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Non-Negative in Value, but Absolute in Function by a Magnum and Parabolin—the Cogent Value Function

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Abstract

The absolute value function is a fundamental mathematical concept taught in elementary algebra. In differential calculus, the absolute value function has certain well-known mathematical properties that are often used to illustrate such concepts of—a continuous function, differentiability or the existence of a derivative, the limit, and etcetera.

An alternative to the classical definition of absolute value is given to define a *new* function that is mathematically equivalent to the absolute value, yet the different mathematically. This new mathematical formalism, the cogent value function, does not have the same mathematical properties of the absolute value function. Two other new mathematical functions are used in the definition of the cogent value function—the parabolin function, and the magnum function.

The cogent value function and the absolute value function have the same domain and range, but both are mathematically very different. The cogent value function demonstrates that the same mathematical concept when formally defined by an alternative method has different mathematical properties. The functions by operation are mathematically similar, but in mathematical formalism each is unique.

Keywords: absolute value, absolute value function, cogent function, cogent value function, ceiling function, continuous function, derivative, derivative of function, differentiability, differential of function, greatest integer function, floor function, indeterminate value, L'Hopital's rule, least integer function, limit, limit from left, limit from right, magnitude of a number function, magnum function, parabolin, trichotomy, undefined value

Acknowledgements

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1. Introduction

There are four fundamental functions to consider which are:

1. Absolute value function
2. Signum function
3. Ceiling function
4. Floor function

The first two functions are in relation to the overall concept of the absolute value function. The last two are the ceiling and floor functions, that are in relation to the cogent value function, and magnitude of a number function.

1.1 Absolute Value Function

The absolute value function [Wiki 2018a] is a fundamental function for the basis of mathematics. One simple definition of the absolute value function given by Martin-Gay [Mart 2012] is:

> "The absolute value of a number *a* denoted by | *a* |, is the distance between a and 0 on the number line."

The simple definition is too simple—a formal mathematical definition is required in order to examine the mathematical properties of the absolute value function.

1.1.1 Absolute Value Function Definition

Mathematically in terms of domain and range, the absolute value function *f* is defined as:

$$\forall\, x \in R \wedge y \in R,\ f : x \rightarrow y \text{ where } -\infty < x < +\infty \wedge 0 \leq y < +\infty. \qquad \textbf{(1.1.1.1)}$$

Another definition that is conceptually elegant and much simpler is using the magnitude, the sign of a number, sign, or signum function. The sign of a number or signum definition for the absolute value function is:

$$|x| = sgn(x) \cdot x \ \text{ where } x \in R. \qquad \textbf{(1.1.1.2)}$$

A typical definition for the absolute value function is in the form of an if conditional statement. This definition of the absolute value function is more computational than mathematical. The conditional definition for the absolute value function is:

$$|x| := \begin{cases} +x & if\ x \geq 0, \\ -x & if\ x < 0. \end{cases} \text{ where } x \in R. \tag{1.1.1.3}$$

Two important mathematical properties of the absolute value function are from the established definition [Sull 2012] using exponents, more specifically the square (power of 2) and the square root (power of ½):

$$|x| \quad = \quad \sqrt{x^2} \tag{1.1.1.4}$$

These two mathematical properties are for a continuous function, and the existence of a derivative at 0, or differentiability of the absolute value function at 0.

1.1.2 Absolute Value Function Continuous at Zero

Absolute value is continuous at zero. For the established definition using exponents, consider the value of $x = 0$.

$$|0| \quad = \quad \sqrt{0^2} \tag{1.1.2.1}$$

Simplifying the equation is:

$$|0| \quad = \quad 0 \tag{1.1.2.2}$$

There is a real number value for the absolute value function when $x = 0$, hence the absolute value function is continuous at zero.

1.1.3 Absolute Value Function Not Differentiable at Zero

The absolute value function is not differentiable at zero—there exists no derivative at zero. Keisler states [Keis 2000a] that:

> "The absolute value function y = |x| is continuous but not differentiable at the point x = 0."

The derivative of the absolute value function is:

$$\frac{dy}{dx}|x| \quad = \quad \frac{|x|}{x} \tag{1.1.3.1}$$

Consider the limit as the variable x approaches 0 for then the limit of the derivative of the absolute value function is:

$$\lim_{x\to 0}\frac{|x|}{x}=\frac{0}{0}=\emptyset \tag{1.1.3.2}$$

The result is an indeterminate value $\emptyset$ at 0 indicating that the differential or the derivative at zero is indeterminate or undefined—it does not exist. Varberg, Purcell, and Rigdon [Varb 2007] state:

> "This is easily seen considering f(x) = |x| at the origin...The function is certainly continuous at zero. However it does not have a derivative there..."

1.1.4 Absolute Value Function Limit from Left and Right

For the limit, consider the limit as the variable x approaches from both the left and right of 0.

Thus for $x = 0$ consider the limit is:

$$\left.\frac{d|x|}{dx}\right|_{x=0}=\lim_{x\to 0}\frac{|x|-0}{x-0} \tag{1.1.4.1}$$

For the limit as x approaches 0, take both the limit approaching 0 from the left, and the limit approaching 0 from the right is:

$$\left.\frac{d|x|}{dx}\right|_{x=0} = \begin{cases} \lim\limits_{x\to 0^+}\dfrac{+x}{x} : x>0 \\ \lim\limits_{x\to 0^+}\dfrac{-x}{x} : x<0 \end{cases} \tag{1.1.4.2}$$

Each limit then converges to a value of +1 or -1 respectively, thus the limit is:

$$\left.\frac{d|x|}{dx}\right|_{x=0}=\begin{cases}+1 : x>0\\ -1 : x<0\end{cases} \tag{1.1.4.3}$$

The limit as x approaches 0 from both the left and the right are not equal—they do not converge to a real value. The limit for the absolute value function at 0, or $|x|$ where $x = 0$, does not exist, and thus the absolute value function is not differentiable or has no derivative at 0.

An important mathematical property from differential calculus is that a real number function *f* that is differentiable at *x* is continuous, but not the converse. Larson, Hodgkins, and Falvo [Lars 2010] state:

> "...that continuity is not a strong enough condition to guarantee differentiability. On the other hand, if a function is differentiable at a point, then it must be continuous at that point."

More formally in terms of continuity and differentiability [Lars 2010] is stated:

> "If a function *f* is differentiable at $x = c$, then *f* is continuous at $x = c$."

Another statement of a continuous function that is differentiable [Thom 1996] is:

> "A function is continuous at every point where it has a derivative."

Thus a real number function continuous at *x* is then not necessarily differentiable at *x*. The absolute value function is continuous at $x = 0$, but not differentiable at $x = 0$.

1.2 Signum Function

The signum function or sign function [Wiki 2018c] is the sign of a number function, returning the sign or magnitude of a real numeric value. Oldham, Myland, and Spanier [Oldh 2009] summarize the signum function as:

> "As its name implies, the signum function extracts the sign of its argument..."

Mathematically in terms of domain and range, the signum function *f* is defined as:

$$\forall\ \mathrm{x} \in \mathrm{R} \wedge \mathrm{y} \in \mathrm{Z},\ \mathrm{f} : \mathrm{x} \rightarrow \mathrm{y} \text{ where } -\infty < \mathrm{x} < +\infty \wedge\ \mathrm{y} \in \{ -1, 0, +1 \}. \tag{1.2.1}$$

The domain is any possible real number, but the range is a finite set of integer constants that are multiplicative coefficients for the sign of a number *x* where the form is:

$$x \in R \text{ where } x \equiv \{ x_s \cdot x_v | \, x_s \in \{-1, 0, +1\} \wedge x_v \geq 0 \tag{1.2.2}$$

A typical definition for the signum function is in the form of an if conditional statement. This definition of the signum function is more computational than mathematical. Each case of the Law of Trichotomy [Mars 1993] is given for each conditional.

The if conditional definition for the signum function is:

$$sgn(x) := \begin{cases} +1 & if\ x > 0, \\ 0 & if\ x = 0, \\ -1 & if\ x < 0. \end{cases} \tag{1.2.3}$$

Another definition of the signum function uses the absolute value function. The definition of the signum function using the absolute value function is:

$$sgn(x) \coloneqq \frac{|x|}{x} = \frac{x}{|x|} \text{ where } x \neq 0. \tag{1.2.4}$$

One interesting property of this definition is that if the at 0 the signum is indeterminate, or thus the sign of 0 is ∅.

1.3 Ceiling Function

The ceiling function or greatest integer function, is a fundamental notion in mathematics. The notation and the name were introduced by Iverson [Iver 1962].

1.3.1 Ceiling Function Definition

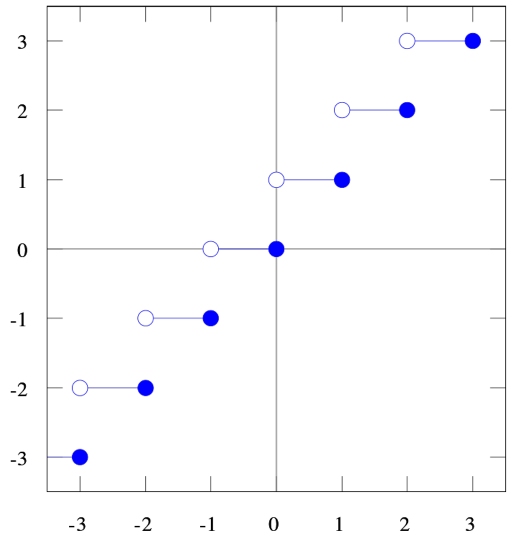
Mathematically in terms of domain and range, the ceiling function *f* is defined as:

$$\forall\ x \in R \wedge y \in Z,\ f : x \to y \text{ where } y \geq x. \tag{1.3.1}$$

The ceiling function *f* takes a real number *x*, and determines the greatest integer that is greater than or equal to *x*. Formally stated that the ceiling function definition is:

$$y = \lceil x \rceil\ where\ x \in R \wedge y \in Z \wedge y \geq x \tag{1.3.2}$$

The graph of the ceiling function [Wiki 2018b] is:



Plot of the Ceiling Function

1.3.2 Ceiling Function Derivative

The derivative of the ceiling function is defined everywhere except where the ceiling function value is an integer. The integer end points of the ceiling function are a point of a discontinuous jump to the next step in the ceiling function. At these discontinuous jump points, the ceiling function is not continuous, and thus is not differentiable. Kiesler [Keis 2000b] explains, "For each integer n, $\lceil n \rceil$ is equal to n. The function $\lceil x \rceil$ is continuous when x is not an integer but is discontinuous when x is an integer n."

But for the real, non-integer numeric points, the ceiling function is defined and thus is differentiable. The derivative for the ceiling function where defined is 0.

The formal mathematical definition is:

$$\frac{dy}{dx}\lceil x \rceil = 0 \; where \;\; i < x < i + 1 \;\wedge\; i \;\in \mathbb{Z} \qquad \textbf{(1.3.2.1)}$$

The derivative exists and is 0 for all non-integer numeric values.

1.4 Floor Function

The floor function or least integer function, is a fundamental notion in mathematics. The notation and the name were introduced by Iverson [Iver 1962].

1.4.1 Floor Function Definition

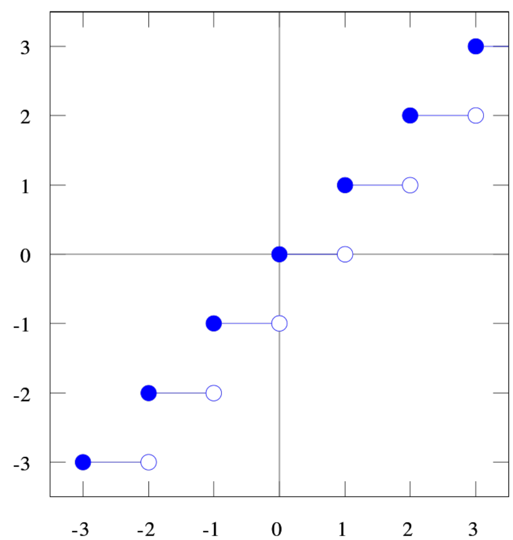
Mathematically in terms of domain and range, the floor function *f* is defined as:

$$\forall\ x \in R \wedge y \in Z,\ f : x \rightarrow y \text{ where } y \leq x. \tag{1.4.1.1}$$

The floor function *f* takes a real number *x*, and determines the least integer that is less than or equal to *x*. Formally stated that the floor function:

$$y = \lfloor x \rfloor\ where\ x\ \in R\ \wedge\ y \in Z\ \wedge\ y \leq x \tag{1.4.1.2}$$

The graph of the floor function [Wiki 2018b] is:



Plot of the Floor Function

1.4.2 Floor Function Derivative

The derivative of the floor function is defined everywhere except where the floor function value is an integer. The integer end points of the ceiling function are a point of a discontinuous jump to the next step in the floor function. At these discontinuous jump points, the floor function is not continuous, and thus is not differentiable.

But for the real, non-integer numeric points, the floor function is defined and thus is differentiable. The derivative for the floor function where defined is 0.

The formal mathematical derivative is:

$$\frac{dy}{dx}\lfloor x \rfloor = 0 \; where \;\; i < x < i + 1 \;\wedge\; i \;\in \mathbb{Z} \qquad \textbf{(1.4.2.1)}$$

The derivative exists and is 0 for all non-integer numeric values.

2. Concept

There are three important concepts that the functions utilize. Each concept is given a name to identify the function. The parabolin function is not special, and is a classic polynomial function—yet has special significance mathematically. Two of the functions, the magnum and cogent value, are analogous to functions of the signum and absolute value, respectively.

2.1 Parabolin Function

The *parabo*la *lin*e function or parabolin function is introduced as a new conceptual function. The parabolin function is a polynomial equation that is defined using the composition of three simpler polynomial functions. The three simpler polynomial functions are:

1. parabola
2. line
3. constant

The equation expressing the parabolin function is:

$$f(x) = \frac{x}{x^2 + c} \tag{2.1.1}$$

Each of the three elements is significant in the properties of the parabolin function.

2.1.1 Parabola Function

The parabola function both squares the numeric value, and assures the resulting value in the denominator is positive.

2.1.2 Line Function

The line function preserves the magnitude or sign of the numeric value, and in the numerator, this mathematically ensures that the resulting ratio is always less than one.

2.1.3 Constant

The constant, or slack constant, is a small positive real value added to the parabola function, that is always greater than zero. This ensures the resulting numeric value for the parabolin function is less than one, but also that there is no indeterminate value at zero. While the slack constant can be any positive real value, the simplest is an integer value of 1. The slack constant $c = 1$ in the development of the magnitude, and the cogent value function.

2.1.4 Properties of Parabolin Function

Mathematically in terms of domain and range, the parabolin function f is defined as:

$$\forall \, x \in R \wedge y \in R, f : x \rightarrow y \text{ where } -\infty < x < +\infty \wedge 0 < y < 1 \wedge \text{sgn}(x) \equiv \text{sgn}(y). \quad \textbf{(2.1.4.1)}$$

The parabolin function takes any real number x, and maps it into another real number y, but within the range bounded from 0 to 1 with the same sign. More commonly, the parabolin function maps a number into a non-integer or fractional number, but preserves the sign of the number.

2.1.5 Limit of Parabolin Function Approaching Infinity

The limit of the parabolin function f as $x \rightarrow \infty$ is:

$$\lim_{x\to\infty} \frac{x}{x^2 + c} = \frac{\infty}{\infty} \quad \textbf{(2.1.5.1)}$$

This is an indeterminate result, but using L'Hopital's Rule [Weis 2018] and taking the derivative of the numerator and denominator:

$$\lim_{x\to\infty} \frac{1}{2x} = \frac{\infty}{\infty} \quad \textbf{(2.1.5.2)}$$

The result is still indeterminate, so again by L'Hopital's Rule and taking the derivative of the numerator and denominator:

$$\lim_{x\to\infty} \frac{0}{2} = 0 \quad \textbf{(2.1.5.3)}$$

Thus the parabolin function is converging to a limit of 0.

2.1.6 Limit of Parabolin Function Approaching Zero

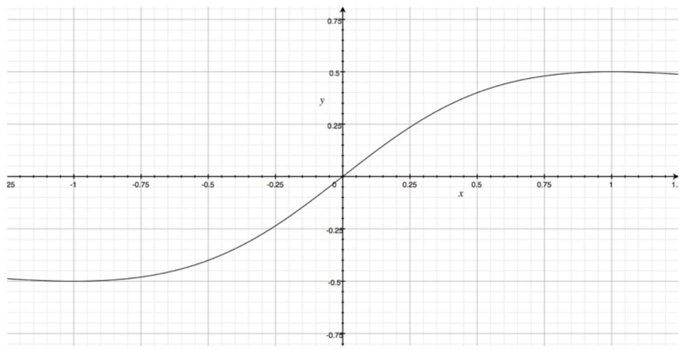
The limit of the parabolin function *f* as $x \rightarrow 0$ is:

$$\lim_{x \to 0} \frac{x}{x^2 + c} = \frac{0}{0 + c} = \frac{0}{c} = 0 \qquad \textbf{(2.1.6.1)}$$

Thus the limit of the parabolin function is converging to a limit of 0.

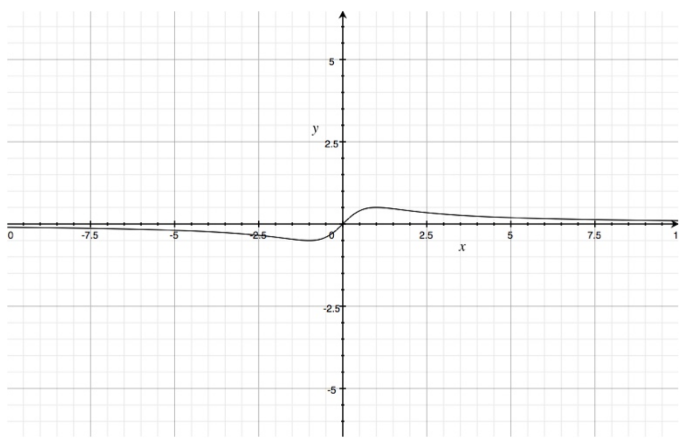
2.1.7 Graph of Parabolin Function

When the slack constant $c = 1$, the parabolin function has a smooth, elongated S-style curve. The maximum values of 0.5 occur when $x = \pm 1$. The graph of the plot of the parabolin function is:



Plot of Parabolin Function where slack constant c = 1.

When the slack constant $c = 1$, the parabolin function graph when extended approaches zero. The graph of the plot of the parabolin function is:



Plot of Parabolin Function where slack constant c = 1.

The limits as the parabolin function approaches infinity and zero both converged to zero. The graph of the parabolin function extended along the x-axis shows for both quadrants that the parabolin function is approaching zero.

2.1.8 Properties of Parabolin Function

The parabolin function is simply a well-known polynomial equation from elementary algebra. However for the cogent value function, it has four important mathematical properties.

The significant properties of the parabolin function are:

1. The parabolin function is continuous. $\forall x \in R \therefore f(x) \in R$
2. The parabolin function converges to a limit of zero. $\lim x \to \infty \ f(x) = 0$
3. The parabolin function is a real fractional value greater than zero. $f(x) \in R \wedge f(x) > 0$
4. The parabolin function is a real fractional value less than one. $f(x) \in R \wedge f(x) < 1$

2.2 Magnum Function

The new function of the magnum function is the *mag*nitude of a *num*ber function is introduced, and is comparable to the signum function. The magnum function *f* is formally defined mathematically as:

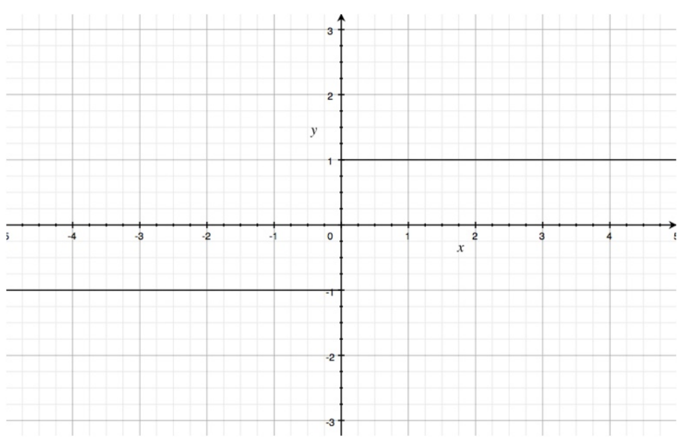
$$f : x \rightarrow \{ -1, 0, +1 \} \text{ where } x \in R. \tag{2.2.1}$$

The magnum function uses parabolin function. The parabolin function preserves the sign in the range of the original numeric value in the domain, and maps a value into a fractional non-integer value.

There are two ranges the magnum function maps a given value from the parabolin function into—the non-positive (or in the range $-\infty$ to 0) and the non-negative (or in the range of 0 to $+\infty$) ranges.

2.2.1 Graph of Magnum Function

The graph of the magnum function is:



Plot of the Magnum Function.

The plot of the magnum function is a constant for the actual sign of the number with the constant values of -1, 0, +1 for the result.

2.2.2 Ceiling of Parabolin Function

The ceiling function of the parabolin function is used to compute the positive magnitude for a given real value *x*.

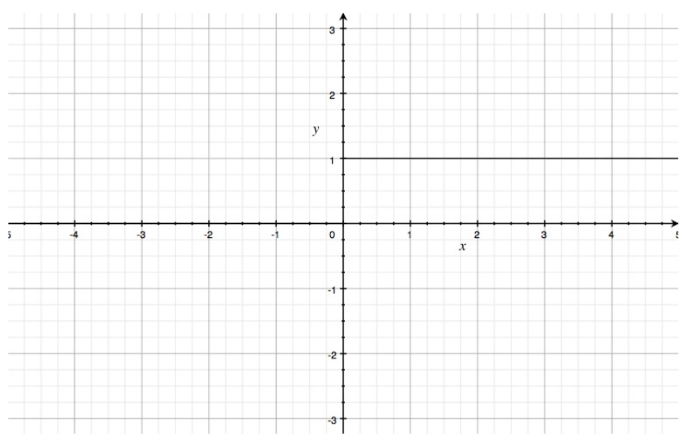
Using the greatest integer function, or ceiling function, the parabolin function is the composition of *f(x)* with *g(x)*, or *f(g(x))* to map the domain into the set { 0, +1 } in the range, or the non-negative range. This ceiling of the parabolin function is formally defined as:

$$f(x) = \left\lceil \frac{x}{x^2 + c} \right\rceil \tag{2.2.2.1}$$

This composition of functions is continuous and has a derivative. The ceiling function derivative is 0 for the non-integer range—and by the properties of the parabolin function range is fractional or non-integer values.

2.2.3 Graph of Ceiling of Parabolin Function

The ceiling of the parabolin function is all positive values less than 0, which by the ceiling function are -1. At 0 the ceiling of the parabolin function is 0. The graph of the ceiling of the parabolin function is:



Plot of Ceiling of Parabolin Function where slack constant *c* = 1.

The plot of the floor of the parabolin function is a constant +1, but at 0 is 0.

2.2.4 Floor of Parabolin Function

The floor function of the parabolin function is used to compute the negative magnitude for a given real value *x*.

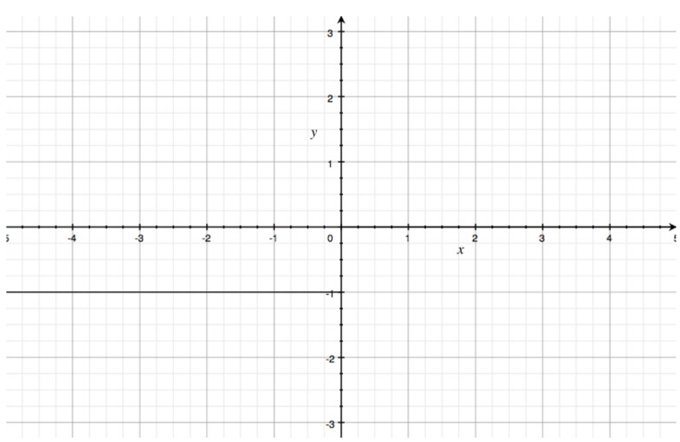
Using the least integer function, or floor function, the parabolin function is composed by the floor function in the form of the composition of *f(x)* with *g(x)*, or *f(g(x))* to map the domain into the set { 0, -1 } in the range, or the non-positive range. This floor of the parabolin function is formally defined as:

$$f(x) = \left\lfloor \frac{x}{x^2 + c} \right\rfloor \tag{2.2.4.1}$$

This composition of functions is continuous and has a derivative. The ceiling function derivative is 0 for the non-integer range—and by the properties of the parabolin function range is fractional or non-integer values.

2.2.5 Graph of Floor of Parabolin Function

The floor of the parabolin function is all negative values less than 0, which by the floor function are -1. At 0 the floor of the parabolin function is 0. The graph of the floor of the parabolin function is:



Plot of Floor of Parabolin Function where slack constant *c* = 1.

The plot of the floor of the parabolin function is a constant -1, but at 0 is 0.

2.2.6 Sum of Ceiling and Floor of Parabolin Function

The sum of the ceiling function and floor function of the parabolin function is used to compute the zero magnitude for a given real value x. When both the ceiling function of the parabolin and the floor function of the parabolin function are zero, the magnitude is zero. This sum of the ceiling and floor of the parabolin function is defined as:

$$f(x) = \left\lceil \frac{x}{x^2 + c} \right\rceil + \left\lfloor \frac{x}{x^2 + c} \right\rfloor \quad \textbf{(2.2.6.1)}$$

This composition of functions is continuous and has a derivative.

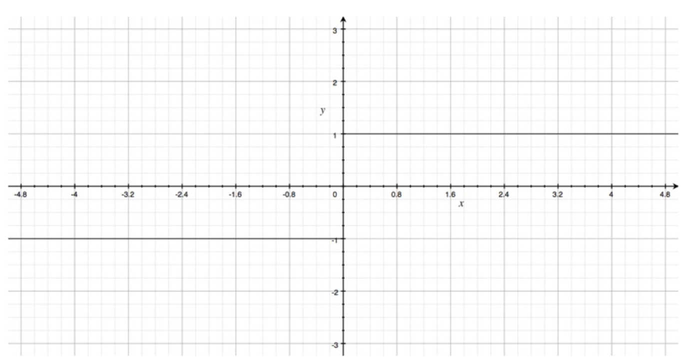
The ceiling function derivative is 0 for the non-integer range—and by the properties of the parabolin function range is fractional or non-integer values.

The floor function derivative is 0 for the non-integer range—and by the properties of the parabolin function range is fractional or non-integer values.

The derivative is simply the sum of the derivatives of the ceiling function and floor function.

2.2.7 Graph of Sum of Ceiling and Floor of Parabolin Function

The graph of the sum of the ceiling and floor of the parabolin function is:



Plot of Sum of Ceiling and Floor of Parabolin Function where slack constant *c* = 1.

The plot of the sum of the ceiling and floor of the parabolin function is a function that is constant with the values -1, 0, +1.

2.2.8 Derivative of Magnum Function

The magnitude of a number function, or magnum function derivative is expressed as:

$$mag(x) = \frac{dy}{dx}\left(\left\lceil\frac{x}{x^2+c}\right\rceil \quad + \quad \left\lfloor\frac{x}{x^2+c}\right\rfloor\right) \tag{2.2.8.1}$$

The derivative is the sum of the two derivatives which is:

$$mag(x) = \frac{dy}{dx}\left(\left\lceil\frac{x}{x^2+c}\right\rceil\right) + \frac{dy}{dx}\left(\left\lfloor\frac{x}{x^2+c}\right\rfloor\right) \tag{2.2.8.2}$$

The parabolin function has the property:

$$0 \leq \quad \frac{x}{x^2+c} \quad < 1 \tag{2.2.8.3}$$

More specifically the real numeric value is always a non-integer. Now consider the derivative of the ceiling and floor function in the magnum function. The respective derivatives for each are:

$$\frac{dy}{dx}\left(\left\lceil\frac{x}{x^2+c}\right\rceil\right) = 0 \qquad \frac{dy}{dx}\left(\left\lfloor\frac{x}{x^2+c}\right\rfloor\right) = 0 \tag{2.2.8.4}$$

The derivatives for both are 0. Thus the derivative of the magnum function is:

$$\frac{dy}{dx}mag(x) = \frac{dy}{dx}\left(\left\lceil\frac{x}{x^2+c}\right\rceil\right) + \frac{dy}{dx}\left(\left\lfloor\frac{x}{x^2+c}\right\rfloor\right) \tag{2.2.8.5}$$

Since the parabolin function is always non-integer, the derivatives of the ceiling and floor function are 0. Hence the derivative for the magnum function is:

$$\frac{dy}{dx}mag(x) = 0 \tag{2.2.8.6}$$

Since the range of the magnum function is a set of three numeric value constants, or three constant lines, the derivative for each constant line is then 0. Thus is logically follows that the derivative of the magnum function is 0.

2.3 Cogent Value Function

The new cogent value function is the mathematical equivalent of the absolute value function, and is defined in an overall similar nature as the absolute value function. But the cogent value function is *not* the absolute value function mathematically.

2.3.1 Cogent Value Function Definition

The definition of the cogent value function is simply the product of the magnitude of the number, or the magnum function and the number x. The mathematical definition is:

$$cog(x) = mag(x) \cdot x \tag{2.3.1.1}$$

Now substituting in the parabolin mathematical function and the sum of the ceiling and floor for the magnum function is then:

$$cog(x) = (\lceil parabolin(x) \rceil + \lfloor parabolin(x) \rfloor) \cdot x \tag{2.3.1.2}$$

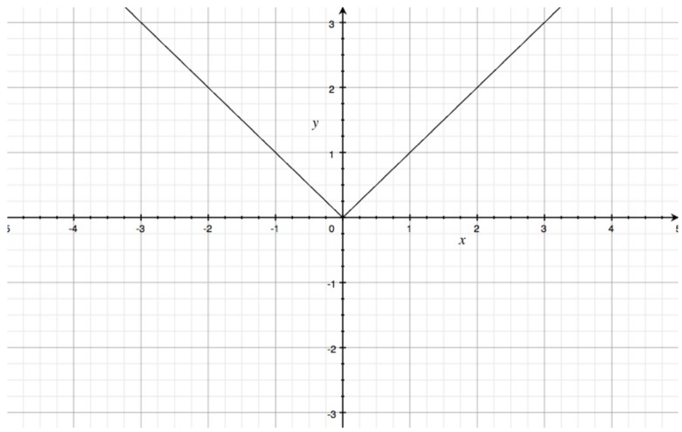
Lastly substituting the polynomial expression for the parabolin function, the cogent value function is:

$$cog(x) = \left(\left\lceil \frac{x}{x^2 + c} \right\rceil + \left\lfloor \frac{x}{x^2 + c} \right\rfloor \right) \cdot x \tag{2.3.1.3}$$

The cogent value function is the product of the magnum function and the numeric value x.

2.3.2 Graph of Cogent Value Function

The graph of the cogent value function is:



Plot of the Cogent Value Function where slack constant c = 1.

The plot of the cogent value function is exactly similar to that of the absolute value function. This illustrates that both functions are mathematically equivalent in terms of the functional mapping from domain to range.

3. Analysis

The analysis of the cogent value function, and the magnitude of a number function or magnum function is to simply use the analysis often found with the absolute value function in mathematics texts.

The analysis uses the limit and definition of a derivative to demonstrate there is a derivative at zero, and then go further, that a limit exists approaching zero from both the left and the right. Both demonstrate that the cogent value function has a derivative at zero—unlike the absolute value function.

3.1 Derivative of the Cogent Value Function

The derivative of the cogent value function is the magnitude of a number of magnum function. Using the definition of the derivative [Apos 1967] using the limit, and using *f(x) = cog(x)* is then:

$$f'(x) = \lim_{h \to x} \frac{f(0+h) - f(0)}{h} \quad \textbf{(3.1.1)}$$

Substituting the cogent value function:

$$f'(x) = \lim_{h \to x} \frac{cog(0+h) - cog(0)}{h} \quad \textbf{(3.1.2)}$$

The cogent value function is defined as:

$$f(x) = cog(x) \equiv x \cdot mag(x) \quad \textbf{(3.1.3)}$$

Using the definition of a derivative, as $h \to x$:

$$f'(x) = \lim_{h \to x} \frac{(0+h) \cdot mag(0+h) - 0 \cdot mag(0)}{h} \quad \textbf{(3.1.4)}$$

Simplify:

$$f'(x) = \lim_{h \to x} \frac{(h) \cdot mag(h) - 0 \cdot mag(0)}{h} \quad \textbf{(3.1.5)}$$

Simplify further:

$$f'(x) = \lim_{h \to x} \frac{h \cdot mag(h) - 0}{h} \tag{3.1.6}$$

Simplify for the limit:

$$f'(x) = \lim_{h \to x} \frac{h \cdot mag(h)}{h} \tag{3.1.7}$$

Thus the limit as $h \to x$, which is the derivative of the cogent value function:

$$f'(x) = \lim_{h \to x} mag(h) \tag{3.1.8}$$

The derivative is the magnum function:

$$f'(x) = mag(x) \tag{3.1.9}$$

Thus the derivative of the cogent value function *cog(x)* is the magnitude of a number or magnum function *mag(x)*.

3.2 Limit of the Cogent Function at Zero

Derivative f' of cogent function or $f(x) = cog(x)$. Take the derivative using the definition with a limit [Apos 1967] is:

$$f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} \tag{3.2.1}$$

Simplify:

$$f'(0) = \lim_{h \to 0} \frac{cog(0 + h) - cog(0)}{h} \tag{3.2.2}$$

Simplifying further:

$$f'(0) = \lim_{h \to 0} \frac{cog(h) - cog(0)}{h} \tag{3.2.3}$$

Substituting the derivative:

$$f'(0) = \lim_{h \to 0} \frac{h \cdot mag(h) - 0 \cdot mag(0)}{h} \tag{3.2.4}$$

Simplify:

$$f'(0) = \lim_{h \to 0} \frac{h \cdot mag(h)}{h} \tag{3.2.5}$$

Simplifying further:

$$f'(0) = \lim_{h \to 0} mag(h) \tag{3.2.6}$$

Solve for the derivative at 0 for $f'(0)$ is:

$$f'(0) = mag(0) \tag{3.2.7}$$

The solution is then:

$$f'(0) = 0 \tag{3.2.8}$$

The limit is 0, thus exists. Therefore, the derivative exists for the cogent function $f'(0) = 0$.

3.3 Limit Approaching Zero from Left and Right

One method of evaluating the absolute value function, which is indeterminate at 0, is to evaluate the limit approaching 0, from both the left and the right for the derivative. This method allows determining if the absolute value function is differentiable, or has a derivative at 0. Stewart [Stew 2008] states:

> "...a two-sided limit exists if and only if both of the one-sided limits exist and are equal."

3.3.1 Limit Approaching Zero from Left

Expressing the limit approach zero from the left using the definition of the derivative with a limit:

$$\lim_{h\to 0^-} \frac{-h\ \cdot mag(h) - 0 \cdot mag(0)}{h} \tag{3.3.1.1}$$

Substituting -1 for the numeric value approached from the left:

$$\lim_{h\to 0^-} \frac{-(-1)\ \cdot mag(-1) - 0}{-1} \tag{3.3.1.2}$$

Simplifying:

$$\lim_{h\to 0^-} \frac{+1\ \cdot mag(-1)}{-1} \tag{3.3.1.3}$$

Evaluating the magnum function:

$$\lim_{h\to 0^-} \frac{+1\ \cdot -1}{-1} \tag{3.3.1.4}$$

Finally simplifying:

$$\lim_{h\to 0^-} \frac{-1}{-1} = +1 \tag{3.3.1.5}$$

The limit approach from the left for the magnum function is +1.

3.3.2 Limit Approaching Zero from Right

Expressing the limit approach zero from the right using the definition of the derivative with a limit:

$$\lim_{h\to 0^+} \frac{+h\ \cdot mag(h) - 0 \cdot mag(0)}{h} \tag{3.3.2.1}$$

Substituting +1 for the numeric value approached from the right:

$$\lim_{h\to 0^+} \frac{+1\ \cdot mag(+1) - 0}{+1} \tag{3.3.2.2}$$

Simplifying:

$$\lim_{h\to 0^+} \frac{+1\ \cdot mag(+1)}{+1} \tag{3.3.2.3}$$

Evaluating the magnum function:

$$\lim_{h\to 0^+} \frac{+1\ \cdot +1}{+1} \tag{3.3.2.4}$$

Finally simplifying:

$$\lim_{h\to 0^+} \frac{+1}{+1} = +1 \tag{3.3.2.5}$$

3.3.3 Limit Approaching Zero from Left and Right Converges

The two one-sided limits approaching from the left and right converge to the same numeric value of +1. Thus for the limit $h \to 0$ is:

$$\lim_{h\to 0} mag(h) \text{ exists.} \tag{3.3.3.1}$$

Thus, the derivative exists for $f'(0)$ the cogent value function $f(x) = cog(x)$ at 0.

4. Application

The application of the parabolin, ceiling of a parabolin, floor of a parabolin, magnum, and cogent value function is to real example numeric values of *x* both negative, positive, and zero. The table shows the application for the numeric value of *x*, and the intermediate forms, resulting in the cogent value function or *cog(x)*.

x	parabolin	⌈parabolin⌉	⌊parabolin⌋	mag(x)	cog(x)
0	0	0	0	0	0
0.5	0.4	1	0	1	0.5
1	0.5	1	0	1	1
2.71828	0.324027303	1	0	1	2.71828
-0.5	-0.4	0	-1	-1	0.5
-1	-0.5	0	-1	-1	1
-1.11111	-0.497237621	0	-1	-1	1.11111

Table of Intermediate Values for Calculating the Magnum and Cogent Value Functions

The table illustrates that the values for the parabolin function are always between -1 and +1, and only 0 at 0. The components of the magnum function, the ceiling and floor of the parabolin function, are either 0, +1, and -1. The cogent value function then calculates the value using the magnum function and the original real numeric value *x*.

5. Future Work

The new definition, as the cogent value function, for the absolute value function is the basis for continued research. Potential areas of future work are:

1. Slack constant value
2. Application of cogent value function
3. Alternative definitions for absolute value

These three possibilities are not exhaustive, and undoubtedly there are other areas of future work to build upon in regard to the absolute value function, or the cogent value and magnum functions.

5.1 Slack Constant Value

Future work around the cogent value function involves the slack constant, and variations in its value. For the slack constant *c*, how do values impact the parabolin function:

1. Fractional Value – when *c* is a real value in the range of $0 < c < 1$.
2. Integral Value – when *c* is a real value in the range of $1 < c < \infty$.

5.2 Cogent Value Function Application

Another area for future work is applying the cogent value function in areas where the existing absolute value function is used. The application of the cogent value function evaluates the mathematical properties in contrast to the absolute value function.

5.3 Alternative Definitions

With the new definitions of the absolute value and signum function, is it possible that there exist, are other definitions with other mathematical properties? This question is an open possibility for future research into alternative definitions of both functions.

6. Conclusion

The parabolin function, cogent value function, and magnum function are both old and new mathematical concepts. Old concepts such as absolute value, sign of a number, and a polynomial expression of a line, parabola, and constant. But new concepts by means of a different mathematical definition with different mathematical properties.

6.1 Cogent Value Function Derivative at Zero

The most important contradistinction for the cogent value function with the absolute value function is that the derivative of the cogent value function is 0 at 0.

The discussion of the existing absolute value function is not applicable for the cogent value function. Guichard [Guic 2009] writes:

> Discuss the derivative of the absolute value function $y = f(x) = |x|$.
>
> If x is positive, then this is the function $y = x$, whose derivative is the constant 1. (Recall that when $y = f(x) = mx+b$, the derivative is the slope m.) If x is negative, then we're dealing with the function $y = -x$, whose derivative is the constant -1. If $x = 0$, then the function has a corner, i.e., there is no tangent line. A tangent line would have to point in the direction of the curve—but there are *two* directions of the curve that come together at the origin. We can summarize this as

$$y' = \begin{cases} 1 & if\ x > 0 \\ -1 & if\ x < 0 \\ undefined & if\ x = 0 \end{cases}$$

This is not correct for the case if $x = 0$, but more simply, the derivative at 0 exists for the cogent value function which is 0—thus is continuous at 0. [Lars 2010][Thom 1996]

6.2 Parabolin Function

The parabolin function is an ordinary polynomial equation involving a line, parabola, and slack constant. The parabolin function is the basis with the ceiling and floor function to create the magnitude of a number function—the magnum function.

But yet the equation for the parabolin function is found in many textbooks of mathematics. For the cogent value function, the parabolin function is the core basis of the magnitude of a number function.

6.3 New Functions for Foundational Mathematics

There two new definitions of old foundational mathematical functions. The two new definitions and the old foundational mathematical functions:

1. Magnitude of a number – magnum function for signum function.
2. Cogent value function – cogent function for absolute value function.

From the mathematical mapping of the domain into the range, the two functions are equivalent. However, the formal mathematical definitions are quite different, and thus the mathematical properties of both functions.

6.4 Mutability of Mathematical Concepts

The cogent value function, and the magnum function illustrate that older, establish mathematical concepts are not immutable—reconsider old, existing mathematical knowledge through a new definition of the same concept.

There is an important distinction between the formal definition in terms of operation over the domain and range, and the definition in terms of a mathematical formalism. The formal definition changes the mathematical properties, while the operation over the mapping over the domain and range remains unchanged. The same thing conceptually, but the concept has new mathematical properties by means of the new definition.

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