$$(((4*(1.2209e+19)^4))) / (((3.6627e+19)^4))$$

Input interpretation:

$$\frac{4(1.2209 \times 10^{19})^4}{(3.6627 \times 10^{19})^4}$$

Result:

0.049382716049382716049382716049382716049382716049...

Repeating decimal:

$\overline{0.049382716}$ (period 9)

$$0.049382716 = \lambda = 4.9382716 * 10^{-2}$$

$$\varphi = \varphi_0 \exp \{-[\sqrt{\lambda} M_p / (6\pi)^{1/2}] t\}$$

$$3.6627e+19 * \exp((((-(((\sqrt(4.9382716e-2)*1.2209e+19)))/(\sqrt(6\pi)*(5.391e-44))))))$$

We note that:

$$\exp((((-(((\sqrt(4.9382716e-2)*1.2209e+19)))/(\sqrt(6\pi)*(5.391e-44))))))$$

Input interpretation:

$$\exp \left\{ -\frac{\sqrt{4.9382716 \times 10^{-2}} \times 1.2209 \times 10^{19}}{\sqrt{6\pi}} \times 5.391 \times 10^{-44} \right\}$$

Result:

0.999999999999999999999999999966311137290625426234743627534526...

0.99999999999999999999999999996631113729....

The final result is:

Input interpretation:

$$3.6627 \times 10^{19} \exp \left\{ -\frac{\sqrt{4.9382716 \times 10^{-2}} \times 1.2209 \times 10^{19}}{\sqrt{6\pi}} \times 5.391 \times 10^{-44} \right\}$$

Result:

$3.66269999999999999999999998766078025543737486699954845... \times 10^{19}$

$$3.66269999... * 10^{19} \text{ GeV} = \varphi$$

We have that:

$$H = (\frac{8}{3}\pi V(\varphi)/M_p^2)^{1/2} = (\frac{2}{3}\pi\lambda)^{1/2}\varphi^2/M_p$$

$$[\sqrt{(2/3*\pi*4.9382716e-2)}*(3.662699999e+19)^2] / (1.2209e+19)$$

Input interpretation:

$$\frac{\sqrt{\frac{2}{3}\pi \times 4.9382716 \times 10^{-2} (3.662699999 \times 10^{19})^2}}{1.2209 \times 10^{19}}$$

Result:

$$3.5337790830642630531826184852627505264368780578171988... \times 10^{19}$$

$$3.533779083.... \times 10^{19} \text{ GeV} = H$$

From H / φ , we obtain:

$$(3.533779083064263053 \times 10^{19}) / (3.662699999e+19)$$

Input interpretation:

$$\frac{3.533779083064263053 \times 10^{19}}{3.662699999 \times 10^{19}}$$

Result:

$$0.964801671998543349168248382113809043086741759654555863066...$$

0.96480167199... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

From:

where ρ is the energy density of relativistic particles,

$$\rho = \frac{1}{30} N \pi^2 T^4 . \quad (10)$$

Here N is the effective number of degrees of freedom (of particles) in the theory. Typically in GUTs $N \gtrsim 200$. By comparison of $\tau \sim T^{-1}$ and t for $N \gtrsim 200$ one concludes that the field φ can be influenced by high-temperature effects at $T \lesssim \frac{1}{50} M_p$ only. However at such a temperature the energy density of relativistic particles becomes negligibly small as compared with $V(\varphi)$ in the domains with $V(\varphi) \sim M_p^4$. Therefore the expansion of such domains becomes exponential much earlier than the temperature decreases down to $\frac{1}{50} M_p$, the temperature inside the domains becomes exponentially small and all high-temperature effects disappear.

$$1/30*200*\pi^2*(1/50*1.2209e+19)^4$$

Input interpretation:

$$\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4$$

Result:

$$2.3391002961569544568926923289657523992345924584517473... \times 10^{71}$$

$$2.33910029... \times 10^{71} = \rho$$

$$(1.2209e+19)^4$$

Scientific notation:

$$2.2218788499821761 \times 10^{76}$$

$$2.221878... \times 10^{76} = V(\varphi)$$

Thence, $V(\varphi) \times \rho$ is equal to:

$$(((1/30*200*\pi^2*(1/50*1.2209e+19)^4))) * 2.2218788499821761 \times 10^{76}$$

Input interpretation:

$$\left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4 \right) \times 2.2218788499821761 \times 10^{76}$$

Result:

$$5.1971974760181814984144378647420379713938444976147238\dots \times 10^{147}$$

$$5.1971974\dots * 10^{147}$$

Multiplying by c^2 , we obtain, for $E = mc^2$:

$$9*10^{16}(((1/30*200*\pi^2*(1/50*1.2209e+19)^4)) * 2.2218788499821761 \times 10^{76}$$

Input interpretation:

$$9 \times 10^{16} \left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4 \right) \times 2.2218788499821761 \times 10^{76}$$

Result:

$$4.6774777284163633485729940782678341742544600478532514\dots \times 10^{164}$$

$$4.6774777\dots * 10^{164}$$

$$9*10^{16}(((1/30*200*\pi^2*(1/50*1.2209e+19)^4)) * 2.2218788499821761 \times 10^{76} * 1/5.1971974e+147$$

Input interpretation:

$$\frac{9 \times 10^{16} \left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4 \right) \times 2.2218788499821761 \times 10^{76}}{5.1971974 \times 10^{147}}$$

Result:

$$9.00000\dots \times 10^{16}$$

$$9*10^{16} = c^2$$

From , $V(\varphi) \times \rho$, we also obtain, performing the 4096th root:

$$((((((1/30*200*\pi^2*(1/50*1.2209e+19)^4))) * 2.2218788499821761 \times 10^{76})))^{1/(64^2)}$$

Input interpretation:

$$\sqrt[64^2]{\left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4 \right) \times 2.2218788499821761 \times 10^{76}}$$

Result:

$$1.086584286426355887430053982677716640254852754845435177352\dots$$

1.086584286... result very near to the value of the following Ramanujan mock theta function:

$$\left(1 + \frac{0.5^{(2+1)\times(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2\times2+1})} \right) =$$

$$= 1.086405529953917050691244239631336405529953917050691244239\dots =$$

$$= 1.0864055\dots$$

Thence, we obtain the following new mathematical connection:

$$\begin{aligned} & \left(\sqrt[64^2]{\left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4 \right) \times 2.2218788499821761 \times 10^{76}} \right) \cong \\ & \cong \left(1 + \frac{0.5^{(2+1)\times(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2\times2+1})} \right) = \\ & = 1.086584286\dots \cong 1.0864055\dots \end{aligned}$$

Further, we have:

$$\ln((((((((((1/30*200*\pi^2*(1/50*1.2209e+19)^4))) * 2.2218788499821761 \times 10^{76}))))^1/(64^2))))$$

Input interpretation:

$$\log \left(\sqrt[64^2]{\left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19} \right)^4 \right) \times 2.2218788499821761 \times 10^{76}} \right)$$

$\log(x)$ is the natural logarithm

Result:

0.083039093799684673597701833627558521154804079043466736464...

0.08303909... $\approx 1/12$

We have also that:

$$\log \text{base} 1.086584286 (((((((1/30*200*\pi^2*(1/50*1.2209e+19)^4))) * 2.2218788499821761 \times 10^{76}))))$$

Input interpretation:

$$\log_{1.086584286}\left(\left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19}\right)^4\right) \times 2.2218788499821761 \times 10^{76}\right)$$

$\log_b(x)$ is the base- b logarithm

Result:

4096.000019354688818500785756230849936270297803536870199560...

4096.00001935... ≈ 4096

And:

$$\sqrt{\log \text{base} 1.086584286 (((((((1/30*200*\pi^2*(1/50*1.2209e+19)^4))) * 2.2218788499821761 \times 10^{76}))))})}$$

Input interpretation:

$$\sqrt{\log_{1.086584286}\left(\left(\frac{1}{30} \times 200 \pi^2 \left(\frac{1}{50} \times 1.2209 \times 10^{19}\right)^4\right) \times 2.2218788499821761 \times 10^{76}\right)}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.00000015120850621591229222018174704964038931305978606565...

64.0000001512... ≈ 64

From:

Numerical study of inflationary preheating with arbitrary power-law potential and a realization of curvaton mechanism

Jie Jiang, Qiuyue Liang, Yi-Fu Cai, Damien A. Easson and Yang Zhang

We have that:

The effective mass of inflaton is

$$m_{\phi_{\text{eff}}}^2 = p \lambda M_p^{4-p} \mu^{p-2} \quad (14)$$

$$M_p = 1/\sqrt{8\pi G}$$

$$1/3 * 3.7e-10 * (1/sqrt(8Pi * 6.67e-11))^{(4-(1/3))} * 0.043^{((1/3)-2)}$$

Input interpretation:

$$\frac{1}{3} \times 3.7 \times 10^{-10} \left(\frac{1}{\sqrt{8 \pi \times 6.67 \times 10^{-11}}} \right)^{4-1/3} \times 0.043^{1/3-2}$$

Result:

$$2.86613... \times 10^8$$

$$2.86613... * 10^8$$

| p | $\phi_1 (M_p)$ | λ | μ_{max} | g | q |
|-----|----------------|-----------------------|--------------------|----------|---------|
| 1/3 | -7.9 | 3.7×10^{-10} | 0.043 | 0.0868 | 1.48968 |
| 1/2 | -9.2 | 3.7×10^{-10} | 0.05 | 0.075864 | 2.17388 |
| 2/3 | -10.2 | 3.2×10^{-10} | 0.058 | 0.057 | 2.8855 |
| 1.0 | -12.1 | 2.0×10^{-10} | 0.078 | 0.0288 | 4.92016 |
| 2.0 | -17.4 | 1.7×10^{-11} | 0 | 0.003291 | \ |

Table 1. Model parameters and initial conditions during inflation.

the resonance area. The equation of motion of $X_k(t) = a^{3/2}(t)\chi_k(t)$ is,

$$\ddot{X}_k + \left(\frac{k^2}{a^2} + g^2 \Phi^2(t) \sin^2 m_{\phi_{\text{eff}}} t \right) X_k = 0 , \quad (25)$$

where we have neglected the $(-\frac{9}{4}H^2 - \frac{3}{2}\dot{H})$ term since the background is matter-dominated. One can straightforwardly rewrite Equation (25) in the form of the Mathieu equation as follows,

$$X_k'' + (A_k - 2q \cos 2z) = 0 ,$$

$$A_k = \frac{k^2/a^2}{m_{\phi_{\text{eff}}}^2} + 2q , \quad q = \frac{g^2 \Phi^2}{4m_{\phi_{\text{eff}}}^2} \approx \frac{g^2 \mu^{4-p}}{400 p \lambda M_p^{4-p}} , \quad (26)$$

where the prime denotes the derivative with respect to $z = m_{\phi_{\text{eff}}} t$. The parameter q is often used to distinguish the narrow resonance ($q \ll 1$) and the broad one ($q \gg 1$).

$$k \in [1 \times 10^{-60}, 1 \times 10^{-57}] l_p^{-1}$$

$$1*10^{-60} * (1.616252e-35)^{-1}$$

Input interpretation:

$$\frac{1 \times 10^{-60}}{1.616252 \times 10^{-35}}$$

Result:

$$6.1871539834134776012651492465283878999066977179301247... \times 10^{-26}$$

$$k = 6.18715398... * 10^{-26}$$

Now, from (26):

$$\frac{g^2 \mu^{4-p}}{400 p \lambda M_p^{4-p}}$$

$$0.0868^2 * (0.043)^{(4-(1/3))} / (((((400*1/3*(3.7e-10)*(1/sqrt(8Pi*6.67e-11)))^{(4-(1/3))})))))$$

Input interpretation:

$$0.0868^2 \times \frac{0.043^{4-1/3}}{400 \times \frac{1}{3} \times 3.7 \times 10^{-10} \left(\frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}} \right)^{4-1/3}}$$

Result:

$$1.21512... \times 10^{-16}$$

$$1.21512.. \cdot 10^{-16} = q$$

$$A_k = \frac{k^2/a^2}{m_{\phi_{\text{eff}}}^2} + 2q$$

$$((((6.18715398e-26)^2 / (2.86613e+8)))) + 2*1.21512e-16$$

Input interpretation:

$$\frac{(6.18715398 \times 10^{-26})^2}{2.86613 \times 10^8} + 2 \times 1.21512 \times 10^{-16}$$

Result:

$2.43024... \times 10^{-16}$

$$X_k'' + (A_k - 2q \cos 2z) = 0$$

$$2.43024\text{e-}16 - ((2 * 1.21512\text{e-}16) \cos(2 * 16929.6)))$$

Input interpretation:

$$2.43024 \times 10^{-16} - (2 \times 1.21512 \times 10^{-16}) \cos(2 \times 16929.6)$$

Result:

$$8.92362\ldots \times 10^{-17}$$

$$8.92362... \times 10^{-17}$$

And:

$$(((2*1.21512e-16) \cos(2*16929.6)))-2.43024e-16$$

Input interpretation:

$$(2 \times 1.21512 \times 10^{-16}) \cos(2 \times 16929.6) - 2.43024 \times 10^{-16}$$

Result:

$$-8.92362 \dots \times 10^{-17}$$

$-8.92362 \dots \times 10^{-17} = X_k''$ that is the equation of motion

From $k = 6.18715398 \dots \times 10^{-26}$, we obtain:

$$(-8.92362 \times 10^{-17}) / (6.18715398 \times 10^{-26})$$

Input interpretation:

$$\frac{-8.92362 \times 10^{-17}}{6.18715398 \times 10^{-26}}$$

Result:

$$-1.4422818680197126757139475620420877257688679666575875 \dots \times 10^9$$

$$-1.442281867 \dots \times 10^9$$

Or:

Input interpretation:

$$\frac{8.92362 \times 10^{-17}}{6.18715398 \times 10^{-26}}$$

Result:

$$1.44228186801971267571394756204208772576886796665758753 \dots \times 10^9$$

Decimal form:

$$1442281868.01971267571394756204208772576886796665758753$$

$$1442281868.01971267 \dots$$

We have:

$$\ln(1442281868.01971267571394)$$

Input interpretation:

$$\log(1.44228186801971267571394 \times 10^9)$$

$\log(x)$ is the natural logarithm

Result:

$$21.08949232690133480861221 \dots$$

21.0894923... result very near to the Fibonacci number 21

We have also:

$$1.962364415 * \sqrt{1.442281868.01971267571394} - (4096/4+8)$$

Where 1.962364415 is a Ramanujan mock theta function

Input interpretation:

$$1.962364415 \sqrt{1.44228186801971267571394 \times 10^9} - \left(\frac{4096}{4} + 8 \right)$$

Result:

73493.47139...

73493.47139...

Indeed, we obtain:

$$((((((73493.47139 + (4096/4+8)))) / (((1.962364415))))))^2$$

Input interpretation:

$$\left(\frac{73493.47139 + \left(\frac{4096}{4} + 8 \right)}{1.962364415} \right)^2$$

Result:

$$1.44228186786372058825903588415351527529494982031675077... \times 10^9$$

$$1.442281867... * 10^9$$

Decimal form:

$$1442281867.86372058825903588415351527529494982031675077$$

$$1442281867.8637....$$

Further, we have the following mathematical connections:

$$\left[1.962364415 \sqrt{1.44228186801971267571394 \times 10^9} - \left(\frac{4096}{4} + 8 \right) \right] = **73493.47139** \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt{_{^{13}} \left(\begin{array}{c} N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS} + \\ \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS} \end{array} \right)} = \right.$$

$$-3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525 \dots \Rightarrow$$

(the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$)

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700\dots$$

$$= 73491.7883254 \dots \Rightarrow$$

(the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane)

$$\left(\begin{array}{c} I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(- \left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leqslant H^{1-\epsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\epsilon_1} \right\} \end{array} \right),$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\dots$$

(the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series)

From:

Unification of constant-roll inflation and dark energy with logarithmic R^2 -corrected and exponential $F(R)$ gravity

S.D.Odintsova, V.K.Oikonomou, L.Sebastiani - Nuclear Physics B 923 (2017) 608–632

We have the following action:

$$I = \int_{\mathcal{M}} d^4 \sqrt{-g} \left[\frac{R}{\kappa^2} + \gamma(R) R^2 + f_{DE}(R) + \mathcal{L}_m \right]. \quad (2.1)$$

$$\kappa^2 = 16\pi/M_{Pl}^2,$$

$H = \dot{a}(t)/a(t)$ being the Hubble parameter.

$$H = 1.00000000000000021978021978022$$

At the beginning of inflation we have $\mathcal{N} \ll N$ and $H \simeq H_{dS}$, while at the end of the early-time acceleration, when $N = 0$, one recovers $H = H_0$.

In the case of a constant value of $\gamma(R) = \gamma_0$, namely for $\gamma_1 = 0$, the de Sitter solution is obtained as an asymptotic limit of the first Friedmann equation, when the R^2 -term is dominates the evolution. This is the so-called Starobinsky model, where the Hilbert–Einstein term guarantees a graceful exit from the accelerated phase. Since in the Starobinsky model $1/\kappa^2 \ll R$, we have a regime of super-Planckian curvature. Here, the R^2 term has the same order of magnitude as the Hilbert–Einstein term during the inflationary era. In this case, the running constant $\gamma(R)$ in (2.5) with $\gamma_1 \neq 0$, determines the value of the de Sitter solution as in Eq. (2.7).

where $R = 4\Lambda$ is the curvature of the Universe when the dark energy is dominant, and Λ is the Cosmological constant. In the following, we will assume that $f_{DE}(R)$ and \mathcal{L}_m in (2.1) are

$$R = 4 * 2.888e-122 = 11.552 * 10^{-122}$$

$$\beta = \frac{1}{2\mathcal{N}}. \quad (2.19)$$

This means that the model at hand satisfies the condition for constant-roll inflation. This fact has

$$\beta = 1/120 = 0.008333333$$

$$n_s = 0.9644 \pm 0.0049, \quad r < 0.10. \quad (2.22)$$

As a consequence, we must require $\mathcal{N} \simeq 60$ in order to obtain a viable inflationary scenario. This

$$H \simeq \sqrt{\frac{N}{18\gamma_0\kappa^2}}. \quad (2.28)$$

When $t = t_i$, the e -foldings number is given by $N = 18\gamma_0\kappa^2 H_{dS}^2$, and it is easy to verify that $H = H_{dS}$.

By imposing $\mathcal{N} \simeq 60$ in Eq. (2.13) we obtain,

$$R_{dS} \simeq R_0 e^{80}, \quad (2.31)$$

and the expansion curvature rate during inflation is defined in this way. The characteristic curvature at the time of inflation is $R_{dS} \simeq 10^{120} \Lambda$, in which case one has $R_0 \simeq 1.8 \times 10^{85} \Lambda$ and from Eq. (2.6) we must require $\gamma_1 \ll 0.005$. Finally, the relation between γ_0 and γ_1 is fixed by Eq. (2.7) and we obtain,

$$\gamma_0 \simeq \frac{e^{-80}}{\gamma_1 R_0 \kappa^2}. \quad (2.32)$$

where we have expanded (2.12) with respect to $H_{dS}|t_f - t_0| \ll 1$. Since $h_0 \simeq 1$, if we use (2.15) we get,

$$t_f \simeq \frac{N}{H_{dS}} - \frac{\sqrt{N}}{\sqrt{6}H_{dS}} + t_i \simeq \frac{N}{H_{dS}}, \quad (2.37)$$

with $\mathcal{N} \ll N$ being the total e -foldings of inflation. By equating (2.12) with (2.35) and by imposing $h_0 \simeq 1$, we can specify the frequency ω of the reheating solution as follows,

$$\omega = \frac{3H_{dS}}{\sqrt{6\mathcal{N}}} = \frac{\sqrt{\beta}}{2\sqrt{\gamma_0\gamma_1\kappa^2}}, \quad (2.38)$$

$$R = 4 * 2.888e-122 = 11.552 * 10^{-122}$$

$$R_{dS} = 1e+120 \times 2.888e-122 = 0.02888$$

$$R_0 = 1.8e+85 \times 2.888e-122 = 5.1984e-37$$

$$R_0 e^{80} = 5.1984e-37 \times e^{(80)} = 0.0288023714 \approx 0.02888$$

$$\gamma_1 = 0.00086$$

$$4.76213\dots \cdot 10^{39} = \gamma_0$$

$H_{ds} = H = 1.00000000000000021978021978022$

$$\beta = 1/120 = 0.008333333$$

From:

$$\gamma_0 \simeq \frac{e^{-80}}{\gamma_1 R_0 \kappa^2}.$$

$$\kappa^2 = 16\pi/M_{\text{Pl}}^2, = 8.477580536653051928 \times 10^{-36}$$

$$\exp(-80)/(((0.00086*5.1984\text{e-}37*(16\pi/2.435\text{e+}18^2))))$$

Input interpretation:

$$\frac{\exp(-80)}{0.00086 \times 5.1984 \times 10^{-37} \left(16 \times \frac{\pi}{(2.435 \times 10^{18})^2} \right)}$$

Result:

$$4.76213... \times 10^{39}$$

$$4.76213\dots \cdot 10^{39} = \gamma_0$$

$$\Delta_{\pm} = \frac{H_{\text{dS}}}{2} \left(-3 \pm \frac{\sqrt{\log \left[\frac{R_{\text{dS}}}{R_0} \right] \left(16 + 9 \log \left[\frac{R_{\text{dS}}}{R_0} \right] \right)}}{\log \left[\frac{R_{\text{dS}}}{R_0} \right]} \right) \quad (2.10)$$

$$(1.00000000000000021978021978022/2)(-3+(\sqrt{((\ln(0.02888 /5.1984e-37)*(16+9\ln(0.02888/5.1984e-37)))))*(((1/\ln(0.02888/5.1984e-37))))}))$$

Input interpretation:

1.000000000000000021978021978022

$$\frac{2}{\left(-3 + \sqrt{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right) \left(16 + 9 \log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right) \right) \times \frac{1}{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right)}} \right)}$$

$\log(x)$ is the natural logarithm

Result:

0.0165745...

$$0.0165745\dots = \Delta_+$$

$$(1.000000000000000021978021978022/2)(-3-(\sqrt{((\ln(0.02888 / 5.1984e-37)*(16+9\ln(0.02888/5.1984e-37))))*((((1/\ln(0.02888/5.1984e-37))))}))$$

Input interpretation:

1.000000000000000021978021978022

$$\frac{2}{\left(-3 - \sqrt{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right) \left(16 + 9 \log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right) \right) \times \frac{1}{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right)}} \right)}$$

$\log(x)$ is the natural logarithm

Result:

-3.016575...

$$-3.016575\dots = \Delta_- \quad (\text{note that } \Delta_+ - \Delta_- \approx -3 =$$

$$= -3.0000000000000006593406593406599999\dots)$$

And:

$$((((((1164.2696)^{1/15} + 0.00246)^{1/8}))) / (((1.000000000000000021978021978022/2)(-3+(\sqrt{((\ln(0.02888 / 5.1984e-37)*(16+9\ln(0.02888/5.1984e-37))))*((((1/\ln(0.02888/5.1984e-37))))))))*(((1/\ln(0.02888/5.1984e-37)))))))$$

Where 1164.2696 is the Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$, and 0.00246 is the sub-multiple of Fibonacci sum $246 = 233+13$

Input interpretation:

$$\left(\sqrt[8]{\sqrt[15]{1164.2696}} + 0.00246 \right) / \left(\frac{1.000000000000000021978021978022}{2} \right)$$

$$\left(-3 + \sqrt{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right) \left(16 + 9 \log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right)\right) \times \frac{1}{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right)}} \right)$$

$\log(x)$ is the natural logarithm

Result:

64.00184024298484715425973861995187751034473484956010662191...

64.00184024...

Further, we also obtain:

$$[((((1164.2696)^{1/15} + 0.00246)^{1/8})) / (((1.00000000000000021978021978022/2)(-3 + (\sqrt{(\ln(0.02888 / 5.1984e-37)) * (16 + 9 \ln(0.02888 / 5.1984e-37)))}) * (((1/\ln(0.02888 / 5.1984e-37)))))))^2]$$

Input interpretation:

$$\left(\sqrt[8]{\sqrt[15]{1164.2696}} + 0.00246 \right) / \left(\frac{1.00000000000000021978021978022}{2} \right)$$

$$\left(-3 + \sqrt{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right) \left(16 + 9 \log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right)\right) \times \frac{1}{\log\left(\frac{0.02888}{5.1984 \times 10^{-37}}\right)}} \right)^2$$

$\log(x)$ is the natural logarithm

Result:

4096.235554488554679024410165770928741687427296891525437197...

4096.235554...

Now, we have that:

with $\mathcal{N} \ll N$ being the total e -foldings of inflation. By equating (2.12) with (2.35) and by imposing $h_0 \simeq 1$, we can specify the frequency ω of the reheating solution as follows,

$$\omega = \frac{3H_{dS}}{\sqrt{6\mathcal{N}}} = \frac{\sqrt{\beta}}{2\sqrt{\gamma_0\gamma_1\kappa^2}}, \quad (2.38)$$

Data:

$$R = 4 * 2.888e-122 = 11.552 * 10^{-122}$$

$$R_{dS} = 1e+120 \times 2.888e-122 = 0.02888$$

$$R_0 = 1.8e+85 \times 2.888e-122 = 5.1984e-37$$

$$R_0 e^{80} = 5.1984e-37 \times e^{(80)} = 0.0288023714 \approx 0.02888$$

$$\gamma_1 = 0.00086; \quad \gamma_0 = 4.76213\dots * 10^{39}$$

$$H_{dS} = H = 1.000000000000000021978021978022$$

$$\beta = 1/120 = 0.008333333$$

$$\kappa^2 = 8.477580536653051928 \times 10^{-36}$$

$$\omega = \frac{3H_{dS}}{\sqrt{6\mathcal{N}}} = \frac{\sqrt{\beta}}{2\sqrt{\gamma_0\gamma_1\kappa^2}},$$

We have that:

$$\text{sqrt}(0.008333333)/(2*\text{sqrt}(4.76213e+39*0.00086*8.477580536653051928*10^{-36}))$$

Input interpretation:

$$\frac{\sqrt{0.008333333}}{2\sqrt{4.76213 \times 10^{39} \times 0.00086 \times 8.477580536653051928 \times 10^{-36}}}$$

Result:

$$0.007746286605155185821203006214751738102243469167085960030\dots$$

$$0.00774628660515\dots$$

We note that:

$$(((\text{sqrt}(0.008333333)/(2*\text{sqrt}(4.76213e+39*0.00086*8.477580536653051928*10^{-36}))))^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{\frac{\sqrt{0.008333333}}{2 \sqrt{4.76213 \times 10^{39} \times 0.00086 \times 8.477580536653051928 \times 10^{-36}}}}$$

Result:

0.998814048109416034716985614604821551191004651977838659829...

0.9988140481094.... result very near to the dilaton value $\mathbf{0.989117352243 = \phi}$

And that:

log base 0.998814048109416034716
 $((\text{sqrt}(0.008333333)/(2*\text{sqrt}(4.76213e+39*0.00086*8.477580536653051928*10^{-36}))))$

Input interpretation:

$$\log_{0.998814048109416034716} \left(\frac{\sqrt{0.008333333}}{2 \sqrt{4.76213 \times 10^{39} \times 0.00086 \times 8.477580536653051928 \times 10^{-36}}} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

4096.00...

4096

And:

$\text{sqrt}(((\text{log base } 0.998814048109416034716 ((\text{sqrt}(0.008333333)/(2*\text{sqrt}(4.76213e+39*0.00086*8.477580536653051928*10^{-36})))))))$

Input interpretation:

$$\sqrt{\log_{0.998814048109416034716} \left(\frac{\sqrt{0.008333333}}{2 \sqrt{4.76213 \times 10^{39} \times 0.00086 \times 8.477580536653051928 \times 10^{-36}}} \right)}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.00000...

64

In the Starobinsky inflationary scenario, the equation (2.33) with solution (2.35) is still valid, but during the reheating era we have,

$$t_r \simeq t_0 + \sqrt{6\gamma_0\kappa^2} \simeq 36\gamma_0\kappa^2 H_{dS} + \sqrt{6\gamma_0\kappa^2} + t_i \simeq 36\gamma_0\kappa^2 H_{dS}, \quad (2.47)$$

or equivalently,

$$t_r \simeq \frac{2N}{H_{dS}}, \quad (2.48)$$

with N being the total number of e -foldings during the inflationary era. In this case, the frequency ω in (2.35) results to be,

$$\omega = \frac{1}{\sqrt{24\gamma_0\kappa^2}}. \quad (2.49)$$

Thus, if we compare the reheating temperature of our model T with the reheating temperature corresponding to the Starobinsky inflation T_{St} , we find,

$$\frac{T}{T_{infl}} = \sqrt{\frac{6\beta}{\gamma_1}}. \quad (2.50)$$

From (2.47), we obtain:

$$t_r \simeq t_0 + \sqrt{6\gamma_0\kappa^2} \simeq 36\gamma_0\kappa^2 H_{dS} + \sqrt{6\gamma_0\kappa^2} + t_i \simeq 36\gamma_0\kappa^2 H_{dS}$$

$$(36*4.76213e+39*8.477580536653051928*10^-36*1.000000000000000021978021978022)$$

Input interpretation:

$$36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36} \times \\ 1.000000000000000021978021978022$$

Result:

$$1.45336826163641753759813499964048129180699049762095473... \times 10^6$$

Decimal form:

$$1453368.26163641753759813499964048129180699049762095473$$

$$1453368.261636$$

And:

$$64*\sqrt{1453368.261636417} - (4096 + 2*276 - 1024 + 64 - 24)$$

Input interpretation:

$$64\sqrt{1.453368261636417 \times 10^6} - (4096 + 2 \times 276 - 1024 + 64 - 24)$$

Result:

73491.66343219896...

73491.663432...

Further, we have the following mathematical connections:

$$\left(64 \sqrt{1.453368261636417 \times 10^6} - (4096 + 2 \times 276 - 1024 + 64 - 24) \right) =$$

$$= 73491.66343219896...$$

$$= \textcolor{blue}{73491.66343 \dots \Rightarrow}$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS}} + \sqrt[13]{\int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS}} \right) =$$

$$-3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= \textcolor{blue}{73490.8437525\dots \Rightarrow}$$

(the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$)

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700\dots$$

$$= \textcolor{blue}{73491.7883254\dots \Rightarrow}$$

(the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane)

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leqslant p^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right. \\ \left. \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\} \right)$$

$$\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} = 73493.30662\dots$$

(the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series)

From:

When $t = t_i$, the e -foldings number is given by $N = 18\gamma_0\kappa^2 H_{\text{dS}}^2$, and it is easy to verify that $H = H_{\text{dS}}$.

$$N = 18\gamma_0\kappa^2 H_{\text{dS}}^2$$

We obtain:

$$18(36*4.76213e+39*8.477580536653051928*10^{-36})*(1.00000000000000021978021978022)^2$$

Input interpretation:

$$18 \left(36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36} \right) \times \\ 1.00000000000000021978021978022^2$$

Result:

$$2.61606287094555157342623172670573266314162453287032072\dots \times 10^7$$

2.616062870945...*10⁷, and considering H_{ds} = 1, we obtain:

Input interpretation:

$$18 \left(36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36} \right)$$

Result:

$$2.616062870945551561927054272 \times 10^7$$

$$2.61606287\dots \times 10^7 = N$$

Dividing this formula by $4096 \times 2048 = 64^3 \times 32$, and adding $24/10^3$, we obtain the following approximation to π :

$$(((18(36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36})))/(64^3 \times 32)) + (24/10^3)$$

Input interpretation:

$$\frac{18(36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36})}{64^3 \times 32} + \frac{24}{10^3}$$

Result:

$$3.1425899626559633754814318084716796875$$

$$3.14258996... \approx \pi$$

Further, we also obtain:

$$(2.616062870945 \times 10^7)/(233+89+21+13)+8$$

Input interpretation:

$$\frac{2.616062870945 \times 10^7}{233 + 89 + 21 + 13} + 8$$

Result:

$$73492.91210519662921348314606741573033707865168539325842696...$$

$$\color{blue}{73492.9121051...}$$

$$\left(\frac{2.616062870945 \times 10^7}{233 + 89 + 21 + 13} + 8 \right) =$$

$$= \color{blue}{73492.9121051 ...} \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{NS}} + \right. \\ \left. \sqrt{ \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D\mathbf{X}^\mu D^2 \mathbf{X}^\mu \right) \right\} |\mathbf{X}^\mu, \mathbf{X}^i = 0\rangle_{NS} } \right) =$$

$$-3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525 \dots \Rightarrow$$

(the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$)

$$\begin{aligned} & \Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow \\ & \Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) = \\ & = 73491.78832548118710549159572042220548025195726563413398700... \\ & = 73491.78832548118710549159572042220548025195726563413398700... \end{aligned}$$

$$= 73491.78832548118710549159572042220548025195726563413398700...$$

(the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane)

$$\begin{aligned} & \left(I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(-\left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right. \\ & \left. \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\} \right) \\ & /(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662... \end{aligned}$$

(the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series)

From (2.48)

$$t_r \simeq \frac{2N}{H_{dS}},$$

we obtain:

$$(2 * 2.616062870945 * 10^7) / (1.00000000000000021978021978022)$$

Input interpretation:

$$\frac{2 \times 2.616062870945 \times 10^7}{1.000000000000000021978021978022}$$

Result:

$$5.2321257418899998850082254529669182279875948286443884... \times 10^7$$

$$5.23212574189... \times 10^7 = t_r$$

$$[1/(((2*2.616062870945*10^7)/(1.000000000000000021978021978022))))]^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{\frac{1}{2 \times 2.616062870945 \times 10^7}}$$

$$\frac{1}{1.000000000000000021978021978022}$$

Result:

$$0.99567031008995876...$$

0.99567031... result very near to the dilaton value **0.989117352243 = ϕ**

Note that:

$$\log \text{base } 0.99567031008995876$$

$$[1/(((2*2.616062870945*10^7)/(1.000000000000000021978021978022))))]$$

Input interpretation:

$$\log_{0.99567031008995876} \left(\frac{1}{2 \times 2.616062870945 \times 10^7} \right)$$

$$\frac{1}{1.000000000000000021978021978022}$$

$\log_b(x)$ is the base- b logarithm

Result:

$$4096.0000000000...$$

4096

And:

Input interpretation:

$$\sqrt{\log_{0.99567031008995876} \left(\frac{1}{2 \times 2.616062870945 \times 10^7} \right)}$$

$$\frac{1}{1.000000000000000021978021978022}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.000000000000...

64

From (2.49)

$$\omega = \frac{1}{\sqrt{24\gamma_0\kappa^2}}.$$

We obtain:

$1/\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}$

Input interpretation:

$$\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}}$$

Result:

0.000169319...

$0.000169319... = \omega$

$1/(((1/\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}})))-(64+8)$

Input interpretation:

$$\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}} - (64 + 8)$$

Result:

5834.00...

5834 result very near to the rest mass of bottom Sigma baryon 5835.1

Further, we can to obtain also:

$1/(((1/\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}})))-5834-8$

Input interpretation:

$$\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}} - 5834 - 8$$

Result:

64.0002...

64.0002... ≈ 64

From which:

$$27 * (((((1 / (((((1 / \sqrt{24 * 36 * 4.76213e+39 * 8.477580536653051928 * 10^{-36}})))) - 5834 - 8))))$$

Input interpretation:

$$27 \left(\frac{\frac{1}{\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}}} - 5834 - 8}{\right)$$

Result:

1728.01...

1728.01

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We have that:

$$(((1 / \sqrt{24 * 36 * 4.76213e+39 * 8.477580536653051928 * 10^{-36}})))^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}}}$$

Result:

0.9978821959...

0.9978821959... result very near to the dilaton value **0.989117352243 = ϕ**

And:

$\text{sqrt}[\log \text{base } 0.9978821959(((1/\text{sqrt}(24*36*4.76213e+39*8.477580536653051928*10^{-36}))))]$

Input interpretation:

$$\sqrt{\log_{0.9978821959}\left(\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}}\right)}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.00000...

64

And again:

$16\text{sqrt}[\log \text{base } 0.9978821959(((1/\text{sqrt}(24*36*4.76213e+39*8.477580536653051928*10^{-36}))))]-5$

Input interpretation:

$$\sqrt[16]{\log_{0.9978821959}\left(\frac{1}{\sqrt{24 \times 36 \times 4.76213 \times 10^{39} \times 8.477580536653051928 \times 10^{-36}}}\right)} - 5$$

$\log_b(x)$ is the base- b logarithm

Result:

1019.000...

1019.000... result very near to the rest mass of Phi meson 1019.445

From (2.50)

$$\frac{T}{T_{\text{infl}}} = \sqrt{\frac{6\beta}{\gamma_1}}.$$

We obtain:

$\text{sqrt}(((6*0.008333333)/(0.00086)))$

Input interpretation:

$$\sqrt{\frac{6 \times 0.008333333}{0.00086}}$$

Result:

7.62493...

7.62493...

$$3+8\sqrt{((6*0.008333333)/(0.00086)))}$$

Input interpretation:

$$3 + 8 \sqrt{\frac{6 \times 0.008333333}{0.00086}}$$

Result:

63.9994...

63.9994... ≈ 64

$$((((1/\sqrt{((6*0.008333333)/(0.00086)))))))^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{\frac{1}{\sqrt{\frac{6 \times 0.008333333}{0.00086}}}}$$

Result:

0.9995041701...

0.9995041701... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\log \text{base } 0.9995041701((((1/\sqrt{((6*0.008333333)/(0.00086)))))))$$

Input interpretation:

$$\log_{0.9995041701}\left(\frac{1}{\sqrt{\frac{6 \times 0.008333333}{0.00086}}}\right)$$

$\log_b(x)$ is the base- b logarithm

Result:

4096.00...

4096**Alternative representation:**

$$\log_{0.999504} \left(\frac{1}{\sqrt{\frac{6 \times 0.00833333}{0.00086}}} \right) = \frac{\log \left(\frac{1}{\sqrt{\frac{0.05}{0.00086}}} \right)}{\log(0.999504)}$$

 $\log(x)$ is the natural logarithm**Series representations:**

$$\log_{0.999504} \left(\frac{1}{\sqrt{\frac{6 \times 0.00833333}{0.00086}}} \right) = - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{1}{\sqrt{58.1395}} \right)^k}{k}}{\log(0.999504)}$$

$$\log_{0.999504} \left(\frac{1}{\sqrt{\frac{6 \times 0.00833333}{0.00086}}} \right) = \log_{0.999504} \left(\frac{1}{\sqrt{57.1395} \sum_{k=0}^{\infty} e^{-4.0455k} \binom{\frac{1}{2}}{k}} \right)$$

$$\log_{0.999504} \left(\frac{1}{\sqrt{\frac{6 \times 0.00833333}{0.00086}}} \right) = \log_{0.999504} \left(\frac{1}{\sqrt{57.1395} \sum_{k=0}^{\infty} \frac{(-0.017501)^k (-\frac{1}{2})_k}{k!}} \right)$$

 $\binom{n}{m}$ is the binomial coefficient $n!$ is the factorial function $(\alpha)_n$ is the Pochhammer symbol (rising factorial)

Integral representations:

$$\log(z) = \int_1^z \frac{1}{t} dt$$

- $\log(1+z) = \frac{1}{2\pi i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s+1)\Gamma(-s)^2}{\Gamma(1-s)z^s} ds \text{ for } (-1 < \gamma < 0 \text{ and } |\arg(z)| < \pi)$

$\Gamma(x)$ is the gamma function

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

$$t_r \simeq -\frac{\sqrt{\mathcal{N}} \sqrt{6\mathcal{N}(1-2h_0+h_0^2)+h_0}}{\sqrt{6}h_0 H_{dS}} + t_0,$$

$$5.23212574189e+7 - \text{sqrt}(60)*\text{sqrt}(((360(1-2+1)+1)))$$

Input interpretation:

$$5.23212574189 \times 10^7 - \sqrt{60} \sqrt{360(1-2+1)+1}$$

Result:

$$5.23212496729\dots \times 10^7$$

$$5.23212496729\dots * 10^7 = t_0$$

We resume the data:

$$R = 4 * 2.888e-122 = 11.552 * 10^{-122}$$

$$R_{dS} = 1e+120 \times 2.888e-122 = 0.02888$$

$$R_0 = 1.8 \times 10^{85} \times 2.888 \times 10^{-122} = 5.1984 \times 10^{-37}$$

$$R_0 e^{80} = 5.1984 \times 10^{-37} \times e^{80} = 0.0288023714 \approx 0.02888$$

$$\gamma_1 = 0.00086; \quad \gamma_0 = 4.76213\dots * 10^{39}$$

$$H_{ds} = H = 1.00000000000000021978021978022$$

$$\beta = 1/120 = 0.008333333$$

$$\kappa^2 = 8.477580536653051928 \times 10^{-36}$$

From:

$$R = 12H^2 + 6\dot{H} \,,$$

$$1/6(11.552*10^{-122}-12*1.00000000000000021978021978022^2)$$

Input interpretation:

$$\frac{1}{6} \left(\frac{11.552}{10^{122}} - 12 \times 1.00000000000000021978021978022^2 \right)$$

Result:

-2.000000000000000879120879120880000966066900132836130902...

$$-2.0000\dots = \dot{H}$$

$$R = 12H^2 + 6\dot{H},$$

$$12*1.00000000000000021978021978022^2 + 6 * (-2)$$

Input interpretation:

$$12 \times 1.00000000000000021978021978022^2 + 6 \times (-2)$$

Result:

$$5.274725274725280005796401400797016785412389808 \times 10^{-17}$$

$$5.2747252747\ldots \cdot 10^{-17} = R$$

We have that:

0.000169319 = ω

$$5.23212496729\dots \cdot 10^7 = t_0$$

$$5.23212574189\ldots \times 10^7 = t_r$$

From

$$R \simeq -\frac{8\omega}{(t - t_r)} \sin [2\omega(t - t_r)] . \quad (2.40)$$

we obtain:

$$-(8*0.000169319) / (5.23212496729e+7 - 5.23212574189e+7) \sin (2*0.000169319((5.23212496729e+7 - 5.23212574189e+7)))$$

Input interpretation:

$$-\frac{8 \times 0.000169319}{5.23212496729 \times 10^7 - 5.23212574189 \times 10^7} \sin(2 \times 0.000169319 (5.23212496729 \times 10^7 - 5.23212574189 \times 10^7))$$

Result:

$$-4.58702\ldots \times 10^{-7}$$

$$-4.58702\ldots \times 10^{-7} = R$$

From

$$F(R) = R - 2\Lambda \left(1 - e^{\frac{R}{b\Lambda}}\right) - \tilde{\gamma} \Lambda \left(\frac{R}{3m^2}\right)^n , \quad (2.63)$$

where $\Lambda = 7.93m^2$, $\tilde{\gamma} = 1/1000$, $m = 1.57 \times 10^{-67}$ eV, b is an arbitrary parameter [43,44,55] and n is a positive real parameter. This model has quite appealing inflationary dynamics in the context of constant-roll inflation, as we now evince. Consider the first equation of Eq. (2.53),

for $b = 2$ and $n = 1/2$, we obtain:

$$-4.58702e-7 - 2*7.93((1-exp(((4.58702e-7)/(2*7.93)))) - 1/1000*7.93(((4.58702e-7)/((3*(1.57e-67)^2))))^{1/2}$$

Input interpretation:

$$\begin{aligned} & -4.58702 \times 10^{-7} - 2 \times 7.93 \left(1 - \exp\left(-\frac{4.58702 \times 10^{-7}}{2 \times 7.93}\right)\right) - \\ & \frac{1}{1000} \times 7.93 \sqrt{-\frac{4.58702 \times 10^{-7}}{3 (1.57 \times 10^{-67})^2}} \end{aligned}$$

Result:

$$-9.17404... \times 10^{-7} - \\ 1.97505... \times 10^{61} i$$

Polar coordinates:

$$r = 1.97505 \times 10^{61} \text{ (radius)}, \quad \theta = -90^\circ \text{ (angle)}$$

$$1.97505 * 10^{61} = F(R)$$

From:

$$432\beta n^2 \dot{H}(t) + 864n^2 \dot{H}(t) - 432\beta n \dot{H}(t) - 864n \dot{H}(t) - 6^n n H(t)^2 \\ + 36n H(t)^2 + 6^n H(t)^2 = 0. \quad (2.65)$$

The differential equation (2.65) can be analytically solved, with the solution being,

$$H(t) = \frac{432(\beta + 2)(1 - n)n}{432(\beta + 2)C_1(n - 1)n + ((6^n - 36)n - 6^n)t}, \quad (2.66)$$

with C_1 being an arbitrary integration constant which plays no role in the dynamics of inflation,

We obtain, for $C_1 = 3$ and $\beta = 1/120 = 0.0083333333$, $n = 1/2$ and
 $5.23212496729... * 10^7 = t_0$:

$$((((432((1/120)+2)(1-1/2)*1/2)))) / (((((432((1/120)+2)*3(1/2-1)*1/2+((6^1/2-36)*1/2-6^1/2))*5.23212496729e+7))))$$

Input interpretation:

$$\frac{432 \left(\frac{1}{120} + 2\right) \left(\left(1 - \frac{1}{2}\right) \times \frac{1}{2}\right)}{\left(432 \left(\frac{1}{120} + 2\right) \times 3 \left(\left(\frac{1}{2} - 1\right) \times \frac{1}{2}\right) + \left(\left(\sqrt{6} - 36\right) \times \frac{1}{2} - \sqrt{6}\right)\right) \times 5.23212496729 \times 10^7}$$

Result:

$$-6.18807293879... \times 10^{-9}$$

$$-6.18807293879... * 10^{-9} = H(t)$$

From

$$v_s = \frac{1}{2} \sqrt{\frac{((\beta + 2)(6^n + 1260)n^2 - n(4\beta(6^n + 297) + 3(6^n + 852)) + (3\beta + 1)6^n)^2}{(-432(\beta + 2)n^2 + n(432\beta - 6^n + 900) + 6^n)^2}}, \quad (2.68)$$

We obtain, from $\beta = 1/120 = 0.0083333333$, $n = 1/2$:

$$[((((1/120)+2)(6^{1/2}+1260)^{1/4}-1/2(((4/120(6^{1/2}+297)+3(6^{1/2}+852))+(3/120+1)6^{1/2}))))]^2$$

Input:

$$\left(\left(\frac{1}{120} + 2\right)\left(\sqrt{6} + 1260\right) \times \frac{1}{4} - \frac{1}{2} \left(\left(\frac{4}{120} \left(\sqrt{6} + 297\right) + 3 \left(\sqrt{6} + 852\right)\right) + \left(\frac{3}{120} + 1\right) \sqrt{6}\right)\right)^2$$

Result:

$$\left(\frac{241}{480} \left(1260 + \sqrt{6}\right) + \frac{1}{2} \left(-\frac{41 \sqrt{\frac{3}{2}}}{20} + \frac{1}{30} \left(-297 - \sqrt{6}\right) - 3 \left(852 + \sqrt{6}\right)\right)\right)^2$$

Decimal approximation:

427801.7763495542488645355793558013281861626115208227079913...

- **Alternate forms:**

$$\frac{16240765345 + 76270116\sqrt{6}}{38400}$$

$$\frac{3248153069}{7680} + \frac{6355843\sqrt{\frac{3}{2}}}{1600}$$

$$\frac{6355843\sqrt{6}}{3200} + \frac{3248153069}{7680}$$

Minimal polynomial:

$$1474560000x^2 - 1247290778496000x + 263727556207785048289$$

$$(((((-432(1/120)+2)^{1/4}+1/2(432/120-6^{1/2}+900)+6^{1/2}))))^2$$

$$\beta = 1/120 = 0.008333333, n = 1/2$$

Input:

$$\left(-432\left(\frac{1}{120} + 2\right) \times \frac{1}{4} + \frac{1}{2} \left(\frac{432}{120} - \sqrt{6} + 900\right) + \sqrt{6}\right)^2$$

- Open code

Result:

$$\left(-\frac{2169}{10} + \sqrt{6} + \frac{1}{2} \left(\frac{4518}{5} - \sqrt{6}\right)\right)^2$$

Decimal approximation:

55754.89514057976853526654202914841388797280106320625181316...

- **Alternate forms:**

$$\frac{3}{100} \left(1839317 + 7830 \sqrt{6} \right)$$

$$\frac{1}{100} \left(5517951 + 23490 \sqrt{6} \right)$$

$$\frac{5517951}{100} + \frac{2349 \sqrt{\frac{3}{2}}}{5}$$

- **Minimal polynomial:**

$$10000x^2 - 1103590200x + 30444472557801$$

$$\begin{aligned} & 1/2((((((1/120)+2)(6^{1/2}+1260)*1/4- \\ & 1/2(((4/120(6^{1/2}+297)+3(6^{1/2}+852))+(3/120+1)6^{1/2}))))])^2 / \\ & 55754.895140579)))^1/2 \end{aligned}$$

Input interpretation:

$$\frac{1}{2} \sqrt{\left(\frac{\left(\left(\frac{1}{120} + 2 \right) (\sqrt{6} + 1260) \times \frac{1}{4} - \frac{1}{2} \left(\left(\frac{4}{120} (\sqrt{6} + 297) + 3 (\sqrt{6} + 852) \right) + \left(\frac{3}{120} + 1 \right) \sqrt{6} \right) \right)^2}{55754.895140579} \right)}$$

Result:

1.3850000670570...

1.3850000670570.... = v_s

And:

$$\begin{aligned} & (((((1/2*((((((1/120)+2)(6^{1/2}+1260)*1/4- \\ & 1/2(((4/120(6^{1/2}+297)+3(6^{1/2}+852))+(3/120+1)6^{1/2}))))])^2 / \\ & 55754.895140579))))^1/2)))^1/192 \end{aligned}$$

Input interpretation:

$$\begin{aligned} & \left(\frac{1}{2} \sqrt{\left(\left(\left(\frac{1}{120} + 2 \right) (\sqrt{6} + 1260) \times \frac{1}{4} - \frac{1}{2} \left(\left(\frac{4}{120} (\sqrt{6} + 297) + 3 (\sqrt{6} + 852) \right) + \left(\frac{3}{120} + 1 \right) \sqrt{6} \right) \right)^2 / \right.} \right. \\ & \left. \left. 55754.895140579 \right)^{1/192} \right) \end{aligned}$$

Result:

1.0016977947704383...

1.00169779477... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}}$$

the spectral index of primordial curvature perturbations n_s

$$n_s = 4 - \sqrt{\frac{(6^n(n-1)(-3\beta + (\beta+2)n-1) + 36n(-33\beta + 35(\beta+2)n-71))^2}{(36n(-12\beta + 12(\beta+2)n-25) + 6^n(n-1))^2}}. \quad (2.69)$$

From (2.69), and $\beta = 1/120 = 0.0083333333$, $n = 1/2$, we obtain:

$$((((((6^{1/2}(1/2-1)(-3/120+(1/120+2)*1/2-1)+36/2*(-33/120+35(1/120+2)*1/2-71))))))^2$$

Input:

$$\left(\sqrt{6}\left(\frac{1}{2}-1\right)\left(-\frac{3}{120}+\left(\frac{1}{120}+2\right)\times\frac{1}{2}-1\right)+\frac{36}{2}\left(-\frac{33}{120}+35\left(\frac{1}{120}+2\right)\times\frac{1}{2}-71\right)\right)^2$$

Result:

$$\left(\frac{1}{16\sqrt{6}}-\frac{26013}{40}\right)^2$$

Decimal approximation:

422889.4195173546777021518719007107685662608280250967073124...

Alternate forms:

$$\frac{16240228081 - 520260\sqrt{6}}{38400}$$

$$\frac{16240228081}{38400} - \frac{8671\sqrt{\frac{3}{2}}}{320}$$

$$\frac{16\ 240\ 228\ 081}{38\ 400} - \frac{8671\sqrt{6}}{640}$$

Minimal polynomial:

$$1\,474\,560\,000 x^2 - 1\,247\,249\,516\,620\,800 x + 263\,745\,006\,498\,878\,136\,961$$

$$4 - \sqrt{(((422889.4195173546777 / (((((36/2(-12/120+12(1/120+2)*1/2-25)+6^{1/2}(1/2-1))))))^2)))})$$

Input interpretation:

$$4 - \sqrt{\frac{422\,889.4195173546777}{\left(\frac{36}{2} \left(-\frac{12}{120} + 12 \left(\frac{1}{120} + 2\right) \times \frac{1}{2} - 25\right) + \sqrt{6} \left(\frac{1}{2} - 1\right)\right)^2}}$$

Result:

1.245949445764186748...

$$1.245949445764186748\dots = n_s$$

We have also:

$$1 / (((((4-\sqrt((((422889.4195173546777 / (((((36/2(-12/120+12(1/120+2)*1/2-25)+6^{1/2}(1/2-1))))))^2)))))))^2)))^{1/1024}$$

Input interpretation:

$$\frac{1}{\sqrt[1024]{4 - \sqrt{\frac{422889.4195173546777}{\left(\frac{36}{2} \left(-\frac{12}{120} + 12 \left(\frac{1}{120} + 2\right) \times \frac{1}{2} - 25\right) + \sqrt{6} \left(\frac{1}{2} - 1\right)\right)^2}}}}$$

Result:

0.9997852790652942645586...

0.999785279065294.... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

We have also that:

$$\frac{1}{16} \log_{0.999785279065294} \left(\frac{1}{\sqrt{4 - \sqrt{\frac{422889.4195173546777}{(\frac{36}{2}(-\frac{12}{120}+12(\frac{1}{120}+2)\times\frac{1}{2}-25)+\sqrt{6}(\frac{1}{2}-1))^2}}}} \right)$$

Input interpretation:

$$\frac{1}{16} \log_{0.999785279065294} \left(\frac{1}{\sqrt{4 - \sqrt{\frac{422889.4195173546777}{(\frac{36}{2}(-\frac{12}{120}+12(\frac{1}{120}+2)\times\frac{1}{2}-25)+\sqrt{6}(\frac{1}{2}-1))^2}}}} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

64.000000000...

64

Alternative representation:

$$\frac{1}{16} \log_{0.9997852790652940000} \left(\frac{1}{\sqrt{4 - \sqrt{\frac{422889.41951735467770000}{(\frac{36}{2}(-\frac{12}{120}+\frac{12}{2}(\frac{1}{120}+2)-25)+\sqrt{6}(\frac{1}{2}-1))^2}}}} \right) = \\ \frac{\log \left(\frac{1}{\sqrt{4 - \sqrt{\frac{422889.41951735467770000}{(-\frac{\sqrt{6}}{2}+18(-25-\frac{12}{120}+6(2+\frac{1}{120}))^2}}}}} \right)}{16 \log(0.9997852790652940000)}$$

$\log(x)$ is the natural logarithm

Series representations:

$$\frac{1}{16} \log_{0.9997852790652940000} \left(\frac{1}{\sqrt{4 - \sqrt{\frac{422889.41951735467770000}{(\frac{36}{2}(-\frac{12}{120}+\frac{12}{2}(\frac{1}{120}+2)-25)+\sqrt{6}(\frac{1}{2}-1))^2}}}} \right) = \\ - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{1}{4 - \sqrt{7.5847944552865901512009}} \right)^k}{k}}{16 \log(0.9997852790652940000)}$$

$$\frac{1}{16} \log_{0.9997852790652940000} \left(\frac{1}{4 - \sqrt{\frac{422889.41951735467770000}{\left(\frac{36}{2} \left(-\frac{12}{120} + \frac{12}{2} \left(\frac{1}{120} + 2 \right) - 25 \right) + \sqrt{6} \left(\frac{1}{2} - 1 \right) \right)^2}}} \right) =$$

$$\frac{1}{16} \log_{0.9997852790652940000} \left(\frac{1}{4 - \sqrt{6.5847944552865901512009} \sum_{k=0}^{\infty} e^{-1.8847631206285896764965 k} \binom{\frac{1}{2}}{k}} \right)$$

$$\frac{1}{16} \log_{0.9997852790652940000} \left(\frac{1}{4 - \sqrt{\frac{422889.41951735467770000}{\left(\frac{36}{2} \left(-\frac{12}{120} + \frac{12}{2} \left(\frac{1}{120} + 2 \right) - 25 \right) + \sqrt{6} \left(\frac{1}{2} - 1 \right) \right)^2}}} \right) =$$

$$\frac{1}{16} \log_{0.9997852790652940000} \left(\frac{1}{4 - \sqrt{6.5847944552865901512009} \sum_{k=0}^{\infty} \frac{(-0.15186502886163012030061)^k \left(-\frac{1}{2} \right)_k}{k!}} \right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

We note that $1.3850000670570 (v_s) + 1.245949445764186748 (n_s)$

$$= 2.630949512821186748.$$

Furthermore, we have:

$$((((((1.3850000670570 + 4 - \text{sqrt}((((((422889.4195173546777 / (((((36/2(-12/120+12(1/120+2)*1/2-25)+6^1/2(1/2-1)))))))^2)))))))^2)))^1/2$$

Input interpretation:

$$\sqrt{1.3850000670570 + 4 - \sqrt{\frac{422889.4195173546777}{\left(\frac{36}{2} \left(-\frac{12}{120} + 12 \left(\frac{1}{120} + 2\right) \times \frac{1}{2} - 25\right) + \sqrt{6} \left(\frac{1}{2} - 1\right)\right)^2}}}$$

Result:

1.62202019494863...

1.62202019494863....

and:

$$(47/10^3 + 2/10^3) + (((((1.3850000670570 + 4 - \sqrt{(((422889.4195173546777 / (((((36/2(-12/120+12(1/120+2)*1/2-25)+6^1/2(1/2-1))))^2)))))))^1/2)$$

Where 2 and 47 are Lucas numbers, we obtain:

Input interpretation:

$$\left(\frac{47}{10^3} + \frac{2}{10^3}\right) + \sqrt{1.3850000670570 + 4 - \sqrt{\frac{422889.4195173546777}{\left(\frac{36}{2} \left(-\frac{12}{120} + 12 \left(\frac{1}{120} + 2\right) \times \frac{1}{2} - 25\right) + \sqrt{6} \left(\frac{1}{2} - 1\right)\right)^2}}}$$

Result:

1.67102019494863...

1.67102019494863....

We note that 1.67102019... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Haramein)

From

Accordingly, by using Eq. (2.61), the scalar-to-tensor ratio is found to be,

$$r = \frac{48 (6^n - (6^n - 36)n)^2}{(6^n - (6^n + 828)n)^2}. \quad (2.70)$$

We obtain, for $n = 1/2$:

$$48(((6^{1/2} - (6^{1/2} - 36)^{1/2}))^2 / (((6^{1/2} - (6^{1/2} + 828)^{1/2}))^2$$

Input:

$$\frac{48 \times \frac{(\sqrt{6} - (\sqrt{6} - 36) \times \frac{1}{2})^2}{(\sqrt{6} - (\sqrt{6} + 828) \times \frac{1}{2})^2}}$$

Exact result:

$$\frac{48 \left(\sqrt{6} + \frac{1}{2} (36 - \sqrt{6}) \right)^2}{\left(\sqrt{6} + \frac{1}{2} (-828 - \sqrt{6}) \right)^2}$$

Decimal approximation:

0.104120225132130406574680357135753264069545933830940507718...

0.10412022513213.....

$$((((48(((6^{1/2} - (6^{1/2} - 36)^{1/2}))^2 / (((6^{1/2} - (6^{1/2} + 828)^{1/2}))^2)))^1/4096$$

Input:

$$\sqrt[4096]{\frac{48 \times \frac{(\sqrt{6} - (\sqrt{6} - 36) \times \frac{1}{2})^2}{(\sqrt{6} - (\sqrt{6} + 828) \times \frac{1}{2})^2}}$$

Exact result:

$$\sqrt[1024]{2} \sqrt[4096]{3} \sqrt[2048]{\frac{\sqrt{6} + \frac{1}{2} (36 - \sqrt{6})}{\frac{1}{2} (828 + \sqrt{6}) - \sqrt{6}}}$$

Decimal approximation:

0.999447855359903366474646677344149852632419167665748890529...

0.9994478553599..... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt{5^3}} - 1}}{\varphi + 1}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

64

Alternate forms:

$$\sqrt[1024]{2} \sqrt[4096]{3} \sqrt[2048]{\frac{4969 + 144\sqrt{6}}{114263}}$$

$$\sqrt[1024]{2} \sqrt[4096]{3} \sqrt[2048]{\frac{36 + \sqrt{6}}{828 - \sqrt{6}}}$$

$$\sqrt[1024]{2} \sqrt[4096]{3} \sqrt[2048]{\frac{18 - \sqrt{\frac{3}{2}} + \sqrt{6}}{414 + \sqrt{\frac{3}{2}} - \sqrt{6}}}$$

We have also:

$$\text{sqrt}(((\log \text{base } 0.9994478553599 (((48(((6^{1/2}-(6^{1/2}-36)*1/2)))^2 / (((6^{1/2}-(6^{1/2}+828)*1/2)))^2))))))$$

Input interpretation:

$$\sqrt{\log_{0.9994478553599} \left(48 \times \frac{\left(\sqrt{6} - (\sqrt{6} - 36) \times \frac{1}{2} \right)^2}{\left(\sqrt{6} - (\sqrt{6} + 828) \times \frac{1}{2} \right)^2} \right)}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.00000000...

64

All 2nd roots of 4096.000000:

$64.00000000 e^0 \approx 64.000$ (real, principal root)

$64.00000000 e^{i\pi} \approx -64.000$ (real root)

Alternative representation:

$$\sqrt{\log_{0.99944785535990000} \left(\frac{48 \left(\sqrt{6} - \frac{1}{2} (\sqrt{6} - 36) \right)^2}{\left(\sqrt{6} - \frac{1}{2} (\sqrt{6} + 828) \right)^2} \right)} = \sqrt{\frac{\log \left(\frac{48 \left(\frac{1}{2} (36 - \sqrt{6}) + \sqrt{6} \right)^2}{\left(\frac{1}{2} (-828 - \sqrt{6}) + \sqrt{6} \right)^2} \right)}{\log(0.99944785535990000)}}$$

$\log(x)$ is the natural logarithm

Series representations:

$$\begin{aligned} & \sqrt{\log_{0.99944785535990000} \left(\frac{48 \left(\sqrt{6} - \frac{1}{2} (\sqrt{6} - 36) \right)^2}{\left(\sqrt{6} - \frac{1}{2} (\sqrt{6} + 828) \right)^2} \right)} = \\ & \sqrt{-\frac{\sum_{k=1}^{\infty} \frac{(-6)^k \left(\frac{1}{(-828 + \sqrt{6})^2} \right)^k (-103849 + 852\sqrt{6})^k}{k}}{\log(0.99944785535990000)}} \\ & \sqrt{\log_{0.99944785535990000} \left(\frac{48 \left(\sqrt{6} - \frac{1}{2} (\sqrt{6} - 36) \right)^2}{\left(\sqrt{6} - \frac{1}{2} (\sqrt{6} + 828) \right)^2} \right)} = \\ & \sqrt{-1 + \log_{0.99944785535990000} \left(\frac{48 (24815377 + 1431072\sqrt{6})}{13056033169} \right)} \\ & \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \log_{0.99944785535990000} \left(\frac{48 (24815377 + 1431072\sqrt{6})}{13056033169} \right) \right)^{-k} \end{aligned}$$

$$\begin{aligned} & \sqrt{\log_{0.99944785535990000} \left(\frac{48 \left(\sqrt{6} - \frac{1}{2} (\sqrt{6} - 36) \right)^2}{\left(\sqrt{6} - \frac{1}{2} (\sqrt{6} + 828) \right)^2} \right)} = \\ & \sqrt{-1 + \log_{0.99944785535990000} \left(\frac{48 (24815377 + 1431072\sqrt{6})}{13056033169} \right)} \\ & \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + \log_{0.99944785535990000} \left(\frac{48 (24815377 + 1431072\sqrt{6})}{13056033169} \right) \right)^{-k} \left(-\frac{1}{2} \right)_k}{k!} \end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

Now, we have that:

$$\Delta H(t) = -\frac{12(\beta+2)(48\beta+95)(96\beta+193)}{(72\beta(128\beta+511)+36721)t}. \quad (2.73)$$

$$f_{\text{DE}}(R) = -\frac{2\Lambda g(R)(1 - e^{-bR/\Lambda})}{\kappa^2} \quad (3.1)$$

$$g(R) = \left[1 - c \left(\frac{R}{4\Lambda} \right) \log \left[\frac{R}{4\Lambda} \right] \right], \quad 0 < c,$$

where c is a real and positive parameter. We need to note that,

$$g(R) = \left[1 - c \left(\frac{R}{4\Lambda} \right) \log \left[\frac{R}{4\Lambda} \right] \right] \quad (3.2)$$

$$R/\kappa^2 \sim M_{Pl}^4.$$

For the following data:

$$\beta = 1/120 = 0.008333333; \quad 5.23212496729... * 10^7 = t_0; \quad b = 2; \quad c = 3/2 \text{ and}$$

$$\kappa^2 = 8.477580536653051928 \times 10^{-36}$$

$$H_{ds} = H = 1.000000000000000021978021978022$$

$$\beta = 1/120 = 0.008333333$$

$$\kappa^2 = 8.477580536653051928 \times 10^{-36}$$

$$R = 4 * 2.888e-122 = 11.552 * 10^{-122}$$

$$1.12456... * 10^{21} = f_{\text{DE}}(R)$$

From:

$$R = 12H^2 + 6\dot{H},$$

$$\frac{1}{6} \left(\frac{11.552}{10^{122}} - 12 \times 1.00000000000000021978021978022^2 \right)$$

$$-2.000000000000000879120879120880000966066900132836130902\dots$$

$$-2.0000\dots = \dot{H}$$

$$R = 12H^2 + 6\dot{H},$$

$$12 \times 1.00000000000000021978021978022^2 + 6 \times (-2)$$

$$12 \times 1.00000000000000021978021978022^2 + 6 \times (-2)$$

$$5.274725274725280005796401400797016785412389808 \times 10^{-17}$$

$$5.2747252747... \times 10^{-17} = R$$

Thence:

$$g(R) = \left[1 - c \left(\frac{R}{4\Lambda} \right) \log \left[\frac{R}{4\Lambda} \right] \right]$$

$$1 - \frac{3}{2} \left[\left(\frac{(5.2747252747e-17)}{(4 * 2.888e-122)} \right) \right] * \ln \left(\frac{(5.2747252747e-17)}{(4 * 2.888e-122)} \right)$$

Input interpretation:

$$1 - \frac{3}{2} \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \log \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \right) \right)$$

$\log(x)$ is the natural logarithm

Result:

$$-1.65055... \times 10^{107}$$

$$-1.65055... \times 10^{107} = g(R)$$

We have also that:

Input interpretation:

$$-\log \left(-\frac{1}{1 - \frac{3}{2} \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \log \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \right) \right)} \right)$$

$\log(x)$ is the natural logarithm

Result:

246.87771...

246.87771...

And that:

```
7 colog [-1/((((1-3/2[((((5.2747252747e-17)/(4*2.888e-122)))))) * ln
((((5.2747252747e-17)/(4*2.888e-122))))])])])])]
```

Input interpretation:

$$7 \left(-\log \left(-\frac{1}{1 - \frac{3}{2} \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \log \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \right) \right)} \right) \right)$$

$\log(x)$ is the natural logarithm

Result:

1728.1440...

1728.1440...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729.

And:

Input interpretation:

$$\sqrt[15]{7 \left(-\log \left(-\frac{1}{1 - \frac{3}{2} \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \log \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \right) \right)} \right) \right)}$$

$\log(x)$ is the natural logarithm

Result:

1.643760961...

$$1.643760961 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$(((6 * (((((7 \operatorname{colog} [-1 / (((1 - 3 / 2) [((((5.2747252747e-17) / (4 * 2.888e-122))))) * \ln (((((5.2747252747e-17) / (4 * 2.888e-122)))))])]))]))^1 / 15)))^1 / 2$$

Input interpretation:

$$\sqrt[6]{\sqrt[15]{7 \left(-\log \left(-\frac{1}{1 - \frac{3}{2} \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \log \left(\frac{5.2747252747 \times 10^{-17}}{4 \times \frac{2.888}{10^{122}}} \right) \right)} \right) \right)}}$$

$\log(x)$ is the natural logarithm

Result:

3.140472219775346957274454013679778310231037074350371451380...

$$3.1404722197753469 \dots \approx \pi$$

From:

$$f_{\text{DE}}(R) = -\frac{2\Lambda g(R)(1 - e^{-bR/\Lambda})}{\kappa^2}$$

We obtain:

$$-\frac{(-((2 * 2.888e-122 * (-1.65055e+107) * (1 - \exp(-2 * 5.2747252747e-17 / 2.888e-122)))) / (8.477580536653051928e-36))}{}$$

Input interpretation:

$$-\frac{2 \times \frac{2.888}{10^{122}}(-(1.65055 \times 10^{107})) \left(1 - \exp\left(-2 \times \frac{5.2747252747 \times 10^{-17}}{\frac{2.888}{10^{122}}}\right)\right)}{8.477580536653051928 \times 10^{-36}}$$

Result:

$$1.12456\dots \times 10^{21}$$

$$1.12456\dots \times 10^{21} = f_{DE}(R)$$

We have also that

$$-1.65055\dots \times 10^{107} = g(R) \quad 1.12456\dots \times 10^{21} = f_{DE}(R)$$

$$9 \ln \frac{-1.65055 \times 10^{107}}{1.12456 \times 10^{21}}$$

Input interpretation:

$$9 \log\left(-\left(-\frac{1.65055 \times 10^{107}}{1.12456 \times 10^{21}}\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

$$1785.654\dots$$

1785.654.... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

And:

$$((((9 \ln \frac{-1.65055 \times 10^{107}}{1.12456 \times 10^{21}}))))^{1/15}$$

Input interpretation:

$$\sqrt[15]{9 \log\left(-\left(-\frac{1.65055 \times 10^{107}}{1.12456 \times 10^{21}}\right)\right)}$$

$\log(x)$ is the natural logarithm

Result:

$$1.6473523\dots$$

$$1.6473523\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$\text{sqrt}(((6*((((9 \ln -(-1.65055e+107 / 1.12456e+21)))))^{1/15})))$$

Input interpretation:

$$\sqrt[6]{\sqrt[15]{9 \log \left(-\left(\frac{1.65055 \times 10^{107}}{1.12456 \times 10^{21}} \right) \right)}}$$

$\log(x)$ is the natural logarithm

Result:

$$3.143901069313508131920498066776797819227387306232333525301\dots$$

$$3.1439010693135\dots \approx \pi$$

From:

$$\Delta H(t) = -\frac{12(\beta + 2)(48\beta + 95)(96\beta + 193)}{(72\beta(128\beta + 511) + 36721)t}. \quad (2.73)$$

We obtain:

$$-\left(((12(1/120+2)(48/120+95)(96/120+193))) / (((72/120(128/120+511)+36721)*5.23212496729e+7)) \right)$$

Input interpretation:

$$-\frac{12 \left(\frac{1}{120} + 2 \right) \left(\frac{48}{120} + 95 \right) \left(\frac{96}{120} + 193 \right)}{\left(\frac{72}{120} \left(\frac{128}{120} + 511 \right) + 36721 \right) \times 5.23212496729 \times 10^7}$$

Result:

$$-2.299895013536644321170403953096328257374373974281860\dots \times 10^{-7}$$

$$-2.299895013536\dots \times 10^{-7} = \Delta H(t)$$

Now, from this equation, we obtain:

$$((((((24.1*95.4*193.8)-(((307.24*-2.29989501353664432117e-7*5.23212496729e+7)))))))^*-1)/(((((-2.29989501353664432117e-7*5.23212496729e+7))))))$$

Input interpretation:

$$-\left(\left(24.1 \times 95.4 \times 193.8 - 307.24\right.\right. \\ \left.\left. - \left(2.29989501353664432117 \times 10^{-7}\right) \times 5.23212496729 \times 10^7\right) \times (-1)\right) / \\ \left(2.29989501353664432117 \times 10^{-7} \times 5.23212496729 \times 10^7\right)$$

Result:

$$36721.0000000000000000650363260564002768038377345472574045\dots$$

36721

We have the following formula concerning the 7th order mock theta functions:

$$\exp(\text{Pi} * \sqrt{2 * n / 21}) / (2^{(3/2)} * \sin(2 * \text{Pi} / 7) * \sqrt{7 * n})$$

and we have the following value for n and a(n):

$$152 \approx 2308; 153 \approx 2376; 154 \approx 2479; 155 \approx 2554; 156 \approx 2660$$

We observe that, for n = 155.963, we obtain:

$$\exp(\text{Pi} * \sqrt{2 * 155.963 / 21}) / (2^{(3/2)} * \sin(2 * \text{Pi} / 7) * \sqrt{7 * 155.963})$$

Input interpretation:

$$\frac{\exp\left(\pi \sqrt{2 \times \frac{155.963}{21}}\right)}{2^{3/2} \sin\left(2 \times \frac{\pi}{7}\right) \sqrt{7 \times 155.963}}$$

Result:

$$2481.09\dots$$

Alternative representations:

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{\exp\left(\pi \sqrt{\frac{311.926}{21}}\right)}{\cos\left(\frac{\pi}{2} - \frac{2\pi}{7}\right) 2^{3/2} \sqrt{1091.74}}$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = - \frac{\exp\left(\pi \sqrt{\frac{311.926}{21}}\right)}{\cos\left(\frac{\pi}{2} + \frac{2\pi}{7}\right) 2^{3/2} \sqrt{1091.74}}$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{\exp\left(\pi \sqrt{\frac{311.926}{21}}\right)}{\frac{2^{3/2} (-e^{-(2i\pi)/7} + e^{(2i\pi)/7}) \sqrt{1091.74}}{2i}}$$

i is the imaginary unit

Series representations:

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \exp\left(\pi \exp\left(i\pi \left[\frac{\arg(14.8536 - x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (14.8536 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) / \\ \left(4\sqrt{2} \exp\left(i\pi \left[\frac{\arg(1091.74 - x)}{2\pi}\right]\right) \sqrt{x} \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{2\pi}{7}\right)\right.\right. \\ \left.\left. \sum_{k=0}^{\infty} \frac{(-1)^k (1091.74 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \exp\left(\pi \exp\left(i\pi \left[\frac{\arg(14.8536 - x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (14.8536 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) / \\ \left(2\sqrt{2} \exp\left(i\pi \left[\frac{\arg(1091.74 - x)}{2\pi}\right]\right) \sqrt{x} \left(\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}\right)^{-1-2k} e^{ik\pi} \pi^{1+2k}}{(1+2k)!}\right.\right. \\ \left.\left. \sum_{k=0}^{\infty} \frac{(-1)^k (1091.74 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} =$$

$$\left(\exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(14.8536 - z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(14.8536 - z_0)/(2\pi) \rfloor)} \right. \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (14.8536 - z_0)^k z_0^{-k}}{k!} \right)$$

$$\left. \left(\frac{1}{z_0} \right)^{-1/2 \lfloor \arg(1091.74 - z_0)/(2\pi) \rfloor} z_0^{-1/2 - 1/2 \lfloor \arg(1091.74 - z_0)/(2\pi) \rfloor} \right) /$$

$$\left(4\sqrt{2} \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k} \left(\frac{2\pi}{7}\right) \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1091.74 - z_0)^k z_0^{-k}}{k!} \right)$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} =$$

$$\left(\exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(14.8536 - z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(14.8536 - z_0)/(2\pi) \rfloor)} \right. \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (14.8536 - z_0)^k z_0^{-k}}{k!} \right)$$

$$\left. \left(\frac{1}{z_0} \right)^{-1/2 \lfloor \arg(1091.74 - z_0)/(2\pi) \rfloor} z_0^{-1/2 - 1/2 \lfloor \arg(1091.74 - z_0)/(2\pi) \rfloor} \right) /$$

$$\left(2\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}\right)^{-1-2k} e^{ik\pi} \pi^{1+2k}}{(1+2k)!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1091.74 - z_0)^k z_0^{-k}}{k!} \right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$J_n(z)$ is the Bessel function of the first kind

\mathbb{R} is the set of real numbers

Integral representations:

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{7 \exp\left(\pi \sqrt{14.8536}\right)}{4 \sqrt{2} \pi \sqrt{1091.74} \int_0^1 \cos\left(\frac{2\pi t}{7}\right) dt}$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{7 i \exp\left(\pi \sqrt{14.8536}\right)}{\sqrt{2} \sqrt{1091.74} \sqrt{\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-\pi^2/(49s)+s}}{s^{3/2}} ds} \text{ for } \gamma > 0$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{i \pi \exp\left(\pi \sqrt{14.8536}\right)}{\sqrt{2} \sqrt{1091.74} \sqrt{\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\pi^{-1+2s} \pi^{1-2s} \Gamma(s)}{\Gamma(\frac{3}{2}-s)} ds}$$

for $0 < \gamma < 1$

$\Gamma(x)$ is the gamma function

Multiple-argument formulas:

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{\exp\left(\pi \sqrt{14.8536}\right)}{4 \sqrt{2} \cos\left(\frac{\pi}{7}\right) \sin\left(\frac{\pi}{7}\right) \sqrt{1091.74}}$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{\exp\left(\pi \sqrt{14.8536}\right)}{\sqrt{2} (6 \sin\left(\frac{2\pi}{21}\right) \sqrt{1091.74} - 8 \sin^3\left(\frac{2\pi}{21}\right) \sqrt{1091.74})}$$

$$\frac{\exp\left(\pi \sqrt{\frac{2 \times 155.963}{21}}\right)}{2^{3/2} \sin\left(\frac{2\pi}{7}\right) \sqrt{7 \times 155.963}} = \frac{\exp\left(\pi \sqrt{14.8536}\right)}{\sqrt{2} (6 \cos^2\left(\frac{2\pi}{21}\right) \sin\left(\frac{2\pi}{21}\right) \sqrt{1091.74} - 2 \sin^3\left(\frac{2\pi}{21}\right) \sqrt{1091.74})}$$

From the sum of the previous results:

$$\left(\frac{-(((24.1 \times 95.4 \times 193.8 - -307.24 \\ (-2.29989501353664432117 \times 10^{-7}) \times 5.23212496729 \times 10^7) \times (-1)) / \\ (2.29989501353664432117 \times 10^{-7} \times 5.23212496729 \times 10^7))}{\exp(\pi \sqrt{2 \times \frac{155.963}{21}})} \right) = 36721$$

$$\left(\frac{\exp(\pi \sqrt{2 \times \frac{155.963}{21}})}{2^{3/2} \sin(2 \times \frac{\pi}{7}) \sqrt{7 \times 155.963}} \right) = 2481.09$$

We have:

$$\left(\frac{-(((24.1 \times 95.4 \times 193.8 - -307.24 \\ (-2.29989501353664432117 \times 10^{-7}) \times 5.23212496729 \times 10^7) \times (-1)) / \\ (2.29989501353664432117 \times 10^{-7} \times 5.23212496729 \times 10^7))}{\exp(\pi \sqrt{2 \times \frac{155.963}{21}})} \right) +$$

$$+ \left(\frac{\exp(\pi \sqrt{2 \times \frac{155.963}{21}})}{2^{3/2} \sin(2 \times \frac{\pi}{7}) \sqrt{7 \times 155.963}} \right) = 39202.09$$

This result is practically equal to the value that is in the fundamental Ramanujan equation already above analyzed:

$$64 \{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\} \\ = 2508927.99839293; \quad \textcolor{blue}{64 * 39202 = 2508928}; \quad 2508928 \div \textcolor{blue}{4096} = 612.53125$$

We have obtained $-6.18807293879 \times 10^{-9} = H(t)$. Dividing $H(t)$ and $\Delta H(t)$ we obtain:

$$(((((-6.18807293879e-9)))/((-2.299895013536e-7))))$$

Input interpretation:

$$\frac{-6.18807293879 \times 10^{-9}}{-2.299895013536 \times 10^{-7}}$$

Result:

$$0.026905893105425174160109414807527718076392534493074967814\dots$$

$$0.0269058931054\dots$$

And performing the 4096th root:

$$(((((-6.18807293879e-9)))/((-2.299895013536e-7))))^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{\frac{-6.18807293879 \times 10^{-9}}{-2.299895013536 \times 10^{-7}}}$$

Result:

$$0.9991177209943739\dots$$

0.9991177209943739... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

And:

$$[\log \text{base } 0.9991177209943739 ((((-6.18807293879e-9)))/((-2.299895013536e-7))))]^{1/2}$$

Input interpretation:

$$\sqrt{\log_{0.9991177209943739} \left(\frac{-6.18807293879 \times 10^{-9}}{-2.299895013536 \times 10^{-7}} \right)}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.00000000000...

64

For the following data:

$$\beta = 1/120 = 0.008333333; \quad 5.23212496729... * 10^7 = t_0; \quad b = 2; \quad c = 3/2 \text{ and}$$

$$\kappa^2 = 8.477580536653051928 \times 10^{-36}$$

$$H_{dS} = H = 1.00000000000000021978021978022$$

$$\beta = 1/120 = 0.008333333$$

$$\kappa^2 = 8.477580536653051928 * 10^{-36}$$

$$R = 4 * 2.888e-122 * 10^4 = 11.552 * 10^{-118}$$

$$1.12456... * 10^{21} = f_{DE}(R)$$

$$-1.65055... * 10^{107} = g(R)$$

Now, we have that:

$$F(R) \simeq R + \kappa^2 f_{DE}(R) \quad (3.3)$$

$$0 < F_{RR}(R) \simeq \frac{c}{2R}. \quad (3.8)$$

$$R \simeq 4\Lambda \times 10^4$$

$$F(R) = \kappa_0^2 \left[\frac{R}{\kappa^2} + \gamma(R) R^2 + f_{DE}(R) \right], \quad (3.11)$$

$$\nu = \frac{1}{2\pi} \sqrt{\frac{1}{\Lambda F_{RR}(4\Lambda)} - \frac{25}{4}}. \quad (3.16)$$

$$v \simeq \frac{1}{2\pi \sqrt{RF_{RR}(R)}(z+1)}. \quad (3.18)$$

$$z \ll 1.$$

From (3.3), we obtain:

$$F(R) \simeq R + \kappa^2 f_{\text{DE}}(R)$$

11.552e-118+ 8.477580536653051928e-36 * 1.12456e+21

Input interpretation:

$$\frac{11.552}{10^{118}} + 8.477580536653051928 \times 10^{-36} \times 1.12456 \times 10^{21}$$

Result:

$$9.533547968298556 \dots \times 10^{-15}$$

From (3.8), we have:

$$0 < F_{RR}(R) \simeq \frac{c}{2R}.$$

$$1.5 / (2 * 11.552 \times 10^{-18})$$

Input interpretation:

$$2 \times \frac{11.552}{10^{118}}$$

$$2 \times \frac{11.552}{10^{118}}$$

Result:

$$6.4923822714681440443213296398891966759002770083102493... \times 10^{116}$$

$$6.492382271468144 \dots \times 10^{116}$$

From (3.16), we have:

$$v = \frac{1}{2\pi} \sqrt{\frac{1}{\Lambda F_{RR}(4\Lambda)} - \frac{25}{4}}.$$

$$1/(2\pi)*\text{sqrt}(((1/(2.888e-122*6.492382271468144e-116)-25/4)))$$

Input interpretation:

$$\frac{1}{2\pi} \sqrt{\frac{1}{\frac{2.888}{10^{122}} \times \frac{6.492382271468144}{10^{116}}} - \frac{25}{4}}$$

Result:

$$3.67553\dots \times 10^{117}$$

$$3.67553\dots * 10^{117}$$

Series representations:

$$\frac{\sqrt{\frac{1}{\frac{2.888 \times 6.4923822714681440000}{10^{122} \times 10^{116}}} - \frac{25}{4}}}{2\pi} = \frac{\sqrt{5.33333 \times 10^{236}} \sum_{k=0}^{\infty} e^{-545.084k} \binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{\sqrt{\frac{1}{\frac{2.888 \times 6.4923822714681440000}{10^{122} \times 10^{116}}} - \frac{25}{4}}}{2\pi} = \frac{\sqrt{5.33333 \times 10^{236}} \sum_{k=0}^{\infty} \frac{(-1.875 \times 10^{-237})^k (-\frac{1}{2})_k}{k!}}{2\pi}$$

$$\frac{\sqrt{\frac{1}{\frac{2.888 \times 6.4923822714681440000}{10^{122} \times 10^{116}}} - \frac{25}{4}}}{2\pi} = \frac{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-545.084s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{4\pi \sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=0} f$ is a complex residue

We have, dividing:

$$v = 3.67553\dots \times 10^{117}; \quad F_{RR}(R) = 6.492382271468144\dots \times 10^{116}$$

$$(3.67553e+116) / (6.492382271468144e+117)$$

Input interpretation:

$$\frac{3.67553 \times 10^{116}}{6.492382271468144 \times 10^{117}}$$

Result:

$$0.05661296341333333719811163568355558193910876626640610603\dots$$

$$0.0566129634133\dots$$

And:

$$(((3.67553e+116) / (6.492382271468144e+117)))^{1/1024}$$

Input interpretation:

$$\sqrt[1024]{\frac{3.67553 \times 10^{116}}{6.492382271468144 \times 10^{117}}}$$

Result:

$$0.997199712\dots$$

0.997199712... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}}{\varphi + 1}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Further:

$$1/16 \log \text{base } 0.997199712 (((3.67553e+116) / (6.492382271468144e+117)))$$

Input interpretation:

$$\frac{1}{16} \log_{0.997199712} \left(\frac{3.67553 \times 10^{116}}{6.492382271468144 \times 10^{117}} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

64.0000...

64

We have also:

$$((((((6.49238227e+117 + 3.67553e+116))))))^{1/24} - (64^2 + 64*2^5 + 64*2^4 + 64*2^3 + 64 + 2^4)$$

Input interpretation:

$$\sqrt[24]{6.49238227 \times 10^{117} + 3.67553 \times 10^{116}} - (64^2 + 64 \times 2^5 + 64 \times 2^4 + 64 \times 2^3 + 64 + 2^4)$$

Result:

73494.3588...

73494.3588...

Further, we have the following mathematical connections:

$$\left(\sqrt[24]{6.49238227 \times 10^{117} + 3.67553 \times 10^{116}} - (64^2 + 64 \times 2^5 + 64 \times 2^4 + 64 \times 2^3 + 64 + 2^4) \right) =$$

$$= 73494.3588 \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS} + \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS}} \right) =$$

$$-3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525 \dots \Rightarrow$$

(the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$)

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700...$$

$$= \color{blue}{73491.7883254...} \Rightarrow$$

(the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane)

$$\left(\begin{array}{l} I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(- \left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leqslant p^{1-\epsilon_2}} \frac{a(\lambda)}{\lambda} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\epsilon_1} \right\} \end{array} \right)$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = \color{blue}{73493.30662...}$$

(the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series)

From:

f(T) teleparallel gravity and cosmology

Yi-Fu Cai, Salvatore Capozziello, Mariafelicia De Laurentis and Emmanuel N. Saridakis - arXiv:1511.07586v2 [gr-qc] 8 Sep 2016

We have that:

We consider the background matter component to be a homogeneous and isotropic fluid, and hence one acquires $T^\rho_\sigma = \text{diag}(\rho, -p, -p, -p)$ in the comoving frame. Then the highly symmetric background ansatz eventually leads to only two independent background equations of motion, which are a first order equation:

$$\left(1 - \frac{12H^2}{N^2\lambda}\right)^{-\frac{1}{2}} - 1 = \frac{16\pi G}{\lambda} N^2 \rho, \quad (392)$$

which results from varying with respect to e_0^0 ($H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter), and a second order one:

$$\left(\frac{16H^2}{N^2\lambda} + \frac{4H^2}{N^2\lambda}q - 1\right)\left(1 - \frac{12H^2}{N^2\lambda}\right)^{-\frac{3}{2}} + 1 = \frac{16\pi G}{\lambda} p, \quad (393)$$

which results from varying with respect to e_σ^A ($q = -\ddot{a}a/\dot{a}^2$ is the deceleration parameter). The above two equations can also be derived by varying the Lagrangian (389) with respect to the lapse function $N(t)$ and the scale factor $a(t)$. Note that $S_\mu^{\nu\rho} T^\mu_{\nu\rho} = -6H(t)^2/N(t)^2$, and thus λ in (389) will prevent the Hubble parameter from becoming infinite. Note also that Eq. (392) is not a dynamical equation for $N(t)$, but a constraint for $a(t)$, and therefore one has the freedom to fix $N(t)$, namely to set $N(t) = 1$.

Combining Eqs. (392) and (393), one can derive

$$1 + q = \frac{3}{2} \frac{(1 + \omega)}{\left(1 + \frac{16\pi G}{\lambda} \rho\right) \left(1 + \frac{8\pi G}{\lambda} \rho\right)}. \quad (396)$$

In the limit of GR (i.e., $\lambda \rightarrow \infty$) an accelerated expansion ($q < 0$) is only possible if $\omega < -1/3$ (negative pressure). However, it is interesting to observe that in Born-Infeld modified teleparallelism an accelerated expansion can be realized without resorting to negative pressure, since a large energy density ρ is sufficient:

$$\frac{32\pi G}{\lambda} \rho > -3 + \sqrt{13 + 12\omega}, \quad (397)$$

which can be achieved in early universe. Note that for $\rho \rightarrow \infty$ in (396) one gets $q \rightarrow -1$, and the expansion becomes exponential.

$$1 - 0.5 = \frac{3}{2} \times \frac{1 - \frac{1}{4}}{\left(1 + \frac{16\pi \times 6.67 \times 10^{-11}}{12752043} x\right) \left(1 + \frac{8\pi \times 6.67 \times 10^{-11}}{12752043} x\right)}$$

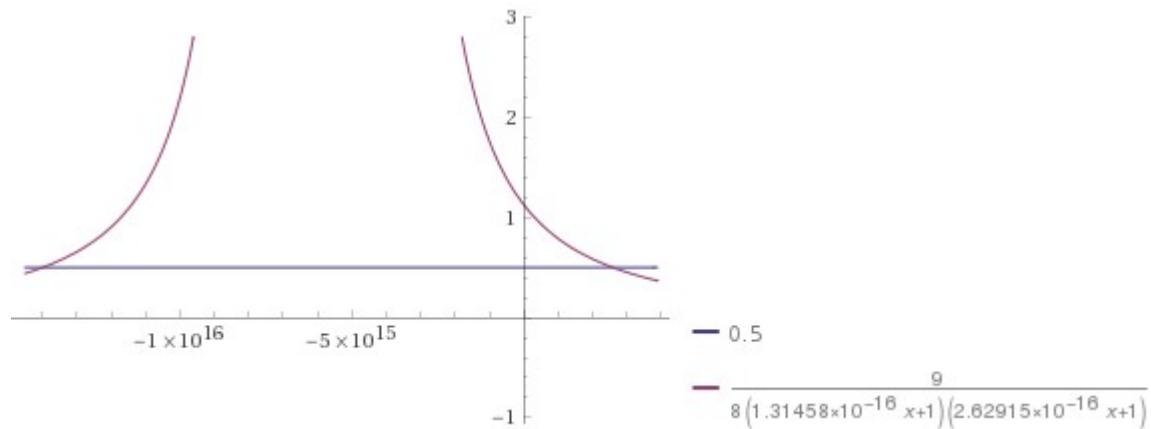
Input interpretation:

$$1 - 0.5 = \frac{3}{2} \times \frac{1 - \frac{1}{4}}{\left(1 + \frac{16\pi \times 6.67 \times 10^{-11}}{12752043} x\right) \left(1 + \frac{8\pi \times 6.67 \times 10^{-11}}{12752043} x\right)}$$

Result:

$$0.5 = \frac{9}{8(1.31458 \times 10^{-16} x + 1)(2.62915 \times 10^{-16} x + 1)}$$

Plot:



Alternate forms:

$$0.5 = \frac{1.125}{(3.45622 \times 10^{-32} x + 3.94373 \times 10^{-16}) x + 1} \quad (\text{ignoring removable singularities})$$

$$0.5 = \frac{1.125}{3.45622 \times 10^{-32} x^2 + 3.94373 \times 10^{-16} x + 1}$$

$$0.5 = \frac{9}{8(1.31458 \times 10^{-16} x + 1)(2.62915 \times 10^{-16} x + 1)}$$

Alternate form assuming x is positive:

$$1.31458 \times 10^{-16} x = 0.339725 \quad (\text{for } x \neq -7.60701 \times 10^{15} \text{ and } x \neq -3.80351 \times 10^{15})$$

Solutions:

$$x = -13994807290084946$$

$$x = 2584289985153896$$

Integer solutions:

$$x = -13994807290084946$$

$$x = 2584289985153896$$

$$x = 2584289985153896$$

And:

$$(32*\pi*2584289985153896*6.67e-11) / (1729*10^4)$$

Input interpretation:

$$\frac{32\pi \times 2584289985153896 \times 6.67 \times 10^{-11}}{1729 \times 10^4}$$

Result:

$$1.002240471988735911797990879070699988973702353240636211174\dots$$

1.00224047198... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$-3 + \sqrt{13 + 12 \times 0.25}$$

Input:

$$-3 + \sqrt{13 + 12 \times 0.25}$$

Result:

$$\begin{matrix} 1 \\ 1 \end{matrix}$$

We note that:

$$1 / (((32*\pi*2584289985153896*6.67e-11) / (1729*10^4)))$$

Input interpretation:

$$\frac{1}{\frac{32\pi \times 2584289985153896 \times 6.67 \times 10^{-11}}{1729 \times 10^4}}$$

Result:

0.997764536504607357089471683787111558438678289366435156578...

0.9977645365046..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt{5^3}} - 1}}{\varphi + 1}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Mathematical connections between Ramanujan mathematics and some sectors of Theoretical Physics: Dilaton value and associated Spectral Index

From:

<http://empslocal.ex.ac.uk/people/staff/mrwatkin/zeta/surprising.htm>

To treat the occurrence of a prime number as a kind of "random event" is to apply to the pure, eternal world of number a type of thinking inspired by the ever changing physical world. Hardy and Littlewood commented that: "*Probability is not a notion of pure mathematics, but of physics or philosophy.*"

From:

SOME PROBLEMS OF 'PARTITIO NUMERORUM'; III: ON THE EXPRESSION OF A NUMBER AS A SUM OF PRIMES.

BY

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We have that:

The formula (5. 26) is equivalent, when $k = 2$, to

$$(5. 321) \quad \sum_{m < n} A(m) A(m+2) \sim 2 C_2 n;$$

and, when we pass from this formula to one for the number of prime-pairs, the formula which arises most naturally is not (5. 311) but¹

$$(5. 322) \quad P_2(n) \sim 2 C_2 \int_{2}^n \frac{dx}{(\log x)^2};$$

indeed it is not unreasonable to expect this approximation to be a really good one, and much better than the formulae of 4. 4. The formula (5. 322) is naturally equivalent to (5. 311). But

$$\int_{2}^n \frac{dx}{(\log x)^2} = \frac{n}{(\log n)^2} \left(1 + \frac{2!}{\log n} + \frac{3!}{(\log n)^2} + \dots \right),^1$$

and the second factor on the right hand side is (for such values of n as we have to consider) far from negligible. It is for this reason that Brun, when he

We therefore take the formula (5. 322) as our basis for comparison, choosing the lower limit to be 2. For our statistics as to the actual number of prime-pairs we are indebted to (a) a count up to 100,000 made by GLAISHER in 1878² and (b) a much more extensive count made for us recently by Mrs. G. A. STREATFIELD. The results obtained by Mrs. Streatfield are as follows.

| n | $P_2(n)$ | $2 C_2 \int_{2}^n \frac{dx}{(\log x)^2}$ | Ratio |
|---------|----------|--|-------|
| 100000 | 1224 | 1246.3 | 1.018 |
| 200000 | 2159 | 2179.5 | 1.009 |
| 300000 | 2992 | 3035.4 | 1.015 |
| 400000 | 3801 | 3846.1 | 1.012 |
| 500000 | 4562 | 4625.6 | 1.014 |
| 600000 | 5328 | 5381.5 | 1.010 |
| 700000 | 6058 | 6118.7 | 1.010 |
| 800000 | 6763 | 6840.2 | 1.011 |
| 900000 | 7469 | 7548.6 | 1.011 |
| 1000000 | 8164 | 8245.6 | 1.010 |

We obtain, from the ratio between the values of the second and the third column, the following results:

$$1224 / 1246.3 = 0.9821070368290138$$

$$2159 / 2179.5 = 0.990594172975453085$$

$$2992 / 3035.4 = 0.9857020491533241$$

$$3801 / 3846.1 = 0.988273835833701671$$

$$4562 / 4625.6 = 0.9862504323763403$$

$$5328 / 5381.5 = 0.990058533866022484$$

$$6058 / 6118.7 = 0.9900795920702109$$

$$6763 / 6840.2 = 0.988713780298821671$$

$$7469 / 7548.6 = 0.9894549982778263$$

$$8164 / 8245.6 = 0.990103812942660327$$

And the mean is: **0.98813382147**

Furthermore, we have that:

5. 41. Of the four problems mentioned by Landau in his Cambridge address, two were Goldbach's problem and the problem of the prime-pairs. The third was that of *the existence of an infinity of primes of the form $m^2 + 1$.*¹

Our method is applicable to this problem also. We have now to consider the integral

$$J(R) = \frac{1}{2\pi} \int_0^{2\pi} f(Re^{i\psi}) \vartheta(Re^{-i\psi}) e^{-i\psi} d\psi,$$

where $f(x)$ is the same function as before and

$$\vartheta(x) = \sum_{m=1}^{\infty} x^{m^2}.$$

The approximation for $\vartheta(\bar{x}) = \vartheta(Re^{-i\psi})$ on $\xi_{p,q}$ is

$$\vartheta(Re^{-i\psi}) \sim \frac{1}{2} V\pi \frac{\bar{S}_{p,q}}{q} \left(\frac{1}{n} + i \left(\psi - \frac{2p\pi}{q} \right) \right)^{-\frac{1}{2}},$$

where

$$S_{p,q} = \sum_{h=1}^q e_q(h^2 p)$$

and $\bar{S}_{p,q}$ is the conjugate of $S_{p,q}$; and we find, as an approximation for $J(R)$,

$$\frac{1}{4V\pi} \sum_{p,q} \frac{\mu(q)}{q \varphi(q)} \bar{S}_{p,q} e_q(-p) \int_{-\theta'_{p,q}}^{\theta_{p,q}} \frac{e^{-iu} du}{\left(\frac{1}{n} - iu \right) \sqrt{\frac{1}{n} + iu}}.$$

We replace the integral here by

$$\int_{-\infty}^{\infty} \frac{du}{\left(\frac{1}{n} - iu \right) \sqrt{\frac{1}{n} + iu}} = \pi \sqrt{2n};$$

Thence:

$$\begin{aligned} & \frac{1}{4V\pi} \sum_{p,q} \frac{\mu(q)}{q \varphi(q)} \bar{S}_{p,q} e_q(-p) \int_{-\theta'_{p,q}}^{\theta_{p,q}} \frac{e^{-iu} du}{\left(\frac{1}{n} - iu \right) \sqrt{\frac{1}{n} + iu}} = \\ & \int_{-\infty}^{\infty} \frac{du}{\left(\frac{1}{n} - iu \right) \sqrt{\frac{1}{n} + iu}} = \pi \sqrt{2n}; \end{aligned}$$

For $n = 8$, we obtain:

$$\text{Pi} * \text{sqrt}(2 * 8)$$

Input:

$$\pi \sqrt{2 \times 8}$$

Result:

4π

Decimal approximation:

12.56637061435917295385057353311801153678867759750042328389...

12.5663706.....

Property:

4π is a transcendental number

Series representations:

$$\pi \sqrt{2 \times 8} = 16 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\pi \sqrt{2 \times 8} = \sum_{k=0}^{\infty} -\frac{16 (-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$\pi \sqrt{2 \times 8} = 4 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

And:

$((((1/((Pi*sqrt(2*8)))))))^{1/64}$

Input:

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}}$$

Exact result:

$$\frac{1}{\sqrt[32]{2} \sqrt[64]{\pi}}$$

Decimal approximation

0.961224531349038176643226169241200977045073369851812216905...

0.961224531349.... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}$$

Property:

$\frac{1}{\sqrt[32]{2} \sqrt[64]{\pi}}$ is a transcendental number

All 64th roots of $1/(4\pi)$:

$$\frac{e^0}{\sqrt[32]{2} \sqrt[64]{\pi}} \approx 0.961225 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/32}}{\sqrt[32]{2} \sqrt[64]{\pi}} \approx 0.956596 + 0.09422i$$

$$\frac{e^{(i\pi)/16}}{\sqrt[32]{2} \sqrt[64]{\pi}} \approx 0.94275 + 0.18753i$$

$$\frac{e^{(3i\pi)/32}}{\sqrt[32]{2} \sqrt[64]{\pi}} \approx 0.91983 + 0.27903i$$

$$\frac{e^{(i\pi)/8}}{\sqrt[32]{2} \sqrt[64]{\pi}} \approx 0.88806 + 0.36784i$$

Series representations:

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}} = \frac{1}{\sqrt[16]{2} \sqrt[64]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}$$

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}} = \frac{1}{\sqrt[16]{2} \sqrt[64]{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}}$$

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}} = \frac{1}{\sqrt[32]{2} \sqrt[64]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}$$

Integral representations:

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}} = \frac{1}{\sqrt[16]{2} \sqrt[64]{\int_0^1 \sqrt{1-t^2} dt}}$$

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}} = \frac{1}{2^{3/64} \sqrt[64]{\int_0^\infty \frac{1}{1+t^2} dt}}$$

$$\sqrt[64]{\frac{1}{\pi \sqrt{2 \times 8}}} = \frac{1}{2^{3/64} \sqrt[64]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}$$

From:

Integer Partitions

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From:

$$\begin{aligned}
\sum_{n=0}^{\infty} p(n, 4)q^n &= \frac{1}{(1-q)(1-q^2)(1-q^3)(1-q^4)} \\
&= \frac{1/24}{(1-q)^4} + \frac{1/8}{(1-q)^3} + \frac{(5/12)^2}{(1-q)^2} + \frac{1/8}{(1-q^2)^2} \\
&\quad + \frac{1/16}{1-q^2} + \frac{(2+q)/9}{1-q^3} + \frac{1/4}{1-q^4} \\
&= \sum_{n \geq 0} \left(\left(\frac{1}{24} \binom{n+3}{3} \right) + \frac{1}{8} \binom{n+2}{2} + \left(\frac{5}{12} \right)^2 (n+1) \right) q^n \\
&\quad + \left(\frac{1}{8}(n+1) + \frac{1}{16} \right) q^{2n} \\
&\quad + \sum_{n \geq 0} \left(-\frac{1}{16} q^{2n} + \frac{2}{9} q^{3n} + \frac{1}{9} q^{3n+1} + \frac{1}{4} q^{4n} \right) \\
&= \sum_{n \geq 0} \left(\frac{1}{24} \binom{n+3}{3} + \frac{1}{8} \binom{n+2}{2} + \left(\frac{5}{12} \right)^2 (n+1) \right) q^n \\
&\quad + \left(\frac{1}{8} \left(\frac{n}{2} + 1 \right) + \frac{1}{16} \right) \left((n+1) - 2 \left\lfloor \frac{n+1}{2} \right\rfloor \right) q^n \\
&\quad + \sum_{n \geq 0} \left(-\frac{1}{16} q^{2n} + \frac{2}{9} q^{3n} + \frac{1}{9} q^{3n+1} + \frac{1}{4} q^{4n} \right) \tag{6.11}
\end{aligned}$$

We now note that the power series represented by the final sum in (6.11) has coefficients that lie in the closed interval $[-\frac{1}{16}, \frac{17}{36}]$. That is, each of these coefficients is strictly less than $\frac{1}{2}$ in absolute value.

Consequently, given that $p(n, 4)$ is obviously an integer, we may conclude from (6.11) by the uniqueness of the Maclaurin series expansion that

$$\begin{aligned}
p(n, 4) &= \left\{ \frac{1}{24} \binom{n+3}{3} + \frac{1}{8} \binom{n+2}{2} + \frac{25}{144} (n+1) \right. \\
&\quad \left. + \frac{1}{8} (n+4) \left(\frac{n+1}{2} - \left\lfloor \frac{n+1}{2} \right\rfloor \right) \right\} \\
&= \left\{ (n+1)(n^2 + 23n + 85)/144 - (n+4) \left\lfloor \frac{n+1}{2} \right\rfloor / 8 \right\}.
\end{aligned}$$

From

$$\begin{aligned}
 p(n, 4) &= \left\{ \frac{1}{24} \binom{n+3}{3} + \frac{1}{8} \binom{n+2}{2} + \frac{25}{144}(n+1) \right. \\
 &\quad \left. + \frac{1}{8}(n+4) \left(\frac{n+1}{2} - \left\lfloor \frac{n+1}{2} \right\rfloor \right) \right\} \\
 &= \left\{ (n+1)(n^2 + 23n + 85)/144 - (n+4) \left\lfloor \frac{n+1}{2} \right\rfloor / 8 \right\}
 \end{aligned}$$

For $n = 3$, we obtain:

$(((((3+1)(9+23*3+85)/144-((3+4)((3+1)/2)))/8))))$

Input:

$$(3+1)\left(\frac{1}{144}(9+23\times 3+85)\right)-\frac{1}{8}\left((3+4)\times \frac{3+1}{2}\right)$$

Exact result:

25
9

Decimal approximation:

We note that the 576th of the result is:

$$((((((3+1)(9+23*3+85)/144-(((3+4)((3+1)/2))/8))))))^{1/576}$$

Input:

$$\sqrt[576]{(3+1)\left(\frac{1}{144}(9+23\times 3+85)\right)-\frac{1}{8}\left((3+4)\times \frac{3+1}{2}\right)}$$

Result:

$$288\sqrt{\frac{5}{3}}$$

Decimal approximation:

1.001775274018938324672439322149113083481925561851267650466...

1.001775274..... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

$$\approx 1.0018674362$$

Alternate form:

$$\frac{1}{3} \sqrt[288]{5} 3^{287/288}$$

And the inverse of the expression is:

$$1 / (((((3+1)(9+23*3+85)/144-((3+4)((3+1)/2))/8)))))^{1/576}$$

Input:

$$\frac{1}{\sqrt[576]{(3+1)\left(\frac{1}{144}(9+23\times 3+85)\right) - \frac{1}{8}\left((3+4)\times \frac{3+1}{2}\right)}}$$

Result:

$$\sqrt[288]{\frac{3}{5}}$$

Decimal approximation:

$$0.998227871993869192326563557834620902891007558518441497193\dots$$

0.998227871993869.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Alternate form:

$$\frac{1}{5} \sqrt[288]{3} 5^{287/288}$$

We note that, with regard the previous expression, is possible to obtain:

$$1/9 \log \text{base } 1.001775274018938 (((((3+1)(9+23*3+85)/144-((3+4)((3+1)/2))/8))))$$

Input interpretation:

$$\frac{1}{9} \log_{1.001775274018938} \left((3+1) \left(\frac{1}{144} (9 + 23 \times 3 + 85) \right) - \frac{1}{8} \left((3+4) \times \frac{3+1}{2} \right) \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

$$64.00000000001\dots$$

64

Alternative representation:

$$\frac{1}{9} \log_{1.0017752740189380000} \left(\frac{1}{144} (3+1)(9+23 \times 3+85) - \frac{(3+4)(3+1)}{2 \times 8} \right) = \\ \frac{\log \left(-\frac{14}{8} + \frac{652}{144} \right)}{9 \log(1.0017752740189380000)}$$

$\log(x)$ is the natural logarithm

Series representation:

$$\frac{1}{9} \log_{1.0017752740189380000} \left(\frac{1}{144} (3+1)(9+23 \times 3+85) - \frac{(3+4)(3+1)}{2 \times 8} \right) = \\ 62.643702470238464 \log \left(\frac{25}{9} \right) - \\ 0.1111111111111111 \log \left(\frac{25}{9} \right) \sum_{k=0}^{\infty} 0.0017752740189380000^k G(k) \\ \text{for } \left(G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

Now, we have that:

$$\begin{aligned}
\sum_{n=0}^{\infty} p(n, 3)q^n &= \frac{1/6}{(1-q)^3} + \frac{1/4}{(1-q)^2} + \frac{1/4}{1-q^2} + \frac{1/3}{1-q^3} \\
&= \frac{1}{6} \sum_{n=0}^{\infty} \binom{n+2}{2} q^n + \frac{1}{4} \sum_{n=0}^{\infty} (n+1)q^n + \frac{1}{4} \sum_{n=0}^{\infty} q^{2n} + \frac{1}{3} \sum_{n=0}^{\infty} q^{3n} \\
&= \sum_{n=0}^{\infty} \left(\frac{(n+3)^2}{12} q^n - \frac{1}{3} q^n + \frac{1}{4} q^{2n} + \frac{1}{3} q^{3n} \right) \\
&= \sum_{n=0}^{\infty} \left(\frac{(n+3)^2}{12} + \epsilon(n) \right) q^n,
\end{aligned} \tag{6.7}$$

where $\epsilon(n)$ takes only the values $-\frac{1}{3}, -\frac{1}{12}, 0, \frac{1}{4}$.

We obtain, for $q = 0.5$ and $\epsilon(n) = -1/12$:

((((sum (((((n+3)^2)/12-1/12))*0.5^n, n=0..infinity))))

Infinite sum:

$$\sum_{n=0}^{\infty} \left(\frac{1}{12} (n+3)^2 - \frac{1}{12} \right) 0.5^n = 2.83333$$

2.83333

Convergence tests:

By the ratio test, the series converges.

Partial sum formula:

$$\sum_{n=0}^m \left(\frac{1}{12} (n+3)^2 - \frac{1}{12} \right) 0.5^n \approx 0.333333 \times 2^{-m-2} (-m^2 - 10m + 17 \times 2^{m+1} - 26)$$

We have also:

((((sum (((((n+3)^2)/12-1/12))*0.5^n, n=0..infinity))))^1/576

Input interpretation:

$$\sqrt[576]{\sum_{n=0}^{\infty} \left(\frac{1}{12} (n+3)^2 - \frac{1}{12} \right) \times 0.5^n}$$

Result:

1.00181

1.00181 result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

And the inverse of the expression:

$$1 / (((\text{sum } (((n+3)^2/12 - 1/12)) * 0.5^n, n=0..infinity)))^{1/576}$$

Input interpretation:

$$\frac{1}{\sqrt[576]{\sum_{n=0}^{\infty} \left(\frac{1}{12} (n+3)^2 - \frac{1}{12} \right) \times 0.5^n}}$$

Result:

0.998194

0.998194 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt{5^3}} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Further:

$$1/9 \log \text{base } 1.00181(((\text{sum } (((n+3)^2/12 - 1/12)) * 0.5^n, n=0..infinity)))$$

Input interpretation:

$$\frac{1}{9} \log_{1.00181} \left(\sum_{n=0}^{\infty} \left(\frac{1}{12} (n+3)^2 - \frac{1}{12} \right) \times 0.5^n \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

63.9899

63.9899 ≈ 64

And:

$3 \log_{1.00181}(((\text{sum } (((n+3)^2)/12 - 1/12) * 0.5^n, n=0..infinity)))$

Input interpretation:

$$3 \log_{1.00181} \left(\sum_{n=0}^{\infty} \left(\frac{1}{12} (n+3)^2 - \frac{1}{12} \right) \times 0.5^n \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

1727.73

1727.73

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Now, we have that:

$$\begin{aligned} 1 + \sum_{j=1}^{\infty} \frac{q^{j^2}}{(1-q)(1-q^2)\cdots(1-q^j)} &= \lim_{n \rightarrow \infty} s_n(q) = \lim_{n \rightarrow \infty} \sigma_n(q) \\ &= \prod_{n=0}^{\infty} \frac{1}{(1-q^{5n+1})(1-q^{5n+4})} \end{aligned}$$

product $1/(((1-q^{(5n+1)})(1-q^{(5n+4)})))$, $n=0$ to infinity

Input interpretation:

$$\prod_{n=0}^{\infty} \frac{1}{(1-q^{5n+1})(1-q^{5n+4})}$$

Partial product formula:

$$\prod_{n=0}^m \frac{1}{(1-q^{1+5n})(1-q^{4+5n})} = \frac{1}{(q; q^5)_{m+1} (q^4; q^5)_{m+1}}$$

$(a; q)_n$ gives the q -Pochhammer symbol

For $q < 1$, $q = 0.38$, we obtain:

product $1/(((1-0.38^{5n+1}))(1-0.38^{5n+4})))$, $n=0$ to infinity

Input interpretation:

$$\prod_{n=0}^{\infty} \frac{1}{(1-0.38^{5n+1})(1-0.38^{5n+4})}$$

Approximated product:

$$\prod_{n=0}^{\infty} \frac{1}{(1-0.38^{1+5n})(1-0.38^{4+5n})} \approx 1.65254$$

1.65254

For $q = 0.0018$, we obtain:

product $1/(((1-0.0018^{5n+1}))(1-0.0018^{5n+4})))$, $n=0$ to infinity

Input interpretation:

$$\prod_{n=0}^{\infty} \frac{1}{(1-0.0018^{5n+1})(1-0.0018^{5n+4})}$$

Approximated product:

$$\prod_{n=0}^{\infty} \frac{1}{(1-0.0018^{1+5n})(1-0.0018^{4+5n})} \approx 1.0018$$

1.0018 result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}}$$

$$\approx 1.0018674362$$

For $q = 1/496$, we obtain:

product $1/(((1-(1/496)^{5n+1}))(1-(1/496)^{5n+4})))$, $n=0$ to infinity

Input interpretation:

$$\prod_{n=0}^{\infty} \frac{1}{\left(1 - \left(\frac{1}{496}\right)^{5n+1}\right) \left(1 - \left(\frac{1}{496}\right)^{5n+4}\right)}$$

Infinite product:

$$\prod_{n=0}^{\infty} \frac{1}{(1 - 496^{-5n-4})(1 - 496^{-5n-1})} = \\ \frac{1}{\left(\frac{1}{60\ 523\ 872\ 256}; \frac{1}{300\ 19\ 840\ 638\ 976}\right)_\infty \left(\frac{1}{496}; \frac{1}{300\ 19\ 840\ 638\ 976}\right)_\infty}$$

$(a; q)_n$ gives the q -Pochhammer symbol

Decimal approximation:

1.002020202036757872259641979422821242328127995918809106475...

1.00202020..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

For $q = 1/512$, we obtain:

1/ product $1/(((1-(1/512)^{(5n+1)})(1-(1/512)^{(5n+4)})))$, $n=0$ to infinity

Input interpretation:

$$\frac{1}{\prod_{n=0}^{\infty} \left(1 - \left(\frac{1}{512}\right)^{5n+1}\right) \left(1 - \left(\frac{1}{512}\right)^{5n+4}\right)}$$

Result:

$$\left(\frac{1}{68\ 719\ 476\ 736}; \frac{1}{35\ 184\ 372\ 088\ 832}\right)_\infty \left(\frac{1}{512}; \frac{1}{35\ 184\ 372\ 088\ 832}\right)_\infty \approx \\ 0.998046874985476$$

$(a; q)_n$ gives the q -Pochhammer symbol

0.99804687498.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

We have also:

$$1 + \sum_{j=1}^{\infty} \frac{q^{j^2+j}}{(1-q)(1-q^2)\cdots(1-q^j)} = \lim_{n \rightarrow \infty} t_n(q) = \lim_{n \rightarrow \infty} \tau_n(q)$$

$$= \prod_{n=0}^{\infty} \frac{1}{(1-q^{5n+2})(1-q^{5n+3})}.$$

For $q = 1/24$, we obtain:

$$\text{product } 1/(((1-(1/24)^{(5n+2)})(1-(1/24)^{(5n+3)}))), \text{ n=0 to infinity}$$

Input interpretation:

$$\prod_{n=0}^{\infty} \frac{1}{\left(1 - \left(\frac{1}{24}\right)^{5n+2}\right) \left(1 - \left(\frac{1}{24}\right)^{5n+3}\right)}$$

Infinite product:

$$\prod_{n=0}^{\infty} \frac{1}{(1 - 24^{-5n-3})(1 - 24^{-5n-2})} = \frac{1}{\left(\frac{1}{13824}; \frac{1}{7962624}\right)_\infty \left(\frac{1}{576}; \frac{1}{7962624}\right)_\infty}$$

$(a; q)_n$ gives the q -Pochhammer symbol

Decimal approximation:

$$1.001811599672687839308309275955341989141661853887001724129\dots$$

1.00181159967.... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}} - \varphi} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

And:

1/ product $1/(((1-(1/24)^{(5n+2)})(1-(1/24)^{(5n+3)})))$, n=0 to infinity

Input interpretation:

$$\frac{1}{\prod_{n=0}^{\infty} \left(1 - \left(\frac{1}{24}\right)^{5n+2}\right) \left(1 - \left(\frac{1}{24}\right)^{5n+3}\right)}$$

Result:

$$\left(\frac{1}{13824}; \frac{1}{7962624}\right)_{\infty} \left(\frac{1}{576}; \frac{1}{7962624}\right)_{\infty} \approx 0.998191676285961$$

0.998191676.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^4\sqrt{5^3}-1}}}-\varphi+1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Now, we have that:

$$\begin{aligned} 1 + \sum_{j=1}^{\infty} \frac{q^{j^2}}{(1-q)(1-q^2)\cdots(1-q^j)} \\ &= \lim_{n \rightarrow \infty} s_n(q) = \lim_{n \rightarrow \infty} \sigma_n(q) \\ &= \sum_{j=-\infty}^{\infty} (-1)^j q^{j(5j+1)/2} \frac{1}{\prod_{m=1}^{\infty} (1-q^m)} \quad (\text{by (7.8)}) \\ &= \frac{\prod_{m=1}^{\infty} (1-q^{5m})(1-q^{5m-2})(1-q^{5m-3})}{\prod_{m=1}^{\infty} (1-q^m)} \\ &\quad (\text{by Theorem 11 with } q \text{ replaced by } q^5 \text{ and } z \text{ by } -q^{-2}) \\ &= \prod_{m=1}^{\infty} \frac{1}{(1-q^{5m-4})(1-q^{5m-1})}, \end{aligned}$$

proving the first Rogers-Ramanujan identity.

For $q = 1/512$, we obtain:

$$\text{product } 1/(((1-(1/512)^{(5n-4)})(1-(1/512)^{(5n-1)}))), \text{ n=1 to infinity}$$

Input interpretation:

$$\prod_{n=1}^{\infty} \frac{1}{\left(1 - \left(\frac{1}{512}\right)^{5n-4}\right) \left(1 - \left(\frac{1}{512}\right)^{5n-1}\right)}$$

Infinite product:

$$\prod_{n=1}^{\infty} \frac{1}{(1 - 512^{1-5n})(1 - 512^{4-5n})} = \frac{35\ 115\ 652\ 611\ 585}{(512; \frac{1}{35\ 184\ 372\ 088\ 832})_{\infty} (68\ 719\ 476\ 736; \frac{1}{35\ 184\ 372\ 088\ 832})_{\infty}}$$

$(a; q)_n$ gives the q -Pochhammer symbol

Decimal approximation:

$$1.001956947177007062658986087903902264665529951238845130084\dots$$

1.001956947177007..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

And:

$$1/\text{ product } 1/(((1-(1/512)^{(5n-4)})(1-(1/512)^{(5n-1)}))), \text{ n=1 to infinity}$$

Input interpretation:

$$\frac{1}{\prod_{n=1}^{\infty} \left(1 - \left(\frac{1}{512}\right)^{5n-4}\right) \left(1 - \left(\frac{1}{512}\right)^{5n-1}\right)}$$

Result:

$$\frac{(512; \frac{1}{35\ 184\ 372\ 088\ 832})_{\infty} (68\ 719\ 476\ 736; \frac{1}{35\ 184\ 372\ 088\ 832})_{\infty}}{35\ 115\ 652\ 611\ 585} \approx 0.998047$$

$(a; q)_n$ gives the q -Pochhammer symbol

0.998047 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

We have that also:

$$\begin{aligned} 1 + \sum_{j=1}^{\infty} \frac{q^{j^2+j}}{(1-q)(1-q^2)\cdots(1-q^j)} \\ &= \lim_{n \rightarrow \infty} t_n(q) = \lim_{n \rightarrow \infty} \tau_n(q) \\ &= \sum_{j=-\infty}^{\infty} (-1)^j q^{j(5j-3)/2} \frac{1}{\prod_{m=1}^{\infty} (1-q^m)} \quad (\text{by (7.8)}) \\ &= \frac{\prod_{m=1}^{\infty} (1-q^{5m})(1-q^{5m-1})(1-q^{5m-4})}{\prod_{m=1}^{\infty} (1-q^m)} \\ &\quad (\text{by Theorem 11 with } q \text{ replaced by } q^5 \text{ and } z \text{ by } -q^{-1}) \\ &= \prod_{m=1}^{\infty} \frac{1}{(1-q^{5m-3})(1-q^{5m-2})}, \end{aligned}$$

proving the second Rogers-Ramanujan identity.

For $q = 1/24$, we obtain:

product $1/(((1-(1/24)^{(5n-3)})(1-(1/24)^{(5n-2)})))$, $n=1$ to infinity

Input interpretation:

$$\prod_{n=1}^{\infty} \frac{1}{\left(1 - \left(\frac{1}{24}\right)^{5n-3}\right) \left(1 - \left(\frac{1}{24}\right)^{5n-2}\right)}$$

Infinite product:

$$\prod_{n=1}^{\infty} \frac{1}{(1 - 24^{2-5n})(1 - 24^{3-5n})} = \frac{7948225}{(576; \frac{1}{7962624})_{\infty} (13824; \frac{1}{7962624})_{\infty}}$$

$(a; q)_n$ gives the q -Pochhammer symbol

- **Approximated product:**

$$\prod_{n=1}^{\infty} \frac{1}{(1 - 24^{2-5n})(1 - 24^{3-5n})} \approx 1.00181$$

- **Decimal approximation:**

- More digits

1.001811599672687839308309275955341989141661853887001724129...

1.001811599672687839.... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-2\pi}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

And:

1/ product $1/(((1-(1/24)^{(5n-3)})(1-(1/24)^{(5n-2)})))$, n=1 to infinity

Input interpretation:

$$\frac{1}{\prod_{n=1}^{\infty} \frac{1}{(1 - (\frac{1}{24})^{5n-3})(1 - (\frac{1}{24})^{5n-2})}}$$

Result:

$$\frac{(576; \frac{1}{7962624})_{\infty} (13824; \frac{1}{7962624})_{\infty}}{7948225} \approx 0.998192$$

$(a; q)_n$ gives the q -Pochhammer symbol

0.998192 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}-1}}-\varphi+1}}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}$$

From:

Partitions: At the Interface of q -Series and Modular Forms

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In memory of Robert A. Rankin - Received February 10, 2003; Accepted February 20, 2003

We have that:

Rademacher's Conjecture [32, p. 302].

$$\lim_{N \rightarrow \infty} C_{hkj}(N)$$

exists and is given by

$$C_{hkj}(\infty) = -2\pi \left(\frac{\pi}{12}\right)^{3/2} \frac{w_{hk} e^{\frac{2\pi i h j}{k}}}{k^{5/2}} \Delta_{\alpha}^{j-1} L_{3/2} \left(-\frac{\pi^2}{6k^2}(\alpha + 1)\right).$$

where $\alpha = 1/24$, and

$$L_{3/2}(-y^2) = -\frac{1}{2\sqrt{\pi}y} \frac{d}{dy} \left(\frac{\sin 2y}{y} \right) = -\frac{1}{2\sqrt{\pi}y^2} \left(2\cos 2y - \frac{\sin 2y}{y} \right).$$

Rademacher goes on to provide a small table of values (reprinted here with one correction):

| N | C_{011} | C_{012} | C_{121} |
|-----|--|----------------------------------|------------------------------|
| 1 | -1 | 0 | 0 |
| 2 | $-\frac{1}{4} = -0.25$ | $\frac{1}{2} = 0.5$ | $\frac{1}{4} = 0.25$ |
| 3 | $-\frac{17}{72} = -0.23611\dots$ | $\frac{1}{4} = 0.25$ | $\frac{1}{8} = 0.125$ |
| 4 | $-\frac{17}{72} = -0.23611\dots$ | $\frac{59}{288} = 0.204861\dots$ | $\frac{1}{8} = 0.125$ |
| 5 | $-\frac{20831}{86400} = -0.24101\dots$ | $\frac{3}{16} = 0.1875$ | $\frac{13}{128} = 0.1015625$ |

and he lists explicitly three of the conjectured limits:

$$C_{011}(\infty) = -\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) = -0.273339\dots,$$

$$C_{012}(\infty) = \frac{24}{25 \cdot 49} \left(6 + \frac{109\sqrt{3}}{35\pi} \right) = 0.15119\dots,$$

$$C_{121}(\infty) = -\frac{\sqrt{6}}{25} \left(\cos \frac{5\pi}{12} - \frac{12}{5\pi} \sin \frac{5\pi}{12} \right) = 0.046941.$$

We have that:

$$((-6/25(1+(2\sqrt{3})/(5\pi)))+24/(25*49)(6+(109\sqrt{3})/(35\pi)))-\\((\sqrt{6}/25(((\cos(5\pi/12)-12/(5\pi)\sin(5\pi/12))))))$$

Input:

$$-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24}{25 \times 49} \left(6 + \frac{109\sqrt{3}}{35\pi} \right) - \frac{\sqrt{6}}{25} \left(\cos \left(5 \times \frac{\pi}{12} \right) - \frac{12}{5\pi} \sin \left(5 \times \frac{\pi}{12} \right) \right)$$

Exact result:

$$-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi} \right)$$

Decimal approximation:

$$-0.09479602732514528632300216597029201526869493022063806176\dots$$

$$-0.094796027\dots$$

Property:

$$-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \frac{1}{25} \sqrt{6} \left(\frac{-1+\sqrt{3}}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi} \right)$$

is a transcendental number

Alternate forms:

$$\frac{12348 + 1116\sqrt{3} - 15645\pi + 1715\sqrt{3}\pi}{85750\pi}$$

$$-\frac{447}{2450} + \frac{\sqrt{3}}{50} + \frac{18}{125\pi} + \frac{558\sqrt{3}}{42875\pi}$$

$$\frac{49\sqrt{3} - 447}{2450} + \frac{18(343 + 31\sqrt{3})}{42875\pi}$$

Alternative representations:

$$\begin{aligned} & \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi}\right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi}\right) = \\ & -\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) + \frac{24\left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} - \frac{1}{25} \left(\cosh\left(-\frac{5i\pi}{12}\right) + \frac{12 \cos\left(\frac{\pi}{2} + \frac{5\pi}{12}\right)}{5\pi}\right) \sqrt{6} \end{aligned}$$

$$\begin{aligned} & \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi}\right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi}\right) = \\ & -\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) + \frac{24\left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} - \frac{1}{25} \left(\cosh\left(-\frac{5i\pi}{12}\right) - \frac{12 \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)}{5\pi}\right) \sqrt{6} \end{aligned}$$

$$\begin{aligned} & \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi}\right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi}\right) = \\ & -\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) + \frac{24\left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} - \frac{1}{25} \left(\cosh\left(\frac{5i\pi}{12}\right) - \frac{12 \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)}{5\pi}\right) \sqrt{6} \end{aligned}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations:

$$\begin{aligned}
& \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& - \frac{1}{42875\pi} \left[5250\pi + 1500 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \\
& \quad 8232 \exp\left(i\pi \left\lfloor \frac{\arg(6-x)}{2\pi} \right\rfloor\right) \sqrt{x} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (6-x)^{k_2} x^{-k_2} J_{1+2k_1}\left(\frac{5\pi}{12}\right) \left(-\frac{1}{2}\right)_{k_2}}{k_2!} + \\
& \quad \left. 1715\pi \exp\left(i\pi \left\lfloor \frac{\arg(6-x)}{2\pi} \right\rfloor\right) \sqrt{x} \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(\frac{5}{12}\right)^{2k_1} \pi^{2k_1} (6-x)^{k_2} x^{-k_2} \left(-\frac{1}{2}\right)_{k_2}}{(2k_1)! k_2!} \right]
\end{aligned}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\begin{aligned}
& \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& - \frac{1}{42875\pi} \left[5250\pi + 1500 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 1715\pi \exp\left(i\pi \left\lfloor \frac{\arg(6-x)}{2\pi} \right\rfloor\right) \sqrt{x} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(\frac{5}{12}\right)^{2k_1} \pi^{2k_1} (6-x)^{k_2} x^{-k_2} \left(-\frac{1}{2}\right)_{k_2}}{(2k_1)! k_2!} - \\
& \quad \left. 4116 \exp\left(i\pi \left\lfloor \frac{\arg(6-x)}{2\pi} \right\rfloor\right) \sqrt{x} \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(\frac{12}{5}\right)^{-1-2k_1} \pi^{1+2k_1} (6-x)^{k_2} x^{-k_2} \left(-\frac{1}{2}\right)_{k_2}}{(1+2k_1)! k_2!} \right]
\end{aligned}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\begin{aligned}
& \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& - \frac{1}{6125\pi} \left(750\pi - \right. \\
& 6125\pi \sum_{k=0}^{\infty} - \frac{1}{8575\pi k!} (-1)^k x^{-k} \left(300(3-x)^k \exp\left(i\pi\left[\frac{\arg(3-x)}{2\pi}\right]\right) + 343\pi \right. \\
& (6-x)^k J_0\left(\frac{5\pi}{12}\right) \exp\left(i\pi\left[\frac{\arg(6-x)}{2\pi}\right]\right) \left(-\frac{1}{2}\right)_k \sqrt{x} + 490\pi \exp\left(i\pi\left[\frac{\arg(6-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (6-x)^{k_2} x^{-k_2} J_{2k_1}\left(\frac{5\pi}{12}\right) \left(-\frac{1}{2}\right)_{k_2}}{k_2!} - \\
& \left. \left. 1176 \exp\left(i\pi\left[\frac{\arg(6-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (6-x)^{k_2} x^{-k_2} J_{1+2k_1}\left(\frac{5\pi}{12}\right) \left(-\frac{1}{2}\right)_{k_2}}{k_2!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$J_n(z)$ is the Bessel function of the first kind

\mathbb{R} is the set of real numbers

Integral representations:

$$\begin{aligned}
& \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& - \frac{6}{49} + \int_0^1 \frac{1}{300} \left(12 \cos\left(\frac{5\pi t}{12}\right) - \pi \sin\left(-\frac{1}{12}\pi(-6+t)\right) \right) \sqrt{6} dt - \frac{12\sqrt{3}}{343\pi}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& - \frac{6}{49} + \int_0^1 \frac{1}{300} \left(12 \cos\left(\frac{5\pi t}{12}\right) + 5\pi \sin\left(\frac{5\pi t}{12}\right) \right) \sqrt{6} dt - \frac{12\sqrt{3}}{343\pi} - \frac{\sqrt{6}}{25}
\end{aligned}$$

$$\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) =$$

$$-\frac{6}{49} + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{e^{-(25\pi^2)/(576s)+s}}{100i\pi s^{3/2}} (-1+2s)\sqrt{6}\sqrt{\pi} ds - \frac{12\sqrt{3}}{343\pi} \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) =$$

$$-\frac{6}{49} - \frac{12\sqrt{3}}{343\pi} + \frac{\sqrt{6}}{25} - \frac{2}{25} \cos^2\left(\frac{5\pi}{24}\right) \sqrt{6} + \frac{24 \cos\left(\frac{5\pi}{24}\right) \sin\left(\frac{5\pi}{24}\right) \sqrt{6}}{125\pi}$$

$$\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) =$$

$$-\frac{6}{49} - \frac{12\sqrt{3}}{343\pi} - \frac{\sqrt{6}}{25} + \frac{24 \cos\left(\frac{5\pi}{24}\right) \sin\left(\frac{5\pi}{24}\right) \sqrt{6}}{125\pi} + \frac{2}{25} \sin^2\left(\frac{5\pi}{24}\right) \sqrt{6}$$

$$\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) =$$

$$\frac{\frac{3}{25} \cos\left(\frac{5\pi}{36}\right) \sqrt{6} - \frac{4}{25} \cos^3\left(\frac{5\pi}{36}\right) \sqrt{6} -}{42875\pi}$$

$$\frac{6 \left(875\pi + 250\sqrt{3} - 2058 \sin\left(\frac{5\pi}{36}\right) \sqrt{6} + 2744 \sin^3\left(\frac{5\pi}{36}\right) \sqrt{6} \right)}{42875\pi}$$

And:

$$((((((-6/25(1+(2\sqrt{3})/(5\pi))))+24/(25*49)(6+(109\sqrt{3})/(35\pi)))-((\sqrt{6}/25(((\cos(5\pi/12)-12/(5\pi)\sin(5\pi/12)))))))))^{1/64}$$

Input:

$$\sqrt[64]{\left(-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24}{25 \times 49} \left(6 + \frac{109\sqrt{3}}{35\pi} \right) \right) - \frac{\sqrt{6}}{25} \left(\cos\left(5 \times \frac{\pi}{12}\right) - \frac{12}{5\pi} \sin\left(5 \times \frac{\pi}{12}\right) \right)}$$

Exact result:

$$\sqrt[64]{-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi} \right)}$$

Decimal approximation:

$$0.9626954159707981084457306148670751161630235399801940480\dots + \\ 0.04729419307415218534995032979641518576616912859022226531\dots i$$

Property:

$$\sqrt[64]{-\frac{6}{25} \left(1+\frac{2\sqrt{3}}{5\pi}\right) + \frac{24\left(6+\frac{109\sqrt{3}}{35\pi}\right)}{1225} - \frac{1}{25}\sqrt{6} \left(\frac{-1+\sqrt{3}}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi}\right)}$$

is a transcendental number

Polar coordinates:

$$r \approx 0.963856 \text{ (radius)}, \quad \theta \approx 2.8125^\circ \text{ (angle)}$$

0.963856 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

Alternate forms:

$$\frac{\sqrt[64]{-\frac{-12348-1116\sqrt{3}+15645\pi-1715\sqrt{3}\pi}{2\pi}}}{35^{3/64}}$$

$$\frac{\sqrt[64]{\frac{35}{2}(49\sqrt{3}-447)+\frac{6174+558\sqrt{3}}{\pi}}}{35^{3/64}}$$

$$\frac{\sqrt[64]{\frac{6\sqrt{2}(686\sqrt{3}(1+\sqrt{3})-500\sqrt{3})-35(447\sqrt{2}-49\sqrt{6})\pi}{\pi}}}{2^{3/128} \times 35^{3/64}}$$

All 64th roots of $\frac{-6}{25}(1 + (2\sqrt{3})/(5\pi)) + (24(6 + (109\sqrt{3})/(35\pi)))/1225 - 1/25\sqrt{6}((\sqrt{3}-1)/(2\sqrt{2}) - (3\sqrt{2}(1+\sqrt{3}))/(5\pi))$:

$$\sqrt[64]{\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} + \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi}\right)} e^{(i\pi)/64}$$

$\approx 0.96270 + 0.04729 i$ (principal root)

$$\sqrt[64]{\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} + \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi}\right)} e^{(3i\pi)/64}$$

$\approx 0.95342 + 0.14143 i$

$$\sqrt[64]{\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} + \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi}\right)} e^{(5i\pi)/64}$$

$\approx 0.93497 + 0.23420 i$

$$\sqrt[64]{\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} + \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi}\right)} e^{(7i\pi)/64}$$

$\approx 0.90751 + 0.32471 i$

$$\sqrt[64]{\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} + \frac{1}{25} \sqrt{6} \left(\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{3\sqrt{2}(1+\sqrt{3})}{5\pi}\right)} e^{(9i\pi)/64}$$

$\approx 0.87132 + 0.41210 i$

Alternative representations:

$$\sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right)(-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi}\right)24}{25 \times 49}\right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi}\right)} =$$

$$\sqrt[64]{-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} - \frac{1}{25} \left(\cosh\left(-\frac{5i\pi}{12}\right) + \frac{12 \cos\left(\frac{\pi}{2} + \frac{5\pi}{12}\right)}{5\pi}\right) \sqrt{6}}$$

$$\sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right)(-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi}\right)24}{25 \times 49}\right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi}\right)} =$$

$$\sqrt[64]{-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi}\right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi}\right)}{1225} - \frac{1}{25} \left(\cosh\left(-\frac{5i\pi}{12}\right) - \frac{12 \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)}{5\pi}\right) \sqrt{6}}$$

$$\begin{aligned} & \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\ & \sqrt[64]{-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \frac{1}{25} \left(\cosh\left(\frac{5i\pi}{12}\right) - \frac{12 \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)}{5\pi} \right) \sqrt{6}} \end{aligned}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations:

$$\begin{aligned} & \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\ & \frac{1}{\sqrt[32]{5} \sqrt[7]{7^{3/64}}} \\ & \left(\left(\frac{1}{\pi} \left(-1050\pi - 300 \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\ & 343\pi \exp\left(i\pi \left[\frac{\arg(6-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!} (-1)^{k_2} (6-x)^{k_2} x^{-k_2} \\ & \left. \left. \left(-\frac{24(-1)^{k_1} J_{1+2k_1}\left(\frac{5\pi}{12}\right)}{5\pi} + \frac{(-1)^{k_1} 5^{2k_1} \times 12^{-2k_1} \pi^{2k_1}}{(2k_1)!} \right) \right. \right) \\ & \left. \left. \left(-\frac{1}{2}\right)_{k_2} \right) \right)^{(1/64)} \text{ for } (x \in \mathbb{R} \text{ and } x < 0) \end{aligned}$$

$$\begin{aligned}
& \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& \frac{1}{\sqrt[32]{5} \sqrt[7]{7^{3/64}}} \left(\left(\frac{1}{\pi} \left(-1050\pi - 300 \exp\left(i\pi \left[\frac{\arg(3-x)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& 343\pi \exp\left(i\pi \left[\frac{\arg(6-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!} (-1)^{k_2} (6-x)^{k_2} x^{-k_2} \\
& \left. \left. \left(\frac{(-1)^{k_1} 5^{2k_1} \times 12^{-2k_1} \pi^{2k_1}}{(2k_1)!} + \frac{(-1)^{1+k_1} \left(\frac{5}{12}\right)^{2k_1} \pi^{2k_1}}{(1+2k_1)!} \right) \right. \right. \\
& \left. \left. \left(-\frac{1}{2}\right)_{k_2} \right) \right)^{(1/64)} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\
& \frac{1}{\sqrt[32]{5} \sqrt[7]{7^{3/64}}} \left(\left(\frac{1}{\pi} \left(-1050\pi - 300 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} \right. \right. \right. \\
& z_0^{1/2+1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} - \\
& 343\pi \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(6-z_0)/(2\pi) \rfloor} z_0^{1/2+1/2 \lfloor \arg(6-z_0)/(2\pi) \rfloor} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!} \\
& (-1)^{k_2} \left(-\frac{24(-1)^{k_1} J_{1+2k_1}\left(\frac{5\pi}{12}\right)}{5\pi} + \frac{(-1)^{k_1} 5^{2k_1} \times 12^{-2k_1} \pi^{2k_1}}{(2k_1)!} \right) \\
& \left. \left. \left. \left(-\frac{1}{2}\right)_{k_2} (6-z_0)^{k_2} z_0^{-k_2} \right) \right) \right)^{(1/64)}
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$J_n(z)$ is the Bessel function of the first kind

\mathbb{R} is the set of real numbers

Integral representations:

$$\begin{aligned} & \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\ & \left(-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \right. \\ & \left. \frac{1}{25} \left(1 + \int_0^1 \left(-\cos\left(\frac{5\pi t}{12}\right) - \frac{5}{12} \pi \sin\left(\frac{5\pi t}{12}\right) \right) dt \right) \sqrt{6} \right)^{(1/64)} \end{aligned}$$

$$\begin{aligned} & \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\ & \left(-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \right. \\ & \left. \frac{\sqrt{6}}{25} \int_0^1 \left(-\cos\left(\frac{5\pi t}{12}\right) + \frac{1}{12} \pi \sin\left(-\frac{1}{12} \pi (-6+t)\right) \right) dt \right)^{(1/64)} \end{aligned}$$

$$\begin{aligned} & \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\ & \left(-\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) + \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} - \right. \\ & \left. \frac{\sqrt{6}}{25} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(25\pi^2)/(576s)+s} (-1+2s)\sqrt{\pi}}{4i\pi s^{3/2}} ds \right)^{(1/64)} \text{ for } \gamma > 0 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) = \\ & \frac{\sqrt[64]{-5250 - \frac{1500\sqrt{3}}{\pi} + 1715\sqrt{6} - 3430 \cos^2\left(\frac{5\pi}{24}\right)\sqrt{6} + \frac{8232 \cos\left(\frac{5\pi}{24}\right) \sin\left(\frac{5\pi}{24}\right) \sqrt{6}}{\pi}}}{35^{3/64}} \end{aligned}$$

$$\sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2 \sqrt{3}}{5 \pi}\right) (-6) + \frac{\left(6 + \frac{109 \sqrt{3}}{35 \pi}\right) 24}{25 \times 49}\right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5 \pi}{12}\right) - \frac{12 \sin\left(\frac{5 \pi}{12}\right)}{5 \pi}\right) =$$

$$\frac{\sqrt[64]{5145 \cos\left(\frac{5 \pi}{36}\right) \sqrt{6} - 6860 \cos^3\left(\frac{5 \pi}{36}\right) \sqrt{6} - \frac{6(875 \pi + 250 \sqrt{3} - 1372 \cos\left(\frac{5 \pi}{24}\right) \sin\left(\frac{5 \pi}{24}\right)) \sqrt{6}}{\pi}}}{35^{3/64}}$$

$$\sqrt[64]{\left(\frac{1}{25} \left(1 + \frac{2 \sqrt{3}}{5 \pi}\right) (-6) + \frac{\left(6 + \frac{109 \sqrt{3}}{35 \pi}\right) 24}{25 \times 49}\right)} - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5 \pi}{12}\right) - \frac{12 \sin\left(\frac{5 \pi}{12}\right)}{5 \pi}\right) =$$

$$\frac{\sqrt[64]{\frac{-5250 \pi - 1500 \sqrt{3} + 1715 \pi \sqrt{6} - 3430 \pi \cos^2\left(\frac{5 \pi}{24}\right) \sqrt{6} + 12348 \sin\left(\frac{5 \pi}{36}\right) \sqrt{6} - 16464 \sin^3\left(\frac{5 \pi}{36}\right) \sqrt{6}}{\pi}}{35^{3/64}}$$

From which:

$$\log_{0.963856} -(((\left(\left(\left(\left(-\frac{6}{25} \left(1 + \frac{2 \sqrt{3}}{5 \pi}\right) + \frac{24}{25 \times 49} \left(6 + \frac{109 \sqrt{3}}{35 \pi}\right)\right) - \frac{\sqrt{6}}{25} \left(\cos\left(5 \times \frac{\pi}{12}\right) - \frac{12}{5 \pi} \sin\left(5 \times \frac{\pi}{12}\right)\right)\right)\right)\right)\right)\right)$$

$$6/25(1+(2\text{sqrt}(3))/(5\text{Pi}))) + 24/(25*49)(6+(109\text{sqrt}(3))/(35\text{Pi}))) - ((\text{sqrt}(6)/25((((\cos(5\text{Pi}/12)-12/(5\text{Pi})\sin(5\text{Pi}/12))))))))))$$

Input interpretation:

$$\log_{0.963856} \left(- \left(\left(\left(\left(- \frac{6}{25} \left(1 + \frac{2 \sqrt{3}}{5 \pi} \right) + \frac{24}{25 \times 49} \left(6 + \frac{109 \sqrt{3}}{35 \pi} \right) \right) - \frac{\sqrt{6}}{25} \left(\cos\left(5 \times \frac{\pi}{12}\right) - \frac{12}{5 \pi} \sin\left(5 \times \frac{\pi}{12}\right)\right) \right) \right) \right) \right) \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

63.9992...

63.9992... ≈ 64

Alternative representations:

$$\frac{\log_{0.963856} \left(-\left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\ \frac{\log \left(\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} + \frac{1}{25} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \sqrt{6} \right)}{\log(0.963856)}$$

$$\log_{0.963856} \left(-\left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\ \log_{0.963856} \left(\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} + \frac{1}{25} \left(\cosh\left(-\frac{5i\pi}{12}\right) + \frac{12 \cos\left(\frac{\pi}{2} + \frac{5\pi}{12}\right)}{5\pi} \right) \sqrt{6} \right)$$

$$\log_{0.963856} \left(-\left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\ \log_{0.963856} \left(\frac{6}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) - \frac{24 \left(6 + \frac{109\sqrt{3}}{35\pi} \right)}{1225} + \frac{1}{25} \left(\cosh\left(-\frac{5i\pi}{12}\right) - \frac{12 \cos\left(\frac{\pi}{2} - \frac{5\pi}{12}\right)}{5\pi} \right) \sqrt{6} \right)$$

$\log(x)$ is the natural logarithm

$\cosh(x)$ is the hyperbolic cosine function

Series representations:

$$\log_{0.963856} \left(- \left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\ - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{43}{49} + \frac{12\sqrt{3}}{343\pi} + \frac{1}{25} \cos\left(\frac{5\pi}{12}\right) \sqrt{6} - \frac{12 \sin\left(\frac{5\pi}{12}\right) \sqrt{6}}{125\pi} \right)^k}{k}}{\log(0.963856)}$$

$$\log_{0.963856} \left(- \left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\ \log_{0.963856} \left(\frac{6}{25} \left(1 + \frac{2 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} }{5\pi} \right) - \right. \\ \left. 24 \left(6 + \frac{109 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} }{35\pi} \right) \right) + \frac{1}{25} \exp\left(i\pi \left\lfloor \frac{\arg(6-x)}{2\pi} \right\rfloor\right) \\ \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!} (-1)^{k_2} (6-x)^{k_2} x^{-k_2} \left(\frac{(-1)^{k_1} 5^{2k_1} \times 12^{-2k_1} \pi^{2k_1}}{(2k_1)!} + \right. \\ \left. \frac{(-1)^{1+k_1} \left(\frac{5}{12}\right)^{2k_1} \pi^{2k_1}}{(1+2k_1)!} \right) \left(-\frac{1}{2} \right)_{k_2} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\begin{aligned}
& \log_{0.963856} \left(- \left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\
& \log_{0.963856} \left(\frac{6}{25} \left(1 + \frac{2 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} }{5\pi} \right) - \right. \\
& \left. \frac{24 \left(6 + \frac{109 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} }{35\pi} \right)}{1225} + \frac{1}{25} \exp\left(i\pi \left\lfloor \frac{\arg(6-x)}{2\pi} \right\rfloor\right) \right. \\
& \left. \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!} (-1)^{k_2} (6-x)^{k_2} x^{-k_2} \left(-\frac{24(-1)^{k_1} J_{1+2k_1}\left(\frac{5\pi}{12}\right)}{5\pi} + \right. \right. \\
& \left. \left. \frac{(-1)^{k_1} 5^{2k_1} \times 12^{-2k_1} \pi^{2k_1}}{(2k_1)!} \right) \left(-\frac{1}{2} \right)_{k_2} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

i is the imaginary unit

\mathbb{R} is the set of real numbers

$J_n(z)$ is the Bessel function of the first kind

Integral representations:

$$\log_{0.963856} \left(-\left[\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right] - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right] = \log_{0.963856} \left(\int_0^1 \frac{1}{300} \left(-12 \cos\left(\frac{5\pi t}{12}\right) + \pi \sin\left(-\frac{1}{12}\pi(-6+t)\right) \right) \sqrt{6} dt + \frac{6}{343} \left(7 + \frac{2\sqrt{3}}{\pi} \right) \right)$$

$$\log_{0.963856} \left(-\left[\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right] - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right] = \log_{0.963856} \left(\frac{6}{49} + \int_0^1 -\frac{1}{300} \left(12 \cos\left(\frac{5\pi t}{12}\right) + 5\pi \sin\left(\frac{5\pi t}{12}\right) \right) \sqrt{6} dt + \frac{12\sqrt{3}}{343\pi} + \frac{\sqrt{6}}{25} \right)$$

$$\log_{0.963856} \left(-\left[\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right] - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right] = \log_{0.963856} \left(\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{343 e^{-(25\pi s)^2/(576s)+s} (-1+2s)\sqrt{6}\sqrt{\pi}}{s^{3/2}} ds + 600i(7\pi+2\sqrt{3})}{34300i\pi} \right) \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\log_{0.963856} \left(-\left[\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right] - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right] = \log_{0.963856} \left(\frac{6}{49} + \frac{12\sqrt{3}}{343\pi} - \frac{\sqrt{6}}{25} + \frac{2}{25} \cos^2\left(\frac{5\pi}{24}\right) \sqrt{6} - \frac{24 \cos\left(\frac{5\pi}{24}\right) \sin\left(\frac{5\pi}{24}\right) \sqrt{6}}{125\pi} \right)$$

$$\log_{0.963856} \left(-\left(\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right) \right) = \\ \log_{0.963856} \left(\frac{6}{49} + \frac{12\sqrt{3}}{343\pi} + \frac{\sqrt{6}}{25} - \frac{24 \cos\left(\frac{5\pi}{24}\right) \sin\left(\frac{5\pi}{24}\right) \sqrt{6}}{125\pi} - \frac{2}{25} \sin^2\left(\frac{5\pi}{24}\right) \sqrt{6} \right)$$

$$\log_{0.963856} \left(- \left[\left(\frac{1}{25} \left(1 + \frac{2\sqrt{3}}{5\pi} \right) (-6) + \frac{\left(6 + \frac{109\sqrt{3}}{35\pi} \right) 24}{25 \times 49} \right) - \frac{1}{25} \sqrt{6} \left(\cos\left(\frac{5\pi}{12}\right) - \frac{12 \sin\left(\frac{5\pi}{12}\right)}{5\pi} \right) \right] \right) =$$

$$\log_{0.963856} \left(- \frac{3}{25} \cos\left(\frac{5\pi}{36}\right) \sqrt{6} + \frac{4}{25} \cos^3\left(\frac{5\pi}{36}\right) \sqrt{6} + \frac{6 \left(875\pi + 250\sqrt{3} - 2058 \sin\left(\frac{5\pi}{36}\right) \sqrt{6} + 2744 \sin^3\left(\frac{5\pi}{36}\right) \sqrt{6} \right)}{42875\pi} \right)$$

Now, from:

$$p(n, 5) = \left\{ (n+8) \left(n^3 + n + 22n^2 + 44n + 248 + 180 \left\lfloor \frac{n}{2} \right\rfloor \right) / 2880 \right\}, \quad (3.9)$$

For n = 3

$$((((((3+8)(3^3+3+22*3^2+44*3+248+180(3/2)))/2880)))$$

Input:

$$\frac{(3+8) \times 3^3 + 3 + 22 \times 3^2 + 44 \times 3 + 248 + 180 \times \frac{3}{2}}{2880}$$

Exact result:

4829
1440

Decimal approximation:

3.3534722222....

And:

$$1/((((((3+8)((((3^3+3+22*3^2+44*3+248+180(3/2))/2880))))))^1/64)$$

Input:

$$\frac{1}{\sqrt[64]{(3+8) \times \frac{3^3+3+22 \times 3^2+44 \times 3+248+180 \times \frac{3}{2}}{2880}}}$$

Result:

$$\sqrt[64]{\frac{5}{4829}} 2^{5/64} \sqrt[32]{3}$$

Decimal approximation:

0.981271408952399325407221341642924690479895899900713243783...

0.98127140895... result very near to the dilaton value **0.989117352243 = ϕ**

Alternate forms:

$$\frac{2^{5/64} \sqrt[32]{3} \sqrt[64]{5} 4829^{63/64}}{4829}$$

root of $4829 x^{64} - 1440$ near $x = 0.981271$

From which:

log base 0.9812714089523993254

$$(((1/((((((3+8)((((3^3+3+22*3^2+44*3+248+180(3/2))/2880)))))))))))$$

Input interpretation:

$$\log_{0.9812714089523993254} \left(\frac{1}{\sqrt[64]{(3+8) \times \frac{3^3+3+22 \times 3^2+44 \times 3+248+180 \times \frac{3}{2}}{2880}}} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

64.000000000000000...

64

Alternative representation:

$$\log_{0.98127140895239932540000} \left(\frac{1}{\frac{(3+8)(3^3 + 3 + 22 \times 3^2 + 44 \times 3 + 248 + \frac{180 \times 3}{2})}{2880}} \right) =$$

$$\log \left(\frac{\frac{1}{9658}}{2880} \right)$$

$$\frac{\log(0.98127140895239932540000)}{\log(0.98127140895239932540000)}$$

$\log(x)$ is the natural logarithm

Series representations:

$$\log_{0.98127140895239932540000} \left(\frac{1}{\frac{(3+8)(3^3 + 3 + 22 \times 3^2 + 44 \times 3 + 248 + \frac{180 \times 3}{2})}{2880}} \right) =$$

$$-\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{3389}{4829}\right)^k}{k}}{\log(0.98127140895239932540000)}$$

$$\log_{0.98127140895239932540000} \left(\frac{1}{\frac{(3+8)(3^3 + 3 + 22 \times 3^2 + 44 \times 3 + 248 + \frac{180 \times 3}{2})}{2880}} \right) =$$

$$-52.8942995208980388867 \log\left(\frac{1440}{4829}\right) - 1.0000000000000000000000000000000$$

$$\log\left(\frac{1440}{4829}\right) \sum_{k=0}^{\infty} (-0.01872859104760067460000)^k G(k)$$

for $G(0) = 0$ and $G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$

Integral representations:

$$\log(z) = \int_1^z \frac{1}{t} dt$$

$$\log(1+z) = \frac{1}{2\pi i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s+1) \Gamma(-s)^2}{\Gamma(1-s) z^s} ds \quad \text{for } (-1 < \gamma < 0 \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

Now, we have:

$$p(n, 6) = \left\{ (n+11) \left((6n^4 + 249n^3 + 2071n^2 - 4931n + 40621)/518400 \right. \right. \\ \left. \left. + \left[\frac{n}{2} \right] (n+10)/192 + \left(\left[\frac{n+1}{3} \right] + 2 \left[\frac{n}{3} \right] \right) / 54 \right) \right\}, \quad (3.10)$$

For $n = 1$, we obtain:

$$(((12(6+249+2071-4931+40621)/518400+(1/2*11)/192+(2/3+2/3)/54)))$$

Input:

$$12 \times \frac{6 + 249 + 2071 - 4931 + 40621}{518400} + \frac{1}{192} \left(\frac{1}{2} \times 11 \right) + \frac{1}{54} \left(\frac{2}{3} + \frac{2}{3} \right)$$

Exact result:

$$\frac{241921}{259200}$$

Decimal approximation:

$$0.933337191358024691358024691358024691358024691358...$$

0.933337191358... result near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

We have that:

$$p(n, 7) = \left\{ (n+14) \left((n^5 + 70n^4 + 1785n^3 - 15365n^2 + 9702n + 277032)/3628800 \right. \right. \\ \left. \left. + \left[\frac{n}{2} \right] (n+14)/384 + \left[\frac{n}{3} \right] / 54 \right) \right\}, \quad (3.11)$$

For $n = 1$, we obtain:

$$(((15(1+70+1785-15365+9702+277032)/3628800+(1/2*15)/384+(1/3)/54)))$$

Input:

$$15 \times \frac{1 + 70 + 1785 - 15365 + 9702 + 277032}{3628800} + \frac{1}{384} \left(\frac{1}{2} \times 15 \right) + \frac{1}{3} \times \frac{1}{54}$$

Exact result:

$$\frac{83833}{72576}$$

Decimal approximation:

1.155106371252204585537918871252204585537918871252204585537...

1.1551063712522...

$$1 / (((((15(1+70+1785-15365+9702+277032)/3628800+(1/2*15)/384+(1/3)*1/54))))^{1/64})$$

Input:

$$\frac{1}{\sqrt[64]{15 \times \frac{1+70+1785-15365+9702+277032}{3628800} + \frac{1}{384} \left(\frac{1}{2} \times 15 \right) + \frac{1}{3} \times \frac{1}{54}}}$$

Result:

$$\sqrt[64]{\frac{7}{83833}} \cdot 2^{7/64} \sqrt[16]{3}$$

Decimal approximation:

0.997749529301499252891705190249441947786541404447496015592...

0.9977495293... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

Alternate forms:

$$\frac{2^{7/64} \sqrt[16]{3} \sqrt[64]{7} 83833^{63/64}}{83833}$$

log base 0.99774952930 (((((1/((((15(1+70+1785-15365+9702+277032)/3628800+(1/2*15)/384+(1/3)*1/54)))))))

Input interpretation:

$$\log_{0.99774952930} \left(\frac{1}{15 \times \frac{1+70+1785-15365+9702+277032}{3628800} + \frac{1}{384} \left(\frac{1}{2} \times 15 \right) + \frac{1}{3} \times \frac{1}{54}} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

64.000000...

64

Alternative representation:

$$\begin{aligned} & \log_{0.997749529300000} \left(\frac{1}{\frac{15(1+70+1785-15365+9702+277032)}{3628800} + \frac{15}{384 \times 2} + \frac{1}{54 \times 3}} \right) = \\ & \frac{\log \left(\frac{1}{\frac{1}{3 \times 54} + \frac{15}{2 \times 384} + \frac{4098375}{3628800}} \right)}{\log(0.997749529300000)} \end{aligned}$$

$\log(x)$ is the natural logarithm

Series representations:

$$\begin{aligned} & \log_{0.997749529300000} \left(\frac{1}{\frac{15(1+70+1785-15365+9702+277032)}{3628800} + \frac{15}{384 \times 2} + \frac{1}{54 \times 3}} \right) = \\ & - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{11257}{83833} \right)^k}{k}}{\log(0.997749529300000)} \end{aligned}$$

$$\begin{aligned} & \log_{0.997749529300000} \left(\frac{1}{\frac{15(1+70+1785-15365+9702+277032)}{3628800} + \frac{15}{384 \times 2} + \frac{1}{54 \times 3}} \right) = \\ & -443.851486114 \log \left(\frac{72576}{83833} \right) - \\ & 1.000000000000 \log \left(\frac{72576}{83833} \right) \sum_{k=0}^{\infty} (-0.002250470700000)^k G(k) \\ & \text{for } \left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right) \end{aligned}$$

Integral representations:

$$\log(z) = \int_1^z \frac{1}{t} dt$$

$$\log(1+z) = \frac{1}{2\pi i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s+1) \Gamma(-s)^2}{\Gamma(1-s) z^s} ds \quad \text{for } (-1 < \gamma < 0 \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

Now, we have

$$p(n, 8) = \left\{ (n+18) \left((n^6 + 108n^5 + 4503n^4 + 79911n^3 + 522148n^2 - 202687n \right. \right. \\ \left. \left. + 9441216)/203212800 + \left\lfloor \frac{n}{2} \right\rfloor (n^2 + 36 + 231) \right) / 9216 \\ \left. + \left(\left\lfloor \frac{n+1}{3} \right\rfloor + 2 \left\lfloor \frac{n}{3} \right\rfloor \right) \right) / 162 + \left\lfloor \frac{n}{4} \right\rfloor / 64 \right\}, \quad (3.12)$$

For $n = 1$, we obtain:

$$((((((19(1+108+4503+79911+522148- \\ 202687+9441216)/203212800))+(1/2(1+36+231))/9216+(2/3+2/3)*1/162+1/4*1/64)))$$

Input:

$$19 \times \frac{1 + 108 + 4503 + 79911 + 522148 - 202687 + 9441216}{203212800} + \\ \frac{\frac{1}{2}(1 + 36 + 231)}{9216} + \left(\frac{2}{3} + \frac{2}{3} \right) \times \frac{1}{162} + \frac{1}{4} \times \frac{1}{64}$$

Exact result:

$$\frac{5774395}{6096384}$$

Decimal approximation:

$$0.947183609168976232468295960359452422944486436549928613420\dots$$

0.9471836091689.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}$$

And, in conclusion, we have

$$p(n, 9) = \left\{ (n+22) \left((n^7 + 158n^6 + 10034n^5 + 327352n^4 + 5419144n^3 - 32063602n^2 + 5172096n + 564401888)/14631321600 + \left\lfloor \frac{n}{2} \right\rfloor (2n^2 + 91n + 728) / 36864 + \left((n+20) \left\lfloor \frac{n+1}{3} \right\rfloor + 2(n+23) \left\lfloor \frac{n}{3} \right\rfloor \right) / 2916 + \left(\left\lfloor \frac{n}{4} \right\rfloor + \left\lfloor \frac{n+2}{4} \right\rfloor \right) / 256 \right) \right\}. \quad (3.13)$$

For n = 1, we obtain:

$$((((23(1+158+10034+327352+5419144-32063602+5172096+564401888)/(14631321600)+((1/2(2+91+728)))/(36864)+((21*2/3+2*24*1/3))/(2916)+((1/4+3/4))/(256))))$$

Input:

$$23 \times \frac{1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + 564401888}{14631321600} + \frac{\frac{1}{2}(2 + 91 + 728)}{36864} + \frac{\frac{21}{3} + 2 \times 24 \times \frac{1}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right)$$

Exact result:

$$\frac{12865751683}{14631321600}$$

Decimal approximation:

$$0.879329430022233945018336552728087119621511155902690294224\dots$$

0.87932943.....

We obtain, from the mean of n_s and w_0 cosmological parameters (see previous Table II) 0.9722

$$0.9722 / (((((((((23(1+158+10034+327352+5419144-32063602+5172096+564401888)/(14631321600)+((1/2(2+91+728)))/(36864)+((21*2/3+2*24*1/3))/(2916)+((1/4+3/4)/(256))))))))))$$

Input:

$$0.9722 / \left(23 \times \frac{1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + 564401888}{14631321600} + \frac{\frac{1}{2}(2 + 91 + 728)}{36864} + \frac{21 \times \frac{2}{3} + 2 \times 24 \times \frac{1}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right) \right)$$

Result:

$$1.105615218605179417249910869432408273536861483975836812363...$$

1.1056152186....

result practically equal to the value of Cosmological Constant 1.1056

Furthermore, from:

Ascertaining The Cosmological Constant with Superclusters in f(R; T) Gravity
S. Bhattacharjee and P.K. Sahoo - arXiv:1908.06759v2 [gr-qc] 7 Sep 2019

$$\Lambda \approx \frac{2\rho}{9 \times 10^{16}} (2 \times 10^{-8}) \quad (19)$$

Plugging density of a typical supercluster $\rho \sim 10^{-27} \text{ kg m}^{-3}$, generates $\Lambda \sim 10^{-52} \text{ m}^{-2}$, which agrees well with observations. However, density of superclusters varies and can be as high as $10^{-24} \text{ kg m}^{-3}$ [28]. In such cases, by substituting a small, non-zero λ , the cosmological constant can be ascertained quite accurately.

we obtain, for $\rho = 0.24876 * 10^{-27}$

$$(2 * 0.24876 * 10^{-27} * 2 * 10^{-8}) / (9 * 10^{16})$$

Input interpretation:

$$\frac{2 \times 0.24876 \times 10^{-27} \times 2 \times 10^{-8}}{9 \times 10^{16}}$$

Result:

$$1.1056 \times 10^{-52}$$

$$1.1056 * 10^{-52}$$

From the previous expression, we obtain also:

$$((((((23(1+158+10034+327352+5419144-32063602+5172096+564401888)/(14631321600)+((1/2(2+91+728)))/(36864)+((21*2/3+2*24*1/3))/(2916)+((1/4+3/4)/(256)))))))^1/192$$

Input:

$$\left(\frac{23 \times \frac{1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + 564401888}{14631321600} + \frac{\frac{1}{2}(2+91+728)}{36864} + \frac{21 \times \frac{2}{3} + 2 \times 24 \times \frac{1}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right) }{2916} \right)^{1/192}$$

Result:

$$\frac{\sqrt[192]{12865751683}}{2^{7/96} \sqrt[32]{3} \sqrt[96]{35}}$$

Decimal approximation:

$$0.999330455113369349327923315799690238660397957597194551420\dots$$

0.99933045511... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}-1}}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Alternate forms:

$$\frac{1}{210} \sqrt[192]{12865751683} 2^{89/96} \times 3^{31/32} \times 35^{95/96}$$

$$\sqrt[192]{12865751683} \quad \text{root of } 120960x^{96} - 1 \text{ near } x = 0.885229$$

And:

$$1/3 * \log \text{base } 0.999330455113369349 (((((23(1+158+10034+327352+5419144-32063602+5172096+564401888)/(14631321600)+((1/2(2+91+728)))/(36864)+((21*2/3+2*24*1/3))/(2916)+((1/4+3/4)/(256)))))))$$

Input interpretation:

$$\frac{1}{3} \log_{0.999330455113369349} \left(\frac{23 \times \frac{1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + 564401888}{14631321600}}{\frac{1}{36864} + \frac{\frac{1}{2}(2+91+728)}{2916} + \frac{\frac{21 \times 2}{3} + 2 \times 24 \times \frac{1}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right)} \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

64.0000000000000...

64

Alternative representation:

$$\frac{1}{3} \log_{0.9993304551133693490000} \left(\frac{\frac{1}{14631321600}}{\frac{23(1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + 564401888) + \frac{2+91+728}{2 \times 36864} + \frac{\frac{21 \times 2}{3} + \frac{2 \times 24}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right)}{3 \log(0.9993304551133693490000)}} \right)$$

$\log(x)$ is the natural logarithm

Series representations:

$$\frac{1}{3} \log_{0.9993304551133693490000} \left(\frac{\frac{1}{14631321600} 23(1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + 564401888) + \frac{2+91+728}{2 \times 36864} + \frac{\frac{21 \times 2}{3} + \frac{2 \times 24}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right)}{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{1765569917}{14631321600} \right)^k}{k}}{3 \log(0.9993304551133693490000)}} \right)$$

$$\begin{aligned}
& \frac{1}{3} \log_{0.9993304551133693490000} \left(\frac{1}{14631321600} \right. \\
& \quad \left. 23 (1 + 158 + 10034 + 327352 + 5419144 - 32063602 + 5172096 + \right. \\
& \quad \left. 564401888) + \frac{2+91+728}{2\times36864} + \frac{\frac{21\times2}{3} + \frac{2\times24}{3}}{2916} + \frac{1}{256} \left(\frac{1}{4} + \frac{3}{4} \right) \right) = \\
& -497.683947966156241 \log \left(\frac{12865751683}{14631321600} \right) - 0.33333333333333333333 \\
& \log \left(\frac{12865751683}{14631321600} \right) \sum_{k=0}^{\infty} (-0.0006695448866306510000)^k G(k) \\
& \text{for } \left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)
\end{aligned}$$

We want to point out that all the results marked in red are close to the dilaton and Rogers-Ramanujan continued fractions values

Conclusion

Once again, we want to emphasize that Ramanujan's mathematics is continuing and will continue to surprise us with the results and connections we are getting, with the equations belonging to various fields of Theoretical Physics and Cosmology (see the various results obtained concerning the dilaton value and the associated spectral index. therefore, to conclude, the mathematics of the Indian genius is powerful and very current and we will continue to deepen it and investigate it. A mathematics, that of Ramanujan who, as we are describing in our articles, is bringing and will continue to lead to new and surprising results.

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Integer Partitions

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For the Rogers-Ramanujan continued fractions, see the following link:

<http://www.bitman.name/math/article/102/109/>

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