Proof of the Riemann hypothesis

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Abstract

The real value is zero only when the real part of s is 1/2.
Non-trivial zeros must always have a real value of zero.
The real part of s being 1/2 is the minimum requirement for s to be a non-trivial zeros.

key words

Riemann hypothesis, non-trivial zeros, real part of s is 1/2, minimum requirement, to the limit

1 introduction

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{s\pi}{2}\right)\Gamma(1-s)\zeta(1-s) \tag{1}$$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i14.1347\} = -0.950558 - 0.310547i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i21.022\} = -0.904282 + 0.426936i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i25.0109\} = -0.784761 - 0.619798i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i30.4249\} = -0.475849 + 0.879527i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i32.9351\} = -0.410261 - 0.911968i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i37.58618\} = -0.832147 + 0.554555i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i40.91872\} = -0.917431 + 0.397894i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i43.32707\} = -0.275249 - 0.961373i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i48.00515\} = 0.130432 + 0.991457i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i49.77383\} = -0.579292 - 0.81512i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i52.97032\} = -0.867736 - 0.497025i$

$\{2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)\}, \{s = 1/2 + i56.44625\} = -0.752855 + 0.658186i$

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From the above calculation, in Euler's formula Eq.(1), $\zeta(s)=0$ (s is non-trivial zeros) is not from $2^s\pi^{s-1}\sin(\frac{s\pi}{2})\Gamma(1-s)$ but $\zeta(1-s)=0$.

$$\zeta(s) = \zeta(1-s) = 0 \tag{2}$$

At the non-trivial zeros, $\zeta(s) = \zeta(1-s) = 0$ holds. in this case. Eq.(10)=0, Eq(11)=0, and $\eta(s) = \eta(1-s) = 0$ holds.

$$\eta(1-s) = (1-\frac{2}{2^{1-s}})\zeta(1-s) = \frac{2^{1-s}-2}{2^{1-s}}\zeta(1-s) = \frac{2-2^{s+1}}{2}\zeta(1-s) = (1-2^s)\zeta(1-s) \tag{3}$$

$$\eta(s) + \frac{2}{2^s}\zeta(s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{4}$$

$$\eta(s) = \frac{2^s-2}{2^s}\frac{2^{1-s}}{2^{1-s}-2}\eta(1-s) = \frac{2^s-2}{2^s}\zeta(1-s) = (1-\frac{2}{2^s})\zeta(1-s) = 0 \tag{5}$$

$$\eta(1-s) = \frac{2^{1-s}-2}{2^{1-s}}\frac{2^s}{2^s-2}\eta(s) = \frac{2^{1-s}-2}{2^{1-s}}\zeta(s) = (1-\frac{2}{2^{1-s}})\zeta(s) = 0 \tag{6}$$

$$\zeta(s) = \frac{2^s}{2^s-2}\eta(s) = (\frac{2^s-2+2}{2^s-2})\eta(s) = (1+\frac{2}{2^s-2})\eta(s) \tag{7}$$

$$= (1+\frac{2}{2^s}\frac{2^s}{2^s-2})\eta(s) = \eta(s) + \frac{2}{2^s}\zeta(s) = \eta(s) + \frac{2}{2^s}[\eta(s) + \frac{2}{2^s}\zeta(s)] \tag{8}$$

$$= \eta(s) + \frac{2}{2^s}[\eta(s) + \frac{2}{2^s}(\eta(s) + \frac{2}{2^s}\zeta(s))] = \eta(s) + \frac{2}{2^s}\eta(s) + (\frac{2}{2^s})^2\eta(s) + (\frac{2}{2^s})^3\zeta(s) \tag{9}$$

$$= \eta(s)[1 + \frac{2}{2^s} + (\frac{2}{2^s})^2] + (\frac{2}{2^s})^3\zeta(s) \neq \eta(s)[\frac{1-(\frac{2}{2^s})^k}{1-\frac{2}{2^s}}] + (\frac{2}{2^s})^{k+1}\zeta(s) \tag{10}$$

when k=2 (If the formula is a geometric sequence and is the same up to the k-th term, $=$ holds.)

$$\neq \eta(s)[\frac{1-(\frac{2}{2^s})^2}{1-\frac{2}{2^s}}] + (\frac{2}{2^s})^3\zeta(s) = (1-\frac{2}{2^s})\zeta(s)[\frac{1-(\frac{2}{2^s})^2}{1-\frac{2}{2^s}}] + (\frac{2}{2^s})^3\zeta(s) \tag{11}$$

$$= \zeta(s)[1 - (\frac{2}{2^s})^2 + (\frac{2}{2^s})^3] \tag{12}$$

$$\zeta(1-s) = \frac{2^{1-s}}{2^{1-s}-2}\eta(1-s) = (\frac{2^{1-s}-2+2}{2^{1-s}-2})\eta(1-s) = (1+\frac{2}{2^{1-s}-2})\eta(1-s) \tag{13}$$

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$$= (1 + \frac{2}{2^{1-s}}\frac{2^{1-s}}{2^{1-s}-2})\eta(1-s) = \eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s) \tag{14}$$

$$= \eta(1-s) + \frac{2}{2^{1-s}}[\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)] = \eta(1-s) + \frac{2}{2^{1-s}}\eta(1-s) + (\frac{2}{2^{1-s}})^2\zeta(1-s) \tag{15}$$

$$= \eta(1-s) + \frac{2}{2^{1-s}}\eta(1-s) + (\frac{2}{2^{1-s}})^2[\eta(1-s) + \frac{2}{2^{1-s}}\zeta(1-s)] \tag{16}$$

$$= \eta(1-s) + \frac{2}{2^{1-s}}\eta(1-s) + (\frac{2}{2^{1-s}})^2\eta(1-s) + (\frac{2}{2^{1-s}})^3\zeta(1-s) \tag{17}$$

when $\frac{2}{2^{1-s}} = 2^s$

$$= \eta(1-s) + 2^s\eta(1-s) + (2^s)^2\eta(1-s) + (2^s)^3\zeta(1-s) \tag{18}$$

$$= \eta(1-s)[1 + 2^s + (2^s)^2] + (2^s)^3\zeta(1-s) \tag{19}$$

$$\neq \eta(1-s)[\frac{1-(2^s)^k}{1-2^s}] + (2^s)^{k+1}\zeta(1-s) \tag{20}$$

when k=2

$$\neq \eta(1-s)[\frac{1-2^{2s}}{1-2^s}] + 2^{3s}\zeta(1-s) \tag{21}$$

$$= \zeta(1-s)(1-2^s)[\frac{1-2^{2s}}{1-2^s}] + 2^{3s}\zeta(1-s) \tag{22}$$

$$= \zeta(1-s)[1-2^{2s}] + 2^{3s}\zeta(1-s) \tag{23}$$

$$= \zeta(1-s)[1-2^{2s} + 2^{3s}] \tag{24}$$

from Eq.(12) and Eq.(24)

$$\zeta(s)[1 - (\frac{2}{2^s})^2 + (\frac{2}{2^s})^3] = \zeta(1-s)[1-2^{2s} + 2^{3s}] \tag{25}$$

2 Discussion

Define $\quad 0 < \Re(s) < 1$

from Eq.(25)

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i14.1347\} = -0.000160889i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i15.1347\} = -0.280343i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i16.1347\} = -4.17572i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i16.1347\} = 4.17572i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i17.1347\} = 4.82094i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i17.1347\} = -4.82094i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i21.022\} = -0.0000820167i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i21.022\} = 0.0000820167i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i74879.422\} = 0.00056128i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i74879.8804\} = -0.00111728i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i74891.93\} = 0.0000554776i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74892.5452\} = -0.000641199i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74895.7013\} = 0.00117245i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74896.2133\} = 0.000808722i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74896.6987\} = -0.00106666i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74897.0517\} = 0.000224195i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74898.1134\} = -0.000935263i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74898.9041\} = 0.000102353i$

$\{\zeta(s)[1-\frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 0.49999+i74911.8951\} = 0.000232008+0.000914211i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74911.9\} = 2.7105110^{-20} + 0.000914218i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}], \{s = 0.51 + i74911.8951\} = -0.232499 - 0.00586949i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i74912.4918\} = -0.0000277175i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74916.2765\} = 0.000952275i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74918.7\} = 0.000152616i$

$\{\zeta(s)[1 - \frac{2}{2^s}+(\frac{2}{2^s})^2]-\zeta(1-s)[1-2^s+2^{2s}]\}, \{s = 1/2+i74919.1\} = -8.4703310^{-22} + 0.0000171143i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74920.2598\} = 0.000360484i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i74920.8275\} = 0.0000329616i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i99999.422\} = -9.48598i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i999999.422\} = -0.270142i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i9999999.422\} = -6.86408i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i99999999.422\} = 0.0172762i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i999999999.422\} = -0.0048036i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 + i9999999999.422\} =$ no result

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i99999999.422\} = 0.0172762i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i999999999.422\} = -0.0048036i$

$\{\zeta(s)[1 - \frac{2}{2^s} + (\frac{2}{2^s})^2] - \zeta(1-s)[1 - 2^s + 2^{2s}]\}, \{s = 1/2 - i9999999999.422\} =$ no result

Even if s is a non-trivial zero, the imaginary value is close to 0 but not 0.

For example, the imaginary value 14.1347 contains significant errors.

Originally, there are a lot of digits after the decimal point.

That is, the minimum requirement for the non-trivial zeros is that the real part of s is 1/2.

$$\Re(s) = \frac{1}{2} \tag{26}$$

Proof complete.

3 Postscript

These calculations were performed with WolframAlpha.

References

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Please raise the prize money to son and daughter who are still young.

4 Re-Postscript

I don't think this is the form of pushing academic papers.
And something is missing.
And the passion for mathematics is about to disappear.
Looking for co-authors.