

On the possible mathematical connections between some equations of various topics concerning the Dilaton value, the D-Brane, the Bouncing Cosmology and some sectors of Number Theory (Riemann's functions of S. Ramanujan and Rogers-Ramanujan continued fractions).

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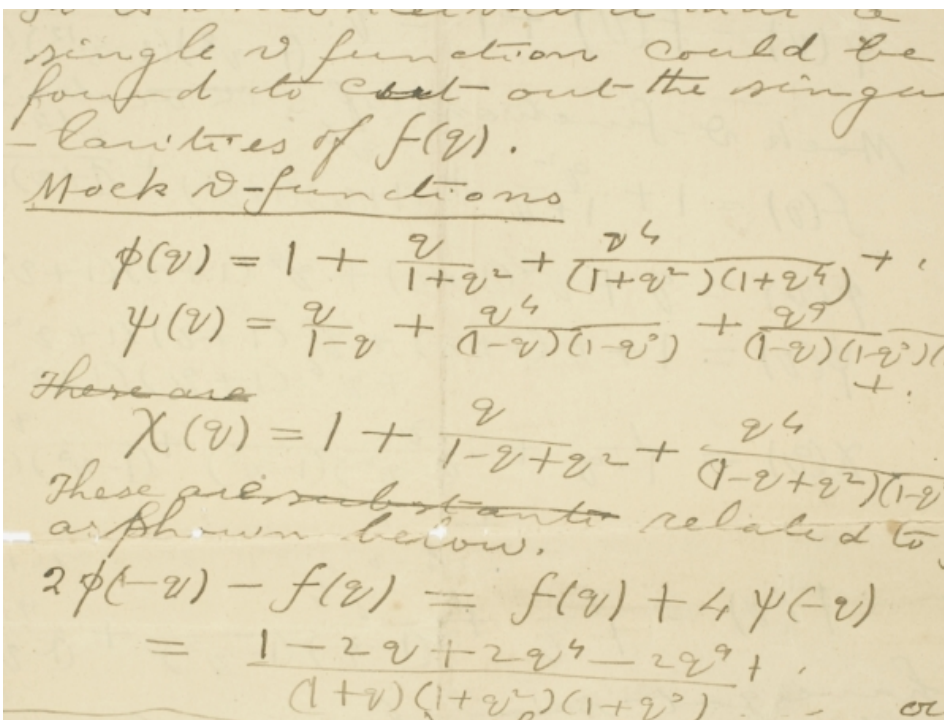
Abstract

In this research thesis, we have described some new mathematical connections between some equations of various topics concerning the Dilaton value, the Bouncing Cosmology and some sectors of Number Theory (Riemann's functions of S. Ramanujan and Rogers-Ramanujan continued fractions).

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<https://www.pinterest.it/pin/444237950734694507/?lp=true>



<https://royalsociety.org/science-events-and-lectures/2018/10/srinivasa-ramanujan/>

From:

New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$ – *Srinivasa Ramanujan*
 Quarterly Journal of Mathematics, XLVI, 1915, 253 – 260

Suppose now that $s = \sigma + it$, where $0 < \sigma < 1$.

t is real

$$n^{-\frac{3}{2}} \int_0^{\infty} v^{-\frac{1}{2}s} dv \int_0^{\infty} x e^{-\pi v x^2/n} \left(\frac{1}{e^{2\pi x} - 1} - \frac{1}{2\pi x} \right) dx$$

$$= -\frac{n^{-\frac{1}{2}(s+1)}}{4\pi\sqrt{\pi}} \Gamma\left(-\frac{s}{2}\right) \Gamma\left(\frac{s-1}{2}\right) \xi(s)$$

And

$$\xi(s) = (s-1)\Gamma\left(1 + \frac{1}{2}s\right)\pi^{-\frac{1}{2}s}\zeta(s)$$

For $\sigma = 0.5$ and $t = 1/4 = 0.25$, we obtain:

$$(((0.5+i*0.25)-1)) \text{ gamma } \left(\left(1 + \frac{1}{2} * (0.5+i*0.25)\right) \right) * \text{Pi}^{(-0.5 * (0.5+i*0.25))} * \text{zeta}(0.5+i*0.25)$$

Input:

$$((0.5 + i \times 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i \times 0.25)\right) \pi^{-0.5(0.5+i \times 0.25)} \zeta(0.5 + i \times 0.25)$$

$\Gamma(x)$ is the gamma function
 $\zeta(s)$ is the Riemann zeta function
 i is the imaginary unit

Result:

0.496403...

(using the principal branch of the logarithm for complex exponentiation)

0.496403...

Alternate form:

0.496403

Alternative representations:

$$((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) =$$

$$(-0.5 + 0.25 i) \exp\left(-\log G\left(1 + \frac{1}{2} (0.5 + 0.25 i)\right) + \log G\left(2 + \frac{1}{2} (0.5 + 0.25 i)\right)\right)$$

$$\pi^{-0.5(0.5+0.25 i)} \zeta(0.5 + 0.25 i, 1)$$

•

$$((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) =$$

$$(-0.5 + 0.25 i) (1) \frac{1}{2} (0.5+0.25 i) \pi^{-0.5(0.5+0.25 i)} \zeta(0.5 + 0.25 i, 1)$$

•

$$((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) =$$

$$(-0.5 + 0.25 i) e^{\log \Gamma(1+1/2 (0.5+0.25 i))} \pi^{-0.5(0.5+0.25 i)} \zeta(0.5 + 0.25 i, 1)$$

$\log G(z)$ gives the logarithm of the Barnes G-function
 $\zeta(s, a)$ is the generalized Riemann zeta function
 $(a)_n$ is the Pochhammer symbol (rising factorial)
 $\log \Gamma(x)$ is the logarithm of the gamma function

Series representations:

$$((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) =$$

$$\pi^{-0.25-0.125 i} \left(\sum_{n=0}^{\infty} \frac{\sum_{k=0}^n (-1)^k (1+k)^{0.5-0.25 i} \binom{n}{k}}{1+n} \right) \sum_{k=0}^{\infty} \frac{(1.25 + 0.125 i - z_0)^k \Gamma^{(k)}(z_0)}{k!}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

•

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) = \\
& - \left(\left(0.25 e^{0.173287i} \pi^{-0.25-0.125i} \right. \right. \\
& \quad \left. \left. \begin{aligned} & \left(-2 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} k_1^{-0.5-0.25i} (1.25 + 0.125i - z_0)^{k_2} \Gamma^{(k_2)}(z_0)}{k_2!} + \right. \right. \\ & \left. \left. i \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} k_1^{-0.5-0.25i} (1.25 + 0.125i - z_0)^{k_2} \Gamma^{(k_2)}(z_0)}{k_2!} \right) \right) / \\
& \quad \left. (-1.41421 + e^{0.173287i}) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) = 0.25 \\
& \pi^{-0.25-0.125i} \left(-2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(0.25 + 0.125i)^{k_1} (0.5 + 0.25i - s_0)^{k_2} \Gamma^{(k_1)}(1) \zeta^{(k_2)}(s_0)}{k_1! k_2!} + \right. \\
& \quad \left. i \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(0.25 + 0.125i)^{k_1} (0.5 + 0.25i - s_0)^{k_2} \Gamma^{(k_1)}(1) \zeta^{(k_2)}(s_0)}{k_1! k_2!} \right) \text{ for } s_0 \neq 1
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

\mathbb{Z} is the set of integers

Integral representations:

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) = \\
& 0.5 \pi^{-0.25-0.125i} \Gamma(1.25 + 0.125i) \\
& \quad \left(1.5 + 0.25i + -2 + i \int_0^{\infty} \frac{(1+t^2)^{-0.25-0.125i} \sin((0.5+0.25i)\tan^{-1}(t))}{-1+e^{2\pi t}} dt \right)
\end{aligned}$$

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) = \\
& \frac{0.5 (-2 + i) \pi^{0.75-0.125i} \mathcal{A} \zeta(0.5 + 0.25i)}{\oint_L e^t t^{-1.25-0.125i} dt}
\end{aligned}$$

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) = \\
& \frac{\pi^{0.75-0.125i} \mathcal{A} \left(1.5 + 0.25i + -2 + i \int_0^{\infty} \frac{(1+t^2)^{-0.25-0.125i} \sin((0.5+0.25i)\tan^{-1}(t))}{-1+e^{2\pi t}} dt \right)}{\oint_L e^t t^{-1.25-0.125i} dt}
\end{aligned}$$

$\tan^{-1}(x)$ is the inverse tangent function

$$= -\frac{n^{-\frac{1}{2}(s+1)}}{4\pi\sqrt{\pi}}\Gamma\left(-\frac{s}{2}\right)\Gamma\left(\frac{s-1}{2}\right)\xi(s)$$

$s = 0.5+0.25*i$; $n = 2$; $\xi(s) = 0.496403$

$[(((((-2^{(-0.5)(0.5+0.25*i+1)}) / (((4\text{Pi}*\text{sqrt}(\text{Pi})))))))) * \text{gamma} (-(0.5+0.25*i)/2)* \text{gamma} ((0.5+0.25*i-1)/2) * 0.496403$

Input interpretation:

$$-\frac{2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}}\Gamma\left(-\frac{1}{2}\times(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)\times 0.496403$$

$\Gamma(x)$ is the gamma function
 i is the imaginary unit

Result:

$-0.243975... + 0.0211919... i$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 0.244894$ (radius), $\theta = 175.036^\circ$ (angle)

Alternative representations:

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} = \frac{0.496403\left(-1+\frac{1}{2}(-0.5-0.25i)\right)!\left(-1+\frac{1}{2}(-0.5+0.25i)\right)!2^{-0.5(1.5+0.25i)}}{4\pi\sqrt{\pi}}$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} = -\frac{1}{4\pi\sqrt{\pi}} \frac{0.496403 \times 2^{-0.5(1.5+0.25i)} \exp\left(\log G\left(1+\frac{1}{2}(-0.5-0.25i)\right) - \log G\left(\frac{1}{2}(-0.5-0.25i)\right)\right) \exp\left(\log G\left(1+\frac{1}{2}(-0.5+0.25i)\right) - \log G\left(\frac{1}{2}(-0.5+0.25i)\right)\right)}{1}$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)^{2^{-0.5(0.5+0.25i+1)}}}{4\pi\sqrt{\pi}} =$$

$$\frac{0.496403 G\left(1+\frac{1}{2}(-0.5-0.25i)\right)G\left(1+\frac{1}{2}(-0.5+0.25i)\right)2^{-0.5(1.5+0.25i)}}{G\left(\frac{1}{2}(-0.5-0.25i)\right)G\left(\frac{1}{2}(-0.5+0.25i)\right)(4\pi\sqrt{\pi})}$$

$n!$ is the factorial function

$\log G(z)$ gives the logarithm of the Barnes G-function

$G(z)$ is the Barnes G-function

Series representations:

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)^{2^{-0.5(0.5+0.25i+1)}}}{4\pi\sqrt{\pi}} =$$

$$\frac{0.0737907 e^{-0.0866434i} \Gamma(-0.25-0.125i) \Gamma(-0.25+0.125i)}{\pi \exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)^{2^{-0.5(0.5+0.25i+1)}}}{4\pi\sqrt{\pi}} =$$

$$\frac{4.72261 e^{-0.0866434i} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-0.25-0.125i)^{k_1} (-0.25+0.125i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!}}{(-2+i)(2+i)\pi\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)^{2^{-0.5(0.5+0.25i+1)}}}{4\pi\sqrt{\pi}} =$$

$$-\left(\left(0.0737907 e^{-0.0866434i} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-0.25-0.125i-z_0)^{k_1} (-0.25+0.125i-z_0)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!}\right) \right) /$$

$$\left(\pi\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}\right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

$\binom{n}{m}$ is the binomial coefficient

\mathbb{Z} is the set of integers

Integral representations:

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} =$$

$$-\frac{1}{\pi\sqrt{\pi}}0.0737907e^{-0.0866434i}\csc((-0.125-0.0625i)\pi)$$

$$\csc((-0.125+0.0625i)\pi)\left(\int_0^\infty t^{-1.25-0.125i}\sin(t)dt\right)\int_0^\infty t^{-1.25+0.125i}\sin(t)dt$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} =$$

$$-\frac{1}{\pi\sqrt{\pi}}0.0737907e^{-0.0866434i}\left(\int_0^\infty e^{-t}t^{-1.25-0.125i}\left(1-e^t\sum_{k=0}^n\frac{(-t)^k}{k!}\right)dt\right)$$

$$\int_0^\infty e^{-t}t^{-1.25+0.125i}\left(1-e^t\sum_{k=0}^n\frac{(-t)^k}{k!}\right)dt \text{ for } (n \in \mathbb{Z} \text{ and } 0 \leq n < 0.25)$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}(0.5+0.25i)\right)\Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right)0.496403\right)(-1)2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} =$$

$$\frac{0.295163e^{-0.0866434i}\pi\mathcal{A}^2}{\sqrt{\pi}\int_L^\infty e^t t^{0.25+0.125i} dt \int_L^\infty e^t t^{0.25-0.125i} dt}$$

csc(x) is the cosecant function

$$6.58 * (((((((((-2^((-0.5)(0.5+0.25*i+1)) / (((4*Pi*sqrt(Pi)))))))))) * gamma (-$$

$$(0.5+0.25*i)/2) * gamma ((0.5+0.25*i-1)/2) * 0.496403))))))$$

Input interpretation:

$$6.58 \left(-\frac{2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} \Gamma\left(-\frac{1}{2}(0.5+0.25i)\right) \Gamma\left(\frac{1}{2}(0.5+0.25i-1)\right) \times 0.496403 \right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$-1.60536... +$$

$$0.139443... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 1.6114 \text{ (radius), } \theta = 175.036^\circ \text{ (angle)}$$

1.6114

Alternative representations:

$$\frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25 i)\right) \left(-2^{-0.5(0.5+0.25 i+1)} \Gamma\left(\frac{1}{2}(0.5 + 0.25 i - 1)\right) 0.496403\right)}{4 \pi \sqrt{\pi}} = \frac{3.26633 \left(-1 + \frac{1}{2}(-0.5 - 0.25 i)\right)! \left(-1 + \frac{1}{2}(-0.5 + 0.25 i)\right)! 2^{-0.5(1.5+0.25 i)}}{4 \pi \sqrt{\pi}}$$

$$\frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25 i)\right) \left(-2^{-0.5(0.5+0.25 i+1)} \Gamma\left(\frac{1}{2}(0.5 + 0.25 i - 1)\right) 0.496403\right)}{4 \pi \sqrt{\pi}} = -\frac{1}{4 \pi \sqrt{\pi}} \frac{3.26633 \times 2^{-0.5(1.5+0.25 i)} \exp\left(\log G\left(1 + \frac{1}{2}(-0.5 - 0.25 i)\right) - \log G\left(\frac{1}{2}(-0.5 - 0.25 i)\right)\right) \exp\left(\log G\left(1 + \frac{1}{2}(-0.5 + 0.25 i)\right) - \log G\left(\frac{1}{2}(-0.5 + 0.25 i)\right)\right)}{1}$$

$$\frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25 i)\right) \left(-2^{-0.5(0.5+0.25 i+1)} \Gamma\left(\frac{1}{2}(0.5 + 0.25 i - 1)\right) 0.496403\right)}{4 \pi \sqrt{\pi}} = \frac{3.26633 G\left(1 + \frac{1}{2}(-0.5 - 0.25 i)\right) G\left(1 + \frac{1}{2}(-0.5 + 0.25 i)\right) 2^{-0.5(1.5+0.25 i)}}{G\left(\frac{1}{2}(-0.5 - 0.25 i)\right) G\left(\frac{1}{2}(-0.5 + 0.25 i)\right) (4 \pi \sqrt{\pi})}$$

$n!$ is the factorial function
 $\log G(z)$ gives the logarithm of the Barnes G-function
 $G(z)$ is the Barnes G-function

Series representations:

$$\frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25 i)\right) \left(-2^{-0.5(0.5+0.25 i+1)} \Gamma\left(\frac{1}{2}(0.5 + 0.25 i - 1)\right) 0.496403\right)}{4 \pi \sqrt{\pi}} = \frac{0.485543 e^{-0.0866434 i} \Gamma(-0.25 - 0.125 i) \Gamma(-0.25 + 0.125 i)}{\pi \exp\left(\pi \mathcal{A}\left[\frac{\text{alg}(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25 i)\right) \left(-2^{-0.5(0.5+0.25 i+1)} \Gamma\left(\frac{1}{2}(0.5 + 0.25 i - 1)\right) 0.496403\right)}{4 \pi \sqrt{\pi}} = \frac{31.0748 e^{-0.0866434 i} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-0.25-0.125 i)^{k_1} (-0.25+0.125 i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!}}{(-2+i)(2+i) \pi \sqrt{-1+i} \sum_{k=0}^{\infty} (-1+i)^{-k} \left(\frac{1}{2}\right)_k}$$

$$\begin{aligned}
& \frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25i)\right) \left(-2^{-0.5(0.5+0.25i+1)}\right) \Gamma\left(\frac{1}{2}(0.5 + 0.25i - 1)\right) 0.496403}{4\pi\sqrt{\pi}} = \\
& - \left(\left(0.485543 e^{-0.0866434i} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-0.25 - 0.125i - z_0)^{k_1} (-0.25 + 0.125i - z_0)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} \right) / \right. \\
& \left. \left(\pi \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} \right) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

$\binom{n}{m}$ is the binomial coefficient

\mathbb{Z} is the set of integers

Integral representations:

$$\begin{aligned}
& \frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25i)\right) \left(-2^{-0.5(0.5+0.25i+1)}\right) \Gamma\left(\frac{1}{2}(0.5 + 0.25i - 1)\right) 0.496403}{4\pi\sqrt{\pi}} = \\
& - \frac{1}{\pi\sqrt{\pi}} 0.485543 e^{-0.0866434i} \csc((-0.125 - 0.0625i)\pi) \\
& \csc((-0.125 + 0.0625i)\pi) \left(\int_0^{\infty} t^{-1.25-0.125i} \sin(t) dt \right) \int_0^{\infty} t^{-1.25+0.125i} \sin(t) dt
\end{aligned}$$

$$\begin{aligned}
& \frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25i)\right) \left(-2^{-0.5(0.5+0.25i+1)}\right) \Gamma\left(\frac{1}{2}(0.5 + 0.25i - 1)\right) 0.496403}{4\pi\sqrt{\pi}} = \\
& - \frac{1}{\pi\sqrt{\pi}} 0.485543 e^{-0.0866434i} \left(\int_0^{\infty} e^{-t} t^{-1.25-0.125i} \left(1 - e^t \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \\
& \int_0^{\infty} e^{-t} t^{-1.25+0.125i} \left(1 - e^t \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \text{ for } (n \in \mathbb{Z} \text{ and } 0 \leq n < 0.25)
\end{aligned}$$

$$\begin{aligned}
& \frac{6.58 \Gamma\left(-\frac{1}{2}(0.5 + 0.25i)\right) \left(-2^{-0.5(0.5+0.25i+1)}\right) \Gamma\left(\frac{1}{2}(0.5 + 0.25i - 1)\right) 0.496403}{4\pi\sqrt{\pi}} = \\
& \frac{1.94217 e^{-0.0866434i} \pi \mathcal{A}^2}{\sqrt{\pi} \int_L^{\infty} e^t t^{0.25+0.125i} dt \int_L^{\infty} e^t t^{0.25-0.125i} dt}
\end{aligned}$$

$\csc(x)$ is the cosecant function

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti arXiv:1711.11494v1 [hep-th] 30 Nov 2017

of a finite new interaction that couples to a massless propagator of zero momentum. In some respect, the phenomenon is thus simpler than the quantum fluctuations that were present in the heterotic $SO(16) \times SO(16)$, which makes coming to terms with it ever more urgent. The new contribution to the $USp(32)$ model is a runaway dilaton potential, which we shall often refer to as a “dilaton tadpole”. The resulting “string frame” effective action reads

$$\mathcal{S} = \frac{1}{2k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] \right. \\ \left. \frac{1}{2(p+2)!} e^{-2\beta_S \phi} \mathcal{H}_{p+2}^2 - T e^{\gamma_S \phi} \right\}, \quad (2.23)$$

with $p = 1$, $\beta_S = 0$ and $\gamma_S = -1$. A term similar to the last one, also with a positive overall coefficient, would be induced in the heterotic $SO(16) \times SO(16)$ model at the torus level, and therefore with $\gamma_S = 0$, while the NS nature of its two-form field ($p = 1$, again), would imply that $\beta_S = 1$. The Weyl rescaling $G \rightarrow g e^{\phi/2}$ turns the action (2.23) into its Einstein-frame form

$$\mathcal{S} = \frac{1}{2k_{10}^2} \int d^{10}x \sqrt{-g} \left\{ -R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2(p+2)!} e^{-2\beta_E^{(p)} \phi} \mathcal{H}_{p+2}^2 - T e^{\gamma_E \phi} \right\}, \quad (2.24)$$

where now $\gamma_E = \frac{3}{2}$ for the orientifold model and $\gamma_E = \frac{5}{2}$ for the heterotic $SO(16) \times SO(16)$ model, while $\beta_E = -\frac{1}{2}$ for the orientifold model and $\beta_E = \frac{1}{2}$ for the heterotic $SO(16) \times SO(16)$ model.

Dilaton potential or “Dilaton tadpole”

0.989117352243 = ϕ

calculating the 128th root, we obtain:

$$\left[\left(\frac{(-2)^{-0.5(0.5+0.25i+1)}}{(4\pi \sqrt{\pi})} \right) \cdot \Gamma(-0.5+0.25i/2) \cdot \Gamma(0.5+0.25i-1/2) \cdot 0.496403 \right]^{1/128}$$

Input interpretation:

$$\sqrt[128]{-\frac{2^{-0.5(0.5+0.25i+1)}}{4\pi\sqrt{\pi}} \Gamma\left(-\frac{1}{2} \times (0.5 + 0.25i)\right) \Gamma\left(\frac{1}{2} (0.5 + 0.25i - 1)\right) \times 0.496403}$$

$\Gamma(x)$ is the gamma function
 i is the imaginary unit

Result:

0.9887868... +
 0.02360365... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 0.989069$ (radius), $\theta = 1.36747^\circ$ (angle)

0.989069

We have that, for $x = 2$:

$$\frac{1}{2\pi x} - \frac{1}{2} + \frac{\pi x}{6} - \frac{\pi^3 x^3}{90} + \frac{\pi^5 x^5}{945} - \frac{\pi^7 x^7}{9450} + \frac{\pi^9 x^9}{93555} - \dots,$$

$$1/(4\pi) - 1/2 + 2\pi/6 - (\pi^3 \cdot 8)/90 + (\pi^5 \cdot 32)/945 - (\pi^7 \cdot 128)/9450 + (\pi^9 \cdot 512)/93555$$

Input:

$$\frac{1}{4\pi} - \frac{1}{2} + 2 \times \frac{\pi}{6} - \frac{1}{90} (\pi^3 \times 8) + \frac{1}{945} (\pi^5 \times 32) - \frac{\pi^7 \times 128}{9450} + \frac{\pi^9 \times 512}{93555}$$

Result:

$$-\frac{1}{2} + \frac{1}{4\pi} + \frac{\pi}{3} - \frac{4\pi^3}{45} + \frac{32\pi^5}{945} - \frac{64\pi^7}{4725} + \frac{512\pi^9}{93555}$$

Decimal approximation:

130.4601932795124433848142962928986160412197953120991357150...

130.4601932...

Property:

$$-\frac{1}{2} + \frac{1}{4\pi} + \frac{\pi}{3} - \frac{4\pi^3}{45} + \frac{32\pi^5}{945} - \frac{64\pi^7}{4725} + \frac{512\pi^9}{93555} \text{ is a transcendental number}$$

Alternate form:

$$\frac{467775 - 935550\pi + 623700\pi^2 - 166320\pi^4 + 63360\pi^6 - 25344\pi^8 + 10240\pi^{10}}{1871100\pi}$$

Alternative representations:

$$\frac{1}{4\pi} - \frac{1}{2} + \frac{2\pi}{6} - \frac{\pi^3}{90} + \frac{\pi^5}{945} - \frac{\pi^7}{9450} + \frac{\pi^9}{93555} = -\frac{1}{2} + \frac{2}{6} \cos^{-1}(-1) - \frac{8}{90} \cos^{-1}(-1)^3 + \frac{32}{945} \cos^{-1}(-1)^5 - \frac{128 \cos^{-1}(-1)^7}{9450} + \frac{512 \cos^{-1}(-1)^9}{93555} + \frac{1}{4 \cos^{-1}(-1)}$$

$$\frac{1}{4\pi} - \frac{1}{2} + \frac{2\pi}{6} - \frac{\pi^3}{90} + \frac{\pi^5}{945} - \frac{\pi^7}{9450} + \frac{\pi^9}{93555} = -\frac{1}{2} + \frac{4E(0)}{6} - \frac{8}{90} (2E(0))^3 + \frac{32}{945} (2E(0))^5 - \frac{128 (2E(0))^7}{9450} + \frac{512 (2E(0))^9}{93555} + \frac{1}{8E(0)}$$

$$\frac{1}{4\pi} - \frac{1}{2} + \frac{2\pi}{6} - \frac{\pi^3}{90} + \frac{\pi^5}{945} - \frac{\pi^7}{9450} + \frac{\pi^9}{93555} = -\frac{1}{2} + \frac{4K(0)}{6} - \frac{8}{90} (2K(0))^3 + \frac{32}{945} (2K(0))^5 - \frac{128 (2K(0))^7}{9450} + \frac{512 (2K(0))^9}{93555} + \frac{1}{8K(0)}$$

$\cos^{-1}(x)$ is the inverse cosine function
 $E(m)$ is the complete elliptic integral of the second kind with parameter $m = k^2$
 $K(m)$ is the complete elliptic integral of the first kind with parameter $m = k^2$

Series representations:

$$\frac{1}{4\pi} - \frac{1}{2} + \frac{2\pi}{6} - \frac{\pi^3}{90} + \frac{\pi^5}{945} - \frac{\pi^7}{9450} + \frac{\pi^9}{93555} = -\frac{1}{2} + \frac{1}{4\pi} + \frac{\pi}{3} + \frac{32\pi^5}{945} - \frac{64\pi^7}{4725} + \frac{512\pi^9}{93555} + \frac{128}{45} \sum_{k=1}^{\infty} \frac{(-1)^k}{(-1+2k)^3}$$

$$\frac{1}{4\pi} - \frac{1}{2} + \frac{2\pi}{6} - \frac{\pi^3}{90} + \frac{\pi^5}{945} - \frac{\pi^7}{9450} + \frac{\pi^9}{93555} = \left(467775 - 3742200 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 9979200 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 - 42577920 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 + 259522560 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^6 - 1660944384 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^8 + 10737418240 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{10} \right) / \left(7484400 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)$$

$$\frac{1}{4\pi} - \frac{1}{2} + \frac{2\pi}{6} - \frac{\pi^3}{90} + \frac{\pi^5}{945} - \frac{\pi^7}{9450} + \frac{\pi^9}{93555} =$$

$$\left(467775 - 935550 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) + \right.$$

$$623700 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 -$$

$$166320 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4 +$$

$$63360 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^6 -$$

$$25344 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^8 +$$

$$10240 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^{10} \Big/$$

$$\left(1871100 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)$$

Now:

$$\xi\left(\frac{1}{2} + \frac{1}{2}it\right) = \Xi\left(\frac{1}{2}t\right),$$

$$t = 0.25 \text{ and } \xi = 0.496403$$

$$\left(\left((0.5 + i \cdot 0.25) - 1 \right) \Gamma\left(1 + \frac{1}{2}(0.5 + i \cdot 0.25)\right) \right) \cdot \pi^{-0.5(0.5 + i \cdot 0.25)} \cdot \zeta(0.5 + i \cdot 0.25) \cdot \left(\frac{1}{2} + \frac{1}{2}i \cdot 0.25 \right)$$

Input:

$$\left((0.5 + i \times 0.25) - 1 \right) \Gamma\left(1 + \frac{1}{2}(0.5 + i \times 0.25)\right)$$

$$\pi^{-0.5(0.5 + i \times 0.25)} \zeta(0.5 + i \times 0.25) \left(\frac{1}{2} + \frac{1}{2}i \times 0.25 \right)$$

$\Gamma(x)$ is the gamma function
 $\zeta(s)$ is the Riemann zeta function
 i is the imaginary unit

Result:

0.248202... +
0.0620504... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 0.25584$ (radius), $\theta = 14.0362^\circ$ (angle)

0.25584

Alternative representations:

$$\begin{aligned} & ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\ & \left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) \\ & \exp\left(-\log G\left(1 + \frac{1}{2} (0.5 + 0.25 i)\right) + \log G\left(2 + \frac{1}{2} (0.5 + 0.25 i)\right)\right) \\ & \pi^{-0.5(0.5+0.25 i)} \zeta(0.5 + 0.25 i, 1) \end{aligned}$$

•

$$\begin{aligned} & ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\ & \left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) (1) \frac{1}{2} (0.5 + 0.25 i) \pi^{-0.5(0.5+0.25 i)} \zeta(0.5 + 0.25 i, 1) \end{aligned}$$

•

$$\begin{aligned} & ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\ & \frac{\left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) G\left(2 + \frac{1}{2} (0.5 + 0.25 i)\right) \pi^{-0.5(0.5+0.25 i)} \zeta(0.5 + 0.25 i, 1)}{G\left(1 + \frac{1}{2} (0.5 + 0.25 i)\right)} \end{aligned}$$

$\log G(z)$ gives the logarithm of the Barnes G-function

$\zeta(s, a)$ is the generalized Riemann zeta function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$G(z)$ is the Barnes G-function

Series representations:

$$\begin{aligned} & ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\ & 0.03125 (4 + i) \pi^{-0.25-0.125 i} \\ & \left(4 + \gamma (-2 + i) + (-2 + i) \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5 + 0.25 i)^k \gamma_k}{k!}\right) \sum_{k=0}^{\infty} \frac{(0.25 + 0.125 i)^k \Gamma^{(k)}(1)}{k!} \end{aligned}$$

•

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\
& 0.03125 (4 + i) \pi^{-0.25-0.125i} \left(4 + \gamma (-2 + i) + (-2 + i) \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5 + 0.25 i)^k \gamma_k}{k!}\right) \\
& \sum_{k=0}^{\infty} \frac{(1.25 + 0.125 i - z_0)^k \Gamma^{(k)}(z_0)}{k!} \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\
& - \left(\left(0.03125 e^{0.173287i} \pi^{-0.25-0.125i} \right. \right. \\
& \quad \left(-8 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} k_1^{-0.5-0.25i} (1.25 + 0.125 i - z_0)^{k_2} \Gamma^{(k_2)}(z_0)}{k_2!} + \right. \\
& \quad \left. 2i \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} k_1^{-0.5-0.25i} (1.25 + 0.125 i - z_0)^{k_2} \Gamma^{(k_2)}(z_0)}{k_2!} + \right. \\
& \quad \left. \left. i^2 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} k_1^{-0.5-0.25i} (1.25 + 0.125 i - z_0)^{k_2} \Gamma^{(k_2)}(z_0)}{k_2!} \right) \right) / \\
& (-1.41421 + e^{0.173287i}) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

$n!$ is the factorial function
 γ_n is the n^{th} Stieltjes constant
 γ is the Euler-Mascheroni constant
 \mathbb{Z} is the set of integers

Integral representations:

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\
& 0.0625 (4 + i) \pi^{-0.25-0.125i} \Gamma(1.25 + 0.125 i) \\
& \left(1.5 + 0.25 i + -2 + i \int_0^{\infty} \frac{(1 + t^2)^{-0.25-0.125i} \sin((0.5 + 0.25 i) \tan^{-1}(t))}{-1 + e^{2\pi t}} dt \right)
\end{aligned}$$

$$\begin{aligned}
& ((0.5 + i 0.25) - 1) \Gamma\left(1 + \frac{1}{2} (0.5 + i 0.25)\right) \pi^{-0.5(0.5+i0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) = \\
& \frac{0.0625 (-2 + i) (4 + i) \pi^{0.75-0.125i} \mathcal{A} \zeta(0.5 + 0.25 i)}{\oint_L e^t t^{-1.25-0.125i} dt}
\end{aligned}$$

1.55950... +
0.389874... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.60749$ (radius), $\theta = 14.0362^\circ$ (angle)

1.60749

Alternative representations:

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) = 2 \left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) \pi \exp\left(-\log G\left(1 + \frac{1}{2}(0.5 + 0.25 i)\right) + \log G\left(2 + \frac{1}{2}(0.5 + 0.25 i)\right)\right) \pi^{-0.5(0.5 + 0.25 i)} \zeta(0.5 + 0.25 i, 1)$$

•

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) = 2 \left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) \pi (1) \frac{1}{2} (0.5 + 0.25 i) \pi^{-0.5(0.5 + 0.25 i)} \zeta(0.5 + 0.25 i, 1)$$

•

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) = \frac{2 \left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) \pi G\left(2 + \frac{1}{2}(0.5 + 0.25 i)\right) \pi^{-0.5(0.5 + 0.25 i)} \zeta(0.5 + 0.25 i, 1)}{G\left(1 + \frac{1}{2}(0.5 + 0.25 i)\right)}$$

$\log G(z)$ gives the logarithm of the Barnes G-function

$\zeta(s, a)$ is the generalized Riemann zeta function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$G(z)$ is the Barnes G-function

Series representations:

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) = 2 \left(\frac{1}{2} + 0.125 i\right) (-0.5 + 0.25 i) \pi^{0.75 - 0.125 i} \left(\gamma + \frac{1}{-0.5 + 0.25 i} + \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5 + 0.25 i)^k \gamma_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(0.25 + 0.125 i)^k \Gamma^{(k)}(1)}{k!}$$

•

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) =$$

$$2 \left(\frac{1}{2} + 0.125 i \right) (-0.5 + 0.25 i) \pi^{0.75 - 0.125 i} \left(\gamma + \frac{1}{-0.5 + 0.25 i} + \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5 + 0.25 i)^k \gamma_k}{k!} \right)$$

$$\sum_{k=0}^{\infty} \frac{(1.25 + 0.125 i - z_0)^k \Gamma^{(k)}(z_0)}{k!} \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) = 0.25 \pi^{-0.125 i}$$

$$\left(4 \pi^{0.75} \left(\sum_{n=0}^{\infty} \frac{\sum_{k=0}^n (-1)^k (1+k)^{0.5-0.25 i} \binom{n}{k}}{1+n} \right) \sum_{k=0}^{\infty} \frac{(1.25 + 0.125 i - z_0)^k \Gamma^{(k)}(z_0)}{k!} + \right.$$

$$i \pi^{0.75} \left(\sum_{n=0}^{\infty} \frac{\sum_{k=0}^n (-1)^k (1+k)^{0.5-0.25 i} \binom{n}{k}}{1+n} \right) \left. \sum_{k=0}^{\infty} \frac{(1.25 + 0.125 i - z_0)^k \Gamma^{(k)}(z_0)}{k!} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$n!$ is the factorial function
 γ_n is the n^{th} Stieltjes constant
 γ is the Euler-Mascheroni constant
 \mathbb{Z} is the set of integers
 $\binom{n}{m}$ is the binomial coefficient

Integral representations:

$$(2\pi)^{(0.5 + i 0.25) - 1} \left(\Gamma\left(1 + \frac{1}{2}(0.5 + i 0.25)\right) \pi^{-0.5(0.5 + i 0.25)} \zeta(0.5 + i 0.25) \left(\frac{1}{2} + \frac{i 0.25}{2}\right) \right) =$$

$$2 \left(\frac{1}{2} + 0.125 i \right) (-0.5 + 0.25 i) \pi^{0.75 - 0.125 i} \Gamma(1.25 + 0.125 i)$$

$$\left(\frac{1}{2} + \frac{1}{-0.5 + 0.25 i} + 2 \int_0^{\infty} \frac{(1+t^2)^{-0.25-0.125 i} \sin((0.5 + 0.25 i) \tan^{-1}(t))}{-1 + e^{2\pi t}} dt \right)$$

$$(2\pi)((0.5 + i0.25) - 1) \frac{\left(\Gamma\left(1 + \frac{1}{2}(0.5 + i0.25)\right)\pi^{-0.5(0.5 + i0.25)} \zeta(0.5 + i0.25) \left(\frac{1}{2} + \frac{i0.25}{2}\right)\right)}{0.125(-2 + i)(4 + i)\pi^{1.75 - 0.125i} \mathcal{A} \zeta(0.5 + 0.25i)} = \int_L \phi e^t t^{-1.25 - 0.125i} dt$$

$$(2\pi)((0.5 + i0.25) - 1) \frac{\left(\Gamma\left(1 + \frac{1}{2}(0.5 + i0.25)\right)\pi^{-0.5(0.5 + i0.25)} \zeta(0.5 + i0.25) \left(\frac{1}{2} + \frac{i0.25}{2}\right)\right)}{1} = \int_L \phi e^t t^{-1.25 - 0.125i} dt \cdot 0.25(4 + i)\pi^{1.75 - 0.125i} \mathcal{A} \left(1.5 + 0.25i + -2 + i \int_0^\infty \frac{(1 + t^2)^{-0.25 - 0.125i} \sin((0.5 + 0.25i) \tan^{-1}(t))}{-1 + e^{2\pi t}} dt\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

From the paper **New expressions for Riemann's functions (S. Ramanujan):**

$$e^{-n} - 4\pi e^{-3n} \int_0^\infty \frac{x e^{-\pi x^2} e^{-4n}}{e^{2\pi x} - 1} dx = \frac{1}{4\pi\sqrt{\pi}} \int_0^\infty \Gamma\left(\frac{-1 + it}{4}\right) \Gamma\left(\frac{-1 - it}{4}\right) \Xi\left(\frac{1}{2}t\right) \cos nt dt.$$

We obtain:

$$1/(4\pi\sqrt{\pi}) * \int_0^\infty [(\gamma(-1 + i*0.25)/4) * (\gamma(-1 - i*0.25)/4) * 0.25584 * \cos(2*0.25)x] dx$$

$$1/(4\pi\sqrt{\pi}) * \int_0^\infty [\gamma((-1 + i*x)/4) * \gamma((-1 - i*x)/4) * 0.25584 * \cos(2x)] dx, [0, \infty)$$

Input:

$$\frac{1}{4\pi\sqrt{\pi}} \int_0^\infty \left(\Gamma\left(\frac{1}{4}(-1 + ix)\right)\Gamma\left(\frac{1}{4}(-1 - ix)\right) \times 0.25584 \cos(2x)\right) x dx$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Computation result:

$$\frac{1}{4\pi\sqrt{\pi}} \int_0^\infty \left(\Gamma\left(\frac{1}{4}(-1 + ix)\right)\Gamma\left(\frac{1}{4}(-1 - ix)\right) 0.25584 \cos(2x)\right) x dx = -0.0349453 + 1.21036 \times 10^{-19} i$$

Input interpretation:

$$-0.0349453 + 1.21036 \times 10^{-19} i$$

i is the imaginary unit

Result:

$$-0.0349453... + 1.21036... \times 10^{-19} i$$

Alternate form:

$$-0.0349453$$

$$-0.0349453$$

$$(((-0.0349453 + 1.21036 \times 10^{-19} i)))^{1/256}$$

Input interpretation:

$$\sqrt[256]{-0.0349453 + 1.21036 \times 10^{-19} i}$$

i is the imaginary unit

Result:

$$0.98690968... + 0.012111812... i$$

Polar coordinates:

$$r = 0.986984 \text{ (radius)}, \quad \theta = 0.703125^\circ \text{ (angle)}$$

0.986984

And:

$$-(((-0.0349453 + 1.21036 \times 10^{-19} i)))^{*47}$$

where 47 is a Lucas number

Input interpretation:

$$-((-0.0349453 + 1.21036 \times 10^{-19} i) \times 47$$

i is the imaginary unit

Result:

$$1.64243... - 5.68869... \times 10^{-18} i$$

Alternate form:

$$1.64243$$

1.64243

From:

$$\int_0^{\infty} \left\{ e^{-z} - 4\pi \int_0^{\infty} \frac{x e^{-3z - \pi x^2 e^{-4z}}}{e^{2\pi x} - 1} dx \right\} \cos tz dz$$

$$= \frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{-1+it}{4}\right) \Gamma\left(\frac{-1-it}{4}\right) \Xi\left(\frac{1}{2}t\right).$$

We obtain:

$$1/((8*\text{sqrt}(\text{Pi}))) * \text{gamma}((-1+i*0.25)/4) * \text{gamma}((-1-i*0.25)/4) * 0.25584$$

Input:

$$\frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1+i \times 0.25)\right) \Gamma\left(\frac{1}{4}(-1-i \times 0.25)\right) \times 0.25584$$

$\Gamma(x)$ is the gamma function
 i is the imaginary unit

Result:

0.403981...
 0.403981...

Alternate form:

0.403981

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}(-1+i 0.25)\right) \Gamma\left(\frac{1}{4}(-1-i 0.25)\right) 0.25584}{8\sqrt{\pi}} =$$

$$\frac{0.25584 \left(-1 + \frac{1}{4}(-1 - 0.25i)\right)! \left(-1 + \frac{1}{4}(-1 + 0.25i)\right)!}{8\sqrt{\pi}}$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i 0.25)\right) \Gamma\left(\frac{1}{4}(-1-i 0.25)\right) 0.25584}{8\sqrt{\pi}} = \frac{1}{8\sqrt{\pi}}$$

$$0.25584 \exp\left(-\log G\left(\frac{1}{4}(-1-0.25i)\right) + \log G\left(1 + \frac{1}{4}(-1-0.25i)\right)\right)$$

$$\exp\left(-\log G\left(\frac{1}{4}(-1+0.25i)\right) + \log G\left(1 + \frac{1}{4}(-1+0.25i)\right)\right)$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} = \frac{0.25584 G\left(1+\frac{1}{4}(-1-0.25i)\right)G\left(1+\frac{1}{4}(-1+0.25i)\right)}{G\left(\frac{1}{4}(-1-0.25i)\right)G\left(\frac{1}{4}(-1+0.25i)\right)(8\sqrt{\pi})}$$

$n!$ is the factorial function
 $\log G(z)$ gives the logarithm of the Barnes G-function
 $G(z)$ is the Barnes G-function

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} = \frac{0.03198 \Gamma(-0.25-0.0625i)\Gamma(-0.25+0.0625i)}{\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} = \frac{8.18688 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-0.25-0.0625i)^{k_1} (-0.25+0.0625i)^{k_2} \Gamma^{(k_1)}(1)\Gamma^{(k_2)}(1)}{k_1!k_2!}}{(-4+i)(4+i)\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} = \frac{0.03198 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{4}-0.0625i-z_0\right)^{k_1} \left(-\frac{1}{4}+0.0625i-z_0\right)^{k_2} \Gamma^{(k_1)}(z_0)\Gamma^{(k_2)}(z_0)}{k_1!k_2!}}{\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$\arg(z)$ is the complex argument
 $[x]$ is the floor function
 $(a)_n$ is the Pochhammer symbol (rising factorial)
 \mathbb{R} is the set of real numbers
 $\binom{n}{m}$ is the binomial coefficient
 \mathbb{Z} is the set of integers

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} = \frac{1}{\sqrt{\pi}}$$

$$0.03198 \operatorname{csc}((-0.125 - 0.03125i)\pi) \operatorname{csc}((-0.125 + 0.03125i)\pi)$$

$$\left(\int_0^\infty t^{-1.25-0.0625i} \sin(t) dt\right) \int_0^\infty t^{-1.25+0.0625i} \sin(t) dt$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} =$$

$$\frac{1}{\sqrt{\pi}} 0.03198 \left(\int_0^\infty e^{-t} t^{-1.25-0.0625i} \left(1 - e^t \sum_{k=0}^n \frac{(-t)^k}{k!}\right) dt \right)$$

$$\int_0^\infty e^{-t} t^{-1.25+0.0625i} \left(1 - e^t \sum_{k=0}^n \frac{(-t)^k}{k!}\right) dt \text{ for } (n \in \mathbb{Z} \text{ and } 0 \leq n < 0.25)$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i0.25)\right)\Gamma\left(\frac{1}{4}(-1-i0.25)\right)0.25584}{8\sqrt{\pi}} =$$

$$\frac{0.12792 \pi^2 \mathcal{A}^2}{\sqrt{\pi} \int_L e^t t^{0.25+0.0625i} dt \int_L e^t t^{1/4-0.0625i} dt}$$

$\operatorname{csc}(x)$ is the cosecant function

And:

$$\left(\left(\left(\frac{1}{(8\sqrt{\pi})}\right) \cdot \Gamma\left(\frac{-1+i0.25}{4}\right) \cdot \Gamma\left(\frac{-1-i0.25}{4}\right) \cdot 0.25584\right)\right)^{1/64}$$

Input:

$$\sqrt[64]{\frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1+i \times 0.25)\right) \Gamma\left(\frac{1}{4}(-1-i \times 0.25)\right) \times 0.25584}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

0.98593751...

0.98593751...

$$4 * \left(\left(\left(\frac{1}{(8\sqrt{\pi})}\right) \cdot \Gamma\left(\frac{-1+i0.25}{4}\right) \cdot \Gamma\left(\frac{-1-i0.25}{4}\right) \cdot 0.25584\right)\right)$$

Where 4 is a Lucas number:

Input:

$$4 \left(\frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1+i \times 0.25)\right) \Gamma\left(\frac{1}{4}(-1-i \times 0.25)\right) \times 0.25584 \right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

1.61592...

1.61592...

Alternate form:

1.61592

Alternative representations:

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i 0.25)\right) \left(\Gamma\left(\frac{1}{4}(-1-i 0.25)\right) 0.25584\right)}{8\sqrt{\pi}} = \frac{1.02336 \left(-1 + \frac{1}{4}(-1-0.25i)\right)! \left(-1 + \frac{1}{4}(-1+0.25i)\right)!}{8\sqrt{\pi}}$$

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i 0.25)\right) \left(\Gamma\left(\frac{1}{4}(-1-i 0.25)\right) 0.25584\right)}{8\sqrt{\pi}} = \frac{1}{8\sqrt{\pi}} \frac{1.02336 \exp\left(-\log G\left(\frac{1}{4}(-1-0.25i)\right) + \log G\left(1 + \frac{1}{4}(-1-0.25i)\right)\right)}{\exp\left(-\log G\left(\frac{1}{4}(-1+0.25i)\right) + \log G\left(1 + \frac{1}{4}(-1+0.25i)\right)\right)}$$

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i 0.25)\right) \left(\Gamma\left(\frac{1}{4}(-1-i 0.25)\right) 0.25584\right)}{8\sqrt{\pi}} = \frac{1.02336 G\left(1 + \frac{1}{4}(-1-0.25i)\right) G\left(1 + \frac{1}{4}(-1+0.25i)\right)}{G\left(\frac{1}{4}(-1-0.25i)\right) G\left(\frac{1}{4}(-1+0.25i)\right) (8\sqrt{\pi})}$$

$n!$ is the factorial function

$\log G(z)$ gives the logarithm of the Barnes G-function

$G(z)$ is the Barnes G-function

Series representations:

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i0.25)\right) \Gamma\left(\frac{1}{4}(-1-i0.25)\right) 0.25584}{8 \sqrt{\pi}} =$$

$$\frac{0.12792 \Gamma(-0.25-0.0625i) \Gamma(-0.25+0.0625i)}{\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \binom{-\frac{1}{2}}{k}}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i0.25)\right) \Gamma\left(\frac{1}{4}(-1-i0.25)\right) 0.25584}{8 \sqrt{\pi}} =$$

$$\frac{32.7475 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-0.25-0.0625i)^{k_1} (-0.25+0.0625i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!}}{(-4+i)(4+i) \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i0.25)\right) \Gamma\left(\frac{1}{4}(-1-i0.25)\right) 0.25584}{8 \sqrt{\pi}} =$$

$$\frac{0.12792 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{4}-0.0625i-z_0\right)^{k_1} \left(-\frac{1}{4}+0.0625i-z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!}}{\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

$\binom{n}{m}$ is the binomial coefficient

\mathbb{Z} is the set of integers

Integral representations:

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i0.25)\right) \Gamma\left(\frac{1}{4}(-1-i0.25)\right) 0.25584}{8 \sqrt{\pi}} = \frac{1}{\sqrt{\pi}}$$

$$0.12792 \csc((-0.125-0.03125i)\pi) \csc((-0.125+0.03125i)\pi)$$

$$\left(\int_0^{\infty} t^{-1.25-0.0625i} \sin(t) dt \right) \int_0^{\infty} t^{-1.25+0.0625i} \sin(t) dt$$

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i0.25)\right) \Gamma\left(\frac{1}{4}(-1-i0.25)\right) 0.25584}{8 \sqrt{\pi}} =$$

$$\frac{1}{\sqrt{\pi}} 0.12792 \left(\int_0^\infty e^{-t} t^{-1.25-0.0625i} \left(1 - e^t \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right)$$

$$\int_0^\infty e^{-t} t^{-1.25+0.0625i} \left(1 - e^t \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \text{ for } (n \in \mathbb{Z} \text{ and } 0 \leq n < 0.25)$$

$$\frac{4 \Gamma\left(\frac{1}{4}(-1+i0.25)\right) \Gamma\left(\frac{1}{4}(-1-i0.25)\right) 0.25584}{8 \sqrt{\pi} \pi^2 \mathcal{A}^2} =$$

$$\frac{\int_L \sqrt{\pi} \oint e^t t^{0.25+0.0625i} dt \int_L \sqrt{\pi} \oint e^t t^{1/4-0.0625i} dt}{\int_L \sqrt{\pi} \oint e^t t^{0.25+0.0625i} dt \int_L \sqrt{\pi} \oint e^t t^{1/4-0.0625i} dt}$$

$\text{csc}(x)$ is the cosecant function

From:

$$\xi\left(\frac{1}{2} + \frac{1}{2}it\right) = \Xi\left(\frac{1}{2}t\right),$$

$$r = 0.25584 \text{ (radius), } \theta = 14.0362^\circ \text{ (angle)}$$

$$0.25584$$

we obtain: $\Xi = 2.04672$;

then $t = 0.25$ and $\xi = 0.496403$; $s = (0.5+i*0.25)$; $\alpha = 3$ and $\beta = 5$

From:

$$\frac{\zeta(1-s)}{4 \cos \frac{1}{2}\pi s} \frac{\alpha^{\frac{1}{2}(s-1)}}{s-1-t} + \frac{\zeta(-s)}{8 \sin \frac{1}{2}\pi s} \frac{\alpha^{\frac{1}{2}(s+1)}}{s+1-t} + \alpha^{\frac{1}{2}(s+1)} \int_0^\infty \int_0^\infty \left\{ \frac{\alpha xy}{1!(s+3-t)} - \frac{(\alpha xy)^3}{3!(s+7-t)} \right.$$

$$\left. + \frac{(\alpha xy)^5}{5!(s+11-t)} - \dots \right\} \frac{x^s dx dy}{(e^{2\pi x} - 1)(e^{2\pi y} - 1)} + \frac{\zeta(1-s)}{4 \cos \frac{1}{2}\pi s} \frac{\beta^{\frac{1}{2}(s-1)}}{s-1+t} + \frac{\zeta(-s)}{8 \sin \frac{1}{2}\pi s} \frac{\beta^{\frac{1}{2}(s+1)}}{s+1+t}$$

$$+ \beta^{\frac{1}{2}(s+1)} \int_0^\infty \int_0^\infty \left\{ \frac{\beta xy}{1!(s+3+t)} - \frac{(\beta xy)^3}{3!(s+7+t)} + \frac{(\beta xy)^5}{5!(s+11+t)} - \dots \right\} \frac{x^s dx dy}{(e^{2\pi x} - 1)(e^{2\pi y} - 1)}$$

$$= \left(\frac{\alpha}{\beta}\right)^{\frac{1}{4}t} \frac{2^{\frac{1}{2}(s-3)} \Gamma\left\{\frac{1}{4}(s-1+t)\right\} \Gamma\left\{\frac{1}{4}(s-1-t)\right\}}{\pi (s+1)^2 - t^2} \times \xi\left(\frac{1+s+t}{2}\right) \xi\left(\frac{1+s-t}{2}\right). \quad (16)$$

we obtain:

$$\left(\left(\left(\frac{3}{5} \right)^{0.0625} * 2^{(0.5*(0.5+i*0.25-3))} * \text{gamma}(0.25(0.5+i*0.25-1+0.25)) * \text{gamma}(0.25(0.5+i*0.25-1-0.25)) \right) \right) / \left(\left(\text{Pi} * (0.5+i*0.25+1)^2 - 0.25^2 \right) \right)$$

Input:

$$\left(\left(\left(\frac{3}{5} \right)^{0.0625} * 2^{0.5(0.5+i*0.25-3)} \Gamma(0.25(0.5+i*0.25-1+0.25)) \Gamma(0.25(0.5+i*0.25-1-0.25)) \right) \right) / \left(\pi(0.5+i*0.25+1)^2 - 0.25^2 \right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$2.78507... + 2.64506... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 3.84096 \text{ (radius), } \theta = 43.523^\circ \text{ (angle)}$$

3.84096 partial result

Alternative representations:

$$\frac{\left(\frac{3}{5} \right)^{0.0625} \left(2^{0.5(0.5+i*0.25-3)} \Gamma(0.25(0.5+i*0.25-1+0.25)) \Gamma(0.25(0.5+i*0.25-1-0.25)) \right)}{\pi(0.5+i*0.25+1)^2 - 0.25^2} = \frac{(-1+0.25(-0.75+0.25i))!(-1+0.25(-0.25+0.25i))! 2^{0.5(-2.5+0.25i)} \left(\frac{3}{5} \right)^{0.0625}}{-0.25^2 + \pi(1.5+0.25i)^2}$$

- $$\frac{\left(\frac{3}{5} \right)^{0.0625} \left(2^{0.5(0.5+i*0.25-3)} \Gamma(0.25(0.5+i*0.25-1+0.25)) \Gamma(0.25(0.5+i*0.25-1-0.25)) \right)}{\pi(0.5+i*0.25+1)^2 - 0.25^2} = \left(2^{0.5(-2.5+0.25i)} \exp(\log G(1+0.25(-0.75+0.25i)) - \log G(0.25(-0.75+0.25i))) \frac{\exp(\log G(1+0.25(-0.25+0.25i)) - \log G(0.25(-0.25+0.25i))) \left(\frac{3}{5} \right)^{0.0625}}{(-0.25^2 + \pi(1.5+0.25i)^2)} \right)$$

- $$\frac{\left(\frac{3}{5} \right)^{0.0625} \left(2^{0.5(0.5+i*0.25-3)} \Gamma(0.25(0.5+i*0.25-1+0.25)) \Gamma(0.25(0.5+i*0.25-1-0.25)) \right)}{\pi(0.5+i*0.25+1)^2 - 0.25^2} = \frac{G(1+0.25(-0.75+0.25i)) G(1+0.25(-0.25+0.25i)) 2^{0.5(-2.5+0.25i)} \left(\frac{3}{5} \right)^{0.0625}}{G(0.25(-0.75+0.25i)) G(0.25(-0.25+0.25i)) (-0.25^2 + \pi(1.5+0.25i)^2)}$$

$n!$ is the factorial function

$\log G(z)$ gives the logarithm of the Barnes G-function

$G(z)$ is the Barnes G-function

$$3.84096 * ((0.496403(((1+(0.5+i*0.25)+0.25)/2))))*((0.496403(((1+(0.5+i*0.25)-0.25)/2))))$$

Input interpretation:

$$3.84096 \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) + 0.25) \right) \right) \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) - 0.25) \right) \right)$$

i is the imaginary unit

Result:

$$0.502814... + 0.177464... i$$

Polar coordinates:

$$r = 0.533212 \text{ (radius), } \theta = 19.44^\circ \text{ (angle)}$$

0.533212 final result

$$((((3.84096 * ((0.496403(((1+(0.5+i*0.25)+0.25)/2))))*((0.496403(((1+(0.5+i*0.25)-0.25)/2))))))))^1/16$$

Input interpretation:

$$\left(3.84096 \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) + 0.25) \right) \right) \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) - 0.25) \right) \right) \right)^{1/16}$$

i is the imaginary unit

Result:

$$0.9612439... + 0.02038699... i$$

Polar coordinates:

$$r = 0.96146 \text{ (radius), } \theta = 1.215^\circ \text{ (angle)}$$

0.96146

$$10 * (((3.84096 * ((0.496403(((1+(0.5+i*0.25)+0.25)/2))))*((0.496403(((1+(0.5+i*0.25)-0.25)/2))))))))))$$

Input interpretation:

$$10 \left(3.84096 \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) + 0.25) \right) \right) \right. \\ \left. \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) - 0.25) \right) \right) \right)$$

i is the imaginary unit

Result:

$$5.02814... + 1.77464... i$$

Polar coordinates:

$$r = 5.33212 \text{ (radius), } \theta = 19.44^\circ \text{ (angle)}$$

5.33212

$$13/2 * (((3.84096 * ((0.496403(((1+(0.5+i*0.25)+0.25)/2))))*((0.496403(((1+(0.5+i*0.25)-0.25)/2))))))))))$$

Input interpretation:

$$\frac{13}{2} \left(3.84096 \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) + 0.25) \right) \right) \right. \\ \left. \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) - 0.25) \right) \right) \right)$$

i is the imaginary unit

Result:

$$3.26829... + 1.15351... i$$

Polar coordinates:

$$r = 3.46588 \text{ (radius), } \theta = 19.44^\circ \text{ (angle)}$$

3.46588

$$11/2 * (((3.84096 * ((0.496403(((1+(0.5+i*0.25)+0.25)/2))))*((0.496403(((1+(0.5+i*0.25)-0.25)/2))))))))))$$

Input interpretation:

$$\frac{11}{2} \left(3.84096 \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) + 0.25) \right) \right) \right. \\ \left. \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) - 0.25) \right) \right) \right)$$

i is the imaginary unit

Result:

2.76548... +
0.976051... *i*

Polar coordinates:

$r = 2.93267$ (radius), $\theta = 19.44^\circ$ (angle)

2.93267

$29/6 * (((3.84096 * ((0.496403(((1+(0.5+i*0.25)+0.25)/2))))*((0.496403(((1+(0.5+i*0.25)-0.25)/2))))))))$

Input interpretation:

$$\frac{29}{6} \left(3.84096 \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) + 0.25) \right) \right) \right. \\ \left. \left(0.496403 \left(\frac{1}{2} (1 + (0.5 + i \times 0.25) - 0.25) \right) \right) \right)$$

i is the imaginary unit

Result:

2.43027... +
0.857742... *i*

Polar coordinates:

$r = 2.57719$ (radius), $\theta = 19.44^\circ$ (angle)

2.57719

From:

$$\begin{aligned}
& \frac{\zeta(1-s)}{4 \cos \frac{1}{2} \pi s} \frac{s-1}{(s-1)^2+t^2} \left\{ \alpha^{\frac{1}{2}(s-1)} + \beta^{\frac{1}{2}(s-1)} \right\} + \frac{\zeta(-s)}{8 \sin \frac{1}{2} \pi s} \frac{s+1}{(s+1)^2+t^2} \left\{ \alpha^{\frac{1}{2}(s+1)} + \beta^{\frac{1}{2}(s+1)} \right\} \\
& + \alpha^{\frac{1}{2}(s+1)} \int_0^\infty \int_0^\infty \left\{ \frac{\alpha xy}{1!} \frac{s+3}{(s+3)^2+t^2} - \frac{(\alpha xy)^3}{3!} \frac{s+7}{(s+7)^2+t^2} + \dots \right\} \frac{x^s dx dy}{(e^{2\pi x}-1)(e^{2\pi y}-1)} \\
& + \beta^{\frac{1}{2}(s+1)} \int_0^\infty \int_0^\infty \left\{ \frac{\beta xy}{1!} \frac{s+3}{(s+3)^2+t^2} - \frac{(\beta xy)^3}{3!} \frac{s+7}{(s+7)^2+t^2} + \dots \right\} \frac{x^s dx dy}{(e^{2\pi x}-1)(e^{2\pi y}-1)} \\
& = \frac{2^{\frac{1}{2}(s-3)} \Gamma\left\{\frac{1}{4}(s-1+it)\right\} \Gamma\left\{\frac{1}{4}(s-1-it)\right\}}{\pi (s+1)^2+t^2} \\
& \quad \times \Xi\left(\frac{t+is}{2}\right) \Xi\left(\frac{t-is}{2}\right) \cos\left(\frac{1}{4}t \log \frac{\alpha}{\beta}\right); \tag{17}
\end{aligned}$$

For $\Xi = 2.04672$; $t = 0.25$ and $\xi = 0.496403$; $s = (0.5+i*0.25)$; $\alpha = 3$ and $\beta = 5$,
we obtain:

$$\left(\left(\left(\left(2^{0.5(0.5+i*0.25-3)} \right) * \text{gamma}(0.25(0.5+i*0.25-1+i*0.25)) \right) * \text{gamma}(0.25(0.5+i*0.25-1-i*0.25)) \right) \right) * 1 / \left(\left(\text{Pi} * (0.5+i*0.25+1)^2 + 0.25^2 \right) \right)$$

Input:

$$\left(2^{0.5(0.5+i*0.25-3)} \Gamma(0.25(0.5+i*0.25-1+i*0.25)) \Gamma(0.25(0.5+i*0.25-1-i*0.25)) \right) \times \frac{1}{\pi(0.5+i*0.25+1)^2+0.25^2}$$

$\Gamma(x)$ is the gamma function
 i is the imaginary unit

Result:

2.74105... +
1.30763... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 3.03698$ (radius), $\theta = 25.5037^\circ$ (angle)

3.03698 partial result

Alternative representations:

$$\frac{2^{0.5(0.5+i*0.25-3)} (\Gamma(0.25(0.5+i*0.25-1+i*0.25)) \Gamma(0.25(0.5+i*0.25-1-i*0.25)))}{\pi(0.5+i*0.25+1)^2+0.25^2} = \frac{0.93628 G(1+0.25(-0.5+0.5i)) 2^{0.5(-2.5+0.25i)}}{0.107406 G(0.25(-0.5+0.5i)) (0.25^2+\pi(1.5+0.25i)^2)}$$

$$\frac{2^{0.5(0.5+i0.25-3)} (\Gamma(0.25(0.5+i0.25-1+i0.25)) \Gamma(0.25(0.5+i0.25-1-i0.25)))}{\pi(0.5+i0.25+1)^2+0.25^2} =$$

$$\frac{(2^{0.5(-2.5+0.25i)} e^{2.1653-3.14159i} \exp(\log G(1+0.25(-0.5+0.5i)) - \log G(0.25(-0.5+0.5i))))}{(0.25^2+\pi(1.5+0.25i)^2)}$$

$$\frac{2^{0.5(0.5+i0.25-3)} (\Gamma(0.25(0.5+i0.25-1+i0.25)) \Gamma(0.25(0.5+i0.25-1-i0.25)))}{\pi(0.5+i0.25+1)^2+0.25^2} =$$

$$\frac{(-1.125)!(-1+0.25(-0.5+0.5i))! 2^{0.5(-2.5+0.25i)}}{0.25^2+\pi(1.5+0.25i)^2}$$

$$3.03698 (((((2.04672(((0.25+i*(0.5+i*0.25))/2))))*(2.04672(((0.25-i*(0.5+i*0.25))/2))))*\cos(((0.25/4 \ln(3/5))))))$$

Input interpretation:

$$3.03698 \left(\left(2.04672 \left(\frac{1}{2} (0.25 + i(0.5 + i \times 0.25)) \right) \right) \left(2.04672 \left(\frac{1}{2} (0.25 - i(0.5 + i \times 0.25)) \right) \right) \cos \left(\frac{0.25}{4} \log \left(\frac{3}{5} \right) \right) \right)$$

log(x) is the natural logarithm

i is the imaginary unit

Result:

$$0.794726... + 0.794726... i$$

Polar coordinates:

$$r = 1.12391 \text{ (radius), } \theta = 45^\circ \text{ (angle)}$$

1.12391 final result

Alternative representations:

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i0.25)))$$

$$\left((2.04672 (0.25 - i(0.5 + i0.25))) \cos \left(\frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right) \right) =$$

$$3.18052 (0.25 - i(0.5 + 0.25i)) (0.25 + (0.5 + 0.25i)i) \cosh \left(\frac{1}{4} \times 0.25 i \log \left(\frac{3}{5} \right) \right)$$

- $$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cosh\left(\frac{1}{4} (-0.25 i) \log\left(\frac{3}{5}\right)\right)$$

- $$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = 1.59026$$

$$(0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \left(e^{1/4 (-0.25) i \log(3/5)} + e^{1/4 \times 0.25 i \log(3/5)} \right)$$

cosh(x) is the hyperbolic cosine function

Series representations:

- $$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-0.198783 (1 + i)^2 (-1 + 2 i + i^2) \cos\left(-0.0625 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{2}{5}\right)^k}{k}\right)$$

- $$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-0.198783 (1 + i)^2 (-1 + 2 i + i^2) \sum_{k=0}^{\infty} \frac{(-1)^k e^{-5.54518 k} \log^{2k}\left(\frac{3}{5}\right)}{(2 k)!}$$

- $$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$0.198783 (1 + i)^2 (-1 + 2 i + i^2) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{\pi}{2} + 0.0625 \log\left(\frac{3}{5}\right)\right)^{1+2k}}{(1 + 2 k)!}$$

n! is the factorial function

Integral representations:

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = 0.795131 (1 + i)^2$$

$$(0.25 - 0.5 i - 0.25 i^2) \left(1 - 0.0625 \log\left(\frac{3}{5}\right) \int_0^1 \sin\left(0.0625 t \log\left(\frac{3}{5}\right)\right) dt \right)$$

•

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$0.198783 (1 + i)^2 (-1 + 2 i + i^2) \int_{\frac{\pi}{2}}^{0.0625 \log\left(\frac{3}{5}\right)} \sin(t) dt$$

•

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-\frac{0.0993914 (1 + i)^2 (-1 + 2 i + i^2) \sqrt{\pi}}{\pi \mathcal{A}} \int_{-\mathcal{A} \infty + \gamma}^{\mathcal{A} \infty + \gamma} \frac{e^{s - (0.000976563 \log^2\left(\frac{3}{5}\right))/s}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$0.795131 (1 + i)^2 (0.25 - 0.5 i - 0.25 i^2) \left(-1 + 2 \cos^2\left(0.03125 \log\left(\frac{3}{5}\right)\right) \right)$$

•

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$0.795131 (1 + i)^2 (0.25 - 0.5 i - 0.25 i^2) \left(1 - 2 \sin^2\left(0.03125 \log\left(\frac{3}{5}\right)\right) \right)$$

•

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

0.9981013... -
0.01224916... i

Polar coordinates:

$r = 0.998176$ (radius), $\theta = -0.703125^\circ$ (angle)

0.998176 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Alternative representations:

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}}} = \frac{1}{\sqrt[64]{3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cosh\left(\frac{1}{4} \times 0.25 i \log\left(\frac{3}{5}\right)\right)}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}}} = \frac{1}{\sqrt[64]{3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cosh\left(\frac{1}{4} (-0.25 i) \log\left(\frac{3}{5}\right)\right)}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} =$$

$$1 / \left((1.59026 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \right. \\ \left. \left(e^{1/4 (-0.25) i \log(3/5)} + e^{1/4 \times 0.25 i \log(3/5)} \right) \right)^{\wedge} (1/64)$$

cosh(x) is the hyperbolic cosine function

Series representations:

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} =$$

$$\frac{1.02556}{\sqrt[64]{-(1+i)^2 \left((-1 + 2i + i^2) \cos\left(-0.0625 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{2}{5}\right)^k}{k}\right) \right)}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} =$$

$$\frac{1.02556}{\sqrt[64]{-(1+i)^2 \left((-1 + 2i + i^2) \sum_{k=0}^{\infty} \frac{(-1)^k e^{-5.54518 k} \log^2 k \left(\frac{3}{5}\right)}{(2k)!} \right)}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} =$$

$$\frac{1.02556}{\sqrt[64]{(1+i)^2 (-1 + 2i + i^2) \sum_{k=0}^{\infty} (-1)^k J_{2k}(0.0625) T_{2k}\left(\log\left(\frac{3}{5}\right)\right) (-2 + \delta_k)}}$$

n! is the factorial function

$J_n(z)$ is the Bessel function of the first kind

$T_n(x)$ is the Chebyshev polynomial of the first kind

Integral representations:

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} = \frac{1.02556}{\sqrt[64]{(1+i)^2 (-1+2i+i^2) \int_2^{0.0625 \log\left(\frac{3}{5}\right)} \sin(t) dt}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} = \frac{1.00359}{\sqrt[64]{(1+i)^2 (0.25 - 0.5i - 0.25i^2) \left(1 - 0.0625 \log\left(\frac{3}{5}\right)\right) \int_0^1 \sin\left(0.0625 t \log\left(\frac{3}{5}\right)\right) dt}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} = \frac{1.03673}{\pi \mathcal{A} \sqrt[64]{\frac{(1+i)^2 \left((-1+2i+i^2) \sqrt{\pi} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-\left(0.000976563 \log^2\left(\frac{3}{5}\right)\right)/s}}{\sqrt{s}} ds \right)}{\sqrt{s}}}} \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} = \frac{1.00359}{\sqrt[64]{(1+i)^2 (0.25 - 0.5i - 0.25i^2) (-1 + 2 \cos^2\left(0.03125 \log\left(\frac{3}{5}\right)\right))}}$$

•

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} = \frac{1.00359}{\sqrt[64]{(1 + i)^2 (0.25 - 0.5 i - 0.25 i^2) (1 - 2 \sin^2(0.03125 \log\left(\frac{3}{5}\right)))}}$$

$$\frac{1}{\sqrt[64]{\frac{3.03698 (2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right)}{2 \times 2}} = \frac{1.02556}{\sqrt[64]{-(1 + i)^2 ((-1 + 2 i + i^2) T_{0.0625}(\cos(\log\left(\frac{3}{5}\right)))}}}$$

Pi(((((((3.03698 (((((2.04672((((0.25+i*(0.5+i*0.25))/2))))*(2.04672(((0.25-i*(0.5+i*0.25))/2))))*cos(((0.25/4 ln(3/5))))))))))))))

Input interpretation:

$$\pi \left(3.03698 \left(\left(2.04672 \left(\frac{1}{2} (0.25 + i(0.5 + i \times 0.25)) \right) \right) \left(2.04672 \left(\frac{1}{2} (0.25 - i(0.5 + i \times 0.25)) \right) \right) \cos\left(\frac{0.25}{4} \log\left(\frac{3}{5}\right)\right) \right) \right)$$

log(x) is the natural logarithm

i is the imaginary unit

Result:

2.49671... +
2.49671... i

Polar coordinates:

r = 3.53087 (radius), θ = 45° (angle)

3.53087

Alternative representations:

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \pi \cosh\left(\frac{1}{4} \times 0.25 i \log\left(\frac{3}{5}\right)\right)$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \pi \cosh\left(\frac{1}{4} (-0.25 i) \log\left(\frac{3}{5}\right)\right)$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 1.59026 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \pi \\ \left(e^{1/4 (-0.25) i \log(3/5)} + e^{1/4 \times 0.25 i \log(3/5)} \right)$$

cosh(x) is the hyperbolic cosine function

Series representations:

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ -0.198783 (1 + i)^2 (-1 + 2 i + i^2) \pi \cos\left(-0.0625 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{2}{5}\right)^k}{k}\right)$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ -0.198783 (1 + i)^2 (-1 + 2 i + i^2) \pi \sum_{k=0}^{\infty} \frac{(-1)^k e^{-5.54518 k} \log^{2k}\left(\frac{3}{5}\right)}{(2 k)!}$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 0.198783 (1 + i)^2 (-1 + 2i + i^2) \pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{\pi}{2} + 0.0625 \log\left(\frac{3}{5}\right)\right)^{1+2k}}{(1 + 2k)!}$$

$n!$ is the factorial function

Integral representations:

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = 0.795131 (1 + i)^2 \\ (0.25 - 0.5i - 0.25i^2) \pi \left(1 - 0.0625 \log\left(\frac{3}{5}\right) \int_0^1 \sin\left(0.0625 t \log\left(\frac{3}{5}\right)\right) dt \right)$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 0.198783 (1 + i)^2 (-1 + 2i + i^2) \pi \int_{\frac{\pi}{2}}^{0.0625 \log\left(\frac{3}{5}\right)} \sin(t) dt$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ - \frac{0.0993914 (1 + i)^2 (-1 + 2i + i^2) \sqrt{\pi}}{\mathcal{A}} \int_{-\mathcal{A} \infty + \gamma}^{\mathcal{A} \infty + \gamma} \frac{e^{s - (0.000976563 \log^2\left(\frac{3}{5}\right))/s}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i (0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 0.795131 (1 + i)^2 (0.25 - 0.5i - 0.25i^2) \pi \left(-1 + 2 \cos^2\left(0.03125 \log\left(\frac{3}{5}\right)\right) \right)$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i(0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 0.795131 (1 + i)^2 (0.25 - 0.5 i - 0.25 i^2) \pi \left(1 - 2 \sin^2\left(0.03125 \log\left(\frac{3}{5}\right)\right) \right)$$

$$\frac{1}{2 \times 2} \pi 3.03698 \left((2.04672 (0.25 + i(0.5 + i 0.25))) \right. \\ \left. (2.04672 (0.25 - i(0.5 + i 0.25))) \cos\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) = \\ 0.795131 (1 + i)^2 (0.25 - 0.5 i - 0.25 i^2) \pi \cos\left(0.0208333 \log\left(\frac{3}{5}\right)\right) \\ \left(-3 + 4 \cos^2\left(0.0208333 \log\left(\frac{3}{5}\right)\right) \right)$$

From:

$$\frac{\zeta(1-s)}{4 \cos \frac{1}{2} \pi s} \frac{1}{(s-1)^2 + t^2} \{ \alpha^{\frac{1}{2}(s-1)} - \beta^{\frac{1}{2}(s-1)} \} + \frac{\zeta(-s)}{8 \sin \frac{1}{2} \pi s} \frac{1}{(s+1)^2 + t^2} \{ \alpha^{\frac{1}{2}(s+1)} - \beta^{\frac{1}{2}(s+1)} \} \\ + \alpha^{\frac{1}{2}(s+1)} \int_0^\infty \int_0^\infty \left\{ \frac{\alpha xy}{1!} \frac{1}{(s+3)^2 + t^2} - \frac{(\alpha xy)^3}{3!} \frac{1}{(s+7)^2 + t^2} + \dots \right\} \frac{x^s dx dy}{(e^{2\pi x} - 1)(e^{2\pi y} - 1)} \\ - \beta^{\frac{1}{2}(s+1)} \int_0^\infty \int_0^\infty \left\{ \frac{\beta xy}{1!} \frac{1}{(s+3)^2 + t^2} - \frac{(\beta xy)^3}{3!} \frac{1}{(s+7)^2 + t^2} + \dots \right\} \frac{x^s dx dy}{(e^{2\pi x} - 1)(e^{2\pi y} - 1)} \\ = \frac{2^{\frac{1}{2}(s-3)} \Gamma\{\frac{1}{4}(s-1+it)\} \Gamma\{\frac{1}{4}(s-1-it)\}}{\pi (s+1)^2 + t^2} \\ \times \Xi\left(\frac{t+is}{2}\right) \Xi\left(\frac{t-is}{2}\right) \sin\left(\frac{1}{4} t \log \frac{\alpha}{\beta}\right). \quad (18)$$

((((2^(0.5*(0.5+i*0.25-3)) * gamma (0.25(0.5+i*0.25-1+i*0.25)) * gamma (0.25(0.5+i*0.25-1-i*0.25)))) * 1/(((Pi*(0.5+i*0.25+1)^2+0.25^2)))) = 3.03698 as above

$$3.03698 \left(\left(\left(\left(2.04672 \left(\frac{1}{2} (0.25 + i(0.5 + i \cdot 0.25)) \right) \right) \right) \right) \cdot \left(2.04672 \left(\frac{1}{2} (0.25 - i(0.5 + i \cdot 0.25)) \right) \right) \cdot \sin \left(\frac{0.25}{4} \ln \left(\frac{3}{5} \right) \right) \right) \right)$$

Input interpretation:

$$3.03698 \left(\left(2.04672 \left(\frac{1}{2} (0.25 + i(0.5 + i \cdot 0.25)) \right) \right) \left(2.04672 \left(\frac{1}{2} (0.25 - i(0.5 + i \cdot 0.25)) \right) \right) \sin \left(\frac{0.25}{4} \log \left(\frac{3}{5} \right) \right) \right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$-0.0253815... - 0.0253815... i$$

Polar coordinates:

$$r = 0.0358949 \text{ (radius), } \theta = -135^\circ \text{ (angle)}$$

$$0.0358949$$

Alternative representations:

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i \cdot 0.25))) \left((2.04672 (0.25 - i(0.5 + i \cdot 0.25))) \sin \left(\frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right) \right) = 3.18052 (0.25 - i(0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cos \left(\frac{\pi}{2} - \frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right)$$

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i \cdot 0.25))) \left((2.04672 (0.25 - i(0.5 + i \cdot 0.25))) \sin \left(\frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right) \right) = -3.18052 (0.25 - i(0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cos \left(\frac{\pi}{2} + \frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right)$$

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i(0.5 + i \cdot 0.25))) \left((2.04672 (0.25 - i(0.5 + i \cdot 0.25))) \sin \left(\frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right) \right) = \frac{1}{2 i} 3.18052 (0.25 - i(0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \left(-e^{1/4 (-0.25) i \log(3/5)} + e^{1/4 \times 0.25 i \log(3/5)} \right)$$

Series representations:

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-0.198783 (1 + i)^2 (-1 + 2i + i^2) \sin\left(-0.0625 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{2}{5}\right)^k}{k}\right)$$

•

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-0.397566 (1 + i)^2 (-1 + 2i + i^2) \sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(0.0625 \log\left(\frac{3}{5}\right)\right)$$

•

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-0.198783 (1 + i)^2 (-1 + 2i + i^2) \sum_{k=0}^{\infty} \frac{(-1)^k 0.0625^{1+2k} \log^{1+2k}\left(\frac{3}{5}\right)}{(1 + 2k)!}$$

$J_n(x)$ is the Bessel function of the first kind

$n!$ is the factorial function

Integral representations:

$$\frac{1}{2 \times 2} 3.03698 (2.04672 (0.25 + i (0.5 + i 0.25)))$$

$$\left((2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) =$$

$$-0.0124239 (1 + i)^2 (-1 + 2i + i^2) \log\left(\frac{3}{5}\right) \int_0^1 \cos\left(0.0625 t \log\left(\frac{3}{5}\right)\right) dt$$

•

i is the imaginary unit

Result:

0.98704556... -
0.0090849101... i

Polar coordinates:

$r = 0.987087$ (radius), $\theta = -0.527344^\circ$ (angle)
0.987087

$$(1/11)/(((((((3.03698 (((((2.04672((((0.25+i*(0.5+i*0.25))/2))))*(2.04672(((0.25-i*(0.5+i*0.25))/2))))*sin(((0.25/4 \ln(3/5))))))))))))))$$

Input interpretation:

$$\frac{1}{11} / \left(3.03698 \left(\left(2.04672 \left(\frac{1}{2} (0.25 + i(0.5 + i \times 0.25)) \right) \right) \left(2.04672 \left(\frac{1}{2} (0.25 - i(0.5 + i \times 0.25)) \right) \right) \sin \left(\frac{0.25}{4} \log \left(\frac{3}{5} \right) \right) \right) \right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

- 1.79085... +
1.79085... i

Polar coordinates:

$r = 2.53265$ (radius), $\theta = 135^\circ$ (angle)
2.53265

Alternative representations:

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i \times 0.25))) (2.04672 (0.25 - i (0.5 + i \times 0.25))) \sin \left(\frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right) \right) 11}{2 \times 2}}{1}} = 11 \left(3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cos \left(\frac{\pi}{2} - \frac{1}{4} \times 0.25 \log \left(\frac{3}{5} \right) \right) \right)$$

•

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{2 \times 2}} =$$

$$\frac{1}{11 \left(-3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \cos\left(\frac{\pi}{2} + \frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right)}$$

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{2 \times 2}} =$$

$$\frac{1}{11 \left(3.18052 (0.25 - i (0.5 + 0.25 i)) (0.25 + (0.5 + 0.25 i) i) \left(-e^{1/4 (-0.25) i \log(3/5)} + e^{1/4 \times 0.25 i \log(3/5)} \right) \right)}$$

Series representations:

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{2 \times 2}} =$$

$$\frac{0.457329}{(1 + i)^2 (-1 + 2 i + i^2) \sin\left(-0.0625 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{2}{5}\right)^k}{k}\right)}$$

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{2 \times 2}} =$$

$$\frac{0.228664}{(1 + i)^2 (-1 + 2 i + i^2) \sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(0.0625 \log\left(\frac{3}{5}\right)\right)}$$

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{2 \times 2}} =$$

$$\frac{0.457329}{(1 + i)^2 (-1 + 2 i + i^2) \sum_{k=0}^{\infty} \frac{(-1)^k 0.0625^{1+2k} \log^{1+2k}\left(\frac{3}{5}\right)}{(1+2k)!}}$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

Integral representations:

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{7.31726^{2 \times 2}}} =$$

$$\frac{1}{(1 + i)^2 (-1 + 2i + i^2) \log\left(\frac{3}{5}\right) \int_0^1 \cos\left(0.0625 t \log\left(\frac{3}{5}\right)\right) dt}$$

- $$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{29.269 \pi \mathcal{A}^{2 \times 2}}} =$$

$$\frac{1}{(1 + i)^2 (-1 + 2i + i^2) \log\left(\frac{3}{5}\right) \sqrt{\pi} \int_{-\mathcal{A} \infty + \gamma}^{\mathcal{A} \infty + \gamma} \frac{e^{-s \left(0.000976563 \log^2\left(\frac{3}{5}\right)\right) / s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{0.228664^{2 \times 2}}} =$$

$$\frac{1}{(1 + i)^2 (-1 + 2i + i^2) \cos\left(0.03125 \log\left(\frac{3}{5}\right)\right) \sin\left(0.03125 \log\left(\frac{3}{5}\right)\right)}$$

- $$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{0.114332^{2 \times 2}}} =$$

$$\frac{1}{(1 + i)^2 (-1 + 2i + i^2) \left(-0.75 \sin\left(0.0208333 \log\left(\frac{3}{5}\right)\right) + \sin^3\left(0.0208333 \log\left(\frac{3}{5}\right)\right) \right)}$$

- $$\frac{1}{\frac{(3.03698 \left((2.04672 (0.25 + i (0.5 + i 0.25))) (2.04672 (0.25 - i (0.5 + i 0.25))) \sin\left(\frac{1}{4} \times 0.25 \log\left(\frac{3}{5}\right)\right) \right) 11}{0.457329^{2 \times 2}}} =$$

$$\frac{1}{(1 + i)^2 (-1 + 2i + i^2) U_{-0.9375}\left(\cos\left(\log\left(\frac{3}{5}\right)\right)\right) \sin\left(\log\left(\frac{3}{5}\right)\right)}$$

We have also:

$$\int_0^{\infty} \Gamma\left(\frac{-1+it}{4}\right) \Gamma\left(\frac{-1-it}{4}\right) \left\{\Xi\left(\frac{1}{2}t\right)\right\}^2 \frac{\cos nt}{1+t^2} dt$$

$$= \pi \sqrt{\pi} \int_0^{\infty} \left(\frac{1}{e^{xe^n} - 1} - \frac{1}{xe^n}\right) \left(\frac{1}{e^{xe^{-n}} - 1} - \frac{1}{xe^{-n}}\right) dx.$$

For $n = 1/2$ and $x = 2$

We obtain:

$$\text{Pi}*\text{sqrt}(\text{Pi}) * \text{integrate} [((((1/(((\exp(2*\text{e}^{0.5}))-1))))-1/(2*\text{e}^{0.5})))) * (((1/(((\exp(2*\text{e}^{-0.5}))-1))))-1/(2*\text{e}^{-0.5}))))]x$$

Input:

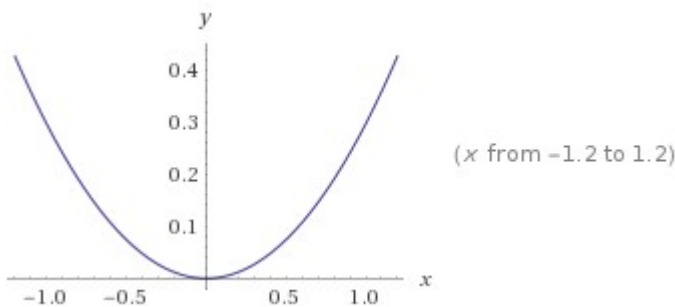
$$\pi \sqrt{\pi} \int \left[\left(\frac{1}{\exp(2\sqrt{e}) - 1} - \frac{1}{2\sqrt{e}} \right) \left(\frac{1}{\exp\left(\frac{2}{e^{0.5}}\right) - 1} - \frac{1}{e^{0.5}} \right) \right] x dx$$

Result:

$$0.295938 x^2$$

$$0.295938$$

Plot:



- **Alternate form assuming x is real:**

$$0.295938 x^2 + 0$$

- **Indefinite integral assuming all variables are real:**

$$0.098646 x^3 + \text{constant}$$

$$\text{Pi}*\text{sqrt}(\text{Pi}) * \text{integrate} [((((1/(((\exp(2*\text{e}^{0.5}))-1))))-1/(2*\text{e}^{0.5})))) * (((1/(((\exp(2*\text{e}^{-0.5}))-1))))-1/(2*\text{e}^{-0.5}))))]x, [0, \text{infinity}]$$

Definite integral:

$$\pi^{3/2} \int_0^{\infty} 0.106293 x dx = (\text{integral does not converge})$$

Alternate form:

$$2.532443524542434 \times 10^{55894}$$

•

Alternate form assuming x is real:

$$\infty$$

•

Indefinite integral:

$$\pi^{3/2} \int 0.106293 x dx = 0.295938 x^2 + \text{constant}$$

$$0.295938$$

For $x = 8/5$, we obtain:

$$0.295938 (8/5)^2$$

Input interpretation:

$$0.295938 \left(\frac{8}{5}\right)^2$$

Result:

$$0.75760128$$

$$0.75760128$$

From:

Generalized dilaton-axion models of inflation, de Sitter vacua and spontaneous SUSY breaking in supergravity

Yermek Aldabergenov, Auttakit Chatrabhuti and Sergei V. Ketov - arXiv:1907.10373v1 [hep-th] 24 Jul 2019

3.2 The case $\alpha > 3$: vacuum solutions

If the axion field is fixed at its VEV, $t_0 = -\omega_2/(2|\mu|^2)$, we can rewrite the scalar potential (8) and (11) for $\alpha > 3$ as

$$V = \frac{(\alpha-3)\omega_1^2}{2^{\alpha+2}|\mu|^2} e^{\sqrt{2\alpha}\phi} + \frac{(\alpha-5)\omega_1}{2^\alpha} e^{(\alpha-1)\sqrt{\frac{2}{\alpha}}\phi} - \frac{(\alpha^2-7\alpha+4)|\mu|^2}{2^\alpha\alpha} e^{(\alpha-2)\sqrt{\frac{2}{\alpha}}\phi} + \frac{1}{2}g^2\xi^2, \quad (28)$$

assuming $\omega_1 \neq 0$. The vacuum equations (12) and (13) then take the form

$$V_0 = \frac{(\alpha-3)\omega_1^2}{2^{\alpha+2}|\mu|^2} x^\alpha + \frac{(\alpha-5)\omega_1}{2^\alpha} x^{\alpha-1} + \frac{(\alpha^2-7\alpha+4)|\mu|^2}{2^\alpha\alpha} x^{\alpha-2} + \frac{1}{2}g^2\xi^2, \quad (29)$$

$$V_x = \frac{\alpha(\alpha-3)\omega_1^2}{2^{\alpha+2}|\mu|^2} x^{\alpha-1} - \frac{(\alpha-1)(\alpha-5)\omega_1}{2^\alpha} x^{\alpha-2} + \frac{(\alpha-2)(\alpha^2-7\alpha+4)|\mu|^2}{2^\alpha\alpha} x^{\alpha-3} = 0, \quad (30)$$

where $x \equiv e^{\phi_0/\sqrt{2}}$ as before. Eq. (30) has two solutions,

$$x_+ = \frac{2(-\alpha^2+7\alpha-4)|\mu|^2}{\alpha(\alpha-3)\omega_1}, \quad x_- = \frac{2(2-\alpha)|\mu|^2}{\alpha\omega_1}, \quad (31)$$

that we parametrize as

$$x_\pm = \gamma_\pm \frac{|\mu|^2}{\omega_1} \quad \begin{cases} \gamma_+ \equiv \frac{2(-\alpha^2+7\alpha-4)}{\alpha(\alpha-3)}, \\ \gamma_- \equiv \frac{2(2-\alpha)}{\alpha}. \end{cases} \quad (32)$$

The positivity of x requires γ/ω_1 to be positive. Since we have $\alpha > 3$, the γ_- is always negative, while the sign of γ_+ depends on the choice of α . More specifically, we find ⁶

$$3 < \alpha < \frac{1}{2}(7 + \sqrt{33}) \longrightarrow \gamma_+ > 0. \quad (33)$$

4 Inflation

In order to study inflation, let us restore the gravitational constant $\kappa \equiv \sqrt{8\pi G} = M_P^{-1}$. We choose the Kähler potential and the chiral field T to be dimensionless, whereas the superpotential has the mass dimension three, $[W] = M^3$. It follows that $[\lambda] = [\mu] = M^3$ and $[\omega_1] = M^6$, where [...] stands for the mass dimension of the corresponding quantity. We also set $[g\xi] = M^0$ and $[\phi] = [\varphi] = M$.

It is convenient to express the FI constant $g\xi$ in terms of the cosmological constant V_0 by using Eq. (29) and the general x -solution (32). Restoring κ results in the potential

$$V = V_0 + \kappa^2 \left(\frac{\gamma}{2}\right)^\alpha \frac{|\mu|^{2(\alpha-1)}}{\omega_1^{\alpha-2}} \left[\frac{\alpha-3}{4} e^{\sqrt{2\alpha}\kappa\varphi} + \frac{\alpha-5}{\gamma} e^{(\alpha-1)\sqrt{\frac{2}{\alpha}}\kappa\varphi} + \frac{\alpha^2-7\alpha+4}{\alpha\gamma^2} e^{(\alpha-2)\sqrt{\frac{2}{\alpha}}\kappa\varphi} - \frac{\alpha(\gamma+2)^2}{4\gamma^2} + \frac{(\gamma+2)(3\gamma+14)}{4\gamma^2} - \frac{4}{\alpha\gamma^2} \right]. \quad (51)$$

In what follows we neglect the cosmological constant V_0 .

We use the standard definitions of the slow-roll parameters,

$$\epsilon \equiv \frac{1}{2\kappa^2} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2, \quad \eta \equiv \frac{1}{\kappa^2} \frac{V''(\varphi)}{V(\varphi)}. \quad (52)$$

Inflation ends when $\epsilon = 1$ that translates into the value of the inflaton field at the end of inflation, φ_f . The scalar spectral index and the tensor-to-scalar ratio are related to the slow-roll parameters as

$$n_s = 1 + 2\eta_\xi - 6\epsilon_\xi, \quad r = 16\epsilon_\xi, \quad (53)$$

respectively. Here the subscript i means evaluation at the initial value of the inflaton, φ_i i.e., at the horizon crossing. The number of e-foldings between φ_i and φ_f is given by

$$N_e = \kappa^2 \int_{\varphi_f}^{\varphi_i} d\varphi \frac{V}{V'} . \quad (54)$$

Another important observable is the amplitude of scalar perturbations given by

$$A_s = \frac{\kappa^4 V(\varphi_i)}{24\pi^2 \epsilon_i} . \quad (55)$$

According to the PLANCK data (2018), the observed values of n_s , r , and A_s are [44]

$$n_s = 0.9649 \pm 0.0042 \text{ (68\%CL)} , \quad r < 0.064 \text{ (95\%CL)} , \quad (56)$$

$$\log(10^{10} A_s) = 2.975 \pm 0.056 \text{ (68\%CL)} \Rightarrow A_s \approx 1.96 \times 10^{-9} . \quad (57)$$

In our models, n_s and r depend only on α and $\text{sgn}(\omega_1)$ (and not on the value of ω_1) which determine the shape of the scalar potential. The observed value of A_s ($\sim 10^{-9}$) can be used to fix the composite parameter $|\mu|^{2(\alpha-1)}/\omega_1^{\alpha-2}$ that is related to the inflaton mass via Eq. (48).

First, we numerically evaluate n_s as a function of α for $N_e = 50$ to 60. The results of the evaluation are presented in Fig. 3. Fig. 3a shows the tilt $n_s(\alpha)$ evaluated for a positive ω_1 and $3 < \alpha < \alpha_*$, while Fig. 3b shows the tilt $n_s(\alpha)$ evaluated for a negative ω_1 and $3 \leq \alpha \leq 7.6$. The $\omega_1 > 0$ case, in part due to its limited domain of validity ($3 < \alpha < \alpha_*$), is fully compatible with the observations of the spectral tilt n_s . However, in the $\omega_1 < 0$ case, if α is greater than the certain value around 7.2 (let us call this value α_{max}), the predicted value of n_s becomes smaller than the lower observational limit $n_s = 0.9607$.⁸ A more precise value of α_{max} can be derived by finding φ_i that solves the condition $n_s(\varphi_i) = 0.9607$ and substituting this value in Eq. (54) to solve $N_e(\alpha) = 60$. This results in

$$\alpha_{\text{max}} \approx 7.235 . \quad (58)$$

Therefore, when $\omega_1 < 0$ we exclude the models with $\alpha > \alpha_{\text{max}}$.

As we show below, the tensor-to-scalar ratio r decreases with increasing α and is always compatible with the limit $r < 0.064$.

4.1 The case $3 \leq \alpha \leq \alpha_*$: Starobinsky-like inflation

Let us divide our models into two classes for $3 \leq \alpha \leq \alpha_*$ and $\alpha_* < \alpha \leq \alpha_{\max}$, respectively. The reason is that in the range $3 \leq \alpha \leq \alpha_*$ the inflationary potential is truly Starobinsky-like and has a single extremum, namely, the global minimum and the infinite plateau asymptotically approaching a constant positive height. In contrast, if $\alpha > \alpha_*$ the potential has a local maximum, which means that we get the hilltop inflationary models.

For simplicity, we restrict ourselves to integer α , and proceed with calculating the inflationary parameters n_s and r for $3 \leq \alpha \leq \alpha_*$ by setting $N_c = 55$. In this Subsection, we take $\alpha = 3, 4, 5, 6$ ($\alpha = 3$ is the Starobinsky case) and, in addition, we include the upper limit $\alpha = \alpha_* \equiv (7 + \sqrt{33})/2$. The results of our numerical calculations of n_s and r are in Table 1, and the corresponding scalar potentials for the chosen values of α are in Fig. 4.

α	3	4		5	6		α_*
$\text{sgn}(\omega_1)$	−	+	−	+/-	+	−	−
n_s	0.9650	0.9649	0.9640	0.9639	0.9634	0.9637	0.9632
r	0.0035	0.0010	0.0013	0.0007	0.0005	0.0004	0.0003
$-\kappa\varphi_i$	5.3529	3.5542	3.9899	3.2657	3.0215	2.7427	2.5674
$-\kappa\varphi_f$	0.9402	0.7426	0.8067	0.7163	0.6935	0.6488	0.6276

Table 1: The predictions for the inflationary parameters (n_s , r), and the values of φ at the horizon crossing (φ_i) and at the end of inflation (φ_f), in the case $3 \leq \alpha \leq \alpha_*$ with both signs of ω_1 . The α parameter is taken to be integer, except of the upper limit $\alpha_* \equiv (7 + \sqrt{33})/2$.

The relation to the amplitude A_s of CMB scalar perturbations in Eq. (55) is conveniently described by the composite parameter

$$\Lambda^6 \equiv \frac{|\mu|^{2(\alpha-1)}}{|\omega_1|^{\alpha-2}}, \quad (59)$$

where Λ has units of mass. When $\omega_1 < 0$ and $3 \leq \alpha \leq \alpha_*$, Eq. (57) yields $\Lambda \sim 10^{101/6}$ GeV $\sim 10^{16.8}$ GeV, whereas in the case of $\omega_1 > 0$ and $3 < \alpha < \alpha_*$ we find

$$\lim_{\alpha \rightarrow 3} \Lambda = 0, \quad \lim_{\alpha \rightarrow \alpha_*} \Lambda = \infty, \quad (60)$$

due to the behavior of $\gamma_+(\alpha)$ (see Eq. (32)) in the scalar potential (51). Given $\alpha = 4, 5, 6$, the parameter Λ is of the order $10^{16.5}, 10^{16.8}, 10^{17.5}$ GeV, respectively.

The inflaton mass is $m_\varphi \sim 10^{13}$ GeV irrespectively of the choice of α and $\text{sgn}(\omega_1)$.

From the above Table:

α	3	4		5	6		α_*
$\text{sgn}(\omega_1)$	-	+	-	+/-	+	-	-
n_s	0.9650	0.9649	0.9640	0.9639	0.9634	0.9637	0.9632
r	0.0035	0.0010	0.0013	0.0007	0.0005	0.0004	0.0003
$-\kappa\varphi_i$	5.3529	3.5542	3.9899	3.2657	3.0215	2.7427	2.5674
$-\kappa\varphi_f$	0.9402	0.7426	0.8067	0.7163	0.6935	0.6488	0.6276

Table 1: The predictions for the inflationary parameters (n_s, r), and the values of φ at the horizon crossing (φ_i) and at the end of inflation (φ_f), in the case $3 \leq \alpha \leq \alpha_*$ with both signs of ω_1 . The α parameter is taken to be integer, except of the upper limit $\alpha_* \equiv (7 + \sqrt{33})/2$.

We can to obtain some mathematical connections with the following results obtained from the previous Ramanujan's mathematical equations, that has been analyzed.

0.989069 0.994689 0.986984 0.98593751 0.96146 0.998176 0.987087

The mean of the values of the inflationary parameter n_s in the Table, is equal to: 0.9640142857..., and the mean of results obtained from the Ramanujan expressions is:

$$1/7(0.989069 + 0.994689 + 0.986984 + 0.98593751 + 0.96146 + 0.998176 + 0.987087)$$

Input interpretation:

$$\frac{1}{7}(0.989069 + 0.994689 + 0.986984 + 0.98593751 + 0.96146 + 0.998176 + 0.987087)$$

Result:

0.986200358571428571428571428571428571428571428571428...

Repeating decimal:

0.98620035857142 (period 6)

0.98620035857142

From the product of the above results, we obtain:

Input interpretation:

$$0.989069 \times 0.994689 \times 0.986984 \times 0.98593751 \times 0.96146 \times 0.998176 \times 0.987087$$

Result:

0.9069162509405600538919798873669259890917888

Repeating decimal:

0.90691625094056005389197988736692598909178880
0.90691625...

From the mean between the previous means obtained from sum and product, we obtain:

$$1/2(0.90691625094056 + 0.98620035857142)$$

Input interpretation:

$$\frac{1}{2} (0.90691625094056 + 0.98620035857142)$$

Result:

0.94655830475599

Repeating decimal:

0.946558304755990

0.9465583...result very near to the mean 0.9640142857

0.75760128 result very near to the value of (ϕ_f) in the Table 0.7426

$$1.64243 - 1 = 0.64243$$

$$1.60749 - 1 = 0.60749$$

$$1.6114 - 1 = 0.6114$$

$$1.61592 - 1 = 0.61592$$

Results very near to the two numbers in the Table 0.6488 and 0.6276 regarding the values of ϕ at the end of inflation (ϕ_f) . The mean is 0.61931 and the mean of the values in the Table is 0.6382

The following results are very near to the values in the Tables regarding the values of ϕ and at the horizon crossing (ϕ_i) .

$$5.33212 \approx 5.3529; 3.46588 \approx 3.2657; 3.53087 \approx 3.5542; 2.93267 \approx 3.0215$$

$$2.57719; 2.53265 \approx 2.5674$$

The mean is: 3.55234 very near to the value 3.53087. Further, we note that the highest values of (ϕ_i) and (ϕ_f) are 0.9402 and 5.3529, where 0.9402 is near to the value 0.9465583 obtained from the results of the Ramanujan expressions

Thus we can conclude that Ramanujan's expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$ can be connected to the inflationary parameter n_s and the values of ϕ at the horizon crossing (ϕ_i) and at the end of inflation (ϕ_f). This with regard to generalized dilaton-axion models of inflation, de Sitter vacua and spontaneous SUSY breaking in supergravity.

α is a positive real constant, λ and μ are complex parameters.

$$\mu = 1/2 + it;$$

$$x_{\pm} = \gamma_{\pm} \frac{|\mu|^2}{\omega_1} \quad \begin{cases} \gamma_+ \equiv \frac{2(-\alpha^2 + 7\alpha - 4)}{\alpha(\alpha - 3)}, \\ \gamma_- \equiv \frac{2(2 - \alpha)}{\alpha}. \end{cases}$$

$$3 < \alpha < \frac{1}{2}(7 + \sqrt{33}) \rightarrow \gamma_+ > 0$$

$$\alpha < (5 + \sqrt{33})/2 \approx 5.37$$

The inflaton mass can be read off from Eq. (28) after using $\varphi = \phi - \phi_0$ and substituting the general x solution (32). We find

$$m_{\varphi}^2 = 2 \left(\frac{\gamma}{2}\right)^{\alpha} \left[\frac{\alpha(\alpha - 3)}{4} + \frac{(\alpha - 5)(\alpha - 1)^2}{\alpha\gamma} + \frac{(\alpha^2 - 7\alpha + 4)(\alpha - 2)^2}{\alpha^2\gamma^2} \right] \frac{|\mu|^{2(\alpha-1)}}{\omega_1^{\alpha-2}}. \quad (48)$$

It is noteworthy that the choice of $\alpha = 5$ leads to $\gamma_+ = -\gamma_- = 6/5$, so that the scalar potentials in the cases $\omega_1 > 0$ and $\omega_1 < 0$ exactly coincide.

$$\omega_1 = 3$$

$$2(-5.37^2 + 7*5.37 - 4) * 1/(5.37(5.37 - 3))$$

Input:

$$2(-5.37^2 + 7 \times 5.37 - 4) \times \frac{1}{5.37(5.37 - 3)}$$

Result:

$$0.746937588886531676999112116854850749200512300717378151788\dots$$

$$0.746937588\dots = \gamma_+$$

$$0.746937588^{5.37} [(((5.37(5.37-3)/4)))+(((5.37-5)(5.37-1)^2)))/((5.37*0.746937588))+(((5.37^2-7*5.37+4)(5.37-2)^2)) / ((5.37^2*0.746937588^2))] * (((1/2+5i)^{(2*(5.37-1))}) / ((3^{(5.37-2))}))$$

Input interpretation:

$$\frac{0.746937588^{5.37} \left(5.37 \left(\frac{1}{4} (5.37 - 3) \right) + \frac{(5.37 - 5) (5.37 - 1)^2}{5.37 \times 0.746937588} + \frac{(5.37^2 - 7 \times 5.37 + 4) (5.37 - 2)^2}{5.37^2 \times 0.746937588^2} \right) \times \left(\frac{1}{2} + 5i \right)^{2(5.37-1)}}{3^{5.37-2}}$$

i is the imaginary unit

Result:

10512.6... +
3151.82... *i*

Polar coordinates:

r = 10974.9 (radius), *θ* = 16.6894° (angle)

10974.9

sqrt(10974.9)

Input interpretation:

$$\sqrt{10974.9}$$

Result:

104.761...
104.761...

We note that:

$$(76+29-2)/\text{sqrt}(10974.9)$$

Where 2, 29 and 76 are Lucas numbers

Input interpretation:

$$\frac{76 + 29 - 2}{\sqrt{10974.9}}$$

Result:

0.983189...

0.983189... result very near to the dilaton value **0.989117352243 = φ** and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and:

$$(76+29)/(\sqrt{10974.9})$$

Input interpretation:

$$\frac{76 + 29}{\sqrt{10974.9}}$$

Result:

1.002279882002793718919768602567641880122383385296777572303...

1.002279882... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

From:

Non-perturbative scalar potential inspired by type IIA strings on rigid CY

Sergei Alexandrov, Sergei V. Ketov and Yuki Wakimoto – arXiv:1607.05293v2 [hep-th] 10 Nov 2016

Figure 3: The profile of the potential on the plane $\gamma-(r/|c|)$. There is a local maximum at $\gamma \approx 0.27$, $r \approx 5.18|c|$ and a saddle point at $\gamma \approx 0.14$, $r \approx 2.66|c|$. The profile corresponds to the choice $f = 26$, and the potential is rescaled by the factor $\frac{3\lambda_2|c|C}{h^2}$.

$$\gamma > 1 + \frac{4c}{r} \Rightarrow \left(1 + \frac{c}{r}\right) \left(1 - \frac{C}{8V}\right) < \frac{3}{4}. \quad (4.10)$$

For $\gamma = 0.27$; $r = 5.18$, we have:

$$0.27 > (5.18 + 4c)/5.18$$

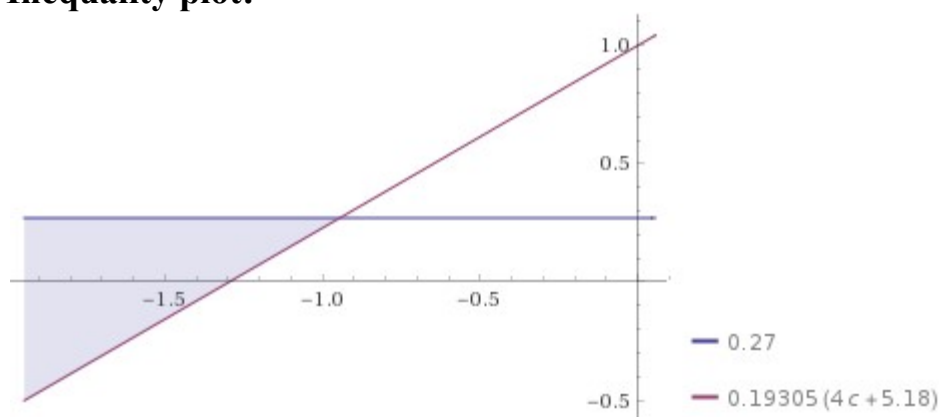
Input:

$$0.27 > \frac{5.18 + 4c}{5.18}$$

Result:

$$0.27 > 0.19305(4c + 5.18)$$

Inequality plot:



•

Alternate forms:

$$c < -0.94535$$

$$c = -0.94535$$

$$c + 0.94535 < 0$$

•

$$0.27 > 0.772201(c + 1.295)$$

•

Expanded form:

$$0.27 > 0.772201c + 1$$

•

Alternate form assuming $c > 0$:

$$c + 0.94535 < 0$$

•

Solution:

- Approximate form

$$c < -\frac{18907}{20000}$$

Interval notation:

$$\left(-\infty, -\frac{18907}{20000}\right)$$

For $\gamma = 0.27$; $r = 5.18$ and $c = -0.94535$, we obtain:

$$\frac{r(et)^2}{(r + 2c)^2} \approx \frac{2\tilde{h}^2 (2\gamma^3 + 9\gamma^2 + 10\gamma - 5)r - 8c}{\lambda_2 (1 + \gamma)^2 (\gamma(r + 2c) + c)}, \tag{4.5}$$

From the formula:

$$\frac{e^{-\mathcal{K}} \kappa^{ij} e_i e_j}{(et)^2} \approx \frac{4}{3 + \gamma}, \tag{4.11}$$

and previous data, we obtain:

$$\frac{r(et)^2}{(r + 2c)^2}$$

$$((5.18(3+0.27))/((5.18+2*(-0.94535))^2))$$

Input:

$$\frac{5.18 (3 + 0.27)}{(5.18 + 2 \times (-0.94535))^2}$$

Result:

1.565562976685798931443422732405310370466300778161401882649...
 1.56556297...

And:

$$\frac{(2\gamma^3 + 9\gamma^2 + 10\gamma - 5)r - 8c}{(1 + \gamma)^2 (\gamma(r + 2c) + c)}$$

$$\frac{(((2*0.27^3+9*0.27^2+10*0.27-5)*5.18 - 8*(-0.94535)))/((((1+0.27)^2((0.27(5.18+2*(-0.94535))-0.94535))))))$$

Input:

$$\frac{(2 \times 0.27^3 + 9 \times 0.27^2 + 10 \times 0.27 - 5) \times 5.18 - 8 \times (-0.94535)}{(1 + 0.27)^2 (0.27 (5.18 + 2 \times (-0.94535)) - 0.94535)}$$

Result:

-0.73346641475357843287816716824348320274295551660471031193...
-0.733466414...

The value of $\frac{\tilde{h}^2}{\lambda_2}$ is:

$$1.56556297 = 2x * (-0.733466414)$$

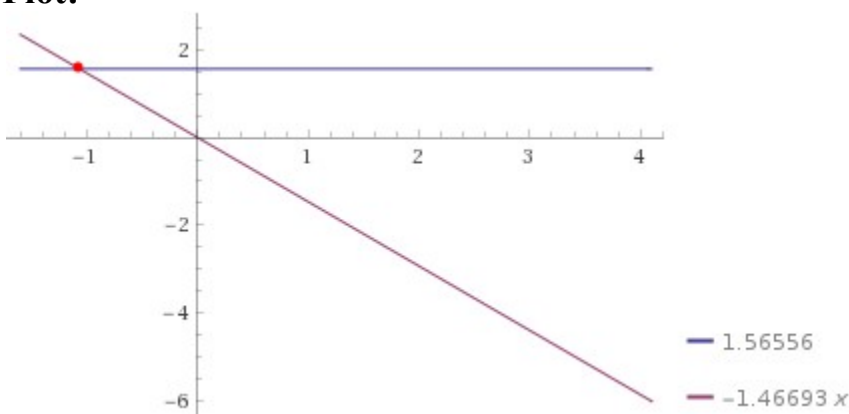
Input interpretation:

$$1.56556297 = 2x * (-0.733466414)$$

Result:

$$1.56556 = -1.46693x$$

Plot:



Alternate form:

$$1.46693x + 1.56556 = 0$$

Alternate form assuming x is real:

$$1.56556 = 0 - 1.46693x$$

Solution:

$$x \approx -1.06724$$

$$\frac{\tilde{h}^2}{\lambda_2} = -1.06724$$

Indeed, we have:

$$2 * (-1.06724) * (-0.733466414)$$

Input interpretation:

$$2 \times (-1.06724) \times (-0.733466414)$$

Result:

$$1.56556939135472$$

$$1.56556939135472$$

This is the result of a cubic equation of the dilaton r (see eq. 4.5). Thence, we can to obtain:

$$(((2 * (-1.06724) * (-0.733466414))))^{1/3}$$

Input interpretation:

$$\sqrt[3]{2 \times (-1.06724) \times (-0.733466414)}$$

Result:

$$1.16116\dots$$

$$1.16116\dots$$

Note that:

$$1/1.16116$$

Input interpretation:

$$\frac{1}{1.16116}$$

Result:

$$0.861207757759481897412931895690516380171552585345688793964\dots$$

$$0.86120775775\dots$$

Repeating decimal:

$$0.861207757759481897412931895690516380171552585345688793964\dots$$

(period 84)

Rational approximation:

$$\frac{25\,000}{29\,029}$$

•

Possible closed forms:

$$\frac{31}{36} \approx 0.8611111111$$

$$\frac{\pi^3}{36} \approx 0.8612854633$$

$$\frac{17}{2\pi^2} \approx 0.8612300610$$

The four-dimensional dilaton $r = e^\phi$

Thence we obtain ϕ :

$$\ln(1.1611565435454298619247472985297363949361782362143326)$$

Input interpretation:

$$\log(1.1611565435454298619247472985297363949361782362143326)$$

$\log(x)$ is the natural logarithm

Result:

$$0.1494165287214362089089060465146575974045433943037009\dots$$

$$0.14941652872\dots$$

Alternative representations:

$$\begin{aligned} \log(1.16115654354542986192474729852973639493617823621433260000) = \\ \log_e(1.16115654354542986192474729852973639493617823621433260000) \end{aligned}$$

•

$$\begin{aligned} \log(1.16115654354542986192474729852973639493617823621433260000) = \log(a) \\ \log_a(1.16115654354542986192474729852973639493617823621433260000) \end{aligned}$$

•

$$\begin{aligned} \log(1.16115654354542986192474729852973639493617823621433260000) = \\ -\text{Li}_1(-0.16115654354542986192474729852973639493617823621433260000) \end{aligned}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\log(1.16115654354542986192474729852973639493617823621433260000) = -\sum_{k=1}^{\infty} \frac{(-0.16115654354542986192474729852973639493617823621433260000)^k}{k}$$

- $$\log(1.16115654354542986192474729852973639493617823621433260000) = 2i\pi \left\lfloor \frac{1}{2\pi} \arg(1.16115654354542986192474729852973639493617823621433260000 - x) \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (1.16115654354542986192474729852973639493617823621433260000 - x)^k x^{-k} \text{ for } x < 0$$

- $$\log(1.16115654354542986192474729852973639493617823621433260000) = \left\lfloor \frac{1}{2\pi} \arg(1.16115654354542986192474729852973639493617823621433260000 - z_0) \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{1}{2\pi} \arg(1.16115654354542986192474729852973639493617823621433260000 - z_0) \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (1.16115654354542986192474729852973639493617823621433260000 - z_0)^k z_0^{-k}$$

arg(z) is the complex argument

[x] is the floor function

i is the imaginary unit

We have that:

$$(0.1494165287214362)^{1/64}$$

Input interpretation:

$$\sqrt[64]{0.1494165287214362}$$

Result:

0.970733413745089624...

0.970733413745089624... result very near to the value of $n_s = 0.9650$ (see previous Table)

Now, we have that, for $e^{\mathcal{K}} = 1$:

$$\begin{aligned}
V_{cr}^{(\varphi)} &\approx \frac{e^{\mathcal{K}}}{r} \left[\frac{\tilde{h}^2}{\lambda_2} \frac{1-\gamma}{1+\gamma} + \frac{c(et)^2}{2(r+2c)^2} \right] \\
&\approx \frac{e^{\mathcal{K}} \tilde{h}^2}{\lambda_2 r^2} \frac{\gamma(1-\gamma^2)r^2 - 4c(1-3\gamma-2\gamma^2)r - 8c^2}{(1+\gamma)^2(\gamma(r+2c)+c)}.
\end{aligned} \tag{4.6}$$

And for $\gamma = 0.27$; $r = 5.18$ and $c = -0.94535$, $\frac{\tilde{h}^2}{\lambda_2} = -1.06724$, we obtain:

$$\begin{aligned}
&(((((-1.06724*(1/(5.18^2))*0.27(1-0.27^2)*5.18^2 - 4(-0.94535)*(1-3*0.27- \\
&2*0.27^2)*5.18 - 8*(-0.94535)^2)))) * 1/(((1+0.27)^2 (0.27(5.18+2*(-0.94535)- \\
&0.94535))))))
\end{aligned}$$

Input interpretation:

$$\begin{aligned}
&\left(-1.06724 \times \frac{1}{5.18^2} \times 0.27 ((1 - 0.27^2) \times 5.18^2) - \right. \\
&\quad \left. 4 \times (-0.94535) (1 - 3 \times 0.27 - 2 \times 0.27^2) \times 5.18 - 8 (-0.94535)^2 \right) \times \\
&\quad \frac{1}{(1 + 0.27)^2 (0.27 (5.18 + 2 \times (-0.94535) - 0.94535))}
\end{aligned}$$

Result:

-6.41769743007888909595077648297732638453806083986648215179...
-6.41769743... = perturbative potential at critical points

$$f - \frac{|c| \tilde{h}^2}{\lambda_2 e_1^2} \left(\frac{4\kappa_{111}}{3C} \right)^{2/3} - \frac{\pi \tilde{h}^2}{24 \lambda_2 e_1^2} \left(\frac{\kappa_{111}}{3\zeta(3)} \right)^{2/3}, \tag{4.18}$$

$$\kappa_{111} = 344,$$

$$f = 26,$$

$$26 = \text{Pi} * (-1.06724) * 1 / (24x) * ((344 / (3 * \text{zeta}(3)))^{2/3})$$

Input interpretation:

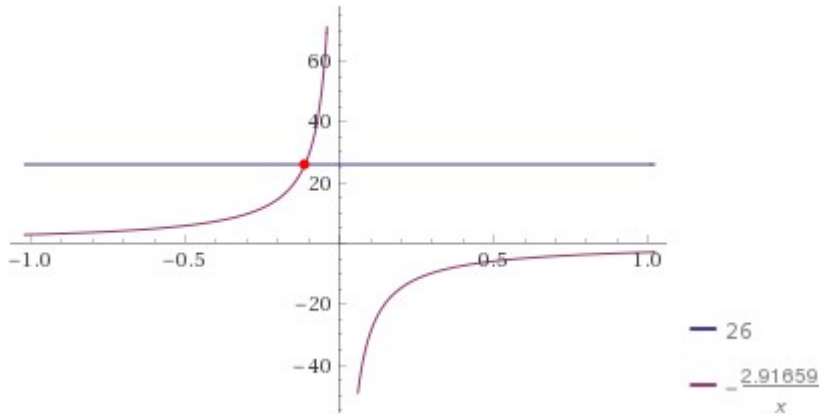
$$26 = \pi \times (-1.06724) \times \frac{1}{24x} \left(\frac{344}{3\zeta(3)} \right)^{2/3}$$

$\zeta(s)$ is the Riemann zeta function

Result:

$$26 = -\frac{2.91659}{x}$$

Plot:



Alternate form assuming x is real:

$$26 = 0 - \frac{2.91659}{x}$$

Solution:

$$x \approx -0.112177$$

$$e_1^2 = -0.112177$$

Note that:

$$26 / (((-1.06724) * 1 / (24 * -0.112177) * ((344 / (3 * \zeta(3)))^{2/3})))$$

Input interpretation:

$$\frac{26}{1.06724 \times \frac{1}{24 \times (-0.112177)} \left(\frac{344}{3 \zeta(3)} \right)^{2/3}}$$

$\zeta(s)$ is the Riemann zeta function

Result:

3.141604734859565113026178890740543092874079622443730552406...

3.14160473... $\approx \pi$

Alternative representations:

$$\frac{26}{\frac{\left(\frac{344}{3 \zeta(3)}\right)^{2/3} (-1) 1.06724}{24 (-0.112177)}} = -\frac{26}{\frac{1.06724 \left(\frac{344}{3 \zeta(3,1)}\right)^{2/3}}{2.69225}}$$

$$\frac{26}{\frac{\left(\frac{344}{3\zeta(3)}\right)^{2/3} (-1) 1.06724}{24 (-0.112177)}} = -\frac{26}{\frac{1.06724 \left(\frac{344}{3S_{2,1}(1)}\right)^{2/3}}{2.69225}}$$

$$\frac{26}{\frac{\left(\frac{344}{3\zeta(3)}\right)^{2/3} (-1) 1.06724}{24 (-0.112177)}} = -\frac{26}{\frac{1.06724 \left(-\frac{344}{3\text{Li}_3(-1)} \right)^{2/3}}{\frac{3}{4}}}{2.69225}}$$

$\zeta(s, a)$ is the generalized Riemann zeta function
 $S_{n,p}(x)$ is the Nielsen generalized polylogarithm function
 $\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{26}{\frac{\left(\frac{344}{3\zeta(3)}\right)^{2/3} (-1) 1.06724}{24 (-0.112177)}} = \frac{2.77887}{\left(\sum_{k=1}^{\infty} \frac{1}{k^3}\right)^{2/3}}$$

$$\frac{26}{\frac{\left(\frac{344}{3\zeta(3)}\right)^{2/3} (-1) 1.06724}{24 (-0.112177)}} = \frac{3.36636}{\left(-\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}\right)^{2/3}}$$

$$\frac{26}{\frac{\left(\frac{344}{3\zeta(3)}\right)^{2/3} (-1) 1.06724}{24 (-0.112177)}} = \frac{3.03759}{\left(\sum_{k=0}^{\infty} \frac{1}{(1+2k)^3}\right)^{2/3}}$$

From the result of f , we obtain:

$$1/(1+1/26)$$

Input:

$$\frac{1}{1 + \frac{1}{26}}$$

Exact result:

$$\frac{1}{10^2} + \frac{1}{\left(\frac{\pi(-1) 1.06724 \left(\frac{344}{3 \zeta(3)} \right)^{2/3}}{24(-0.112177)} \right)^{-1/6.4177}} = \frac{1}{10^2} + \frac{1}{\left(\frac{-1.06724 \pi \left(\frac{344}{3 S_{2,1}(1)} \right)^{2/3}}{-2.69225} \right)^{-1/6.4177}}$$

$$\frac{1}{10^2} + \frac{1}{\left(\frac{\pi(-1) 1.06724 \left(\frac{344}{3 \zeta(3)} \right)^{2/3}}{24(-0.112177)} \right)^{-1/6.4177}} = \frac{1}{10^2} + \frac{1}{\left(\frac{-1.06724 \pi \left(\frac{\frac{344}{3 \operatorname{Li}_3(-1)}}{\frac{3}{4}} \right)^{2/3}}{-2.69225} \right)^{-1/6.4177}}$$

$\zeta(s, a)$ is the generalized Riemann zeta function
 $S_{n,p}(x)$ is the Nielsen generalized polylogarithm function
 $\operatorname{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{1}{10^2} + \frac{1}{\left(\frac{\pi(-1) 1.06724 \left(\frac{344}{3 \zeta(3)} \right)^{2/3}}{24(-0.112177)} \right)^{-1/6.4177}} = 0.01 + 1.41683 \left(\pi \left(\frac{1}{\sum_{k=1}^{\infty} \frac{1}{k^3}} \right)^{2/3} \right)^{0.155819}$$

$$\frac{1}{10^2} + \frac{1}{\left(\frac{\pi(-1) 1.06724 \left(\frac{344}{3 \zeta(3)} \right)^{2/3}}{24(-0.112177)} \right)^{-1/6.4177}} = 0.01 + 1.39731 \left(\pi \left(\frac{1}{\sum_{k=0}^{\infty} \frac{1}{(1+2k)^3}} \right)^{2/3} \right)^{0.155819}$$

$$\frac{1}{10^2} + \frac{1}{\left(\frac{\pi(-1) 1.06724 \left(\frac{344}{3 \zeta(3)} \right)^{2/3}}{24(-0.112177)} \right)^{-1/6.4177}} = 0.01 + 1.37511 \left(\pi \left(-\frac{1}{\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}} \right)^{2/3} \right)^{0.155819}$$

Now, we have:

$$\begin{aligned} \kappa_{ijk} \kappa^{il} e_l \kappa^{jm} e_m \kappa^{kn} e_n &\approx \frac{8re^{2\mathcal{K}}(et)}{r+2c} (2(et)^2 - e^{-\mathcal{K}} \kappa^{ij} e_i e_j) + \frac{64\tilde{h}^2}{\lambda_2} e^{2\mathcal{K}}(et) \left(\frac{2(r+c)}{(1+\gamma)^2} - r \right) \quad (4.23) \\ &\approx \frac{32\tilde{h}^2}{\lambda_2} \frac{(et)e^{2\mathcal{K}} (5 - 10\gamma - 13\gamma^2 - 2\gamma^3)r^3 - 2c(1 - 8\gamma + 3\gamma^2)r^2 - 4c^2(9 - 8\gamma)r - 8c^3(3 - 2\gamma)}{r+2c (1+\gamma)^2 (c+\gamma(r+2c))}, \end{aligned}$$

Now, we have that, for $e^{\mathcal{K}} = 1$, and for $\gamma = 0.27$; $r = 5.18$ and $c = -0.94535$,
 $\frac{\tilde{h}^2}{\lambda_2} = -1.06724$, $(et) = 1.808314132$, we obtain:

$$32*(-1.06724)*1.808314132*1/((5.18-2*0.94535))*(((5-10*0.27-13*0.27^2-2*0.27^3)*5.18^3+2*0.94535(1-8*0.27+3*0.27^2)*5.18^2+4*0.94535^2(9-8*0.27)*5.18-8*(-0.94535)^3(3-2*0.27)))$$

Input interpretation:

$$32 \times (-1.06724) \times 1.808314132 \times \frac{1}{5.18 - 2 \times 0.94535} \\ ((5 - 10 \times 0.27 - 13 \times 0.27^2 - 2 \times 0.27^3) \times 5.18^3 + \\ 2 \times 0.94535 ((1 - 8 \times 0.27 + 3 \times 0.27^2) \times 5.18^2) + \\ 4 \times 0.94535^2 ((9 - 8 \times 0.27) \times 5.18) - 8 (-0.94535)^3 (3 - 2 \times 0.27))$$

Result:

-5219.80305719387788750109577474015748031496062992125984251...
-5219.80305719... partial result

-

$$5219.80305719387788750109577474015748031496062992125984251 * 1 / (((1 + 0.27)^2 (-0.94535 + 0.27(5.18 - 2 * 0.94535))))$$

Input interpretation:

$$-5219.80305719387788750109577474015748031496062992125984251 \times \\ \frac{1}{(1 + 0.27)^2 (-0.94535 + 0.27(5.18 - 2 \times 0.94535))}$$

Result:

56539.84814600080972992846910480937504433885840729247283117...
56539.848146... final result

From:

From Ramanujan:

Modular equations and approximations to π - Srinivasa Ramanujan
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

From the previous final result 56539.848146, we obtain, with the following Ramanujan equations, the new mathematical connection:

$$64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

$$= 2508927.99839293$$

$$2508927.9983929391347126585602054 \div 64 = 39201.9999748$$

$$39202 \times \sqrt{2} = 55440.00007215$$

$$55440 + 4372/4 = 56533$$

Or:

$$55440 - 4372 + 24 + 4096 + 64 + 4*276 + 192 = 56548$$

From:

Classically stable non-singular cosmological bounces

Anna Ijjas and Paul J. Steinhardt

arXiv:1606.08880v2

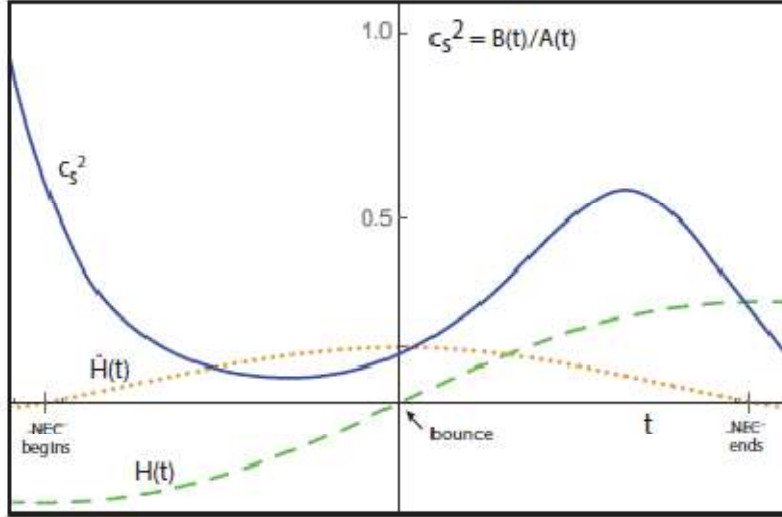


FIG. 1. A plot of the sound speed c_S^2 (solid blue curve) for co-moving curvature perturbations as a function of time t . The time coordinate is given in Planck units and the value of c_S^2 is given in units where the speed of light is unity. Superimposed for illustration purposes are the shapes of the background solutions for $H(t)$ (dashed green curve; also shown in Figs. 2 and 3) and $\dot{H}(t)$ (dotted red curve). More specifically, the results correspond to $H(t) = H_0 t e^{-F(t-t^*)^2}$ and $\gamma(t) = \gamma_0 e^{3\Theta t} + H(t)$ with the parameter values $H_0 = 3 \times 10^{-5}$, $t^* = 0.5$, $F = 9 \times 10^{-5}$, $\gamma_0 = -0.0044$, $\Theta = 0.0046$. Notably, throughout, the sound speed is real ($A(t), B(t) > 0$) and sub-luminal, with $0 < c_S^2 < 1$. The characteristic energy scale $\sim H^2$ is well below the Planck scale, and the NEC violating phase lasts ~ 150 Planck times; it starts when \dot{H} becomes positive at $t_{\text{beg}} \simeq -74 M_{\text{Pl}}^{-1}$ and ends when \dot{H} becomes negative at $t_{\text{end}} \simeq 75 M_{\text{Pl}}^{-1}$; the bounce ($H(t) = 0$) occurs at $t = 0$. Note that the bounce stage occurs well within the classical regime.

We have that:

$$M_{\text{Pl}} = 1.220910 \times 10^{19} \text{ GeV};$$

$$\text{Note that } t_{\text{beg}} - t_{\text{end}} = -74 * (1.220910 * 10^{19})^{-1} - 75 * (1.220910 * 10^{19})^{-1}$$

Input interpretation:

$$\frac{74}{1.220910 \times 10^{19}} - \frac{75}{1.220910 \times 10^{19}}$$

Result:

$$\begin{aligned} & -1.220401176171871800542218509144818209368421914801254... \times 10^{-17} \\ & -1.220401176171871800... * 10^{-17} \end{aligned}$$

And:

$$\left(\left(\left(-74 \cdot (1.220910 \cdot 10^{19})^{-1} - 75 \cdot (1.220910 \cdot 10^{19})^{-1}\right)\right)\right)^{1/(64^2 \cdot 16)}$$

Input interpretation:

$$64^2 \times 16 \sqrt{-\left(-\frac{74}{1.220910 \times 10^{19}} - \frac{75}{1.220910 \times 10^{19}}\right)}$$

Result:

0.99940592655...

0.99940592655... result that is very near to the result of the following Rogers-Ramanujan continued fraction

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

We have also that:

$$\left(\left(\left(-74 \cdot (1.220910 \cdot 10^{19})^{-1} + 75 \cdot (1.220910 \cdot 10^{19})^{-1}\right)\right)\right)$$

Input interpretation:

$$4096 \times 8 \sqrt{\frac{1}{-\frac{74}{1.220910 \times 10^{19}} + \frac{75}{1.220910 \times 10^{19}}}}$$

Result:

1.001342108112153108381733270190881564609680512429590435095...

1.001342108112153.....

And:

$$\left(\left(\left(-74 \cdot (1.220910 \cdot 10^{19})^{-1} + 75 \cdot (1.220910 \cdot 10^{19})^{-1}\right)\right)\right)^{1/(4096 \cdot 8)}$$

Input interpretation:

$$4096 \sqrt[8]{-\frac{74}{1.220910 \times 10^{19}} + \frac{75}{1.220910 \times 10^{19}}}$$

Result:

0.99865969...

0.99865969....

The two results 1.001342108.... and 0.99865969.... are very near to the following values of the Rogers-Ramanujan continued fractions:

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

For:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

We have that:

$$(((4096 * e^{(-\pi * \sqrt{22})})))$$

Input:

$$4096 e^{-\pi \sqrt{22}}$$

Exact result:

$$4096 e^{-\sqrt{22} \pi}$$

Decimal approximation:

0.001632554151233203712134121016646012876814066194393771538...

0.00163255415123.....

Property:

$4096 e^{-\sqrt{22} \pi}$ is a transcendental number

Series representations:

$$4096 e^{-\pi \sqrt{22}} = 4096 e^{-\pi \sqrt{21} \sum_{k=0}^{\infty} 21^{-k} \binom{1/2}{k}}$$

•

$$4096 e^{-\pi \sqrt{22}} = 4096 \exp\left(-\pi \sqrt{21} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{21}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

•

$$4096 e^{-\pi \sqrt{22}} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 21^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

Res_f is a complex residue
 $\Rightarrow 0$

And:

$$(((4096 * e^{(-\pi * \sqrt{22})}))) * 1/64$$

Input:

$$\left(4096 e^{-\pi \sqrt{22}}\right) \times \frac{1}{64}$$

Exact result:

$$64 e^{-\sqrt{22} \pi}$$

Decimal approximation:

0.000025508658613018808002095640885093951200219784287402680...

0.000025508658613.....

Property:

$64 e^{-\sqrt{22} \pi}$ is a transcendental number

Series representations:

$$\frac{4096}{64} e^{-\pi \sqrt{22}} = 64 e^{-\pi \sqrt{21} \sum_{k=0}^{\infty} 21^{-k} \binom{1/2}{k}}$$

•

$$\frac{4096}{64} e^{-\pi \sqrt{22}} = 64 \exp\left(-\pi \sqrt{21} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{21}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

•

$$\frac{4096}{64} e^{-\pi \sqrt{22}} = 64 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 21^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{s=0} f$ is a complex residue

Indeed:

0.000025508658613018808002095640885093951200219784287402680 * 64

Input interpretation:

0.000025508658613018808002095640885093951200219784287402680 × 64

Result:

0.00163255415123320371213412101664601287681406619439377152...

0.0016325541512332....

We note that:

$$1 / (((((((4096 * e^{(-\pi * \sqrt{22})}) * 1/64))))))$$

Input:

$$\frac{1}{(4096 e^{-\pi \sqrt{22}}) \times \frac{1}{64}}$$

Exact result:

$$\frac{e^{\sqrt{22} \pi}}{64}$$

Decimal approximation:

39202.37497277225729946139365815136191369391773231572555827...

39202.374972... that is about equal to previous result

Property:

$\frac{e^{\sqrt{22} \pi}}{64}$ is a transcendental number

Series representations:

$$\frac{1}{\frac{4096}{64} e^{-\pi \sqrt{22}}} = \frac{1}{64} e^{\pi \sqrt{21} \sum_{k=0}^{\infty} 21^{-k} \binom{1/2}{k}}$$

•

$$\frac{1}{\frac{4096}{64} e^{-\pi \sqrt{22}}} = \frac{1}{64} e^{\pi \sqrt{21} \sum_{k=0}^{\infty} \frac{(-\frac{1}{21})^k \binom{-1}{2/k}}{k!}}$$

•

$$\frac{e^{\left(\sqrt{\frac{11}{2}} \pi\right)/2048}}{2^{3/2048}} \text{ is a transcendental number}$$

All 4096th roots of $e^{(\sqrt{22}) \pi}/64$:

$$\frac{e^{\left(\sqrt{\frac{11}{2}} \pi\right)/2048} e^0}{2^{3/2048}} \approx 1.002585 \text{ (real, principal root)}$$

•

$$\frac{e^{\left(\sqrt{\frac{11}{2}} \pi\right)/2048} e^{(i \pi)/2048}}{2^{3/2048}} \approx 1.002584 + 0.0015379 i$$

•

$$\frac{e^{\left(\sqrt{\frac{11}{2}} \pi\right)/2048} e^{(i \pi)/1024}}{2^{3/2048}} \approx 1.002581 + 0.0030759 i$$

•

$$\frac{e^{\left(\sqrt{\frac{11}{2}} \pi\right)/2048} e^{(3 i \pi)/2048}}{2^{3/2048}} \approx 1.002575 + 0.0046138 i$$

•

$$\frac{e^{\left(\sqrt{\frac{11}{2}} \pi\right)/2048} e^{(i \pi)/512}}{2^{3/2048}} \approx 1.002567 + 0.006152 i$$

Series representations:

$$64^2 \sqrt{\frac{1}{\frac{4096}{64} e^{-\pi \sqrt{22}}}} = \frac{4096 \sqrt{e^{\pi \sqrt{21} \sum_{k=0}^{\infty} 21^{-k} \binom{1/2}{k}}}}{2^{3/2048}}$$

•

$$64^2 \sqrt{\frac{1}{\frac{4096}{64} e^{-\pi \sqrt{22}}}} = \frac{4096 \sqrt{e^{\pi \sqrt{21} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{21}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}}{2^{3/2048}}$$

•

$$64^2 \sqrt{\frac{1}{\frac{4096}{64} e^{-\pi \sqrt{22}}}} = \frac{4096 \sqrt{\exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 21^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)}}{2^{3/2048}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

$\operatorname{Re}(z)$ is the real part of z

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

From:

$$-74 \cdot (1.220910 \cdot 10^{19})^{-1}$$

Input interpretation:

$$-\frac{74}{1.220910 \times 10^{19}}$$

Result:

$$-6.061052821256276056384172461524600502903571925858580... \times 10^{-18}$$

$$-6.061052821... \cdot 10^{-18} \text{ GeV}$$

specifically, the results correspond to $H(t) = H_0 t e^{-F(t-t^*)^2}$ and $\gamma(t) = \gamma_0 e^{3\Theta t} + H(t)$ with the parameter values $H_0 = 3 \times 10^{-5}$, $t^* = 0.5$, $F = 9 \times 10^{-5}$, $\gamma_0 = -0.0044$, $\Theta = 0.0046$.

And:

$$N(t) = \exp\left(\int_{t_0}^t H(t) dt\right) = \frac{a(t)}{a(t_0)}.$$

$$H(t) = -1.818274935... \times 10^{-22}$$

$$3 \times 10^{-5} (-6.061052821 \times 10^{-18}) \exp((-9 \times 10^{-5} (-6.061052821 \times 10^{-18} - 0.5)^2))$$

Input interpretation:

$$3 \times 10^{-5} (-6.061052821 \times 10^{-18}) \exp(-9 \times 10^{-5} (-6.061052821 \times 10^{-18} - 0.5)^2)$$

Result:

$$-1.818274935... \times 10^{-22}$$

$$-1.818274935... \times 10^{-22}$$

And from:

$$75 \times (1.220910 \times 10^{19})^{-1}$$

Input interpretation:

$$\frac{75}{1.220910 \times 10^{19}}$$

Result:

$$6.1429589404624419490380126299235815907806472221539671... \times 10^{-18}$$

$$6.14295894 \times 10^{-18}$$

We obtain:

$$\exp \text{ integrate } [-1.818274935 \times 10^{-22}]x, [-6.061052821 \times 10^{-18}, 6.14295894 \times 10^{-18}]$$

$$((((\text{integrate } [-1.818274935 \times 10^{-22}]x, [-6.061052821 \times 10^{-18}, 6.14295894 \times 10^{-18}]))))))$$

Definite integral:

$$\begin{array}{c}
 1 \\
 \hline
 1 + \frac{1}{2384 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{9 + \frac{1}{1 + \frac{1}{22 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{16 + \frac{1}{2 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{65 + \frac{1}{3 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}
 \end{array}$$

Possible closed forms:

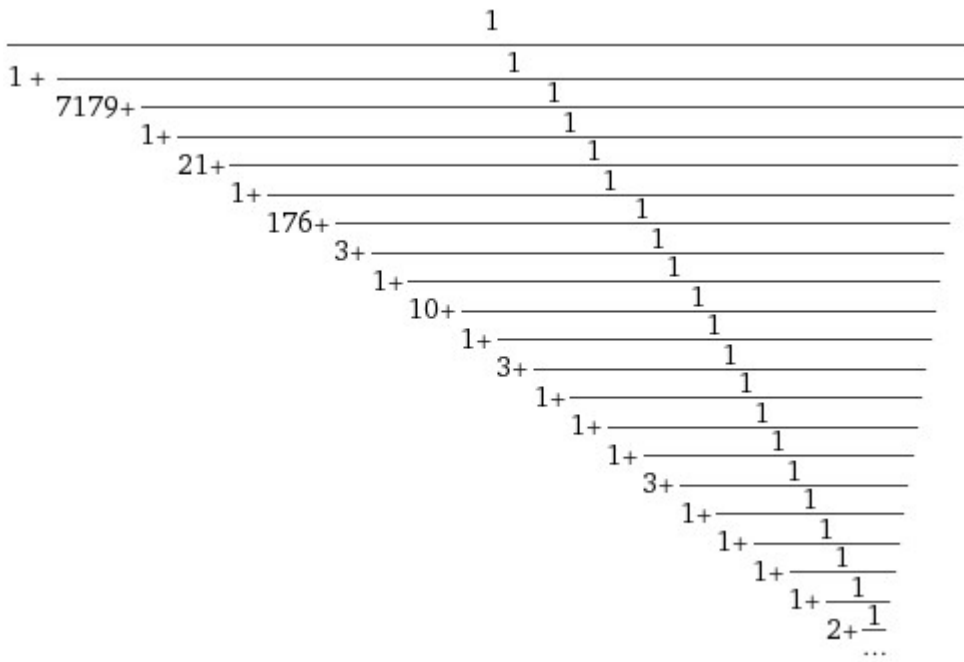
$$\pi \sqrt{\text{root of } 53848x^3 - 36503x^2 + 4667x + 476 \text{ near } x = 0.318176} \approx 0.99958082290127399366641$$

$$\frac{3567182686 \pi}{11211334455} \approx 0.9995808229012739934858905$$

$$\sqrt{\text{root of } 2882x^4 - 812x^3 - 4418x^2 + 2134x + 215 \text{ near } x = 0.999581} \approx 0.999580822901273993490563$$

With 0.99986074278566397 ..., we have:

Continued fraction:



Possible closed forms:

$$\frac{991\,863\,407\pi}{3\,116\,464\,783} \approx 0.99986074278566397710746$$

$$\left(\frac{4\,974\,527}{154\,273\,785} \right)^{2/3} \pi^2 \approx 0.999860742785663991964$$

root of $30\,365x^3 + 4760x^2 - 43\,154x + 8037$ near $x = 0.999861$ \approx
 0.999860742785663977079208

For:

$$(M_{PI} \equiv 1)$$

We obtain: $t_{beg} = -74^{-1}$

Exact result:

$$-\frac{1}{74} \text{ (irreducible)}$$

Decimal approximation:

-0.01351...
 -0.0135135135....

And:

$$t_{fin} = 75^{-1}$$

$$\frac{32768\sqrt{\frac{149}{222}}}{16384\sqrt{5}}$$

Decimal approximation:

0.999889605492701564131702714095786478007353000854370763366...

0.9998896054927.....

$$1/((((75^{-1})+(74^{-1}))))^{1/(4096*8)}$$

Input:

$$\frac{1}{4096 \times 8 \sqrt{\frac{1}{75} + \frac{1}{74}}}$$

Result:

$$32768\sqrt{\frac{222}{149}} \quad 16384\sqrt{5}$$

Decimal approximation:

1.000110406695591198107003912527718644342139791745090581474...

1.00011040669559.....

Alternate form:

$$\frac{1}{149} \quad 16384\sqrt{5} \quad 32768\sqrt{222} \quad 149^{32767/32768}$$

$$1/((((222/149)^{(1/32768)} 5^{(1/16384)}))))$$

Input:

$$\frac{1}{32768\sqrt{\frac{222}{149}} \quad 16384\sqrt{5}}$$

Result:

$$\frac{32768\sqrt{\frac{149}{222}}}{16384\sqrt{5}}$$

Decimal approximation:

0.999889605492701564131702714095786478007353000854370763366...

0.9998896054927....

The two results 0.9998896054927 are similar and very near to the following value of Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

From:

shape) and $\gamma(t) = \gamma_0 e^{3\Theta t} + H(t)$ (dotted red curve) for the parameter values $H_0 = 3 \times 10^{-5}$, $t_* = 0.5$, $F = 7 \times 10^{-5}$, $\gamma_0 = -0.0044$, and $\Theta = 4.6 \times 10^{-6}$. After the bounce and shortly

$$\gamma_0 = 2, t = 0.5$$

$$\gamma(t) = 2 * \exp((3 * (4.6 * 10^{-6}) * 0.5)) - 1.818274935e-22$$

Input interpretation:

$$2 \exp(3 \times 4.6 \times 10^{-6} \times 0.5) - 1.818274935 \times 10^{-22}$$

Result:

2.000013800047610109503007065442172191272970715463296571597...

2.00001380004761....

$$\phi(t) = \phi_0 + \int_{t_0}^t \sqrt[3]{2(H - \gamma)} dt$$

$$= \text{integrate} [(2 * (3 * 10^{-5} - 2))^{1/3}]x, [-6.061052821e-18, 0.5]$$

Definite integral:

$$\int_{-6.061052821 \times 10^{-18}}^{0.5} \sqrt[3]{2(3 \times 10^{-5} - 2)} x dx = 0.0992121 + 0.17184 i$$

Indefinite integral:

$$\int \sqrt[3]{2(3 \times 10^{-5} - 2)} x dx = \frac{\sqrt[3]{-\frac{199997}{2} x^2}}{20 \times 5^{2/3}} + \text{constant}$$

Input interpretation:

0.0992121 + 0.17184 *i*

i is the imaginary unit

Result:

0.0992121... +
0.17184... *i*

Polar coordinates:

r = 0.198424 (radius), *θ* = 59.9999° (angle)

0.198424

Possible closed forms:

$$\log\left(\frac{1}{8} (4 - e + 6 e^2 + 4 \pi - 5 \pi^2)\right) + \frac{i \sqrt{2}}{7 \sqrt[4]{\theta_t}} \approx$$

0.0992120319912360 + 0.1718393517833607 *i*

$$\log\left(\frac{1}{8} (4 - e + 6 e^2 + 4 \pi - 5 \pi^2)\right) + i \cos^2\left(\frac{2}{5} (\pi - 6)\right) \approx$$

0.0992120319912360 + 0.1718404847387741 *i*

$$\frac{1}{\text{root of } x^3 - 10x^2 - x + 2 \text{ near } x = 10.0795} + \frac{537 i}{3125} \approx$$

0.0992110207122695 + 0.1718400000000000 *i*

log(*x*) is the natural logarithm

θ_t is the tetrahedral angle

$$\phi(t) = \phi_0 + 0.198424$$

Thence, from:

$$H(t) = -1.818274935e-22; H_0 = 3 * 10^{-5}$$

$$k(t) = -2 (3H^2 - 2V + 2\dot{H} + \dot{\gamma} + 3H\gamma) / \dot{\phi}^2(t), \quad (18)$$

$$q(t) = \frac{4}{3} (2\dot{H} + \dot{\gamma} + 9H\gamma - 3V) / \dot{\phi}^4(t) + \frac{2}{3} b', \quad (19)$$

We obtain:

$$4/3((((((2*3*10^{-5})+9*2(-1.818274935e-22)+ 2.00001380004761-3)))) / (((((((\phi + 0.198424)^4))) + 2/3))))$$

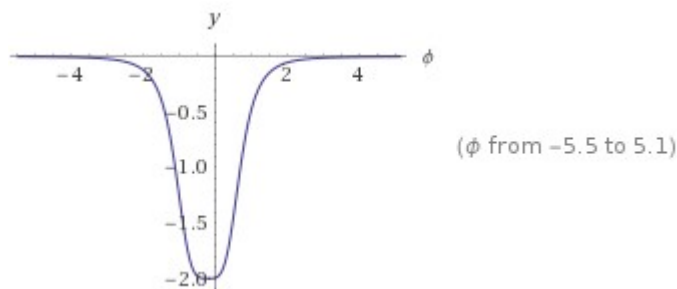
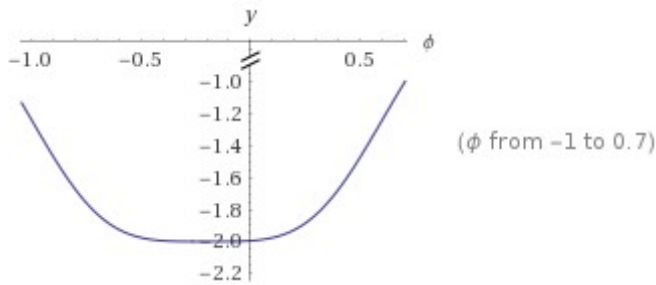
Input interpretation:

$$\frac{4}{3} \times \frac{2 \times 3 \times 10^{-5} + 9 \times 2 (-1.818274935 \times 10^{-22}) + 2.00001380004761 - 3}{(\phi + 0.198424)^4 + \frac{2}{3}}$$

Result:

$$\frac{1.33323}{(\phi + 0.198424)^4 + \frac{2}{3}}$$

Plots:



Alternate forms:

$$\frac{1.33323}{\phi (\phi (\phi (\phi + 0.793696) + 0.236233) + 0.0312495) + 0.668217}$$

$$\frac{1.33323}{(\phi^2 - 0.881038 \phi + 0.602305)(\phi^2 + 1.67473 \phi + 1.10943)}$$

$$\frac{3.9997}{3 \phi^4 + 2.38109 \phi^3 + 0.708698 \phi^2 + 0.0937484 \phi + 2.00465}$$

Partial fraction expansion:

$$\frac{-0.638896 \phi - 0.943209}{\phi^2 + 1.67473 \phi + 1.10943} + \frac{0.638896 \phi - 0.689664}{\phi^2 - 0.881038 \phi + 0.602305}$$

Expanded form:

$$\frac{1.33323}{\phi^4 + 0.793696 \phi^3 + 0.236233 \phi^2 + 0.0312495 \phi + 0.668217}$$

Alternate form assuming ϕ is real:

$$0 - \frac{1.33323}{(\phi + 0.198424)^4 + \frac{2}{3}}$$

Roots:

(no roots exist)

Series expansion at $\phi = 0$:

$$-1.99521 + 0.0933071 \phi + 0.700998 \phi^2 + 2.30411 \phi^3 + 2.51947 \phi^4 + O(\phi^5)$$

(Taylor series)

Series expansion at $\phi = \infty$:

$$-\frac{1.33323}{\phi^4} + \frac{1.05818}{\phi^5} - \frac{0.524922}{\phi^6} + \frac{0.208314}{\phi^7} + O\left(\left(\frac{1}{\phi}\right)^8\right)$$

(Laurent series)

Derivative:

$$\frac{d}{d\phi} \left(-\frac{1.33323}{(\phi + 0.198424)^4 + \frac{2}{3}} \right) = \frac{5.33294 (\phi + 0.198424)^3}{\left((\phi + 0.198424)^4 + \frac{2}{3} \right)^2}$$

Indefinite integral:

$$\int \frac{4(23 \times 10^{-5} + 9 \times 2(-1.818274935 \times 10^{-22}) + 2.00001380004761 - 3)}{3 \left((\phi + 0.198424)^4 + \frac{2}{3} \right)} d\phi =$$

$$0.638896 \tan^{-1}(0.68945 - 1.56508 \phi) +$$

$$0.319448 \log(2.44949 \phi^2 - 2.15809 \phi + 1.47534) -$$

$$0.319448 \log(2.44949 \phi^2 + 4.10224 \phi + 2.71754) -$$

$$0.638896 \tan^{-1}(1.56508 \phi + 1.31055) + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Global minimum:

$$\min \left\{ -\frac{1.33323}{(\phi + 0.198424)^4 + \frac{2}{3}} \right\} = -\frac{274274973}{137147608} \text{ at } \phi = -\frac{24803}{125000}$$

Limit:

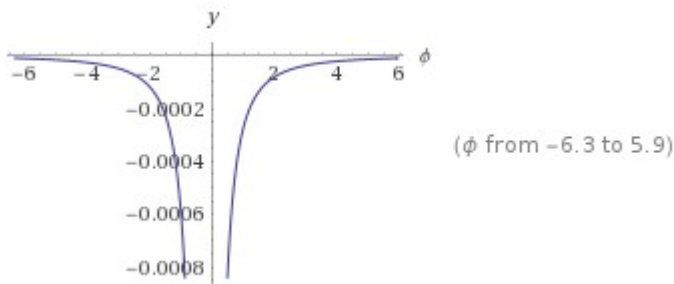
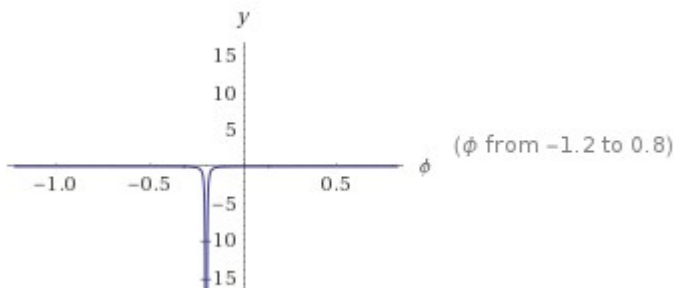
Input interpretation:

$$\frac{-2 \times 3(3 \times 10^{-5})^2 - 2 + 2(-1.818274935 \times 10^{-22}) + 2.00001380004761 + 3 \times 3 \times 10^{-5} \times 2}{(\phi + 0.198424)^2}$$

Result:

$$\frac{0.000387605}{(\phi + 0.198424)^2}$$

Plots:



Alternate form:

$$\frac{0.000387605}{\phi(\phi + 0.396848) + 0.0393721}$$

Partial fraction expansion:

$$\frac{497537.}{\phi + 0.198424} - \frac{497537.}{\phi + 0.198424} \quad (\text{for } \phi \neq -0.198424)$$

Expanded form:

$$\frac{0.000387605}{\phi^2 + 0.396848\phi + 0.0393721}$$

Alternate form assuming ϕ is real:

$$0 - \frac{0.000387605}{(\phi + 0.198424)^2}$$

Roots:

(no roots exist)

•

Series expansion at $\phi = 0$:

$$-0.00984468 + 0.0992287 \phi - 0.750126 \phi^2 + 5.04056 \phi^3 - 31.7537 \phi^4 + O(\phi^5)$$

(Taylor series)

•

Series expansion at $\phi = \infty$:

$$-\frac{0.000387605}{\phi^2} + \frac{0.00015382}{\phi^3} - \frac{0.0000457825}{\phi^4} + \frac{0.0000121125}{\phi^5} + O\left(\left(\frac{1}{\phi}\right)^6\right)$$

(Laurent series)

•

Derivative:

$$\frac{d}{d\phi} \left(-\frac{0.000387605}{(\phi + 0.198424)^2} \right) = \frac{0.000775211}{(\phi + 0.198424)^3}$$

•

Indefinite integral:

$$\int -\frac{1}{(\phi + 0.198424)^2} \left(2(3(3 \times 10^{-5})^2 - 2) + 2(-1.818274935 \times 10^{-22}) + 2.00001380004761 + 3 \times 10^{-5} \right) d\phi = \frac{0.000387605}{\phi + 0.198424} + \text{constant}$$

•

Limit:

$$\lim_{\phi \rightarrow \pm\infty} -\frac{0.000387605}{(0.198424 + \phi)^2} = 0 \approx 0$$

For $\phi = 0.989117352243$, i.e. the calculate dilaton value **0.989117352243 = ϕ** , we obtain:

Input interpretation:

$$-\frac{0.000387605}{(0.198424 + 0.989117352243)^2}$$

Result:

-0.00027484756126673728872995019392604557727020029176948306...

-0.0002748475...

$$-1/(((-0.000387605 / (0.198424 + 0.989117352243)^2)))$$

Input interpretation:

$$\frac{-1}{\frac{0.000387605}{(0.198424 + 0.989117352243)^2}}$$

Result:

3638.380473128914748599860682911727144902671534164935952838...

3638.3804731... result very near to the rest mass of double charmed Xi baryon
3621.40

Multiplying the two results and performing some calculations, we obtain:

$$-64 + ((((((((-1.33323 / (2/3 + (0.198424 + 0.989117352243)^4)))))) / ((((-0.000387605 / (0.198424 + 0.989117352243)^2))))))))))$$

Input interpretation:

$$-64 + \frac{-\frac{1.33323}{\frac{2}{3} + (0.198424 + 0.989117352243)^4}}{\frac{0.000387605}{(0.198424 + 0.989117352243)^2}}$$

Result:

1762.709337168208681224683828716910631596634483005397152576...

1762.709337168... result in the range of the mass of candidate “glueball” $f_0(1710)$ and the hypothetical mass of Gluino (“glueball” = 1760 ± 15 MeV; gluino = 1785.16 GeV).

From:

$$A(t) = \frac{k\dot{\phi}^2 + (3q - 2b')\dot{\phi}^4 + 6Hb\dot{\phi}^3 + \frac{3}{2}b^2\dot{\phi}^6}{2\left(H - \frac{1}{2}b\dot{\phi}^3\right)^2}, \quad (12)$$

$$A(t) = 3 + \frac{6H^2 - 4V + 2\dot{H} + \dot{\gamma} + 3H\gamma}{\gamma^2}.$$

We obtain, for $H = 3 \times 10^{-5}$; $H_0 = -1.818274935e-22$; $\gamma(t) = 2.00001380004761$ $\gamma = 2$

$$3 + \frac{1}{4} \left(\left(\left(\left(6 \times (3 \times 10^{-5})^2 - 4 + 2 \times (-1.818274935e-22) + 2.00001380004761 + 3 \times 2 \times (3 \times 10^{-5}) \right) \right) \right) \right)$$

Input interpretation:

$$3 + \frac{1}{4} \left(6(3 \times 10^{-5})^2 - 4 + 2(-1.818274935 \times 10^{-22}) + 2.00001380004761 + 3 \times 2 \times 3 \times 10^{-5} \right)$$

Result:

2.500048451361902499999990908625325

2.5000484513...

From:

$$\left(\left(\left(\left(\left(-1.33323 / \left(\frac{2}{3} + (0.198424 + 0.989117352243)^4 \right) \right) \right) \right) \right) + \left(\left(\left(-0.000387605 / (0.198424 + 0.989117352243)^2 \right) \right) \right) \right)$$

Input interpretation:

$$\frac{1.33323}{\frac{2}{3} + (0.198424 + 0.989117352243)^4} - \frac{0.000387605}{(0.198424 + 0.989117352243)^2}$$

Result:

-0.50234145402512703595629321187329147822301217264265048881...

-0.502341454...

(822.978)/((-
0.50234145402512703595629321187329147822301217264265048881/-
((((3+1/4((((6*(3*10^-5)^2-4+2*(-1.818274935e-22)+
2.00001380004761+3*2*(3*10^-5)))))))))))))

Input interpretation:

$$\frac{822.978}{\frac{0.50234145402512703595629321187329147822301217264265048881}{3+\frac{1}{4}(6(3 \times 10^{-5})^2-4+2(-1.818274935 \times 10^{-22})+2.00001380004761+3 \times 2 \times 3 \times 10^{-5})}}$$

Result:

4095.789542986832109079505327511044544530213198693027778803...
4095.7895... ≈ 4096 = 64²

Where 822.978 is the result of the following 5th order Ramanujan mock theta function $\psi(q)$:

(from OEIS – sequence A282537)

$$\text{Sum}_{\{k \geq 0\}} x^{(5*k^2)} / ((1 - x^2) * (1 - x^3) * (1 - x^7) * (1 - x^8)...(1 - x^{(5*k+2)})).$$

For k = 0 to ∞, we obtain:

$$\text{sum} (((((0.913899^{(5*k^2)} / ((1 - 0.913899^2) * (1 - 0.913899^3) * (1 - 0.913899^7) * (1 - 0.913899^8)(1 - 0.913899^{(5*k+2)}))))))), k = 0..infinity$$

Input interpretation:

$$\sum_{k=0}^{\infty} \frac{0.913899^{5k^2}}{(1 - 0.913899^2)(1 - 0.913899^3)(1 - 0.913899^7)(1 - 0.913899^8)(1 - 0.913899^{5k+2})}$$

Approximated sum:

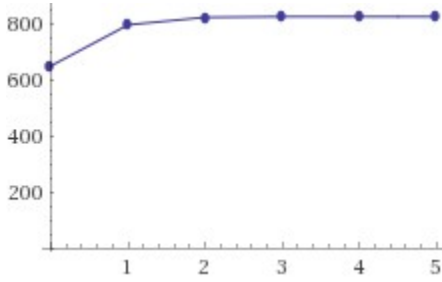
$$\sum_{k=0}^{\infty} \frac{0.913899^{5k^2}}{(1 - 0.913899^2)(1 - 0.913899^3)(1 - 0.913899^7)(1 - 0.913899^8)(1 - 0.913899^{5k+2})} \approx 822.978$$

822.978

Convergence tests:

By the ratio test, the series converges.

Partial sums:



From:

On the Cosmological Implications of the String Swampland

Prateek Agrawal, Georges Obied, Paul J. Steinhardt, Cumrun Vafa

arXiv:1806.09718v2 [hep-th] 19 Jul 2018

We have that:

The experimental bound we have found for single exponential potential with constant λ agrees reasonably with the analysis in [29] based on older data. A good analytical approximation for the limiting trajectory can be found,

$$x \approx \frac{c}{\sqrt{6}} \left(1 - \frac{1-\Omega}{\sqrt{\Omega}} \tanh^{-1}(\sqrt{\Omega}) \right) \sim \frac{2}{3} \frac{c\Omega}{\sqrt{6}} \quad (15)$$

where in the last term we have used a first order approximation that will be more convenient. This gives a lower bound on $1+w$ for today:

$$1+w(z=0) \gtrsim \frac{4}{27} c^2 \Omega_\phi^0 \quad (16)$$

The above derivation assumes that the net field excursion in ϕ up until the present is less than Δ , the maximum allowed by Criterion 1. Indeed for the limiting exponential potential we find

$$\Delta\phi = \sqrt{6} \int x dN \simeq \frac{1}{3} c \Omega_\phi^0 \quad (17)$$

Interestingly, this provides an observational restriction on the Swampland criterion, namely,

$$\Delta \gtrsim \frac{1}{3} c \Omega_\phi^0. \quad (18)$$

From (17) and (18), we have:

$$64^2 \sqrt{\frac{2 \times 0.6 \times 0.7}{3 \sqrt{6}}}$$

Result:

0.999470636888088737227855633735773360829694093271130491302...

0.999470636888.... result very near to the following value of Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

If $\lambda(\phi)$ stays below $\sqrt{3}$, x increases approaching the value $\lambda/\sqrt{6}$. Since the field keeps rolling, Criterion 1 is violated after a finite time. This would imply a breakdown of the effective field theory. The universe would enter a new phase in which a large number of previously massive states become light. This happens when,

$$\Delta\phi = \sqrt{6} \int x dN = \Delta \text{ or } N < \frac{\Delta}{\sqrt{6}x(N=0)} \quad (19)$$

$$\Rightarrow N \lesssim \frac{3\Delta}{2c \Omega_{\phi}^0} \quad (20)$$

$$3 \times 0.14 / 2 \times 0.6 \times 0.7$$

$$(3 \times 0.14) / (2 \times 0.6 \times 0.7)$$

Input:

$$\frac{3 \times 0.14}{2 \times 0.6 \times 0.7}$$

Result:

0.5

0.5

From this result, we can to obtain ϕ , that, here, is equal to:

$$(3 \cdot 0.14) / (2 \cdot 0.6 \cdot 0.7) \cdot 1/0.14$$

Input:

$$\frac{3 \times 0.14}{2 \times 0.6 \times 0.7} \times \frac{1}{0.14}$$

Result:

3.571428571428571428571428571428571428571428571428571428571...

Repeating decimal:

3.571428 (period 6)

3.571428

Indeed: $3.571428 \cdot 0.14 = 0.5 = 1/2$

We note that this result is very near to the following sum of the values of Rogers-Ramanujan continued fraction:

$$2,0663656771 + 0,9568666373 + 0,5269391135 = 3,5501714279$$

Indeed:

$$2 \int_0^{\infty} \frac{t^2 dt}{e^{\sqrt{3}t} \sinh t} = \frac{1}{1 + \frac{1}{1 + \frac{1^3}{1 + \frac{2^3}{3 + \frac{2^3}{1 + \frac{3^3}{5 + \frac{3^3}{1 + \frac{3^3}{7 + \dots}}}}}}}} \approx 0.5269391135$$

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

$$\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!} + \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \dots}}}}} \approx 2.0663656771$$

And also:

$$\left(\left(\left(\left(\left(3 \times 0.14\right) / \left(2 \times 0.6 \times 0.7\right) \times \frac{1}{0.14}\right)\right)\right)\right)^{\left(\frac{1}{64 \times 11}\right)}$$

Input:

$$64 \times 11 \sqrt{\frac{3 \times 0.14}{2 \times 0.6 \times 0.7} \times \frac{1}{0.14}}$$

Result:

1.001809825641511493517985570625170027108757554207848466732...

1.00180982564.... result very near to the following value of Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}} \approx 1.0018674362$$

We can parametrize the scalar potential for $V > 0$ without loss of generality as

$$V(\phi) = V_0 \exp \left(- \int_0^\phi \frac{d\phi'}{M(\phi')} \right) \quad (21)$$

such that $|M_{pl} \nabla_\phi V/V| \equiv M_{pl}/M(\phi)$, where we have restored explicit factors of M_{pl} for illustration. Then, assuming that the initial value of the potential is $\mathcal{O}(M_{pl}^4)$, we find that

$$\int_0^\phi \frac{d\phi'}{M(\phi')} = \Delta\phi \left\langle \frac{1}{M(\phi)} \right\rangle = \log \frac{\Lambda}{M_{pl}^4} \simeq 280 \quad (22)$$

From

$$V(\phi) = V_0 \exp \left(- \int_0^\phi \frac{d\phi'}{M(\phi')} \right) \quad (21)$$

$$\int_0^\phi \frac{d\phi'}{M(\phi')} = \Delta\phi \left\langle \frac{1}{M(\phi)} \right\rangle = \log \frac{\Lambda}{M_{pl}^4} \simeq 280 \quad (22)$$

We have that:

Coefficients of the 3rd order mock theta function $f(q)$ (OEIS - sequence A000025)

Formula:

$$1 + \text{Sum}_{\{n>0\}} (q^{(n^2)} / \text{Product}_{\{i=1..n\}} (1 + q^i)^2)$$

From this formula, we obtain, for $n = 41$:

$$1 + (q^{(n^2)} / (1 + q^i)^2)$$

$$1 + (1.0041828^{(41^2)} / (1 + 1.0041828^i)^2)$$

Input interpretation:

$$1 + \frac{1.0041828^{41^2}}{(1 + 1.0041828^i)^2}$$

i is the imaginary unit

Result:

$$279.752... - 1.16354... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 279.755$ (radius), $\theta = -0.238302^\circ$ (angle)

$279.755 \approx 280$

Thence an interesting new mathematical connection between

$$\int_0^\phi \frac{d\phi'}{M(\phi')} = \Delta\phi \left\langle \frac{1}{M(\phi)} \right\rangle = \log \frac{\Lambda}{M_{pl}^4} \simeq 280 \quad (22)$$

and

$$\left(1 + \frac{1.0041828^{41^2}}{(1 + 1.0041828^i)^2} \right) = 279.755 \approx 280$$

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Prateek Agrawal, Georges Obied, Paul J. Steinhardt, Cumrun Vafa

arXiv:1806.09718v2 [hep-th] 19 Jul 2018