

On some new possible mathematical connections between some equations of the Ramanujan's manuscripts, the Rogers-Ramanujan continued fractions and some sectors of Particle Physics, String Theory and D-branes

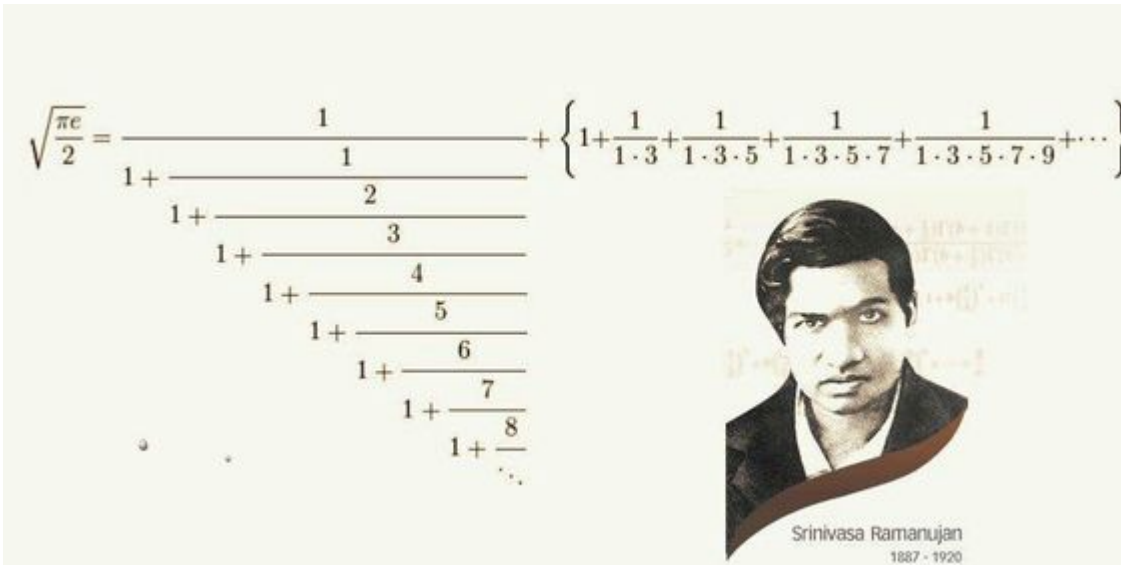
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Abstract

In this research thesis, we have described some new mathematical connections between some equations of the Ramanujan's manuscripts, the Rogers-Ramanujan continued fractions and some sectors of Particle Physics (physical parameters of mesons and dilatons, in particular the values of the masses), String Theory and D-branes.

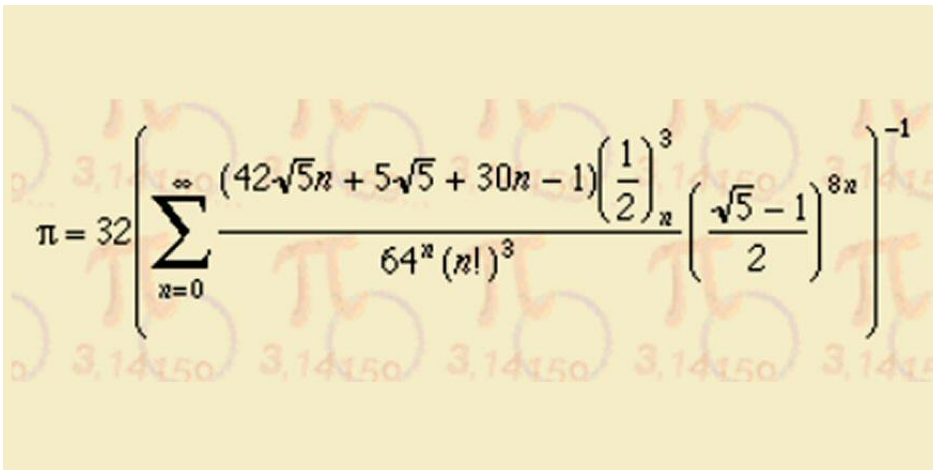
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<https://www.pinterest.nz/pin/132434045270275344/?autologin=true&nic=1a>



From Ramanujan manuscript books:

<https://twitter.com/aarvee18/status/679181194668326912>



For n = 0.1182, we obtain:

$32 * (((((((((((((((((42\sqrt{5} * 0.1182 + 5\sqrt{5} + 30 * 0.1182 - 1) / (8 * 0.1182)))))) / (((64)^{(0.1182)} * (1!)^3))) * (((\sqrt{5} - 1) / 2)^{(8 * 0.1182)))))))))^{-1}$

Input:

$$\frac{32}{\frac{(42\sqrt{5} \times 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1) \times \frac{1}{8 \times 0.1182}}{64^{0.1182} (1!)^3}} \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8 \times 0.1182}$$

$n!$ is the factorial function

Result:

3.14081...

3.14081...

Alternative representations:

$$\frac{32}{\frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8 \times 0.1182} (42\sqrt{5} \times 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)}{(8 \times 0.1182)(64^{0.1182} (1!)^3)}} = \frac{32}{\frac{\left(\frac{1}{2}(-1 + \sqrt{5})\right)^{0.9456} (2.546 + 9.9644\sqrt{5})}{0.9456(64^{0.1182} \Gamma(2)^3)}}$$

•

$$\frac{32}{\frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8 \times 0.1182} (42\sqrt{5} \times 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)}{(8 \times 0.1182)(64^{0.1182} (1!)^3)}} = \frac{32}{\frac{\left(\frac{1}{2}(-1 + \sqrt{5})\right)^{0.9456} (2.546 + 9.9644\sqrt{5})}{0.9456(64^{0.1182} \Gamma(2,0)^3)}}$$

•

$$\frac{32}{\frac{\left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8 \times 0.1182} (42\sqrt{5} \times 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)}{(8 \times 0.1182)(64^{0.1182} (1!)^3)}} = \frac{32}{\frac{\left(\frac{1}{2}(-1 + \sqrt{5})\right)^{0.9456} (2.546 + 9.9644\sqrt{5})}{0.9456(64^{0.1182} ((1)_1)^3)}}$$

$\Gamma(x)$ is the gamma function

$\Gamma(a, x)$ is the incomplete gamma function

$(a)_n$ is the Pochhammer symbol (rising factorial)

Series representations:

$$\frac{32}{\left(\frac{1}{2}(\sqrt{5}-1)\right)^{8 \times 0.1182} (42\sqrt{5} + 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)} = \frac{9.56204 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)^3}{(8 \times 0.1182)(64^{0.1182} (1!)^3)} = \frac{9.56204 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)^3}{(-1 + \sqrt{5})^{0.9456} (0.25551 + \sqrt{5})}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1$

•

$$\frac{32}{\left(\frac{1}{2}(\sqrt{5}-1)\right)^{8 \times 0.1182} (42\sqrt{5} + 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)} = \frac{9.56204 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)^3}{(8 \times 0.1182)(64^{0.1182} (1!)^3)} = \frac{9.56204 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)^3}{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^{0.9456} \left(0.25551 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1$

•

$$\frac{32}{\left(\frac{1}{2}(\sqrt{5}-1)\right)^{8 \times 0.1182} (42\sqrt{5} + 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)} = \frac{9.56204 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)^3}{(8 \times 0.1182)(64^{0.1182} (1!)^3)} = \frac{9.56204 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}\right)^3}{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^{0.9456} \left(0.25551 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}$$

for $(n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0)$ and $n_0 \rightarrow 1$

\mathbb{Z} is the set of integers

$\binom{n}{m}$ is the binomial coefficient

Integral representations:

$$\frac{32}{\left(\frac{1}{2}(\sqrt{5}-1)\right)^{8 \times 0.1182} (42\sqrt{5} + 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)} = \frac{9.56204 \left(\int_0^1 \log\left(\frac{1}{t}\right) dt\right)^3}{(8 \times 0.1182)(64^{0.1182} (1!)^3)} = \frac{9.56204 \left(\int_0^1 \log\left(\frac{1}{t}\right) dt\right)^3}{(-1 + \sqrt{5})^{0.9456} (0.25551 + \sqrt{5})}$$

•

$$\frac{32}{\left(\frac{1}{2}(\sqrt{5}-1)\right)^{8 \times 0.1182} (42\sqrt{5} + 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)} = \frac{9.56204 \left(\int_0^{\infty} e^{-t} t dt\right)^3}{(8 \times 0.1182)(64^{0.1182} (1!)^3)} = \frac{9.56204 \left(\int_0^{\infty} e^{-t} t dt\right)^3}{(-1 + \sqrt{5})^{0.9456} (0.25551 + \sqrt{5})}$$

•

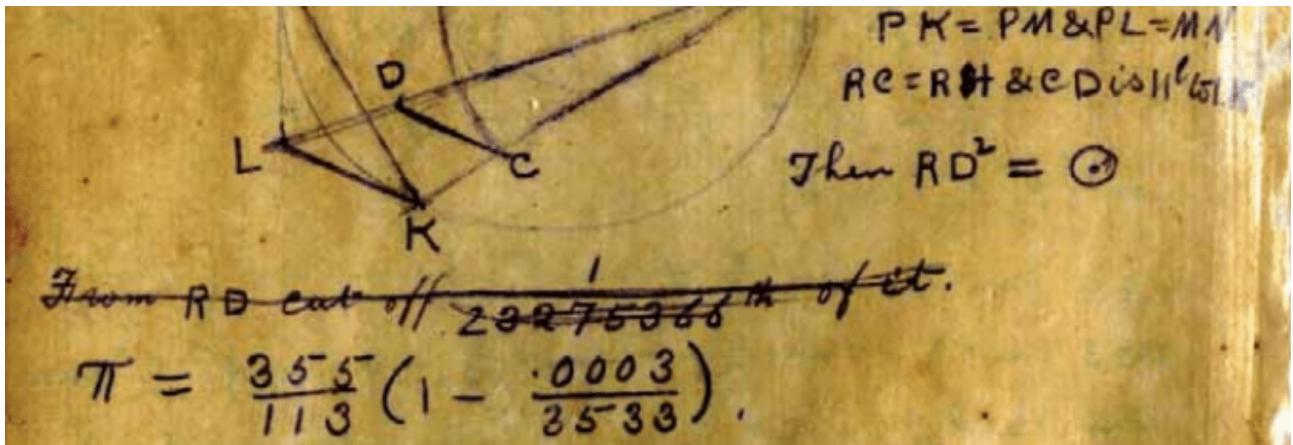
$$\frac{32}{\frac{(\frac{1}{2}(\sqrt{5}-1))^{8 \times 0.1182} (42\sqrt{5} \cdot 0.1182 + 5\sqrt{5} + 30 \times 0.1182 - 1)}{(8 \times 0.1182)(64^{0.1182} (1!)^3)}} = \frac{9.56204 \left(\int_1^\infty e^{-t} t dt + \sum_{k=0}^\infty \frac{(-1)^k}{(2+k)k!} \right)^3}{(-1 + \sqrt{5})^{0.9456} (0.25551 + \sqrt{5})}$$

$\log(x)$ is the natural logarithm

Note that:

${}^{4096}\sqrt{0.1182} = 0.9994788035522$ result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$



$$355/113 (1 - (0.0003/3533))$$

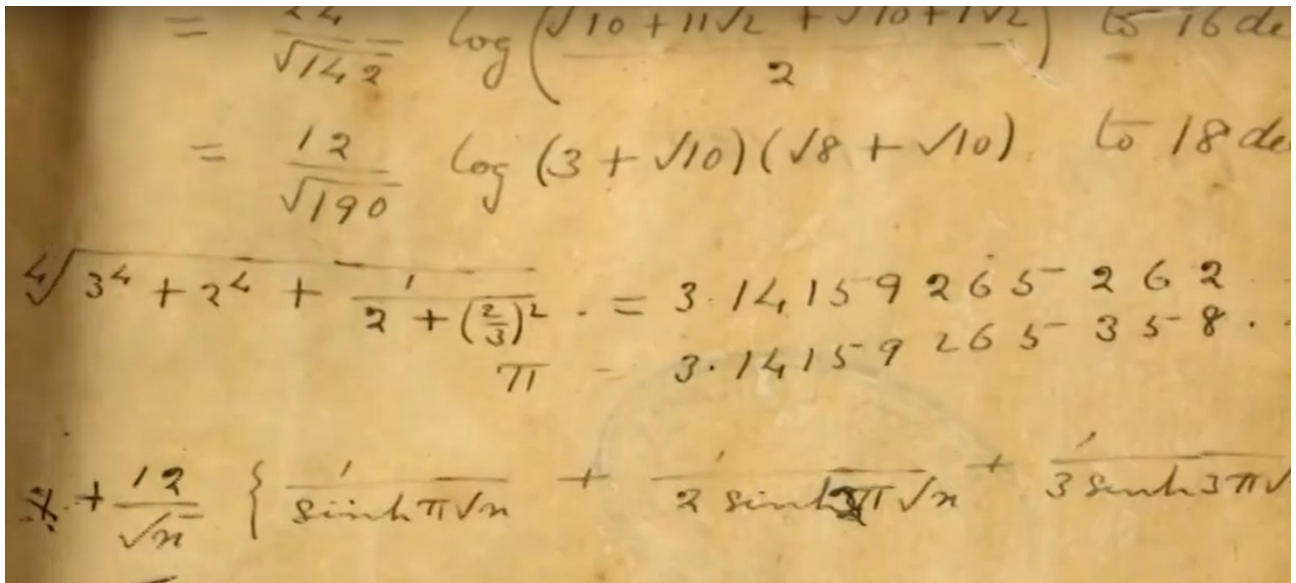
Input:

$$\frac{355}{113} \left(1 - \frac{0.0003}{3533} \right)$$

Result:

3.141592653589794328568315427987445801782936610316384831763...

3.14159265358979....



$$(((3^4+2^4+1/(2+(2/3)^2)))^{1/4})$$

Input:

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}}$$

Result:

$$\sqrt[4]{\frac{2143}{22}}$$

Decimal approximation:

3.141592652582646125206037179644022371557877983160126149695...

3.1415926525826....

Alternate form:

$$\frac{1}{22} \sqrt[4]{2143} 22^{3/4}$$

All 4th roots of 2143/22:

- Polar form

$$\sqrt[4]{\frac{2143}{22}} e^0 \approx 3.14159 \text{ (real, principal root)}$$

-

$$\sqrt[4]{\frac{2143}{22}} e^{(i\pi)/2} \approx 3.14159 i$$

-

$$\sqrt[4]{\frac{2143}{22}} e^{i\pi} \approx -3.1416 \text{ (real root)}$$

-

$$\sqrt[4]{\frac{2143}{22}} e^{-(i\pi)/2} \approx -3.1416 i$$

-

$$((((12/(\sqrt{190})))) * \ln(3+\sqrt{10}) ((\sqrt{8}+\sqrt{10}))))$$

Input:

$$\frac{12}{\sqrt{190}} \log(3 + \sqrt{10}) (\sqrt{8} + \sqrt{10})$$

$\log(x)$ is the natural logarithm

Exact result:

$$6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10})$$

Decimal approximation:

9.483810831223788502416854523775033221924803785680809814645...

9.483810831223...

Property:

$$6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10}) \text{ is a transcendental number}$$

Alternate forms:

$$\frac{12(2 + \sqrt{5}) \sinh^{-1}(3)}{\sqrt{95}}$$

-

$$\frac{12(2 + \sqrt{5}) \log(3 + \sqrt{10})}{\sqrt{95}}$$

- $$\sqrt{\frac{1296}{95} + \frac{576}{19\sqrt{5}}} \log(3 + \sqrt{10})$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = \frac{12 \log_e(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}}$$

- $$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = \frac{12 \log(a) \log_a(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}}$$

- $$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = -\frac{12 \operatorname{Li}_1(-2 - \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}}$$

$\log_b(x)$ is the base- b logarithm
 $\operatorname{Li}_n(x)$ is the polylogarithm function

Series representations:

- $$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = \frac{12(2 + \sqrt{5}) \left(\log(2 + \sqrt{10}) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2+\sqrt{10}}\right)^k}{k} \right)}{\sqrt{95}}$$

- $$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = \frac{12(2 + \sqrt{5}) \left(2i\pi \left[\frac{\arg(3 + \sqrt{10} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 + \sqrt{10} - x)^k x^{-k}}{k} \right)}{\sqrt{95}} \quad \text{for } x < 0$$

- $$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \left(2i\pi \left[\frac{\arg(3 + \sqrt{10} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 + \sqrt{10} - x)^k x^{-k}}{k} \right) \quad \text{for } x < 0$$

$\arg(z)$ is the complex argument

[x] is the floor function

Integral representations:

$$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = \int_1^{3+\sqrt{10}} \frac{12(5 + 2\sqrt{5})}{5\sqrt{19}t} dt$$

$$\frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = -\frac{6i(2 + \sqrt{5})}{\sqrt{95}\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(2 + \sqrt{10})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

We note that the sum of the two expression is:

$$\left(\left(\left(\left(3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2} \right)^{1/4} \right) + \left(\left(\frac{12}{\sqrt{190}} \right) \ln(3 + \sqrt{10}) \right) \right) \right) \left(\sqrt{8} + \sqrt{10} \right)$$

Input:

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12}{\sqrt{190}} \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})$$

$\log(x)$ is the natural logarithm

Exact result:

$$\sqrt[4]{\frac{2143}{22}} + 6\sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10})$$

Decimal approximation:

12.62540348380643462762289170341905559348268176884093596434...

12.625403483806.... result very near to the black hole entropy 12.5664

Property:

$$\sqrt[4]{\frac{2143}{22}} + 6\sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10}) \text{ is a transcendental number}$$

Alternate forms:

$$\sqrt[4]{\frac{2143}{22}} + \frac{12(2 + \sqrt{5}) \sinh^{-1}(3)}{\sqrt{95}}$$

$$\sqrt[4]{\frac{2143}{22}} + \sqrt{\frac{1296}{95} + \frac{576}{19\sqrt{5}}} \log(3 + \sqrt{10})$$

$$\sqrt[4]{\frac{2143}{22}} + \frac{12 \log(3 + \sqrt{10})}{\sqrt{19}} + \frac{24 \log(3 + \sqrt{10})}{\sqrt{95}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{2^4 + 3^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} + \frac{12 \log_e(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}}$$

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{2^4 + 3^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} + \frac{12 \log(a) \log_a(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}}$$

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{2^4 + 3^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} - \frac{12 \text{Li}_1(-2 - \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}}$$

$\log_b(x)$ is the base- b logarithm
 $\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{\frac{2143}{22}} + \frac{12(2 + \sqrt{5}) \left(\log(2 + \sqrt{10}) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2 + \sqrt{10}}\right)^k}{k} \right)}{\sqrt{95}}$$

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \left(\log(2 + \sqrt{10}) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2+\sqrt{10}}\right)^k}{k} \right)$$

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \left(2i\pi \left\lfloor \frac{\arg(3 + \sqrt{10} - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 + \sqrt{10} - x)^k x^{-k}}{k} \right) \text{ for } x < 0$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} =$$

$$\sqrt[4]{\frac{2143}{22}} + \int_1^{3+\sqrt{10}} \frac{12(5 + 2\sqrt{5})}{5\sqrt{19}t} dt$$

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{(\log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})) 12}{\sqrt{190}} = \sqrt[4]{\frac{2143}{22}} -$$

$$\frac{3i \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10})}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(2 + \sqrt{10})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

And:

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12}{\sqrt{190}} \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10}) \right) \right)^2$$

Input:

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12}{\sqrt{190}} \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10}) \right) \right)^2$$

Exact result:

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10}) \right)^2$$

Decimal approximation:

1.671425136759496420858896527649862156286021255496282713692...

1.671425136759.... result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_p = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Hamein)

Property:

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10}) \right)^2$$

is a transcendental number

Alternate forms:

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + \frac{12(2 + \sqrt{5}) \log(3 + \sqrt{10})}{\sqrt{95}} \right)^2$$

•

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + \sqrt{\frac{1296}{95} + \frac{576}{19\sqrt{5}}} \log(3 + \sqrt{10}) \right)^2$$

•

$$\frac{1}{228000} \left(19 \left(132 + 125 \sqrt{\frac{2143}{22}} \right) + 1800 (2\sqrt{2} + \sqrt{10})^2 \log^2(3 + \sqrt{10}) + 300 \sqrt{95} \sqrt[4]{\frac{4286}{11}} (2\sqrt{2} + \sqrt{10}) \log(3 + \sqrt{10}) \right)$$

Alternative representations:

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2 =$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{2^4 + 3^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log_e(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2 =$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{2^4 + 3^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(a) \log_a(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2 =$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{2^4 + 3^4 + \frac{1}{2 + (\frac{2}{3})^2}} - \frac{12 \text{Li}_1(-2 - \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2$$

$\log_b(x)$ is the base- b logarithm
 $\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2 =$$

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \left(\log(2 + \sqrt{10}) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2 + \sqrt{10}}\right)^k}{k} \right) \right)^2$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2 =$$

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \left(2i\pi \left\lfloor \frac{\arg(3 + \sqrt{10} - x)}{2\pi} \right\rfloor + \right. \right.$$

$$\left. \left. \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 + \sqrt{10} - x)^k x^{-k}}{k} \right) \right)^2 \text{ for } x < 0$$

$$\frac{11}{10^3} + \frac{1}{6} \left(\frac{1}{4} \left(\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (\frac{2}{3})^2}} + \frac{12 \log(3 + \sqrt{10})(\sqrt{8} + \sqrt{10})}{\sqrt{190}} \right) \right)^2 =$$

$$\frac{11}{1000} + \frac{1}{96} \left(\sqrt[4]{\frac{2143}{22}} + 6 \sqrt{\frac{2}{95}} (2\sqrt{2} + \sqrt{10}) \left(\log(z_0) + \left\lfloor \frac{\arg(3 + \sqrt{10} - z_0)}{2\pi} \right\rfloor \right) \right.$$

$$\left. \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 + \sqrt{10} - z_0)^k z_0^{-k}}{k} \right)^2$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

From:

Modular equations and approximations to π - Srinivasa Ramanujan
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{2}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \dots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \dots = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \dots = 64\left\{\left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\dots$$

From the following vacuum equations concerning the Brane Supersymmetry Breaking, we can obtain, putting

$4096 e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical solutions that are very near to the originals.

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

With $4096 e^{-\pi\sqrt{18}}$, for

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

we obtain:

$$\left(\frac{-(-1/2 * 0.02390591^2)}{(2.5)} \right) * 4096 * e^{(-\pi * \sqrt{18})}$$

Input interpretation:

$$-\frac{0.02390591^2}{2.5} \times 4096 e^{-\pi\sqrt{18}}$$

Result:

$$7.61802... \times 10^{-7}$$

$$7.61802.... * 10^{-7}$$

Series representations:

$$\frac{(4096 e^{-\pi \sqrt{18}})(-(-0.0239059^2))}{2 \times 2.5} = 0.468167 e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

- $$\frac{(4096 e^{-\pi \sqrt{18}})(-(-0.0239059^2))}{2 \times 2.5} = 0.468167 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

- $$\frac{(4096 e^{-\pi \sqrt{18}})(-(-0.0239059^2))}{2 \times 2.5} = 0.468167 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

With $4096 e^{-\pi \sqrt{18}}$, for

$$16 k' e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2 \beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2 \beta_E^{(p)} \phi}}{(7-p)}$$

We obtain:

$$1/2 * (((((0.02390591^2 * ((5+1-(2*1/2)/2.5))) * 4096 * e^{(-\pi * \text{sqrt}(18))}))$$

Input interpretation:

$$\frac{1}{2} \left(0.02390591^2 \left(\left(5 + 1 - \frac{2 \times \frac{1}{2}}{2.5} \right) \times 4096 e^{-\pi \sqrt{18}} \right) \right)$$

Result:

0.00001066522...

Result:

$1.0665220000 \times 10^{-5}$

$1.066522... * 10^{-5}$

Series representations:

$$\frac{1}{2} \times 0.0239059^2 \left(\left(5 + 1 - \frac{2}{2 \times 2.5} \right) 4096 e^{-\pi \sqrt{18}} \right) = 6.55433 e^{-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

•

$$\frac{1}{2} \times 0.0239059^2 \left(\left(5 + 1 - \frac{2}{2 \times 2.5} \right) 4096 e^{-\pi \sqrt{18}} \right) = 6.55433 \exp \left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17} \right)^k \binom{-\frac{1}{2}}{k}}{k!} \right)$$

•

$$\frac{1}{2} \times 0.0239059^2 \left(\left(5 + 1 - \frac{2}{2 \times 2.5} \right) 4096 e^{-\pi \sqrt{18}} \right) = 6.55433 \exp \left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}} \right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

With $4096 e^{-\pi \sqrt{18}}$, for

$$(A')^2 = k e^{-2A} \left| \frac{\hbar^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)}\psi} \right|$$

we obtain:

$$\exp(-2) + \left(\frac{0.02390591^2}{16 \times 6} \right) \left(7 - 5 + \frac{2 \times \frac{1}{2}}{2.5} \right) \times 4096 \cdot e^{(-\pi \sqrt{18})}$$

Input interpretation:

$$\exp(-2) + \frac{0.02390591^2}{16 \times 6} \left(7 - 5 + \frac{2 \times \frac{1}{2}}{2.5} \right) \times 4096 e^{-\pi \sqrt{18}}$$

Result:

0.1353353784618...

0.13533537...

Alternative representations:

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2} \right) 0.0239059^2 \times 4096 e^{-\pi \sqrt{18}}}{16 \times 6} =$$

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2} \right) 0.0239059^2 \times 4096 z^{-\pi \sqrt{18}}}{16 \times 6} \quad \text{for } z = e$$

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2} \right) 0.0239059^2 \times 4096 e^{-\pi \sqrt{18}}}{16 \times 6} =$$

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2} \right) 0.0239059^2 \times 4096 w^a}{16 \times 6} \quad \text{for } a = -\frac{3 \sqrt{2} \pi}{\log(w)}$$

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2} \right) 0.0239059^2 \times 4096 e^{-\pi \sqrt{18}}}{16 \times 6} =$$

$$\exp(-2) + \frac{4096}{96} \times 0.0239059^2 \left(2 + \frac{1}{2.5} \right) \left(1 + \frac{2}{-1 + \coth\left(-\frac{\pi \sqrt{18}}{2}\right)} \right)$$

$\log(x)$ is the natural logarithm

$\coth(x)$ is the hyperbolic cotangent function

Series representations:

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2}\right) 0.0239059^2 \times 4096 e^{-\pi \sqrt{18}}}{16 \times 6} =$$

$$\exp(-2) + 0.0585208 \sum_{k=0}^{\infty} \frac{(-\pi \sqrt{18})^k}{k!}$$

•

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2}\right) 0.0239059^2 \times 4096 e^{-\pi \sqrt{18}}}{16 \times 6} =$$

$$\exp(-2) + 0.0585208 \sum_{k=-\infty}^{\infty} I_k(-\pi \sqrt{18})$$

•

$$\exp(-2) + \frac{\left(7 - 5 + \frac{2}{2.5 \times 2}\right) 0.0239059^2 \times 4096 e^{-\pi \sqrt{18}}}{16 \times 6} =$$

$$\exp(-2) + 0.0585208 \sum_{k=-\infty}^{\infty} (-1)^k I_k(\pi \sqrt{18})$$

$n!$ is the factorial function

$I_n(z)$ is the modified Bessel function of the first kind

We have obtained, putting

$4096 e^{-\pi \sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

also a new possible mathematical connection between the two exponentials. Thence, also the values concerning p , C , β_E and ϕ correspond to the exponents of e (i.e. of \exp). Thence we can to obtain for $p = 5$ and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

With respect to the exponential of the vacuum equation, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2 , while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

We obtain:

$$\exp((-Pi*\sqrt{18}))$$

Input:

$$\exp(-\pi \sqrt{18})$$

Exact result:

$$e^{-3\sqrt{2}\pi}$$

Decimal approximation:

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

Property:

$e^{-3\sqrt{2}\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{1/2}{k}}$$

•

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \binom{-\frac{1}{2}}{k}}{k!}\right)$$

•

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

Now:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

We have that:

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

and:

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

and:

$(((((\exp(-\pi \sqrt{18})))))) * 1 / 0.000244140625$

Input interpretation:

$$\exp(-\pi \sqrt{18}) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785...

Series representations:

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} 17^{-k} \binom{\frac{1}{2}}{k}\right)$$

•

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\pi \sqrt{17} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{17}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

•

$$\frac{\exp(-\pi \sqrt{18})}{0.000244141} = 4096 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\dots$$

Now, we have that:

$$\ln(0.00666501784619)$$

Input interpretation:

$$\log(0.00666501784619)$$

$\log(x)$ is the natural logarithm

Result:

$$-5.010882647757\dots$$

$$-5.010882647757\dots$$

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

•

$$\log(0.006665017846190000) = \log(a) \log_a(0.006665017846190000)$$

•

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

•

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} dt$$

Now:

$$-6C + \phi = -5.010882647757 \dots$$

for $C = 1$, we obtain:

$$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \phi$$

Indeed, if we put this value of dilaton in the previous three vacuum equations, we obtain the following values:

$$\left(\frac{-(-1/2 * 0.02390591^2)}{(2.5)} \right) * \exp(-2*3 + 2*1/2*0.989117352243)$$

Input interpretation:

$$-\frac{0.02390591^2}{2.5} \exp\left(-2 \times 3 + 2 \times \frac{1}{2} \times 0.989117352243\right)$$

Result:

$$7.61802... \times 10^{-7}$$

$$7.61802... * 10^{-7}$$

$$1/2 * (((((0.02390591^2(((5+1-(2*1/2)/2.5))) * \exp(-2*3+2*1/2*0.989117352243))))))$$

Input interpretation:

$$\frac{1}{2} \left(0.02390591^2 \left(\left(5 + 1 - \frac{2 \times \frac{1}{2}}{2.5} \right) \exp \left(-2 \times 3 + 2 \times \frac{1}{2} \times 0.989117352243 \right) \right) \right)$$

Result:

$$0.00001066522...$$

Result:

$$1.0665220000 \times 10^{-5}$$

$$1.066522... * 10^{-5}$$

$$\exp(-2) + (((0.02390591^2/(16*6))) * ((7-5+(2*1/2)/2.5)) * \exp(-2*3+2*1/2*0.989117352243))))$$

Input interpretation:

$$\exp(-2) + \frac{0.02390591^2}{16 \times 6} \left(7 - 5 + \frac{2 \times \frac{1}{2}}{2.5} \right) \exp \left(-2 \times 3 + 2 \times \frac{1}{2} \times 0.989117352243 \right)$$

Result:

$$0.1353353784618...$$

$$0.1353353...$$

Results practically equal to the previous, as was to be expected

We note that from the inverse, multiplying by 7/32, we obtain:

$$7/32 * 1/((((\exp(-2) + (((0.02390591^2/(16*6))) * ((7-5+(2*1/2)/2.5)) * \exp(-2*3+2*1/2*0.989117352243))))))$$

Input interpretation:

$$\frac{7}{32} \times \frac{1}{\exp(-2) + \frac{0.02390591^2}{16 \times 6} \left(7 - 5 + \frac{2 \times \frac{1}{2}}{2.5} \right) \exp \left(-2 \times 3 + 2 \times \frac{1}{2} \times 0.989117352243 \right)}$$

Result:

1.616354884334...

1.61635488... result practically equal to the value without exponent of the Planck length

The dilaton value obtained

0.989117352243 = ϕ

we note that is very near to the result of the following mean of these Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\phi\sqrt{5}} - \phi} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$\frac{e^{-\frac{2\pi}{\sqrt{5}}}}{\sqrt{5} - \phi} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}} \approx 1.0000007913$$

$$\frac{e^{-\frac{2\pi}{\sqrt{5}}}}{1 + \sqrt[5]{\sqrt{(\phi-1)^5 4\sqrt{5^3}} - 1}} - \phi$$

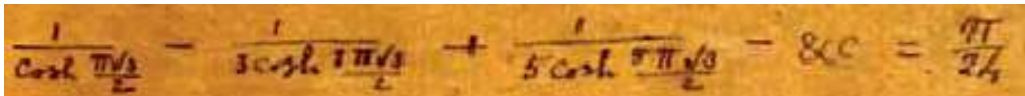
$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Indeed:

$$\begin{aligned} & 1/4 * (1.0018674362 + 1.0000007913 + 0.9568666373 + 0.9991104684) = \\ & = \mathbf{0.9894613333} \text{ result very near to the dilaton value} \end{aligned}$$

From:



$$\begin{aligned} & 1/(\cosh(\frac{\pi\sqrt{3}}{2})) - 1/(3\cosh(\frac{3\pi\sqrt{3}}{2})) + \\ & 1/(5\cosh(\frac{5\pi\sqrt{3}}{2})) - 1/(7\cosh(\frac{7\pi\sqrt{3}}{2})) \end{aligned}$$

Input:

$$\frac{1}{\cosh(\frac{1}{2}(\pi\sqrt{3}))} - \frac{1}{3\cosh(\frac{1}{2}(3\pi\sqrt{3}))} + \frac{1}{5\cosh(\frac{1}{2}(5\pi\sqrt{3}))} - \frac{1}{7\cosh(\frac{1}{2}(7\pi\sqrt{3}))}$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) - \frac{1}{3}\operatorname{sech}\left(\frac{3\sqrt{3}\pi}{2}\right) + \frac{1}{5}\operatorname{sech}\left(\frac{5\sqrt{3}\pi}{2}\right) - \frac{1}{7}\operatorname{sech}\left(\frac{7\sqrt{3}\pi}{2}\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.130899693894434407833394083378623801120617447162657435974...

$$0.13089969389\dots \cong \frac{\pi}{24}$$

Property:

$$\operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) - \frac{1}{3}\operatorname{sech}\left(\frac{3\sqrt{3}\pi}{2}\right) + \frac{1}{5}\operatorname{sech}\left(\frac{5\sqrt{3}\pi}{2}\right) - \frac{1}{7}\operatorname{sech}\left(\frac{7\sqrt{3}\pi}{2}\right)$$

is a transcendental number

Alternate forms:

$$\frac{1}{105}\left(105\operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) - 35\operatorname{sech}\left(\frac{3\sqrt{3}\pi}{2}\right) + 21\operatorname{sech}\left(\frac{5\sqrt{3}\pi}{2}\right) - 15\operatorname{sech}\left(\frac{7\sqrt{3}\pi}{2}\right)\right)$$

$$\frac{2\cosh\left(\frac{\sqrt{3}\pi}{2}\right)}{1+\cosh(\sqrt{3}\pi)} - \frac{2\cosh\left(\frac{3\sqrt{3}\pi}{2}\right)}{3(1+\cosh(3\sqrt{3}\pi))} + \frac{2\cosh\left(\frac{5\sqrt{3}\pi}{2}\right)}{5(1+\cosh(5\sqrt{3}\pi))} - \frac{2\cosh\left(\frac{7\sqrt{3}\pi}{2}\right)}{7(1+\cosh(7\sqrt{3}\pi))}$$

$$\frac{2}{e^{-(\sqrt{3}\pi)/2} + e^{(\sqrt{3}\pi)/2}} - \frac{2}{3\left(e^{-(3\sqrt{3}\pi)/2} + e^{(3\sqrt{3}\pi)/2}\right)} + \frac{2}{5\left(e^{-(5\sqrt{3}\pi)/2} + e^{(5\sqrt{3}\pi)/2}\right)} - \frac{2}{7\left(e^{-(7\sqrt{3}\pi)/2} + e^{(7\sqrt{3}\pi)/2}\right)}$$

Alternative representations:

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5\cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} = \frac{1}{\cos\left(\frac{1}{2}i\pi\sqrt{3}\right)} - \frac{1}{3\cos\left(\frac{3}{2}i\pi\sqrt{3}\right)} + \frac{1}{5\cos\left(\frac{5}{2}i\pi\sqrt{3}\right)} - \frac{1}{7\cos\left(\frac{7}{2}i\pi\sqrt{3}\right)}$$

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3\cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5\cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7\cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} = \frac{1}{\cos\left(-\frac{1}{2}i\pi\sqrt{3}\right)} - \frac{1}{3\cos\left(-\frac{3}{2}i\pi\sqrt{3}\right)} + \frac{1}{5\cos\left(-\frac{5}{2}i\pi\sqrt{3}\right)} - \frac{1}{7\cos\left(-\frac{7}{2}i\pi\sqrt{3}\right)}$$

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5 \cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} =$$

$$\frac{1}{\sec\left(\frac{1}{2}i\pi\sqrt{3}\right)} - \frac{1}{\sec\left(\frac{3}{2}i\pi\sqrt{3}\right)} + \frac{1}{\sec\left(\frac{5}{2}i\pi\sqrt{3}\right)} - \frac{1}{\sec\left(\frac{7}{2}i\pi\sqrt{3}\right)}$$

i is the imaginary unit

$\sec(x)$ is the secant function

Series representations:

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5 \cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} =$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k) \left(\frac{105}{1+k+k^2} - \frac{35}{7+k+k^2} + \frac{21}{19+k+k^2} - \frac{15}{37+k+k^2} \right)}{105\pi}$$

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5 \cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} =$$

$$\sum_{k=0}^{\infty} \frac{4(-1)^k (1+2k) (123886 + 23097k + 24387k^2 + 2599k^3 + 1347k^4 + 57k^5 + 19k^6)}{(105(1+k+k^2)(7+k+k^2)(19+k+k^2)(37+k+k^2)\pi)}$$

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5 \cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} =$$

$$\sum_{k=0}^{\infty} \frac{2}{105} (-1)^k e^{-7/2\sqrt{3}(1+2k)\pi}$$

$$\left(-15 + 21 e^{\sqrt{3}(1+2k)\pi} - 35 e^{2\sqrt{3}(1+2k)\pi} + 105 e^{3\sqrt{3}(1+2k)\pi} \right)$$

Integral representation:

$$\frac{1}{\cosh\left(\frac{\pi\sqrt{3}}{2}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi\sqrt{3}}{2}\right)} + \frac{1}{5 \cosh\left(\frac{5}{2}(\pi\sqrt{3})\right)} - \frac{1}{7 \cosh\left(\frac{7\pi\sqrt{3}}{2}\right)} =$$

$$\int_0^{\infty} \frac{210 t^{i\sqrt{3}} - 70 t^{3i\sqrt{3}} + 42 t^{5i\sqrt{3}} - 30 t^{7i\sqrt{3}}}{105\pi + 105\pi t^2} dt$$

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3\cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5\cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7\cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \dots = \frac{5\pi}{24}$$

Input:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3\cosh\left(3 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{5\cosh\left(5 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{7\cosh\left(7 \times \frac{\pi}{2\sqrt{3}}\right)}$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + \frac{1}{5}\operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - \frac{1}{7}\operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - \frac{1}{3}\operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right)$$

sech(x) is the hyperbolic secant function

Decimal approximation:

0.654442511141622215813893907448716743311205554141382615605...

$$0.6544425111\dots \cong \frac{5\pi}{24}$$

Alternate forms:

$$\frac{1}{105} \left(105 \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + 21 \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - 15 \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - 35 \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) \right)$$

$$\frac{2 \cosh\left(\frac{\pi}{2\sqrt{3}}\right)}{1 + \cosh\left(\frac{\pi}{\sqrt{3}}\right)} + \frac{2 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)}{5 \left(1 + \cosh\left(\frac{5\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)}{7 \left(1 + \cosh\left(\frac{7\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{\sqrt{3}\pi}{2}\right)}{3 \left(1 + \cosh(\sqrt{3}\pi)\right)}$$

$$\frac{2}{e^{-\pi/(2\sqrt{3})} + e^{\pi/(2\sqrt{3})}} + \frac{2}{5 \left(e^{-(5\pi)/(2\sqrt{3})} + e^{(5\pi)/(2\sqrt{3})} \right)} - \frac{2}{7 \left(e^{-(7\pi)/(2\sqrt{3})} + e^{(7\pi)/(2\sqrt{3})} \right)} - \frac{2}{3 \left(e^{-(\sqrt{3}\pi)/2} + e^{(\sqrt{3}\pi)/2} \right)}$$

Alternative representations:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} =$$

$$\frac{1}{\cos\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(\frac{7i\pi}{2\sqrt{3}}\right)}$$

•

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} =$$

$$\frac{1}{\cos\left(-\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(-\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(-\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(-\frac{7i\pi}{2\sqrt{3}}\right)}$$

•

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} =$$

$$\frac{1}{\sec\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \sec\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \sec\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \sec\left(\frac{7i\pi}{2\sqrt{3}}\right)}$$

i is the imaginary unit

$\sec(x)$ is the secant function

Series representations:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} =$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k) \left(-\frac{35}{1+k+k^2} + \frac{315}{1+3k+3k^2} + \frac{63}{7+3k+3k^2} - \frac{45}{13+3k+3k^2} \right)}{105 \pi}$$

•

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} =$$

$$\sum_{k=0}^{\infty} \frac{2}{105} (-1)^k e^{-\frac{7(1+2k)\pi}{2\sqrt{3}}}$$

$$\left(-15 + 21 e^{\frac{(1+2k)\pi}{\sqrt{3}}} - 35 e^{\frac{2(1+2k)\pi}{\sqrt{3}}} + 105 e^{\sqrt{3}(1+2k)\pi} \right)$$

•

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} = \sum_{k=0}^{\infty} (4(-1)^k (1+2k) (6496 + 9495k + 13707k^2 + 8937k^3 + 5751k^4 + 1539k^5 + 513k^6)) / (105(1+k+k^2)(1+3k+3k^2)(7+3k+3k^2)(13+3k+3k^2)\pi)$$

Integral representation:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} = \int_0^{\infty} \frac{2t^{i/\sqrt{3}} \left(-105 + 35t^{(2i)/\sqrt{3}} - 21t^{(4i)/\sqrt{3}} + 15t^{2i/\sqrt{3}}\right)}{105\pi(1+t^2)} dt$$

And:

$$1/(\cosh(\frac{\pi}{2\sqrt{3}})) - 1/(3\cosh(\frac{3\pi}{2\sqrt{3}})) + 1/(5\cosh(\frac{5\pi}{2\sqrt{3}})) - 1/(7\cosh(\frac{7\pi}{2\sqrt{3}})) + 1/(9\cosh(\frac{9\pi}{2\sqrt{3}}))$$

Input:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(3 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(5 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(7 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{1}{2} (9 \pi \sqrt{3})\right)}$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + \frac{1}{5} \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - \frac{1}{7} \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - \frac{1}{3} \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + \frac{1}{9} \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right)$$

sech(x) is the hyperbolic secant function

Decimal approximation:

0.654442511146780749150457301324615572422024535553644738187...

0.6544425111... as above

Alternate forms:

$$\frac{1}{315} \left(315 \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + 63 \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - 45 \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - 105 \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + 35 \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right) \right)$$

$$\frac{2 \cosh\left(\frac{\pi}{2\sqrt{3}}\right)}{1 + \cosh\left(\frac{\pi}{\sqrt{3}}\right)} + \frac{2 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)}{5 \left(1 + \cosh\left(\frac{5\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)}{7 \left(1 + \cosh\left(\frac{7\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{\sqrt{3}\pi}{2}\right)}{3 \left(1 + \cosh(\sqrt{3}\pi)\right)} + \frac{2 \cosh\left(\frac{9\sqrt{3}\pi}{2}\right)}{9 \left(1 + \cosh(9\sqrt{3}\pi)\right)}$$

$$\frac{2}{e^{-\pi/(2\sqrt{3})} + e^{\pi/(2\sqrt{3})}} + \frac{2}{5 \left(e^{-(5\pi)/(2\sqrt{3})} + e^{(5\pi)/(2\sqrt{3})} \right)} - \frac{2}{7 \left(e^{-(7\pi)/(2\sqrt{3})} + e^{(7\pi)/(2\sqrt{3})} \right)} - \frac{2}{3 \left(e^{-(\sqrt{3}\pi)/2} + e^{(\sqrt{3}\pi)/2} \right)} + \frac{2}{9 \left(e^{-(9\sqrt{3}\pi)/2} + e^{(9\sqrt{3}\pi)/2} \right)}$$

Alternative representations:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \frac{1}{\cos\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cos\left(\frac{9}{2}i\pi\sqrt{3}\right)}$$

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \frac{1}{\cos\left(-\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(-\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(-\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(-\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cos\left(-\frac{9}{2}i\pi\sqrt{3}\right)}$$

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \frac{1}{\operatorname{sech}\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \operatorname{sech}\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \operatorname{sech}\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \operatorname{sech}\left(\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \operatorname{sech}\left(\frac{9}{2}i\pi\sqrt{3}\right)}$$

i is the imaginary unit

Series representations:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k) \left(-\frac{105}{1+k+k^2} + \frac{35}{61+k+k^2} + \frac{945}{1+3k+3k^2} + \frac{189}{7+3k+3k^2} - \frac{135}{13+3k+3k^2}\right)}{315\pi}$$

- $$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \sum_{k=0}^{\infty} \frac{2}{315} (-1)^k e^{-9/2\sqrt{3}(1+2k)\pi} \left(35 - 45 e^{(10(1+2k)\pi)/\sqrt{3}} + 63 e^{(11(1+2k)\pi)/\sqrt{3}} + 315 e^{(13(1+2k)\pi)/\sqrt{3}} - 105 e^{4\sqrt{3}(1+2k)\pi}\right)$$

- $$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k) (4758257 + 7043132k + 10258526k^2 + 6864408k^3 + 4523355k^4 + 1329264k^5 + 476226k^6 + 28404k^7 + 7101k^8)}{(315(1+k+k^2)(61+k+k^2)(1+3k+3k^2)(7+3k+3k^2)(13+3k+3k^2)\pi)}$$

Integral representation:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \int_0^{\infty} \frac{2 \left(315 t^{i/\sqrt{3}} + 63 t^{(5i)/\sqrt{3}} - 45 t^{(7i)/\sqrt{3}} - 105 t^{i\sqrt{3}} + 35 t^{9i\sqrt{3}}\right)}{315\pi(1+t^2)} dt$$

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \frac{1}{11 \cosh\left(\frac{11\pi}{2\sqrt{3}}\right)}$$

Input:

$$\frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(3 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(5 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(7 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(9 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{11 \cosh\left(\frac{1}{2} (11 \pi \sqrt{3})\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + \frac{1}{5} \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - \frac{1}{7} \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - \frac{1}{3} \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + \frac{1}{9} \operatorname{sech}\left(\frac{3\sqrt{3}\pi}{2}\right) - \frac{1}{11} \operatorname{sech}\left(\frac{11\sqrt{3}\pi}{2}\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.654505903031951239910081466318636943736421676023195096468...

0.654505903.... as above

Note that:

$$(5\pi/24 - \pi/24)^{1/64}$$

Input:

$$\sqrt[64]{5 \times \frac{\pi}{24} - \frac{\pi}{24}}$$

Exact result:

$$\sqrt[64]{\frac{\pi}{6}}$$

Decimal approximation:

0.989941095379548541423356654369279890237253767559106012445...

0.989941095...

Property:

$\sqrt[64]{\frac{\pi}{6}}$ is a transcendental number

All 64th roots of $\pi/6$:

$$\sqrt[64]{\frac{\pi}{6}} e^0 \approx 0.989941 \text{ (real, principal root)}$$

•

$$\sqrt[64]{\frac{\pi}{6}} e^{(i\pi)/32} \approx 0.985174 + 0.09703 i$$

•

$$\sqrt[64]{\frac{\pi}{6}} e^{(2i\pi)/32} \approx 0.97092 + 0.19313 i$$

•

$$\sqrt[64]{\frac{\pi}{6}} e^{(3i\pi)/32} \approx 0.94731 + 0.28736 i$$

•

$$\sqrt[64]{\frac{\pi}{6}} e^{(4i\pi)/32} \approx 0.91459 + 0.37883 i$$

•

Alternative representations:

$$\sqrt[64]{\frac{5\pi}{24} - \frac{\pi}{24}} = \sqrt[64]{\frac{720^\circ}{24}}$$

•

$$\sqrt[64]{\frac{5\pi}{24} - \frac{\pi}{24}} = \sqrt[64]{-\frac{4}{24} i \log(-1)}$$

•

$$\sqrt[64]{\frac{5\pi}{24} - \frac{\pi}{24}} = \sqrt[64]{\frac{4}{24} \cos^{-1}(-1)}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

$\cos^{-1}(x)$ is the inverse cosine function

Series representations:

$${}^{64}\sqrt{\frac{5\pi}{24} - \frac{\pi}{24}} = {}^{64}\sqrt{\frac{2}{3}} {}^{64}\sqrt{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

•

$${}^{64}\sqrt{\frac{5\pi}{24} - \frac{\pi}{24}} = {}^{64}\sqrt{\frac{2}{3}} {}^{64}\sqrt{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

•

$${}^{64}\sqrt{\frac{5\pi}{24} - \frac{\pi}{24}} = \frac{{}^{64}\sqrt{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}{{}^{64}\sqrt{6}}$$

Integral representations:

$${}^{64}\sqrt{\frac{5\pi}{24} - \frac{\pi}{24}} = \frac{{}^{64}\sqrt{\int_0^{\infty} \frac{1}{1+t^2} dt}}{{}^{64}\sqrt{3}}$$

•

$${}^{64}\sqrt{\frac{5\pi}{24} - \frac{\pi}{24}} = \frac{{}^{64}\sqrt{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}{{}^{64}\sqrt{3}}$$

•

$${}^{64}\sqrt{\frac{5\pi}{24} - \frac{\pi}{24}} = {}^{64}\sqrt{\frac{2}{3}} {}^{64}\sqrt{\int_0^1 \sqrt{1-t^2} dt}$$

Now, we obtain:

Input:

$$1 + \frac{18}{10^3} + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(3 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(5 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(7 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{1}{2} (9\pi\sqrt{3})\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{509}{500} + \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + \frac{1}{5} \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - \frac{1}{7} \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - \frac{1}{3} \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + \frac{1}{9} \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

1.672442511146780749150457301324615572422024535553644738187...

1.672442511... result very near to the proton mass without exponent

Alternate forms:

$$\frac{1}{31500} \left(32067 + 31500 \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + 6300 \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - 4500 \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - 10500 \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + 3500 \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right) \right)$$

$$\frac{509}{500} + \frac{2 \cosh\left(\frac{\pi}{2\sqrt{3}}\right)}{1 + \cosh\left(\frac{\pi}{\sqrt{3}}\right)} + \frac{2 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)}{5 \left(1 + \cosh\left(\frac{5\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)}{7 \left(1 + \cosh\left(\frac{7\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{\sqrt{3}\pi}{2}\right)}{3 \left(1 + \cosh(\sqrt{3}\pi)\right)} + \frac{2 \cosh\left(\frac{9\sqrt{3}\pi}{2}\right)}{9 \left(1 + \cosh(9\sqrt{3}\pi)\right)}$$

$$\frac{509}{500} + \frac{2}{e^{-\pi/(2\sqrt{3})} + e^{\pi/(2\sqrt{3})}} + \frac{2}{5 \left(e^{-(5\pi)/(2\sqrt{3})} + e^{(5\pi)/(2\sqrt{3})} \right)} - \frac{2}{7 \left(e^{-(7\pi)/(2\sqrt{3})} + e^{(7\pi)/(2\sqrt{3})} \right)} - \frac{2}{3 \left(e^{-(\sqrt{3}\pi)/2} + e^{(\sqrt{3}\pi)/2} \right)} + \frac{2}{9 \left(e^{-(9\sqrt{3}\pi)/2} + e^{(9\sqrt{3}\pi)/2} \right)}$$

Alternative representations:

$$1 + \frac{18}{10^3} + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} =$$

$$1 + \frac{1}{\cos\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cos\left(\frac{9}{2}i\pi\sqrt{3}\right)} + \frac{18}{10^3}$$

$$1 + \frac{18}{10^3} + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = 1 + \frac{1}{\cos\left(-\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(-\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(-\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(-\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cos\left(-\frac{9}{2}i\pi\sqrt{3}\right)} + \frac{18}{10^3}$$

$$1 + \frac{18}{10^3} + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} =$$

$$1 + \frac{18}{10^3} + \frac{1}{\sec\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \sec\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \sec\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \sec\left(\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \sec\left(\frac{9}{2}i\pi\sqrt{3}\right)}$$

i is the imaginary unit

$\sec(x)$ is the secant function

$$-(29/10^3 + 7/10^3) + 1 + 1/(\cosh(\frac{\pi}{2\sqrt{3}})) - 1/(3\cosh(\frac{3\pi}{2\sqrt{3}})) + 1/(5\cosh(\frac{5\pi}{2\sqrt{3}})) - 1/(7\cosh(\frac{7\pi}{2\sqrt{3}})) + 1/(9\cosh(\frac{9\pi\sqrt{3}}{2}))$$

Input:

$$-\left(\frac{29}{10^3} + \frac{7}{10^3}\right) + 1 + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(3 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cosh\left(5 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(7 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{1}{2}(9\pi\sqrt{3})\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{241}{250} + \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + \frac{1}{5} \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - \frac{1}{7} \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - \frac{1}{3} \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + \frac{1}{9} \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

1.618442511146780749150457301324615572422024535553644738187...

1.6184425111...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{1}{15750} \left(15183 + 15750 \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + 3150 \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - 2250 \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - 5250 \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + 1750 \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right) \right)$$

- $$\frac{241}{250} + \frac{2 \cosh\left(\frac{\pi}{2\sqrt{3}}\right)}{1 + \cosh\left(\frac{\pi}{\sqrt{3}}\right)} + \frac{2 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)}{5 \left(1 + \cosh\left(\frac{5\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)}{7 \left(1 + \cosh\left(\frac{7\pi}{\sqrt{3}}\right)\right)} - \frac{2 \cosh\left(\frac{\sqrt{3}\pi}{2}\right)}{3 \left(1 + \cosh(\sqrt{3}\pi)\right)} + \frac{2 \cosh\left(\frac{9\sqrt{3}\pi}{2}\right)}{9 \left(1 + \cosh(9\sqrt{3}\pi)\right)}$$

- $$\frac{241}{250} + \frac{2}{e^{-\pi/(2\sqrt{3})} + e^{\pi/(2\sqrt{3})}} + \frac{2}{5 \left(e^{-(5\pi)/(2\sqrt{3})} + e^{(5\pi)/(2\sqrt{3})} \right)} - \frac{2}{7 \left(e^{-(7\pi)/(2\sqrt{3})} + e^{(7\pi)/(2\sqrt{3})} \right)} - \frac{2}{3 \left(e^{-(\sqrt{3}\pi)/2} + e^{(\sqrt{3}\pi)/2} \right)} + \frac{2}{9 \left(e^{-(9\sqrt{3}\pi)/2} + e^{(9\sqrt{3}\pi)/2} \right)}$$

Alternative representations:

$$\begin{aligned}
& -\left(\frac{29}{10^3} + \frac{7}{10^3}\right) + 1 + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \\
& \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \\
& 1 + \frac{1}{\cos\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cos\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cos\left(\frac{9}{2}i\pi\sqrt{3}\right)} - \frac{36}{10^3}
\end{aligned}$$

$$\begin{aligned}
& -\left(\frac{29}{10^3} + \frac{7}{10^3}\right) + 1 + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \\
& \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = 1 + \frac{1}{\cos\left(-\frac{i\pi}{2\sqrt{3}}\right)} - \\
& \frac{1}{3 \cos\left(-\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \cos\left(-\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cos\left(-\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cos\left(-\frac{9}{2}i\pi\sqrt{3}\right)} - \frac{36}{10^3}
\end{aligned}$$

$$\begin{aligned}
& -\left(\frac{29}{10^3} + \frac{7}{10^3}\right) + 1 + \frac{1}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \cosh\left(\frac{3\pi}{2\sqrt{3}}\right)} + \\
& \frac{1}{5 \cosh\left(\frac{5\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \cosh\left(\frac{7\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \cosh\left(\frac{9}{2}(\pi\sqrt{3})\right)} = \\
& 1 - \frac{36}{10^3} + \frac{1}{\sec\left(\frac{i\pi}{2\sqrt{3}}\right)} - \frac{1}{3 \sec\left(\frac{3i\pi}{2\sqrt{3}}\right)} + \frac{1}{5 \sec\left(\frac{5i\pi}{2\sqrt{3}}\right)} - \frac{1}{7 \sec\left(\frac{7i\pi}{2\sqrt{3}}\right)} + \frac{1}{9 \sec\left(\frac{9}{2}i\pi\sqrt{3}\right)}
\end{aligned}$$

i is the imaginary unit

$\sec(x)$ is the secant function

$$\begin{aligned}
& 64/10^3 + \sqrt{2}/(\cosh(\frac{\pi}{2\sqrt{3}})) - \sqrt{2}/(3\cosh(\frac{3\pi}{2\sqrt{3}})) + \\
& \sqrt{2}/(5\cosh(\frac{5\pi}{2\sqrt{3}})) - \sqrt{2}/(7\cosh(\frac{7\pi}{2\sqrt{3}})) + \\
& \sqrt{2}/(9\cosh(\frac{9\pi\sqrt{3}}{2}))
\end{aligned}$$

Input:

$$\begin{aligned}
& \frac{64}{10^3} + \frac{\sqrt{2}}{\cosh\left(\frac{\pi}{2\sqrt{3}}\right)} - \frac{\sqrt{2}}{3 \cosh\left(3 \times \frac{\pi}{2\sqrt{3}}\right)} + \\
& \frac{\sqrt{2}}{5 \cosh\left(5 \times \frac{\pi}{2\sqrt{3}}\right)} - \frac{\sqrt{2}}{7 \cosh\left(7 \times \frac{\pi}{2\sqrt{3}}\right)} + \frac{\sqrt{2}}{9 \cosh\left(\frac{1}{2}(9\pi\sqrt{3})\right)}
\end{aligned}$$

$\cosh(x)$ is the hyperbolic cosine function

Exact result:

$$\frac{8}{125} + \sqrt{2} \operatorname{sech}\left(\frac{\pi}{2\sqrt{3}}\right) + \frac{1}{5} \sqrt{2} \operatorname{sech}\left(\frac{5\pi}{2\sqrt{3}}\right) - \frac{1}{7} \sqrt{2} \operatorname{sech}\left(\frac{7\pi}{2\sqrt{3}}\right) - \frac{1}{3} \sqrt{2} \operatorname{sech}\left(\frac{\sqrt{3}\pi}{2}\right) + \frac{1}{9} \sqrt{2} \operatorname{sech}\left(\frac{9\sqrt{3}\pi}{2}\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

0.989521475057282768723304574514811505631263385802109717743...

0.9895214....

We note that this value is very near to the result of the following mean of these Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

≈ 1.0018674362

$$\frac{e^{-\frac{2\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}}$$

≈ 1.0000007913

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Indeed:

$$\begin{aligned} & 1/4 * (1.0018674362 + 1.0000007913 + 0.9568666373 + 0.9991104684) = \\ & = \mathbf{0.9894613333} \text{ result very near to the dilaton value } \mathbf{0.989117352243} = \phi \\ & \text{(also the various results signed in red, are very near to the dilaton value)} \end{aligned}$$

Now:



$$((2))^{(1/4)} * (34)^{1/24}$$

Input:

$$\sqrt[4]{2} \sqrt[24]{34}$$

Exact result:

$$2^{7/24} \sqrt[24]{17}$$

Decimal approximation:

$$1.377428677129059009089512775001367744410202443192574462815...$$

1.377428677...



$$\chi(x) = \sqrt{2} \cdot \sqrt[24]{(154 \pm 6\sqrt{645})} x$$

$$((2))^{(1/4)} * (154+6\text{sqrt}(645))^{1/24}$$

Input:

$$\sqrt[4]{2} \sqrt[24]{154 + 6\sqrt{645}}$$

Decimal approximation:

1.509564756120062447613814900453139553073691471794999983021...

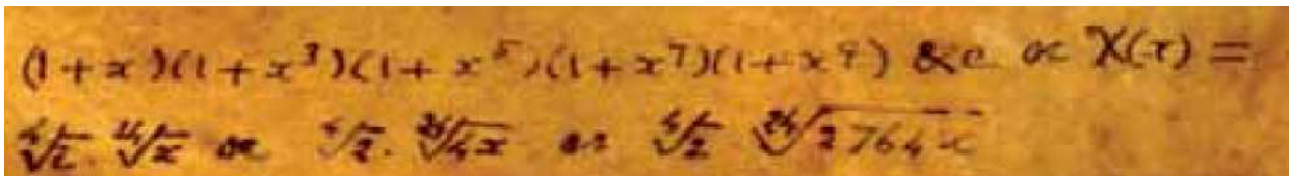
1.50956475612...

Alternate form:

$$2^{7/24} \sqrt[24]{77 + 3\sqrt{645}}$$

Minimal polynomial:

$$x^{48} - 19712x^{24} + 2031616$$



$$(1+x)(1+x^3)(1+x^5)(1+x^7)(1+x^9) \&c \propto \chi(x) = \sqrt{2} \sqrt[24]{2764 \pm 6\sqrt{645}} x$$

$$((2))^{(1/4)} * (2764)^{1/24}$$

Input:

$$\sqrt[4]{2} \sqrt[24]{2764}$$

Result:

$$\sqrt[3]{2} \sqrt[24]{691}$$

Decimal approximation:

1.654454840152195453977606619397495035620525996518436170868...

1.65445484015.... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

$$2^{1/4} * (34)^{1/24} + 2^{1/4} * (154+6\sqrt{645})^{1/24} + 2^{1/4} * (2764)^{1/24}$$

Input:

$$\sqrt[4]{2} \sqrt[24]{34} + \sqrt[4]{2} \sqrt[24]{154+6\sqrt{645}} + \sqrt[4]{2} \sqrt[24]{2764}$$

Exact result:

$$2^{7/24} \sqrt[24]{17} + \sqrt[3]{2} \sqrt[24]{691} + \sqrt[4]{2} \sqrt[24]{154+6\sqrt{645}}$$

Decimal approximation:

4.541448273401316910680934294852002333104419911506010616705...

4.5414482734...

Alternate forms:

$$2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77+3\sqrt{645}} \right)$$

•

$$2^{7/24} \sqrt[24]{17} + \sqrt[3]{2} \sqrt[24]{691} + 2^{7/24} \sqrt[24]{77+3\sqrt{645}}$$

•

$$\left(\left(\left(\left(\left(2^{1/4} * (34)^{1/24} + (2)^{1/4} * (154+6\sqrt{645})^{1/24} + (2)^{1/4} * (2764)^{1/24} \right) \right) \right) \right) \right)^{1/3}$$

Input:

$$\sqrt[3]{\sqrt[4]{2} \sqrt[24]{34} + \sqrt[4]{2} \sqrt[24]{154+6\sqrt{645}} + \sqrt[4]{2} \sqrt[24]{2764}}$$

Exact result:

$$\sqrt[3]{2^{7/24} \sqrt[24]{17} + \sqrt[3]{2} \sqrt[24]{691} + \sqrt[4]{2} \sqrt[24]{154+6\sqrt{645}}}$$

Decimal approximation:

1.656016999614638163657525203417922983897404307813267950236...

1.6560169996.... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Alternate form:

$$2^{7/72} \sqrt[3]{\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}}}$$

$$(5^2/10^3) + \sqrt{11} * (((1/((2)^{1/4}) * (34)^{1/24})) * 1 / (((2)^{1/4})) * ((154 + 6\sqrt{645})^{1/24}))) * 1 / (((2)^{1/4}) * (2764)^{1/24}))))))$$

Input:

$$\frac{5^2}{10^3} + \sqrt{11} \left(\frac{1}{\sqrt[4]{2} \sqrt[24]{34}} \times \frac{1}{\sqrt[4]{2} \sqrt[24]{154 + 6\sqrt{645}}} \times \frac{1}{\sqrt[4]{2} \sqrt[24]{2764}} \right)$$

Exact result:

$$\frac{1}{40} + \frac{\sqrt{11}}{2^{7/8} \sqrt[24]{11747(154 + 6\sqrt{645})}}$$

Decimal approximation:

0.989096613030449618894203505589459482203519403091712613461...

0.989096613...

We note that this value is very near to the result of the following mean of these Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$\frac{e^{\frac{2\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}} \approx 1.0000007913$$

$$\frac{e^{\frac{2\pi}{\sqrt{5}}}}{1 + \sqrt[5]{\sqrt{(\varphi-1)^5 4\sqrt{5^3}} - 1}} - \varphi$$

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3}} - 1}}$$

Indeed:

$$1/4 * (1.0018674362 + 1.0000007913 + 0.9568666373 + 0.9991104684) =$$

$$= 0.9894613333 \text{ result very near to the dilaton value } \mathbf{0.989117352243} = \phi$$

Alternate forms:

$$\frac{1}{40} \left(1 + 20 \sqrt{11} \sqrt[24]{\frac{77 - 3\sqrt{645}}{364157}} \right)$$

- $$\frac{1}{40} + \frac{\sqrt{11}}{2^{11/12} \sqrt[24]{11747(77 + 3\sqrt{645})}}$$

- $$\frac{20 \sqrt[12]{2} \sqrt{11} 11747^{23/24} + 11747 \sqrt[24]{77 + 3\sqrt{645}}}{469880 \sqrt[24]{77 + 3\sqrt{645}}}$$

Considering the χ within the roots, we obtain the following Corollary:

$$2^{1/4} * (34x)^{1/24} + 2^{1/4} * ((154+6\sqrt{645})x)^{1/24} + 2^{1/4} * (2764x)^{1/24}$$

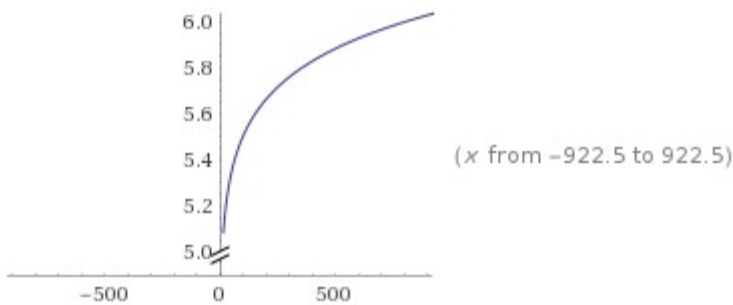
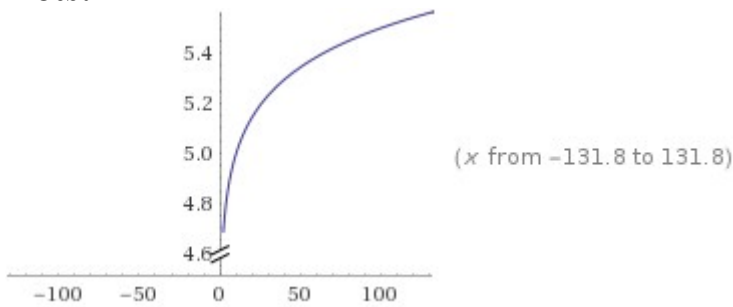
Input:

$$\sqrt[4]{2} \sqrt[24]{34x} + \sqrt[4]{2} \sqrt[24]{(154 + 6\sqrt{645})x} + \sqrt[4]{2} \sqrt[24]{2764x}$$

Exact result:

$$\sqrt[4]{2} \sqrt[24]{154 + 6\sqrt{645}} \sqrt[24]{x} + \sqrt[3]{2} \sqrt[24]{691} \sqrt[24]{x} + 2^{7/24} \sqrt[24]{17} \sqrt[24]{x}$$

Plots:



Alternate forms:

$$2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \sqrt[24]{x}$$

•

$$2^{7/24} \sqrt[24]{77 + 3\sqrt{645}} \sqrt[24]{x} + \sqrt[3]{2} \sqrt[24]{691} \sqrt[24]{x} + 2^{7/24} \sqrt[24]{17} \sqrt[24]{x}$$

•

Integer root:

• Step-by-step solution

$$x = 0$$

•

Properties as a real function:

Domain

$\{x \in \mathbb{R} : x \geq 0\}$ (all non-negative real numbers)

•

Range

$\{y \in \mathbb{R} : y \geq 0\}$ (all non-negative real numbers)

•

Injectivity

injective (one-to-one)

\mathbb{R} is the set of real numbers

Derivative:

$$\frac{d}{dx} \left(\sqrt[4]{2} \sqrt[24]{34x} + \sqrt[4]{2} \sqrt[24]{(154 + 6\sqrt{645})x} + \sqrt[4]{2} \sqrt[24]{2764x} \right) = \frac{\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}}}{12 \times 2^{17/24} x^{23/24}}$$

•

Indefinite integral:

$$\int \left(2^{7/24} \sqrt[24]{17} \sqrt[24]{x} + \sqrt[3]{2} \sqrt[24]{691} \sqrt[24]{x} + \sqrt[4]{2} \sqrt[24]{154 + 6\sqrt{645}} \sqrt[24]{x} \right) dx = \frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) x^{25/24} + \text{constant}$$

•

Global minimum:

$$\min\left\{\sqrt[4]{2} \sqrt[24]{34x} + \sqrt[4]{2} \sqrt[24]{(154+6\sqrt{645})x} + \sqrt[4]{2} \sqrt[24]{2764x}\right\} = 0 \text{ at } x = 0$$

From the result of the above integral, we obtain:

$$\int \left(2^{7/24} \sqrt[24]{17} \sqrt[24]{x} + \sqrt[3]{2} \sqrt[24]{691} \sqrt[24]{x} + \sqrt[4]{2} \sqrt[24]{154+6\sqrt{645}} \sqrt[24]{x} \right) dx =$$

$$\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77+3\sqrt{645}} \right) x^{25/24} + \text{constant}$$

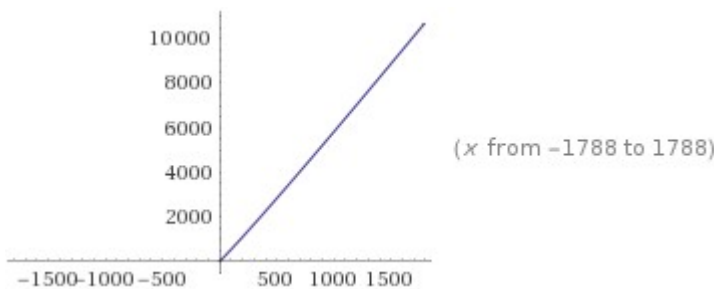
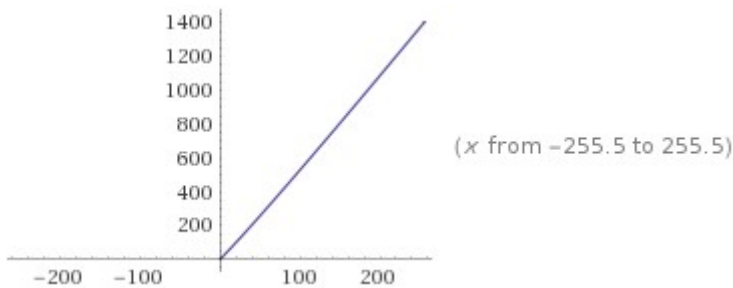
Input:

$$\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77+3\sqrt{645}} \right) x^{25/24}$$

Exact result:

$$\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77+3\sqrt{645}} \right) x^{25/24}$$

Plots:



Alternate form:

$$\left(\frac{24}{25} \times 2^{7/24} \sqrt[24]{77 + 3\sqrt{645}} + 2^{7/24} \left(\frac{24 \sqrt[24]{17}}{25} + \frac{24 \sqrt[24]{1382}}{25} \right) \right) x^{25/24}$$

Expanded form:

$$\frac{24}{25} \times 2^{7/24} \sqrt[24]{77 + 3\sqrt{645}} x^{25/24} + \frac{24}{25} \sqrt[3]{2} \sqrt[24]{691} x^{25/24} + \frac{24}{25} \times 2^{7/24} \sqrt[24]{17} x^{25/24}$$

Numerical root:

$$x \approx -3.2768 \times 10^{-12} - 5.67558 \times 10^{-12} i \dots$$

•

Integer root:

$$x = 0$$

•

Properties as a real function:

Domain

$$\{x \in \mathbb{R} : x \geq 0\} \text{ (all non-negative real numbers)}$$

•

Range

$$\{y \in \mathbb{R} : y \geq 0\} \text{ (all non-negative real numbers)}$$

•

Injectivity

injective (one-to-one)

\mathbb{R} is the set of real numbers

Derivative:

$$\frac{d}{dx} \left(\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) x^{25/24} \right) =$$

$$2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \sqrt[24]{x}$$

•

Indefinite integral:

$$\int \frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) x^{25/24} dx =$$

$$\frac{576 \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) x^{49/24}}{1225} + \text{constant}$$

From the numerical root

$$x \approx -3.2768 \times 10^{-12} - 5.67558 \times 10^{-12} i \dots$$

we obtain, after some calculations:

$$1 - (((-3.2768e-12 + (5.67558e-12i))))^{1/6}$$

Input interpretation:

$$1 - \sqrt[6]{-3.2768 \times 10^{-12} + 5.67558 \times 10^{-12} i}$$

i is the imaginary unit

Result:

$$0.98714519\dots - \\ 0.0046787700\dots i$$

Polar coordinates:

$$r = 0.987156 \text{ (radius), } \theta = -0.271563^\circ \text{ (angle)}$$

0.987156 result very near to the dilaton value $0.989117352243 = \phi$

For $x = 4\pi/(144+11)$ where 144 is a Fibonacci number and 11 is a Lucas number, we obtain:

$$\frac{24}{25} 2^{7/24} (17^{1/24} + 1382^{1/24} + (77 + 3 \sqrt{645})^{1/24}) \\ (4\pi/(144+11))^{25/24}$$

Input:

$$\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \left(4 \times \frac{\pi}{144 + 11} \right)^{25/24}$$

Exact result:

$$\frac{96 \times 2^{3/8} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \pi^{25/24}}{3875 \sqrt[24]{155}}$$

Decimal approximation:

$$0.318332046394654405247601985414294144609326467540023539625\dots$$

$$0.31833204639\dots \cong \frac{1}{\pi}$$

Property:

$$\frac{96 \times 2^{3/8} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \pi^{25/24}}{3875 \sqrt[24]{155}} \text{ is a transcendental number}$$

•
Alternate forms:

$$\left(2^{3/8} \left(\frac{96 \sqrt[24]{\frac{17}{155}}}{3875} + \frac{96 \sqrt[24]{\frac{1382}{155}}}{3875} \right) + \frac{96 \times 2^{3/8} \sqrt[24]{\frac{1}{155} (77 + 3\sqrt{645})}}{3875} \right) \pi^{25/24}$$

$$\frac{96 \sqrt[24]{\frac{17}{155}} 2^{3/8} \pi^{25/24}}{3875} + \frac{96 \times 2^{5/12} \sqrt[24]{\frac{691}{155}} \pi^{25/24}}{3875} + \frac{96 \times 2^{3/8} \sqrt[24]{\frac{1}{155} (77 + 3\sqrt{645})} \pi^{25/24}}{3875}$$

Series representations:

$$\frac{\frac{1}{25} \times 2^{7/24} \times 24 \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \left(\frac{4\pi}{144 + 11} \right)^{25/24}}{3875 \sqrt[24]{155}} = \frac{96 \times 2^{3/8} \pi^{25/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \sum_{k=0}^{\infty} 644^{-k} \binom{\frac{1}{2}}{k} \right)}{3875 \sqrt[24]{155}}$$

$$\frac{\frac{1}{25} \times 2^{7/24} \times 24 \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \left(\frac{4\pi}{144 + 11} \right)^{25/24}}{3875 \sqrt[24]{155}} = \frac{96 \times 2^{3/8} \pi^{25/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{644}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)}{3875 \sqrt[24]{155}}$$

$$\frac{1}{25} \times 2^{7/24} \times 24 \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \left(\frac{4\pi}{144 + 11} \right)^{25/24} =$$

$$\frac{96 \sqrt[24]{\frac{17}{155}} 2^{3/8} \pi^{25/24}}{3875} + \frac{96 \times 2^{5/12} \sqrt[24]{\frac{691}{155}} \pi^{25/24}}{3875} +$$

$$\frac{96 \times 2^{3/8} \pi^{25/24} \sqrt[24]{77 + \frac{3 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 644^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}}}{3875 \sqrt[24]{155}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

For $x = 24\pi/(313) = 0.240888893565990$, where $313 = 322-7-2$ (Lucas numbers)

$$24/25 \cdot 2^{7/24} (17^{1/24} + 1382^{1/24} + (77 + 3\sqrt{645})^{1/24}) \left(\frac{24\pi}{322-7-2} \right)^{25/24}$$

Input:

$$\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \left(\frac{24\pi}{322 - 7 - 2} \right)^{25/24}$$

Exact result:

$$\frac{576 \sqrt[24]{\frac{3}{313}} 2^{5/12} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3\sqrt{645}} \right) \pi^{25/24}}{7825}$$

Decimal approximation:

0.989748261508636966240100268085835953840130038442092343596...

0.9897482615.... result very near to the dilaton value **0.989117352243 = ϕ**

Property:

$$\frac{576 \sqrt[24]{\frac{3}{313}} 2^{5/12} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \pi^{25/24}}{7825}$$

is a transcendental number

Alternate form:

$$\frac{576 \sqrt[24]{\frac{51}{313}} 2^{5/12} \pi^{25/24}}{7825} + \frac{576 \times 2^{11/24} \sqrt[24]{\frac{2073}{313}} \pi^{25/24}}{7825} + \frac{576 \times 2^{5/12} \sqrt[24]{\frac{3}{313}} \left(77 + 3 \sqrt{645} \right) \pi^{25/24}}{7825}$$

Series representations:

$$\frac{\frac{1}{25} \times 2^{7/24} \times 24 \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \left(\frac{24 \pi}{322 - 7 - 2} \right)^{25/24} =}{7825} \frac{576 \sqrt[24]{\frac{3}{313}} 2^{5/12} \pi^{25/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{644} \sum_{k=0}^{\infty} 644^{-k} \binom{\frac{1}{2}}{k} \right)}{7825}$$

$$\frac{\frac{1}{25} \times 2^{7/24} \times 24 \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \left(\frac{24 \pi}{322 - 7 - 2} \right)^{25/24} =}{7825} \frac{576 \sqrt[24]{\frac{51}{313}} 2^{5/12} \pi^{25/24}}{7825} + \frac{576 \times 2^{11/24} \sqrt[24]{\frac{2073}{313}} \pi^{25/24}}{7825} + \frac{576 \sqrt[24]{\frac{3}{313}} 2^{5/12} \pi^{25/24} \sqrt[24]{77 + 3 \sqrt{644} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{644}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{7825}$$

$$\frac{\frac{1}{25} \times 2^{7/24} \times 24 \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \left(\frac{24 \pi}{322 - 7 - 2} \right)^{25/24} =}{7825} \frac{576 \sqrt[24]{\frac{51}{313}} 2^{5/12} \pi^{25/24}}{7825} + \frac{576 \times 2^{11/24} \sqrt[24]{\frac{2073}{313}} \pi^{25/24}}{7825} + \frac{576 \sqrt[24]{\frac{3}{313}} 2^{5/12} \pi^{25/24} \sqrt[24]{77 + \frac{3 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 644^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}}}{7825}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

Furthermore, we note that, for $x = 0.233$, sub-multiple of Fibonacci number 233, we obtain:

$$\frac{24}{25} 2^{7/24} (17^{1/24} + 1382^{1/24} + (77 + 3 \sqrt{645})^{1/24}) ((0.233))^{25/24}$$

Input:

$$\frac{24}{25} \times 2^{7/24} \left(\sqrt[24]{17} + \sqrt[24]{1382} + \sqrt[24]{77 + 3 \sqrt{645}} \right) \times 0.233^{25/24}$$

Result:

0.956007624834173672547336037799822976276065982612157755691...

0.95600762483.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

On the Dilaton

From:

Scale-invariant alternatives to general relativity. III. The inflation–dark-energy connection

Santiago Casas, Georgios K. Karananas, Martin Pauly, Javier Rubio
arXiv:1811.05984v2 [astro-ph.CO] 18 Mar 2019

We have that:

In the opposite limit, i.e. for $|\kappa| \rightarrow 0$ (which should of course be taken with care when $c \rightarrow 0$), the predictions coincide with those of the $m^2\phi^2$ chaotic inflationary scenario,¹⁵

$$n_s \simeq 1 - \frac{4}{1 + 2N_*} \simeq 1 - \frac{2}{N_*}, \quad r \simeq \frac{16}{1 + 2N_*} \simeq \frac{8}{N_*}, \quad \alpha_s = -|\kappa|r, \quad (65)$$

For

$$N_* = 60$$

$$\begin{aligned} \text{We obtain: } 1 - 2/60 &= 1 - 0.03333333333333333333333333333333 = \\ &= 0.96666666666666666666666666666667 = n_s; \end{aligned}$$

and

$$8 / 60 = 0.13333333333333333333333333333333 = r;$$

Now, we note that: $r / n_s = 0.13793103448275862068965517241379$; the 64th root of this result is:

Input interpretation:

$$\sqrt[64]{0.13793103448275862068965517241379}$$

Result:

$$0.9695209960571124331556082537197294\dots$$

0.969520996.... value very near to n_s .

From:

Scalatron the healer: removing the strong-coupling in the Higgs- and Higgs-dilaton inflations

Dmitry Gorbunov, Anna Tokareva
arXiv:1807.02392v2 [hep-ph] 10 Dec 2018

We have that:

We can speak about the Higgs-scalaron inflation as a UV completion of the Higgs inflation if the CMB amplitude is actually defined by parameters of the Higgs sector, λ and ξ , rather than β . The heavy degree of freedom indeed can be integrated out if $\beta < \xi^2/\lambda^3$. In this case, the predictions for the tilt of the scalar perturbation spectrum $n_s - 1$ and tensor-to-scalar ratio r are of the standard form⁴ [1],

$$n_s = 1 - \frac{2}{N_e}, \quad r = \frac{12}{N_e^2} \quad (15)$$

with $N_e = 50 \div 60$ being a number of e-foldings of inflation which depends slightly on the reheating temperature. These predictions fall right in the ballpark of the region allowed by the Planck experiment [2]. No significant isocurvature and non-gaussianity is expected since the mass of the orthogonal direction is significantly larger than the Hubble scale ($\mathcal{H} \sim 1.5 \times 10^{13}$ GeV).

Summarising the bound (14) and perturbativity condition (5) on the parameter β we can write,

$$\frac{\xi^2}{4\pi} < \beta < \frac{\xi^2}{\lambda}. \quad (16)$$

Thus, with typical value of $\lambda \sim 0.01$ at large values of the Higgs field the remaining window for parameter β (which determines the scalaron mass (2)) is about three orders of magnitude. Consequently, for the reference value $\lambda = 10^{-2}$, the scalaron mass is in the interval 5×10^{13} GeV $< m < 1.5 \times 10^{15}$ GeV. The relevant part of the model parameter space is outlined in Fig. 4).

We have, from the eq. (15) that:

$$n_s = 1 - 2 / 60 = 1 - 0.03333333333333333333333333333333 = 0.966666666666666666666666666666667;$$

$$r = 12 / 3600 = 0,0033333333333333333333333333333333$$

We note that: $r / n_s = 0,00344827586206896551724137931034$; the 256th root of this result, is equal to:

$$(0.0034482758620689655)^{1/256}$$

Input interpretation:

$$\sqrt[256]{0.0034482758620689655}$$

Result:

$$0.9780954932486464809...$$

0.9780954932486.... value very near to n_s

Now, we have that:

The inflaton potential looks similar to the R^2 -Higgs case. Again, the inflationary stage can be effectively described as the single field rolling inside the valley, under the conditions (16) on β , see Fig. 5. The predictions for spectral parameters are the same as in the Higgs-dilaton model [25]: the scalar tilt depends on the value of ξ ,

$$n_s = 1 - 8\xi \coth 4\xi N_c. \quad (19)$$

Therefore, in order to satisfy Planck limits [2], we need $\xi \lesssim 0.004$. The CMB amplitude can be obtained under the same condition as in (14).

From eq. (19), we obtain:

$$1 - 8 \cdot 0.004 \coth(4 \cdot 60 \cdot 0.004) = n_s$$

Input:

$$1 - (8 \times 0.004) \coth(4 \times 60 \times 0.004)$$

$\coth(x)$ is the hyperbolic cotangent function

Result:

$$0.957005247101892133872636207817695318790000031804134308960\dots$$

$$0.95700524710189\dots = n_s$$

Percent decrease:

$$1 - (8 \times 0.004) \coth(4 \times 60 \times 0.004) = 0.957005 \text{ is } 4.29948\% \text{ smaller than } 1.$$

•

Alternative representations:

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 1 - 0.032 \left(1 + \frac{2}{-1 + e^{1.92}} \right)$$

•

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 1 - 0.032 i \cot(0.96 i)$$

•

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 1 + 0.032 i \cot(-0.96 i)$$

$\cot(x)$ is the cotangent function

i is the imaginary unit

Series representations:

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 1.032 + 0.064 \sum_{k=1}^{\infty} q^{2k} \text{ for } q = 2.6117$$

•

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 0.966667 - 0.06144 \sum_{k=1}^{\infty} \frac{1}{0.9216 + k^2 \pi^2}$$

•

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 1 - 0.03072 \sum_{k=-\infty}^{\infty} \frac{1}{0.9216 + k^2 \pi^2}$$

Integral representation:

$$1 - \coth(4 \times 60 \times 0.004) 8 \times 0.004 = 1 + 0.032 \int_{\frac{i\pi}{2}}^{0.96} \operatorname{csch}^2(t) dt$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019
Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a [spectral index \$n_s = 0.965 \pm 0.004\$](#) , consistent with the predictions of slow-roll, single-field, inflation.

Now, we have, from:

Dilatonic Inflation and SUSY Breaking in String-inspired Supergravity
Mitsuo J. Hayashi, Tomoki Watanabe, Ichiro Aizawa and Koichi Aketo
 arXiv:hep-ph/0303029v3 12 Nov 2003

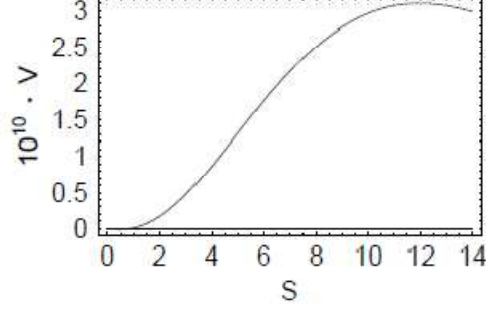


Figure 2: The plot of $V(S)$ minimized with respect to Y . The minimum value of the potential is $V(S_{\min}) \sim -9.3 \times 10^{-13}$.

The value of the scalar potential is $-9.3 * 10^{-13}$

Now:

Because n_s is not equal to 1 and α_s is negligible, our model supports the model with tilted power law spectrum. The value of n_{s*} is consistent with the recent observations; the best fitting of them (WMAPext, 2dFGRS and Lyman α) for power law Λ CDM model suggests $n_s(k_*) = 0.96 \pm 0.02$ [2].

Finally, estimating the spectrum of the density perturbation[16] caused by slow-rolling dilaton:

$$\mathcal{P}_{\mathcal{R}} \sim \frac{1}{12\pi^2} \frac{V^3}{\partial V^2}, \quad (26)$$

we find $\mathcal{P}_{\mathcal{R}*} \sim 2.1 \times 10^{-9}$. This result matches the measurements as well[2, 17]. Incidentally speaking, the energy scale $V \sim 10^{-10}$ is also within the constrained range[17].

inflationary energy scale is $V \sim 10^{-10} \sim (10^{16} \text{ GeV})^4$

From eq. (26), we obtain that the density perturbation caused by slow-rolling dilaton is equal to $\approx 2.1 * 10^{-9}$ and the energy scale is $V \approx 10^{-10}$. We note that, considering $-9.3 * 10^{-13}$ as energy and $c^2 = 9 * 10^{16}$, we obtain the following mass:

$$(-9.3 * 10^{-13} / 9 * 10^{16})$$

Input interpretation:

$$-\frac{9.3 \times 10^{-13}}{9 \times 10^{16}}$$

Result:

$9.67741935483870967741935483870967741935483870967741935\dots \times 10^6$
 $9.677419\dots * 10^6$

Repeating decimal:

$0.967741935483870 \times 10^7$ (period 15)

Input interpretation:

$0.967741935483870967741935483870967741935483870967742 \times 10^4$ TeV
 (teraelectronvolts)

$0.967741935483870 * 10^7$ GeV (1/m)
 $0.967741935483870 * 10^4$ TeV

We note also that:

Number of e -folds at which a comoving scale k crosses the Hubble scale aH during inflation is given by[1]:

$$N(k) \sim 62 - \ln \frac{k}{a_0 H_0} - \frac{1}{4} \ln \frac{(10^{16} \text{ GeV})^4}{V_k} + \frac{1}{4} \ln \frac{V_k}{V_{\text{end}}}, \quad (22)$$

where we assume $V_{\text{end}} = \rho_{\text{reh}}$. We focus the scale $k_* = 0.05 \text{ Mpc}^{-1}$ and the inflationary energy scale is $V \sim 10^{10} \sim (10^{15} \text{ GeV})^4$ as shown in Fig. 2, therefore the number of e -folds which corresponds to our scale must be around 57. On the other hand, using the slow-roll approximation, N is also calculated by:

$$N \sim - \int_{S_1}^{S_2} \frac{V}{\partial V} dS. \quad (23)$$

We could have obtained the number ~ 57 , by integrating from $S_{\text{end}} \sim 1.98$ to $S_* \sim 10.46$, fixing the parameters $c = 183$ and $b = 5.5$, i.e. our potential has the ability to produce the cosmologically plausible number of e -folds. Here S_* is the value corresponding to k_* .

Next, a scalar spectral index standing for a scale dependence of the spectrum of density perturbation and its running are defined by:

$$n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}, \quad \alpha_s = \frac{dn_s}{d \ln k}. \quad (24)$$

These are approximated in the slow-roll paradigm as:

$$n_s(S) \sim 1 - 6\epsilon_S + 2\eta_{SS}, \quad \alpha_s(S) \sim 16\epsilon_S\eta_{SS} - 24\epsilon_S^2 - 2\xi_{(3)}^2, \quad (25)$$

where $\xi_{(3)}$ is an extra slow-roll parameter that includes trivial third derivative of the potential. Substituting S_* into these, we have $n_{s*} \sim 0.95$ and $\alpha_{s*} \sim -4 \times 10^{-4}$. Because n_s is not equal to 1 and α_s is negligible, our model supports the model with tilted power law spectrum. The value of n_{s*} is consistent with the recent observations; the best fitting of them (WMAPext, 2dFGRS and Lyman α) for power law Λ CDM model suggests $n_s(k_*) = 0.96 \pm 0.02$ [2].

For the previous equation, we have that:

$$n_s = 1 - 2 / N = 1 - 2 / 57 = 0,9649122807017543859649122807017 \approx 0.965...$$

For:

$$\Theta_E = \frac{1 - 4c - 2\sqrt{4c^2 - 2c - 2\kappa}}{1 + 8\kappa}, \quad (52)$$

denotes the value of the Θ -field at the end of inflation. As usual, this is defined by the condition $\epsilon(\Theta_E) \equiv 1$. By inverting Eq. (51), we can express the inflationary observables as functions of the model parameters and N_* . For general values of c and κ , this inversion cannot be analytically performed and one must rely on numerical methods. The values of the spectral tilt n_s and the tensor-to-scalar ratio r , following from a numerical treatment of the potential (37), are presented in Fig. 2.

We obtain, for $c = 2$ and $\kappa = 3$:

$$\frac{((1-4*2-2*\sqrt{4*2^2-2*2-2*3}))/((1+8*3))}{1}$$

Input:

$$\frac{1 - 4 \times 2 - 2 \sqrt{4 \times 2^2 - 2 \times 2 - 2 \times 3}}{1 + 8 \times 3}$$

Result:

$$\frac{1}{25} (-7 - 2\sqrt{6})$$

Decimal approximation:

-0.47595917942265424785578272597647131135727579845253361027...

-0.475959179422....

Alternate forms:

$$-\frac{1}{25} (7 + 2\sqrt{6})$$

•

$$-\frac{7}{25} - \frac{2\sqrt{6}}{25}$$

•

Minimal polynomial:

$$25x^2 + 14x + 1$$

•

For $c = 3$ and $\kappa = 5$:

$$\frac{((1-4*3-2*\sqrt{4*3^2-2*3-2*5}))/((1+8*5))}{1}$$

Input:

$$\frac{1 - 4 \times 3 - 2 \sqrt{4 \times 3^2 - 2 \times 3 - 2 \times 5}}{1 + 8 \times 5}$$

Result:

$$\frac{1}{41} (-11 - 4 \sqrt{5})$$

Decimal approximation:

-0.48644565634144289721065108963231963272591398630356348529...

-0.4864456563414...

Alternate forms:

$$-\frac{1}{41} (11 + 4 \sqrt{5})$$

•

$$-\frac{11}{41} - \frac{4 \sqrt{5}}{41}$$

•

Minimal polynomial:

$$41x^2 + 22x + 1$$

•

For $c = 5$ and $\kappa = 8$:

$$\frac{((1 - 4 \cdot 5 - 2 \cdot \sqrt{4 \cdot 5^2 - 2 \cdot 5 - 2 \cdot 8}))}{((1 + 8 \cdot 8))}$$

Input:

$$\frac{1 - 4 \times 5 - 2 \sqrt{4 \times 5^2 - 2 \times 5 - 2 \times 8}}{1 + 8 \times 8}$$

Result:

$$\frac{1}{65} (-19 - 2 \sqrt{74})$$

Decimal approximation:

-0.55699462360131159297629149338614503483161647868586878829...

-0.5569946236...

Alternate forms:

$$-\frac{1}{65} (19 + 2\sqrt{74})$$

•

$$-\frac{19}{65} - \frac{2\sqrt{74}}{65}$$

•

Minimal polynomial:

$$65x^2 + 38x + 1$$

.....

For $c = 144$ and $\kappa = 233$:

$$\left(\frac{1 - 4 \times 144 - 2 \sqrt{4 \times 144^2 - 2 \times 144 - 2 \times 233}}{1 + 8 \times 233}\right)$$

Input:

$$\frac{1 - 4 \times 144 - 2 \sqrt{4 \times 144^2 - 2 \times 144 - 2 \times 233}}{1 + 8 \times 233}$$

Result:

$$\frac{-575 - 2\sqrt{82190}}{1865}$$

Decimal approximation:

-0.61575118895821756370569425911930833964738062111397272996...
 -0.6157511889582...

Alternate forms:

$$-\frac{575 + 2\sqrt{82190}}{1865}$$

•

$$-\frac{115}{373} - \frac{2\sqrt{\frac{16438}{5}}}{373}$$

•

Minimal polynomial:

$$1865x^2 + 1150x + 1$$

And for $c = 121393$ and $\kappa = 196418$:

$$\frac{((1-4*121393-2*\sqrt{4*121393^2-2*121393-2*196418}))/((1+8*196418))$$

Input:

$$\frac{1 - 4 \times 121393 - 2 \sqrt{4 \times 121393^2 - 2 \times 121393 - 2 \times 196418}}{1 + 8 \times 196418}$$

Exact result:

$$\frac{-485571 - 2 \sqrt{58944406174}}{1571345}$$

Decimal approximation:

-0.61803129291082061609299619166189325543364136781504422485...
-0.61803129291082.....

Where 2, 3, 5, 8, 144, 233, 121393 and 196418 are Fibonacci numbers

And for $c = 843$ and $\kappa = 1364$:

$$\frac{((1-4*843-2*\sqrt{4*843^2-2*843-2*1364}))/((1+8*1364))$$

Input:

$$\frac{1 - 4 \times 843 - 2 \sqrt{4 \times 843^2 - 2 \times 843 - 2 \times 1364}}{1 + 8 \times 1364}$$

Result:

$$\frac{-3371 - 38 \sqrt{7862}}{10913}$$

Decimal approximation:

-0.61764693045399572088946392201515130646059866643068169811...
-0.6176469304539.....

Alternate forms:

$$\frac{3371 + 38 \sqrt{7862}}{10913}$$

$$-\frac{3371}{10913} - \frac{38\sqrt{7862}}{10913}$$

• **Minimal polynomial:**

$$10913x^2 + 6742x + 1$$

And for $c = 103682$ and $\kappa = 167761$:

$$\frac{((1-4*103682-2*\sqrt{4*103682^2-2*103682-2*167761}))/((1+8*167761))}{1}$$

Input:

$$\frac{1 - 4 \times 103682 - 2 \sqrt{4 \times 103682^2 - 2 \times 103682 - 2 \times 167761}}{1 + 8 \times 167761}$$

Result:

$$\frac{-414727 - 2 \sqrt{42999285610}}{1342089}$$

Decimal approximation:

-0.61803083249899823020768287984939432601004340381804719779...

-0.61803083249899823.....

• **Alternate forms:**

$$-\frac{414727 + 2 \sqrt{42999285610}}{1342089}$$

$$-\frac{414727}{1342089} - \frac{2 \sqrt{42999285610}}{1342089}$$

• **Minimal polynomial:**

$$1342089x^2 + 829454x + 1$$

Where 843, 1364, 103682 and 167761 are Lucas numbers. We note that, increasing the values of c and κ (inserting Fibonacci or Lucas numbers), the function tends to the reciprocal of the golden ratio and the inverse function to the golden ratio itself! Indeed, for example, we have also that:

$$-1 / (((1-4*121393-2*\sqrt{4*121393^2-2*121393-2*196418}))/((1+8*196418)))$$

Input:

$$\frac{1}{\frac{1-4 \times 121393 - 2 \sqrt{4 \times 121393^2 - 2 \times 121393 - 2 \times 196418}}{1+8 \times 196418}}$$

Exact result:

$$\frac{1571345}{-485571 - 2 \sqrt{58944406174}}$$

Decimal approximation:

1.618041046579005350899213042342540624804890669332490276851...

1.618041046579....

And:

$$-1 / (((1-4*103682-2*\sqrt{4*103682^2-2*103682-2*167761}))/((1+8*167761)))$$

Input:

$$\frac{1}{\frac{1-4 \times 103682 - 2 \sqrt{4 \times 103682^2 - 2 \times 103682 - 2 \times 167761}}{1+8 \times 167761}}$$

Result:

$$\frac{1342089}{-414727 - 2 \sqrt{42999285610}}$$

Decimal approximation:

1.618042251964218801091465806218399506858213240854353744492...

1.6180422519642188.....

Alternate forms:

- $414727 - 2 \sqrt{42999285610}$

- $\frac{1342089}{414727 + 2 \sqrt{42999285610}}$

- $\frac{1}{\frac{414727}{1342089} + \frac{2 \sqrt{42999285610}}{1342089}}$

- **Minimal polynomial:**

$$x^2 - 829454x + 1342089$$

- For the following numbers: 196884, 21493760, 864299970, 20245856256, 333202640600, that are some coefficients of modular function j, we have that:

$$-1/(((1-4*196884-2*\sqrt{4*196884^2-2*196884-2*21493760}))/((1+8*21493760)))$$

Input:

$$-\frac{1}{\frac{1-4 \times 196884 - 2 \sqrt{4 \times 196884^2 - 2 \times 196884 - 2 \times 21493760}}{1+8 \times 21493760}}$$

- Open code

Result:

$$-\frac{171950081}{-787535 - 4 \sqrt{38752464134}}$$

Decimal approximation:

109.1773703379493671686436053542823389540741074642273625459...
109.17737...

Alternate forms:

- $787535 - 4 \sqrt{38752464134}$

- $\frac{171950081}{787535 + 4 \sqrt{38752464134}}$

- $\frac{1}{\frac{787535}{171950081} + \frac{4 \sqrt{38752464134}}{171950081}}$

- **Minimal polynomial:**

$$x^2 - 1575070x + 171950081$$

$$-1/(((1-4*21493760-2*\sqrt{4*21493760^2-2*21493760-2*864299970}))/((1+8*864299970)))$$

Input:

$$\frac{1}{\frac{1-4 \times 21493760 - 2 \sqrt{4 \times 21493760^2 - 2 \times 21493760 - 2 \times 864299970}}{1+8 \times 864299970}}$$

Result:

$$\frac{6914399761}{-85975039 - 12 \sqrt{51331252893415}}$$

Decimal approximation:

40.21167921761073874027200911068165479130135638675841563532...

40.21167921761....

Alternate forms:

$$85975039 - 12 \sqrt{51331252893415}$$

$$\frac{6914399761}{85975039 + 12 \sqrt{51331252893415}}$$

$$\frac{1}{\frac{85975039}{6914399761} + \frac{12 \sqrt{51331252893415}}{6914399761}}$$

Minimal polynomial:

$$x^2 - 171950078x + 6914399761$$

$$-1/\left(\frac{1-4 \times 86429970 - 2 \sqrt{4 \times 86429970^2 - 2 \times 86429970 - 2 \times 20245856256}}{1+8 \times 20245856256}\right)$$

Input:

$$\frac{1}{\frac{1-4 \times 86429970 - 2 \sqrt{4 \times 86429970^2 - 2 \times 86429970 - 2 \times 20245856256}}{1+8 \times 20245856256}}$$

Exact result:

$$\frac{161966850049}{-345719879 - 12 \sqrt{830014394228643}}$$

Decimal approximation:

234.2458660297716811906880345512509269265323749754018659433...

234.2458660297...

$$-1/(((1-4*20245856256-2*\sqrt{4*20245856256^2-2*20245856256-2*333202640600}))/((1+8*333202640600)))$$

Input:

$$\frac{1}{\frac{1-4 \times 20245856256-2 \sqrt{4 \times 20245856256^2-2 \times 20245856256-2 \times 333202640600}}{1+8 \times 333202640600}}$$

Result:

$$\frac{2665621124801}{-80983425023 - 8 \sqrt{102473673840472522277}}$$

Decimal approximation:

16.45781914209481835559277085713599335315273847892244843828...

16.457819142....

Alternate forms:

$$80983425023 - 8 \sqrt{102473673840472522277}$$

•

$$\frac{80983425023 - 8 \sqrt{102473673840472522277}}{2665621124801}$$

$$80983425023 + 8 \sqrt{102473673840472522277}$$

•

Minimal polynomial:

$$x^2 - 161966850046x + 2665621124801$$

•

In this case, we get values that are different every time with regard to the inverse function: 109.17737, 40.21167, 234.2458, 16.4578 . Performing the following calculation on these values, we get:

$$(109.17737 * 1/40.21167 * 1/234.2458 * 1/16.4578)^{1/256}$$

Input interpretation:

$$\sqrt[256]{109.17737 \times \frac{1}{40.21167} \times \frac{1}{234.2458} \times \frac{1}{16.4578}}$$

Result:

0.972045227576573393315122128017655917165475606039234978245...

0.9720452275765....

Regarding instead the results that provide the conjugate of the golden ratio and the golden ratio itself, we have the following interesting ratios:

(1.618041046579 * 1 / 1.6180422519642188)

Input interpretation:

$$1.618041046579 \times \frac{1}{1.6180422519642188}$$

Result:

0.999999255034769743048866997776013651231168900067408561496...

0.9999992550347697....

And:

1/ (1.618041046579 * 1 / 1.6180422519642188)

Input interpretation:

$$\frac{1}{1.618041046579 \times \frac{1}{1.6180422519642188}}$$

Result:

1.000000744965785230558860835911340395012039707729410103743...

1.000000744965.... result that is very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}} \approx 1.0000007913$$

$$\frac{1 + \sqrt[5]{\sqrt{(\varphi-1)^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \varphi$$

Thence the Θ_E -field

$$\Theta_E = \frac{1 - 4c - 2\sqrt{4c^2 - 2c - 2\kappa}}{1 + 8\kappa}$$

can take the following values:

-0.475959179422....; -0.4864456563414....; -0.5569946236....; -0.6157511889582;

-0.61803129291082...; -0.6176469304539....; -0.61803083249899823...;

and the inverse of the Θ_E -field : 1.618041046579; 1.6180422519642188

for c and κ equal to Fibonacci or Lucas numbers

The inverse of the Θ_E -field takes the following values for c and κ equal to numbers corresponding to coefficients of modular function j

109.17737...; 40.21167921761....; 234.2458660297...; 16.457819142....

Thus, for the Fibonacci or Lucas numbers, the Θ_E -field tends to the reciprocal of the golden section and its inverse to the golden ratio itself. Moreover, the inverse of the result ratios which are all close to the value of the golden ratio, provides a value very close to a result of a Rogers-Ramanujan continued fraction. As for the numbers of the modular function j, the 256th root of the result ratios provides a value very close to that of n_s which is also very close to a result of a Rogers-Ramanujan continued fraction. Indeed:

0.9720452275765.... is very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

The value of the dilatonic field could very well be the average of these two results:

1.000000744965.... and 0.9720452275765....

Indeed, we have that:

$$(1.000000744965 + 0.9720452275765)/2$$

Input interpretation:

$$\frac{1.000000744965 + 0.9720452275765}{2}$$

Result:

0.98602298627075

0.98602298627075 result very near to the dilaton value **0.989117352243 = ϕ**

This value is also very close to n_s and also to the Regge slope of the J/ Ψ meson

The dilaton value in kg is:

Input interpretation:

convert 0.989117352243 GeV/c² to kilograms

Result:

$1.76326183987 \times 10^{-27}$ kg (kilograms)

$1.76326183987 * 10^{-27}$ kg

Inserting the value of the dilaton mass $1.763262e-27$ in the Hawking radiation calculator, we obtain:

Mass = $1.763262e-27$

Radius = $2.618182e-54$

Temperature = $6.959847e+49$

Entropy = $3.581180e-38$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}\left[\left[\left[\left[\frac{1}{\left(\left(\left(\left(4 * 1.962364415e+19\right) / \left(5 * 0.0864055^2\right)\right)\right) * \frac{1}{1.763262e-27}\right) * \text{sqrt}\left[\left[\left(\left(6.959847e+49 * 4 * \text{Pi} * \left(2.618182e-54\right)^3 - \left(2.618182e-54\right)^2\right)\right)\right] / \left(\left(6.67 * 10^{-11}\right)\right)\right]\right]\right]\right]\right]$$

Input interpretation:

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.763262 \times 10^{-27}}\right)\sqrt{-\frac{6.959847 \times 10^{49} \times 4 \pi (2.618182 \times 10^{-54})^3 - (2.618182 \times 10^{-54})^2}{6.67 \times 10^{-11}}}\right)}$$

Result:

1.618249323487044216212912712520781720956858289555337951697...

1.618249323...

And:

1/sqrt[[[1/(((((((4*1.962364415e+19)/(5*0.0864055^2))) * 1/(1.763262e-27)) * sqrt[[-(((6.959847e+49 * 4*Pi*(2.618182e-54)^3 - (2.618182e-54)^2)))] / ((6.67*10^-11)))]]]]]]

Input interpretation:

$$1/\left(\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.763262 \times 10^{-27}}\right)\sqrt{-\frac{6.959847 \times 10^{49} \times 4 \pi (2.618182 \times 10^{-54})^3 - (2.618182 \times 10^{-54})^2}{6.67 \times 10^{-11}}}\right)}$$

Result:

0.617951749144053357148066657024289590747583236895637808383...

0.617951749...

From:

Rotating strings confronting PDG mesons

Jacob Sonnenschein and Dorin Weissman - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014

$c\bar{c}$. **The Ψ trajectory:** The left side of figure (15) depicts the Ψ trajectory. Here we use the states $J/\Psi(1S)(3097)1^{--}$, $\chi_{c1}(1P)(3510)1^{++}$, and $\Psi(3770)1^{--}$. Since no $J = 3$ state has been observed, we use three states with $J = 1$, but with increasing orbital angular momentum ($L = 0, 1, 2$) and do the fit to L instead of J . To give an idea of the shifts in mass involved, the $J^{PC} = 2^{++}$ state χ_{c2} has a mass of 3556 MeV, and the $J^{PC} = 3^{--}$ state is expected to lie 30 – 60 MeV above the $\Psi(3770)$ [23].

The best linear fit is

$$\alpha' = 0.418, a = -4.04$$

with $\chi_l^2 = 3.41 \times 10^{-4}$, but the optimal fit is far from the linear, with endpoint masses in the range of the constituent c quark mass:

$$m_c = 1500, \alpha' = 0.979, a = -0.09$$

with $\chi_m^2 = 5 \times 10^{-7}$ ($\chi_m^2/\chi_l^2 = 0.002$). Aside from the improvement in χ^2 , by adding the mass we also get a value for the slope (and to a lesser extent, the intercept) that is much closer to that obtained in fits for the light meson trajectories.

where α' is the Regge slope (string tension)

We know that:

$J/\psi(1S)$ MASS

VALUE (MeV)	EVTs	DOCUMENT ID	TECN	COMMENT
3096.916 ± 0.011 OUR AVERAGE				
3096.917 ± 0.010 ± 0.007		AULCHENKO 03	KEDR	$e^+e^- \rightarrow$ hadrons
3096.89 ± 0.09	502	¹ ARTAMONOV 00	OLYA	$e^+e^- \rightarrow$ hadrons
3096.91 ± 0.03 ± 0.01		² ARMSTRONG 93B	E760	$\bar{p}p \rightarrow e^+e^-$
3096.95 ± 0.1 ± 0.3	193	BAGLIN 87	SPEC	$\bar{p}p \rightarrow e^+e^-X$

$\chi_{c1}(1P)$ MASS

VALUE (MeV)	EVTs	DOCUMENT ID	TECN	COMMENT
3510.67 ± 0.05 OUR AVERAGE				Error includes scale factor of 1.2.
3508.4 ± 1.9 ± 0.7	460	¹ AAIJ	17BB LHCB	$pp \rightarrow b\bar{b}X \rightarrow 2(K^+K^-)X$
3510.71 ± 0.04 ± 0.09	4.8k	² AAIJ	17BI LHCB	$\chi_{c1} \rightarrow J/\psi\mu^+\mu^-$
3510.30 ± 0.14 ± 0.16		ABLIKIM 05G	BES2	$\psi(2S) \rightarrow \gamma\chi_{c1}$
3510.719 ± 0.051 ± 0.019		ANDREOTTI 05A	E835	$p\bar{p} \rightarrow e^+e^-\gamma$
3509.4 ± 0.9		BAI 99B	BES	$\psi(2S) \rightarrow \gamma X$
3510.60 ± 0.087 ± 0.019	513	³ ARMSTRONG 92	E760	$\bar{p}p \rightarrow e^+e^-\gamma$
3511.3 ± 0.4 ± 0.4	30	BAGLIN 86B	SPEC	$\bar{p}p \rightarrow e^+e^-X$
3512.3 ± 0.3 ± 4.0		⁴ GAISER 86	CBAL	$\psi(2S) \rightarrow \gamma X$
3507.4 ± 1.7	91	⁵ LEMOIGNE 82	GOLI	$185\pi^-\text{Be} \rightarrow \gamma\mu^+\mu^-A$
3510.4 ± 0.6		OREGLIA 82	CBAL	$e^+e^- \rightarrow J/\psi 2\gamma$
3510.1 ± 1.1	254	⁶ HIMEL 80	MRK2	$e^+e^- \rightarrow J/\psi 2\gamma$
3509 ± 11	21	BRANDELIK 79B	DASP	$e^+e^- \rightarrow J/\psi 2\gamma$
3507 ± 3		⁶ BARTEL 78B	CNTR	$e^+e^- \rightarrow J/\psi 2\gamma$
3505.0 ± 4 ± 4		^{6,7} TANENBAUM 78	MRK1	e^+e^-
3513 ± 7	367	⁶ BIDDICK 77	CNTR	$\psi(2S) \rightarrow \gamma X$

$\psi(3770)$ MASS (MeV)

OUR FIT includes measurements of $m_{\psi(2S)}$, $m_{\psi(3770)}$, and $m_{\psi(3770)} - m_{\psi(2S)}$.

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
3773.13 ± 0.35 OUR FIT	Error includes scale factor of 1.1.			
3778.1 ± 1.2 OUR AVERAGE				
3779.2 ^{+1.8} / _{-1.7} ^{+0.6} / _{-0.8}	1	ANASHIN	12A	KEDR $e^+e^- \rightarrow D\bar{D}$
3775.5 ± 2.4 ± 0.5	57	AUBERT	08B	BABR $B \rightarrow D\bar{D}K$
3776 ± 5 ± 4	68	BRODZICKA	08	BELL $B^+ \rightarrow D^0\bar{D}^0K^+$
3778.8 ± 1.9 ± 0.9		AUBERT	07BE	BABR $e^+e^- \rightarrow D\bar{D}\gamma$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
3779.8 ± 0.6		² SHAMOV	17	RVUE $e^+e^- \rightarrow D\bar{D}$, hadrons
3772.0 ± 1.9		^{3,4} ABLIKIM	08D	BES2 $e^+e^- \rightarrow$ hadrons
3778.4 ± 3.0 ± 1.3	34	CHISTOV	04	BELL Sup. by BRODZICKA 08

and we know also that mesons have integer spin (thus are bosons). Now, we have that:

$$((((((3778.10 + 3510.67 + 3096.916)/3))))))^{1/4096} =$$

$$((((((3778.10 + 3510.67 + 3096.916)/3))))))^{1/(64)^2}$$

Input interpretation:

$$64^2 \sqrt{\frac{1}{3} (3778.10 + 3510.67 + 3096.916)}$$

Result:

1.001991622130033349221628210235826567561594719190174624421...

1.00199162213... MeV

And that:

$$1/((((((3778.10 + 3510.67 + 3096.916)/3))))))^{1/(64)^2}$$

Input interpretation:

$$\frac{1}{64^2 \sqrt{\frac{1}{3} (3778.10 + 3510.67 + 3096.916)}}$$

Result:

0.998012336544491699666743290242800135486599093656458270531...

0.99801233654449....MeV

We know also that:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 240 - 345 \quad | \quad 0.937 - 1.000$$

$\omega(782)$ MASS

VALUE (MeV)	EVTs	DOCUMENT ID	TECN	COMMENT
782.65 ± 0.12 OUR AVERAGE		Error includes scale factor of 1.9.		See the ideogram below.
783.20 ± 0.13 ± 0.16	18680	AKHMETSHIN 05	CMD2	0.60-1.38 $e^+ e^- \rightarrow \pi^0 \gamma$
782.68 ± 0.09 ± 0.04	11200	¹ AKHMETSHIN 04	CMD2	$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$
782.79 ± 0.08 ± 0.09	1.2M	² ACHASOV 03D	RVUE	0.44-2.00 $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$
782.7 ± 0.1 ± 1.5	19500	WURZINGER 95	SPEC	1.33 $p d \rightarrow {}^3\text{He} \omega$
781.96 ± 0.17 ± 0.80	11k	³ AMSLER 94C	CBAR	0.0 $\bar{p} p \rightarrow \omega \eta \pi^0$
782.08 ± 0.36 ± 0.82	3463	⁴ AMSLER 94C	CBAR	0.0 $\bar{p} p \rightarrow \omega \eta \pi^0$
781.96 ± 0.13 ± 0.17	15k	AMSLER 93B	CBAR	0.0 $\bar{p} p \rightarrow \omega \pi^0 \pi^0$
782.4 ± 0.2	270k	WEIDENAUER 93	ASTE	$\bar{p} p \rightarrow 2\pi^+ 2\pi^- \pi^0$
782.2 ± 0.4	1488	KURDADZE 83B	OLYA	$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$
782.4 ± 0.5	7000	⁵ KEYNE 76	CNTR	$\pi^- p \rightarrow \omega n$

$\omega_3(1670)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1667 ± 4 OUR AVERAGE				
1665.3 ± 5.2 ± 4.5	23400	AMELIN	96	YES 36 $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$
1685 ± 20	60	BAUBILLIER	79	HBC 8.2 $K^- p$ backward
1673 ± 12	430	^{1,2} BALTAY	78E	HBC 15 $\pi^+ p \rightarrow \Delta 3\pi$
1650 ± 12		CORDEN	78B	OMEG 8-12 $\pi^- p \rightarrow N 3\pi$
1669 ± 11	600	² WAGNER	75	HBC 7 $\pi^+ p \rightarrow \Delta^{++} 3\pi$
1678 ± 14	500	DIAZ	74	DBC 6 $\pi^+ n \rightarrow p 3\pi^0$
1660 ± 13	200	DIAZ	74	DBC 6 $\pi^+ n \rightarrow p \omega \pi^0 \pi^0$
1679 ± 17	200	MATTHEWS	71D	DBC 7.0 $\pi^+ n \rightarrow p 3\pi^0$
1670 ± 20		KENYON	69	DBC 8 $\pi^+ n \rightarrow p 3\pi^0$
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
~ 1700	110	¹ CERRADA	77B	HBC 4.2 $K^- p \rightarrow \Lambda 3\pi$
1695 + 20		BARNES	69B	HBC 4.6 $K^- p \rightarrow \omega 2\pi X$
1636 ± 20		ARMENISE	68B	DBC 5.1 $\pi^+ n \rightarrow p 3\pi^0$

¹ Phase rotation seen for $J^P = 3^- \rho\pi$ wave.

² From a fit to $I(J^P) = 0(3^-) \rho\pi$ partial wave.

$\omega_3(1670)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
160 ± 10 OUR AVERAGE				
149 ± 19 ± 7	23400	AMELIN	96	YES 36 $\pi^- p \rightarrow \pi^+ \pi^- \pi^0 n$
160 ± 80	60	³ BAUBILLIER	79	HBC 8.2 $K^- p$ backward
173 ± 16	430	^{4,5} BALTAY	78E	HBC 15 $\pi^+ p \rightarrow \Delta 3\pi$
253 ± 39		CORDEN	78B	OMEG 8-12 $\pi^- p \rightarrow N 3\pi$
173 ± 28	600	^{3,5} WAGNER	75	HBC 7 $\pi^+ p \rightarrow \Delta^{++} 3\pi$
167 ± 40	500	DIAZ	74	DBC 6 $\pi^+ n \rightarrow p 3\pi^0$
122 ± 39	200	DIAZ	74	DBC 6 $\pi^+ n \rightarrow p \omega \pi^0 \pi^0$
155 ± 40	200	³ MATTHEWS	71D	DBC 7.0 $\pi^+ n \rightarrow p 3\pi^0$
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
90 ± 20		BARNES	69B	HBC 4.6 $K^- p \rightarrow \omega 2\pi$
100 ± 40		KENYON	69	DBC 8 $\pi^+ n \rightarrow p 3\pi^0$
112 ± 50		ARMENISE	68B	DBC 5.1 $\pi^+ n \rightarrow p 3\pi^0$

³ Width errors enlarged by us to $4\Gamma/\sqrt{N}$; see the note with the $K^*(892)$ mass.

⁴ Phase rotation seen for $J^P = 3^- \rho\pi$ wave.

⁵ From a fit to $I(J^P) = 0(3^-) \rho\pi$ partial wave.

$\omega(1650)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1670 ± 30 OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1671 ± 6 ± 10	824	¹ AKHMETSHIN 17A	CMD3	1.4–2.0 $e^+e^- \rightarrow \omega\eta$
1660 ± 10	898	² ACHASOV 16B	SND	1.34–2.00 $e^+e^- \rightarrow \omega\eta$
1680 ± 10	13.1k	³ AULCHENKO 15A	SND	1.05–1.80 $c^+c^- \rightarrow \pi^+\pi^-\pi^0$
1667 ± 13 ± 6		AUBERT 07AU	BABR	10.6 $e^+e^- \rightarrow \omega\pi^+\pi^-\gamma$
1645 ± 8	13	AUBERT 06D	BABR	10.6 $e^+e^- \rightarrow \omega\eta\gamma$
1660 ± 10 ± 2		AUBERT,B 04N	BABR	10.6 $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma$
1770 ± 50 ± 60	1.2M	⁴ ACHASOV 03D	RVUE	0.44–2.00 $e^+e^- \rightarrow \pi^+\pi^-\pi^0$
1619 ± 5		⁵ HENNER 02	RVUE	1.2–2.0 $e^+e^- \rightarrow \rho\pi, \omega\pi\pi$
1700 ± 20		EUGENIO 01	SPEC	18 $\pi^-p \rightarrow \omega\eta n$
1705 ± 26	612	⁶ AKHMETSHIN 00D	CMD2	$c^+c^- \rightarrow \omega\pi^+\pi^-$
1820 ⁺¹⁹⁰ ₋₁₅₀		⁷ ACHASOV 98H	RVUE	$e^+e^- \rightarrow \pi^+\pi^-\pi^0$
1840 ⁺¹⁰⁰ ₋₇₀		⁸ ACHASOV 98H	RVUE	$e^+e^- \rightarrow \omega\pi^+\pi^-$
1780 ⁺¹⁷⁰ ₋₃₀₀		⁹ ACHASOV 98H	RVUE	$e^+e^- \rightarrow K^+K^-$
~ 2100		¹⁰ ACHASOV 98H	RVUE	$e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp$
1606 ± 9		¹¹ CLEGG 94	RVUE	
1662 ± 13	750	¹² ANTONELLI 92	DM2	1.34–2.4 $e^+e^- \rightarrow \rho\pi, \omega\pi\pi$
1670 ± 20		ATKINSON 83B	OMEG	20–70 $\gamma p \rightarrow 3\pi X$
1657 ± 13		CORDIER 81	DM1	$e^+e^- \rightarrow \omega 2\pi$
1679 ± 34	21	ESPOSITO 80	FRAM	$e^+e^- \rightarrow 3\pi$
1652 ± 17		COSME 79	OSPK	$e^+e^- \rightarrow 3\pi$

$\omega(1650)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
315 ± 35 OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
113 ± 9 ± 10	824	¹ AKHMETSHIN 17A	CMD3	1.4–2.0 $c^+c^- \rightarrow \omega\eta$
110 ± 20	898	² ACHASOV 16B	SND	1.34–2.00 $e^+e^- \rightarrow \omega\eta$
310 ± 30	13.1k	³ AULCHENKO 15A	SND	1.05–1.80 $e^+e^- \rightarrow \pi^+\pi^-\pi^0$
222 ± 25 ± 20		AUBERT 07AU	BABR	10.6 $e^+e^- \rightarrow \omega\pi^+\pi^-\gamma$
114 ± 14	13	AUBERT 06D	BABR	10.6 $e^+e^- \rightarrow \omega\eta\gamma$
230 ± 30 ± 20		AUBERT,B 04N	BABR	10.6 $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma$
490 ⁻²⁰⁰ ₋₁₅₀ ± 130	1.2M	⁴ ACHASOV 03D	RVUE	0.44–2.00 $e^+e^- \rightarrow \pi^+\pi^-\pi^0$
250 ± 14		⁵ HENNER 02	RVUE	1.2–2.0 $e^+e^- \rightarrow \rho\pi, \omega\pi\pi$
250 ± 50		EUGENIO 01	SPEC	18 $\pi^-p \rightarrow \omega\eta n$
370 ± 25	612	⁶ AKHMETSHIN 00D	CMD2	$e^+e^- \rightarrow \omega\pi^+\pi^-$
113 ± 20		⁷ CLEGG 94	RVUE	
280 ± 24	750	⁸ ANTONELLI 92	DM2	1.34–2.4 $e^+e^- \rightarrow \rho\pi, \omega\pi\pi$
160 ± 20		ATKINSON 83B	OMEG	20–70 $\gamma p \rightarrow 3\pi X$
136 ± 46		CORDIER 81	DM1	$e^+e^- \rightarrow \omega 2\pi$
99 ± 49	21	ESPOSITO 80	FRAM	$e^+e^- \rightarrow 3\pi$
42 ± 17		COSME 79	OSPK	$e^+e^- \rightarrow 3\pi$

The average of the various Regge slope of Omega mesons are:

$$0,91 + 0,918 + 0,988 + 1,18 + 0,937 + 1 = 5,933 \div 6 =$$

$$= 0,988833333333... = \alpha' \text{ of Omega meson.}$$

The masses are 782.65 – 1667 – 1670, thence the average is: 1373.216666 MeV

Thence, we obtain:

$$1/((((((782.65 + 1667 + 1670)/3))))))^{1/192}$$

Input interpretation:

$$\frac{1}{\sqrt[192]{\frac{1}{3}(782.65 + 1667 + 1670)}}$$

Result:

0.963069455372249693187801378180946201236578729042148405048...

0.96306945537.... MeV

$$((((((782.65 + 1667 + 1670)/3))))))^{1/192}$$

Input interpretation:

$$\sqrt[192]{\frac{1}{3}(782.65 + 1667 + 1670)}$$

Result:

1.038346709494047608402529648676420125432963724896142711734...

1.038346709494..... MeV

From:

Dilatonic Dark Matter – A New Paradigm –

Y. M. Cho - arXiv:hep-ph/9810379v1 16 Oct 1998

Now the important question is how one could detect the dilaton. It seems very difficult to detect it through the dilatonic fifth force, because the range of the fifth force would be about 10^{-8} cm (for $\mu = 0.5$ keV) or about 10^{-13} cm (for $\mu = 270$ MeV). Perhaps a more promising

way is to use the two photon decay process, which produces two mono-energetic X-rays of $E \simeq 0.25$ keV or $E \simeq 135$ MeV with the same polarization. With the local halo density of our galaxy $\rho_{HALO} \simeq 0.3$ GeV/cm³ one can easily find the local dilaton number density to be $\bar{n} \sim 5.83 \times 10^5/cm^3$ for $\mu = 0.5$ keV and $\bar{n} \sim 0.11/cm^3$ for $\mu = 270$ MeV. In both cases the local velocity of the dilaton is about $10^{-3} c$. So it is very important to look for the above X-ray signals from the sky (with the Doppler broadening of $\Delta E \simeq 10^{-3} E$) or to perform a Sikivie-type X-ray detection experiment with a strong electromagnetic field to enhance the dilaton conversion, although the long life-time (for $\mu = 0.5$ keV) or the low local number density (for $\mu = 270$ MeV) of the dilaton could make such experiments very difficult. For the $\mu = 270$ MeV dilaton one could also look for the $\mu^+\mu^-$ decay process.

We have that, from the average between 0.25 keV and 270 MeV:

$$(5 \times 10^{-4} + 270)/2$$

Input interpretation:

$$\frac{1}{2} (5 \times 10^{-4} + 270)$$

Exact result:

$$\frac{540001}{4000}$$

Decimal form:

$$135.00025$$

135.00025 MeV that is the energy corresponding to the dilaton mass of 270 MeV

We calculate the 2048th root and obtain:

$$((((5 \times 10^{-4} + 270)/2)))^{1/2048}$$

Input interpretation:

$$\sqrt[2048]{\frac{1}{2} (5 \times 10^{-4} + 270)}$$

Possible closed forms:

$$\tan\left(\operatorname{csch}\left(\frac{13\,311\,997}{52\,815\,242}\right)\right) \approx 0.9976077114894541562349094311253018077070783175558473529016$$

$$\pi \sqrt{\text{root of } 47545x^3 - 75492x^2 - 3820x + 7303 \text{ near } x = 0.317548} \approx 0.9976077114894541559055109214195810261834911129556221910974$$

$$\frac{1\,212\,863\,705 \pi}{3\,819\,460\,958} \approx 0.9976077114894541559510163103853653578007236769452836309879$$

csch(x) is the hyperbolic cosecant function

Further:

Input:

$$1 + \frac{1}{270}$$

Exact result:

$$\frac{271}{270}$$

Decimal approximation:

1.003703...
1.0037037... MeV

And:

Input:

$$\frac{1}{1 + \frac{1}{270}}$$

Exact result:

$$\frac{270}{271}$$

Decimal approximation:

0.99630996309963099630996309963099630996309963099630996309963099...

0.99630996.... MeV result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \varphi + 1 \approx 0.9991104684$$

From the mass equal to 1024 MeV (very near to Phi meson mass 1019 MeV, and where $1024 = 64 * 16 = 4096/4$), we obtain:

Input:

$$1 + \frac{1}{1024}$$

Exact result:

$$\frac{1025}{1024}$$

Decimal form:

1.0009765625
1.0009765625

and also practically the value of above continued fraction:

$$1/(1+1/1024)$$

Input:

$$\frac{1}{1 + \frac{1}{1024}}$$

Exact result:

$$\frac{1024}{1025}$$

Decimal approximation:

0.99902439024390243902439024390243902439024390243902439024390243...
0.9990243902...

We note that:

1024 MeV where

Input interpretation:

1024 MeV (megaelectronvolts)

1.024 GeV (gigaelectronvolts)

And that:

1/ 1.024

Input:

$$\frac{1}{1.024}$$

Result:

0.9765625

0.9765625

•

Rational form:

$$\frac{125}{128}$$

•

Possible closed forms:

$$\frac{1}{2} (e - 2) e \approx 0.9762462210062799$$

$$\frac{42}{43} \approx 0.9767441860465116$$

$$\frac{23 \pi}{74} \approx 0.9764409598995303$$

0.9765625... result practically equal to the following Regge slope meson

$c\bar{c}$. The Ψ trajectory: The left side of figure (15) depicts the Ψ trajectory. Here we use the states $J/\Psi(1S)(3097)1^{--}$, $\chi_{c1}(1P)(3510)1^{++}$, and $\Psi(3770)1^{--}$. Since no $J = 3$ state has been observed, we use three states with $J = 1$, but with increasing orbital angular momentum ($L = 0, 1, 2$) and do the fit to L instead of J . To give an idea of the shifts in mass involved, the $J^{PC} = 2^{++}$ state χ_{c2} has a mass of 3556 MeV, and the $J^{PC} = 3^{--}$ state is expected to lie 30 – 60 MeV above the $\Psi(3770)$ [23].

$$m_c = 1500, \alpha' = 0.979, a = -0.09$$

$\alpha' = 0.979$

In conclusion, the mass of the dilaton can be ≈ 1 GeV, precisely 1.024 GeV , very near to the mass of Phi meson

From:

Ramanujan Lost Notebook part V

Let ζ_5 be a primitive fifth root of unity,,

The 5th root of unity is, by definition, the roots of the polynomial $x^5=1$.
As we can see there is only one real solution, and all of the solutions are given below:

1. 1
2. $\cos(2\pi/5)+i\times\sin(2\pi/5)$
3. $\cos(4\pi/5)+i\times\sin(4\pi/5)$
4. $\cos(6\pi/5)+i\times\sin(6\pi/5)$
5. $\cos(8\pi/5)+i\times\sin(8\pi/5)$

$$(q; q)_{\infty} f_5(q) = (1 - \zeta_5) \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(3n+1)/2}}{1 - \zeta_5 q^n}.$$

For $q = 0.5$, and $\cos(8\pi/5)+i\times\sin(8\pi/5)$, we obtain:

$$\cos(8\pi/5)+i\times\sin(8\pi/5) * \text{sum} (-1^n * 0.5^{((n(3n+1)/2)}) / (((1 - (\cos(8\pi/5)+i\times\sin(8\pi/5))0.5^n))))), n=0..8989$$

Input interpretation:

$$\cos\left(8 \times \frac{\pi}{5}\right) + i \sin\left(8 \times \frac{\pi}{5}\right) \sum_{n=0}^{8989} \frac{1^n \times 0.5^{n(1/2(3n+1))}}{1 - (\cos\left(8 \times \frac{\pi}{5}\right) + i \sin\left(8 \times \frac{\pi}{5}\right)) \times 0.5^n}$$

i is the imaginary unit

Result:

$$1.08563 + 0.696745 i$$

Result:

$$1.08563... + \\ 0.696745... i$$

Polar coordinates:

$r = 1.28998$ (radius), $\theta = 32.6919^\circ$ (angle)

1.28998

$$(1.08563 + 0.696745 i)^2$$

Input interpretation:

$$(1.08563 + 0.696745 i)^2$$

i is the imaginary unit

Result:

0.693139... +
1.51281... i

Polar coordinates:

$r = 1.66405$ (radius), $\theta = 65.3838^\circ$ (angle)

1.66405 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$1/((((((((((1.08563 + 0.696745 i)^2)))))))))^{1/(64^2))}$$

Input interpretation:

$$\frac{1}{\sqrt[64^2]{(1.08563 + 0.696745 i)^2}}$$

i is the imaginary unit

Result:

0.999875640... -
0.000278569393... i

Polar coordinates:

$r = 0.999876$ (radius), $\theta = -0.0159628^\circ$ (angle)

0.999876

And, multiplying by 1/2:

$$1/2 * (((((\cos(8\pi/5)+i \times \sin(8\pi/5) * \sum (-1^n * 0.5^{((n(3n+1)/2)})))/((((1 - (\cos(8\pi/5)+i \times \sin(8\pi/5))0.5^n))))), n=0..8989))))))$$

Input interpretation:

$$\frac{1}{2} \left(\cos\left(8 \times \frac{\pi}{5}\right) + i \sin\left(8 \times \frac{\pi}{5}\right) \sum_{n=0}^{8989} - \frac{1^n \times 0.5^{n(1/2(3n+1))}}{1 - (\cos\left(8 \times \frac{\pi}{5}\right) + i \sin\left(8 \times \frac{\pi}{5}\right)) \times 0.5^n} \right)$$

i is the imaginary unit

Result:

$$0.542815 + 0.348372 i$$

Result:

$$0.542815... + 0.348372... i$$

Polar coordinates:

$$r = 0.644989 \text{ (radius), } \theta = 32.6918^\circ \text{ (angle)}$$

$$0.644989$$

And:

$$1 + 0.644989$$

Input interpretation:

$$1 + 0.644989$$

Result:

$$1.644989$$

$$1.644989 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

For $q = 0.8$, we obtain:

$$\cos(8\pi/5) + i \sin(8\pi/5) * \sum_{n=0}^{8989} \frac{(-1)^n * (0.8)^{n(1/2(3n+1))}}{1 - (\cos(8\pi/5) + i \sin(8\pi/5)) * (0.8)^n}$$

Input interpretation:

$$\cos\left(8 \times \frac{\pi}{5}\right) + i \sin\left(8 \times \frac{\pi}{5}\right) \sum_{n=0}^{8989} - \frac{1^n \times 0.8^{n(1/2(3n+1))}}{1 - (\cos\left(8 \times \frac{\pi}{5}\right) + i \sin\left(8 \times \frac{\pi}{5}\right)) \times 0.8^n}$$

i is the imaginary unit

Result:

$$1.506 + 1.06595 i$$

Result:

1.506 +
1.06595... i

Polar coordinates:

$r = 1.84507$ (radius), $\theta = 35.2909^\circ$ (angle)
1.84507

$1/(1.506 + 1.06595 i)^{1/64}$

Input interpretation:

$$\frac{1}{\sqrt[64]{1.506 + 1.06595 i}}$$

i is the imaginary unit

Result:

0.9904292... -
0.009532295... i

Polar coordinates:

$r = 0.990475$ (radius), $\theta = -0.551421^\circ$ (angle)
0.990475

From:

**Thermodynamics of 4D Dilatonic Black Holes and
the Weak Gravity Conjecture**

Gregory J. Loges, Toshifumi Noumi and Gary Shiu - arXiv:1909.01352v1 [hep-th] 3
Sep 2019

We have that:

We begin by reviewing static, dyonic black hole solutions of 4D EMD theory, for which we write the action as

$$I = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{M_{\text{Pl}}^2}{2} (\partial\phi)^2 - \frac{1}{4} e^{-2\lambda\phi} (F^2) \right]. \quad (2.1)$$

Here and in what follows we use the compact notation $(\partial\phi)^2 = \partial_\mu\phi\partial^\mu\phi$ and $(F^2) = F_{\mu\nu}F^{\mu\nu}$. Going forward we will set $M_{\text{Pl}}^2 = 1/8\pi G_N = 1$. The exponential coupling constant, λ , may take on any real value, and it will be convenient to introduce the associated constant $h = \frac{2}{1+2\lambda^2} \in (0, 2]$. Several special values for λ are of note: $\lambda = 0$ ($h = 2$) gives Einstein-Maxwell theory with a decoupled dilaton; $\lambda^2 = 1/2$ ($h = 1$) corresponds to the low-energy effective action of string theory; $\lambda^2 = 3/2$ ($h = 1/2$) corresponds to the KK reduction of Einstein gravity from 5D to 4D, where the radion plays the role of the dilaton.

We have: $\lambda^2 = 1.5$; $\lambda = \sqrt{1.5} = 1.224744871\dots$

This divergence may be avoided by stabilizing the dilaton with a mass $m_\phi \gtrsim \frac{|\lambda|}{M}$ (for masses below this the solution approaches that of a massless dilaton near the outer horizon and the divergences survive). The classical solution now takes the form [23, 24]

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + R(r)^2 d\Omega^2, \quad (4.19)$$

$$f(r) = \left(1 - \frac{M}{4\pi r} + \frac{q_e^2}{2r^2} \right) - \frac{\lambda^2 q_e^4}{10m_\phi^2 r^6} + \dots, \quad (4.20)$$

$$R(r) = r \left(1 - \frac{\lambda^2 q_e^4}{7m_\phi^4 r^8} + \dots \right), \quad (4.21)$$

$$\phi = -\frac{\lambda^2 q_e^2}{m_\phi^2 r^4} + \dots, \quad (4.22)$$

$$F_{tr} = \frac{q_e}{r^2} \left(1 - \frac{2\lambda^2 q_e^2}{m_\phi^2 r^4} + \dots \right). \quad (4.23)$$

From

Decays of the f0(1370) scalar glueball candidate

Ugo Gastaldi - 2-6 July 2018 Montpellier – Fr

we have:

The clear confined STAR signal of $f_0(1370)$ decays into $\pi^+\pi^-$ pairs confirms features of $f_0(1370)$ decays observed in $p\bar{p}$ annihilations at rest and shows that the σ meson and $f_0(1370)$ are distinct separate objects. The limited width of $f_0(1370)$ and its mass centered around 1370 MeV validate the analysis of data of $p\bar{p}$ annihilations at rest into 3 pseudoscalars that used the hypothesis of the possible existence of $f_0(1370)$ as an individual object [42,51,61,57,59,80].

PxDDBR $f_0(1370) \rightarrow \pi^+\pi^-$

From:

[https://www.semanticscholar.org/paper/A-study-in-depth-of-f0\(1370\)-Bugg/65153a0c27229221d3473e66a1facab06f09e827](https://www.semanticscholar.org/paper/A-study-in-depth-of-f0(1370)-Bugg/65153a0c27229221d3473e66a1facab06f09e827)

01 GeV

	$f_0(1370)$	$f_0(1500)$	σ
M	1.3150	1.5028	0.9128
α_1	3.5320	1.4005	17.051
s_1	2.9876	2.9658	3.0533
w_1	0.8804	0.8129	1.0448
α_2	-0.0427	-0.0135	-0.0536
s_2	-0.4619	-0.2141	-0.0975
w_2	-0.0036	0.0010	0.2801

Table 3. Parameters fitting $m(s)$ in units of GeV

For :

$$m_\phi \gtrsim \frac{|\lambda|}{M}$$

We put $\lambda = 1.3$ and $M = 1.3150$ GeV, corresponding to the mass of scalar glueball candidate $f_0(1370)$ meson, and obtain:

$$1.3/(1.3150)$$

Input interpretation:

$$\frac{1.3}{1.3150}$$

Result:

0.988593155893536121673003802281368821292775665399239543726...

0.9885931... GeV

With regard PxDBR $f_0(1370) \rightarrow \pi^+\pi^-$, from the calculations on the following Rogers-Ramanujan continued fractions:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{\frac{\pi}{\sqrt{5}}}}{\sqrt{5} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!} + \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \dots}}}}} \approx 2.0663656771$$

We obtain:

$$0.9568666373 + 0.9991104684 = 1.9559771057;$$

$$(1.9559771057)^7 = 109.5333499...;$$

$$(2,0663656771)^7 = 160.860811889...$$

From the average of the two result, we obtain the mass of the Pion meson π^0 :

$$(109.5333499 + 160.860811889)/2$$

Input interpretation:

$$\frac{109.5333499 + 160.860811889}{2}$$

Result:

$$135.1970808945$$

135.1970808945 result very near to the rest mass of the Pion meson 134.9766

From:

ASYMPTOTIC FORMULAS FOR TWO CONTINUED FRACTIONS IN RAMANUJAN'S LOST NOTEBOOK

Bruce C. Berndt and Jaebum Sohn

Theorem 3.2. As $x \rightarrow 0+$,

$$\frac{(3x)^{1/3}}{1} - \frac{1}{1+e^x} - \frac{1}{1+e^{2x}} - \frac{1}{1+e^{3x}} - \dots = \frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})} e^{G(x)}, \quad (3.7)$$

where

$$G(x) \sim a_2x^2 + a_4x^4 + a_6x^6 + \dots,$$

with

$$a_\nu = \frac{4\Gamma(\nu)\zeta(\nu)L(\nu+1, \chi)}{(2\pi/\sqrt{3})^{2\nu+1}},$$

where $\chi(n) = (\frac{n}{3})$. In particular,

$$a_2 = \frac{1}{108}, \quad a_4 = \frac{1}{4320}, \quad \text{and} \quad a_6 = \frac{1}{38880}. \quad (3.8)$$

Furthermore, as $x \rightarrow 0+$,

$$\text{the minimum value of } a_\nu x^\nu \sim \frac{3}{\pi} \sqrt{\frac{2x}{\pi}} e^{-\frac{4\pi^2}{3x}}. \quad (3.9)$$

Theorem 3.3. As $x \rightarrow 0+$,

$$\frac{2\sqrt{x}}{1} - \frac{1}{e^x + e^{-x}} - \frac{1}{e^{2x} + e^{-2x}} - \frac{1}{e^{3x} + e^{-3x}} - \dots = \frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})} e^{G(x)}, \quad (3.11)$$

where

$$G(x) \sim a_2 x^2 + a_4 x^4 + a_6 x^6 + \dots,$$

with

$$a_\nu = \frac{4\Gamma(\nu)\zeta(\nu)L(\nu+1, \chi)}{\pi^{2\nu+1}},$$

where χ is the nonprincipal, primitive character modulo 4. Furthermore,

$$a_2 = \frac{1}{48}, \quad a_4 = \frac{1}{1152}, \quad \text{and} \quad a_6 = \frac{61}{362880}, \quad (3.12)$$

and, as $x \rightarrow 0+$,

$$\text{the minimum value of } a_\nu x^\nu \sim \frac{4}{\pi} \sqrt{\frac{2x}{\pi}} e^{-\frac{\pi^2}{x}}. \quad (3.13)$$

Corollary 3.4. As $x \rightarrow 0+$,

$$(6x)^{\frac{2}{3}} \frac{(e^{-5x}; e^{-6x})_\infty}{(e^{-x}; e^{-6x})_\infty} = \frac{\Gamma(\frac{1}{6})}{\Gamma(\frac{5}{6})} e^{G(x)},$$

where

$$G(x) \sim a_2 x^2 + a_4 x^4 + a_6 x^6 + \dots,$$

with

$$a_\nu = \frac{4\Gamma(\nu)\zeta(\nu)L(\nu+1, \chi)}{(2\pi/\sqrt{6})^{2\nu+1}},$$

where χ is the nonprincipal, primitive character modulo 6. Furthermore, as $x \rightarrow 0+$,

$$\text{the minimum value of } a_\nu x^\nu \sim \frac{6}{\pi} \sqrt{\frac{2x}{\pi}} e^{-\frac{2\pi^2}{3x}}.$$

Corollary 3.5. As $x \rightarrow 0+$,

$$(7x) \frac{(e^{-3x}, e^{-5x}, e^{-6x}; e^{-7x})_\infty}{(e^{-x}, e^{-2x}, e^{-4x}; e^{-7x})_\infty} = \frac{\Gamma(\frac{1}{7})\Gamma(\frac{2}{7})\Gamma(\frac{4}{7})}{\Gamma(\frac{3}{7})\Gamma(\frac{5}{7})\Gamma(\frac{6}{7})} e^{G(x)},$$

where

$$G(x) \sim a_2 x^2 + a_4 x^4 + a_6 x^6 + \dots,$$

with

$$a_\nu = \frac{4\Gamma(\nu)\zeta(\nu)L(\nu+1, \chi)}{(2\pi/\sqrt{7})^{2\nu+1}},$$

where χ is the nonprincipal, primitive, real character modulo 7. Furthermore, as $x \rightarrow 0+$,

$$\text{the minimum value of } a_\nu x^\nu \sim \frac{7}{\pi} \sqrt{\frac{2x}{\pi}} e^{-\frac{4\pi^2}{7x}}.$$

We obtain:

$$\left(\frac{\Gamma(1/3)}{\Gamma(2/3)} \right) \exp\left(\left(\frac{1}{108} \left(\frac{1}{12} \right)^2 + \frac{1}{4320} \left(\frac{1}{12} \right)^4 + \frac{1}{38880} \left(\frac{1}{12} \right)^6 \right) \right)$$

Input:

$$\frac{\Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})} \exp\left(\frac{1}{108} \left(\frac{1}{12} \right)^2 + \frac{1}{4320} \left(\frac{1}{12} \right)^4 + \frac{1}{38880} \left(\frac{1}{12} \right)^6 \right)$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{e^{7466257/116095057920} \Gamma(\frac{1}{3})}{\Gamma(\frac{2}{3})}$$

Decimal approximation:

1.978491495476199196806827569059588723106890367497719141884...

1.97849149547619...

Alternate forms:

$$\frac{2 e^{7466257/116095057920} \frac{1}{3}!}{\frac{2}{3}!}$$

$$\frac{e^{7466257/116095057920} \Gamma(\frac{1}{6})}{2^{2/3} \sqrt{\pi}}$$

$$\frac{\sqrt{3} e^{7466257/116095057920} \Gamma\left(\frac{1}{3}\right)^2}{2\pi}$$

$n!$ is the factorial function

Alternative representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{G\left(\frac{4}{3}\right) \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right)}{\frac{G\left(\frac{1}{3}\right)G\left(\frac{5}{3}\right)}{G\left(\frac{2}{3}\right)}}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(-\frac{2}{3}\right)!}{\left(-\frac{1}{3}\right)!}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) e^{-\log G(1/3) + \log G(4/3)}}{e^{-\log G(2/3) + \log G(5/3)}}$$

$G(z)$ is the Barnes G-function

$\log G(z)$ gives the logarithm of the Barnes G-function

Series representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{e^{7466257/116095057920} \sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k c_k}{\sum_{k=1}^{\infty} 3^{-k} c_k}$$

for $\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma c_{-1+k} + \sum_{j=1}^{-2+k} (-1)^{1+j+k} c_j \zeta(-j+k)}{-1+k} \right)$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{2 e^{7.466257/116095057920} \sum_{k=0}^{\infty} \frac{3^{-k} \Gamma^{(k)}(1)}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{2}{3}\right)^k \Gamma^{(k)}(1)}{k!}}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{e^{7.466257/116095057920} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{3}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{2}{3}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{e^{7.466257/116095057920} \sum_{k=0}^{\infty} \left(\frac{2}{3} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}{\sum_{k=0}^{\infty} \left(\frac{1}{3} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

\mathbb{Z} is the set of integers

Integral representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \exp\left(\frac{7.466257}{116095057920} + \frac{\gamma}{3} + \int_0^1 \frac{3 \sqrt[3]{x} - 3x^{2/3} + \log(x)}{3(-1+x)\log(x)} dx\right)$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \exp\left(\frac{7466257}{116095057920} + \int_0^1 \frac{(-1 + \sqrt[3]{x})^2}{3(1 + \sqrt[3]{x} + x^{2/3}) \log(x)} dx\right)$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{2}{3}\right)} = \frac{e^{7466257/116095057920} \int_0^1 \frac{1}{\log^{2/3}\left(\frac{1}{t}\right)} dt}{\int_0^1 \frac{1}{\sqrt[3]{\log\left(\frac{1}{t}\right)}} dt}$$

$\log(x)$ is the natural logarithm

$$\left(\frac{\Gamma(1/6)}{\Gamma(5/6)}\right) * \exp\left(\left(\frac{1}{108} * \left(\frac{1}{12}\right)^2 + \frac{1}{4320} * \left(\frac{1}{12}\right)^4 + \frac{1}{38880} * \left(\frac{1}{12}\right)^6\right)\right)$$

Input:

$$\frac{\Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{1}{4320} \left(\frac{1}{12}\right)^4 + \frac{1}{38880} \left(\frac{1}{12}\right)^6\right)$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{e^{7466257/116095057920} \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)}$$

Decimal approximation:

4.931553822283411721333249773665222342956347077942386162505...

4.9315538222834...

Alternate forms:

$$\frac{5 e^{7466257/116095057920} \frac{1}{6}!}{\frac{5}{6}!}$$

$$\frac{3 e^{7466257/116095057920} \Gamma\left(\frac{1}{3}\right)^4}{2 \times 2^{2/3} \pi^2}$$

$$\frac{16 \sqrt[9]{2} e^{7466257/116095057920} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{\left(2 + \frac{1}{\sqrt{2+\sqrt{3}}}\right) K \left(\frac{\left(1 - \frac{2(\sqrt{2}+\sqrt{3})(5+3\sqrt{3})^2}{(3+\sqrt{2}+\sqrt{3})^2}\right)^2}{\left(1 + \frac{2(\sqrt{2}+\sqrt{3})(5+3\sqrt{3})^2}{(3+\sqrt{2}+\sqrt{3})^2}\right)^2}\right)}{1 + \frac{2(\sqrt{2}+\sqrt{3})(5+3\sqrt{3})}{(3+\sqrt{2}+\sqrt{3})^2}} \right)^{4/3}}{(2 + \sqrt{3})^{4/3}}$$

$n!$ is the factorial function

$K(m)$

is the complete elliptic integral of the first kind with parameter $m = k^2$

Alternative representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(-1 + \frac{1}{6}\right)!}{\left(-1 + \frac{5}{6}\right)!}$$

•

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{G\left(1 + \frac{1}{6}\right) \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right)}{\frac{G\left(\frac{1}{6}\right) G\left(1 + \frac{5}{6}\right)}{G\left(\frac{5}{6}\right)}}$$

•

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) e^{-\log G(1/6) + \log G(1+1/6)}}{e^{-\log G(5/6) + \log G(1+5/6)}}$$

$G(z)$ is the Barnes G-function

$\log G(z)$ gives the logarithm of the Barnes G-function

Series representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{e^{7.466257/116095057920} \sum_{k=1}^{\infty} \left(\frac{5}{6}\right)^k c_k}{\sum_{k=1}^{\infty} 6^{-k} c_k}$$

for $\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma c_{-1+k} + \sum_{j=1}^{-2+k} (-1)^{1+j+k} c_j \zeta(-j+k)}{-1+k} \right)$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{5 e^{7.466257/116095057920} \sum_{k=0}^{\infty} \frac{6^{-k} \Gamma^{(k)}(1)}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{5}{6}\right)^k \Gamma^{(k)}(1)}{k!}}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{e^{7.466257/116095057920} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{6} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{5}{6} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{e^{7.466257/116095057920} \sum_{k=0}^{\infty} \left(\frac{5}{6} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}{\sum_{k=0}^{\infty} \left(\frac{1}{6} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

\mathbb{Z} is the set of integers

Integral representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \exp\left(\frac{7466257}{116095057920} + \frac{2\gamma}{3} + \int_0^1 \frac{3\sqrt[6]{x} - 3x^{5/6} + 2\log(x)}{3(-1+x)\log(x)} dx\right)$$

•

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \exp\left(\frac{7466257}{116095057920} + \int_0^1 \frac{(-1 + \sqrt[6]{x})^2 (2 + \sqrt[6]{x} + 2\sqrt[3]{x})}{3(1 + \sqrt[3]{x} + x^{2/3}) \log(x)} dx\right)$$

•

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{6}\right)}{\Gamma\left(\frac{5}{6}\right)} = \frac{e^{7466257/116095057920} \int_0^1 \frac{1}{\log^{5/6}\left(\frac{1}{t}\right)} dt}{\int_0^1 \frac{1}{\sqrt[6]{\log\left(\frac{1}{t}\right)}} dt}$$

log(x) is the natural logarithm

$((\text{gamma}(1/4)/\text{gamma}(3/4))) * \exp(((1/108 * (1/12)^2 + 1/4320 * (1/12)^4 + 1/38880 * (1/12)^6)))$

Input:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{1}{4320} \left(\frac{1}{12}\right)^4 + \frac{1}{38880} \left(\frac{1}{12}\right)^6\right)$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{e^{7466257/116095057920} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$$

Decimal approximation:

2.958865402388967840952030624014381108602670974333415469806...

2.95886540238896...

Alternate forms:

$$\frac{3 e^{7466257/116095057920} \frac{1}{4}!}{\frac{3}{4}!}$$

$$\frac{e^{7466257/116095057920} \Gamma\left(\frac{1}{4}\right)^2}{\sqrt{2} \pi}$$

$$\frac{4(2 + \sqrt{2}) e^{7466257/116095057920} \sqrt{\frac{2}{\pi}} K\left(\frac{(-2-2\sqrt{2})^2}{(4+2\sqrt{2})^2}\right)}{4 + 2\sqrt{2}}$$

$n!$ is the factorial function

$K(m)$

is the complete elliptic integral of the first kind with parameter $m = k^2$

Alternative representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(-1 + \frac{1}{4}\right)!}{\left(-1 + \frac{3}{4}\right)!}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{G\left(1 + \frac{1}{4}\right) \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right)}{\frac{G\left(\frac{1}{4}\right) G\left(1 + \frac{3}{4}\right)}{G\left(\frac{3}{4}\right)}}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) e^{-\log G(1/4) + \log G(1+1/4)}}{e^{-\log G(3/4) + \log G(1+3/4)}}$$

$G(z)$ is the Barnes G-function

$\log G(z)$ gives the logarithm of the Barnes G-function

Series representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{e^{7466257/116095057920} \sum_{k=1}^{\infty} \left(\frac{3}{4}\right)^k c_k}{\sum_{k=1}^{\infty} 4^{-k} c_k}$$

for $\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma c_{-1+k} + \sum_{j=1}^{-2+k} (-1)^{1+j+k} c_j \zeta(-j+k)}{-1+k} \right)$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{3 e^{7466257/116095057920} \sum_{k=0}^{\infty} \frac{4^{-k} \Gamma^{(k)}(1)}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{3}{4}\right)^k \Gamma^{(k)}(1)}{k!}}$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{e^{7466257/116095057920} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{3}{4} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{e^{7466257/116095057920} \sum_{k=0}^{\infty} \left(\frac{3}{4} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}{\sum_{k=0}^{\infty} \left(\frac{1}{4} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

\mathbb{Z} is the set of integers

Integral representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \exp\left(\frac{7466257}{116095057920} + \frac{\gamma}{2} + \int_0^1 \frac{2\sqrt[4]{x} - 2x^{3/4} + \log(x)}{2(-1+x)\log(x)} dx\right)$$

- $$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \exp\left(\frac{7466257}{116095057920} + \int_0^1 \frac{(-1 + \sqrt[4]{x})^2}{2(1 + \sqrt{x})\log(x)} dx\right)$$

- $$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} = \frac{e^{7466257/116095057920} \int_0^1 \frac{1}{\log^{3/4}\left(\frac{1}{t}\right)} dt}{\int_0^1 \frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}} dt}$$

$\log(x)$ is the natural logarithm

$$\left(\frac{\Gamma\left(\frac{1}{7}\right)\Gamma\left(\frac{2}{7}\right)\Gamma\left(\frac{4}{7}\right)}{\Gamma\left(\frac{3}{7}\right)\Gamma\left(\frac{5}{7}\right)\Gamma\left(\frac{6}{7}\right)}\right) \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{1}{4320} \left(\frac{1}{12}\right)^4 + \frac{1}{38880} \left(\frac{1}{12}\right)^6\right)$$

Input:

$$\frac{\Gamma\left(\frac{1}{7}\right)\Gamma\left(\frac{2}{7}\right)\Gamma\left(\frac{4}{7}\right)}{\Gamma\left(\frac{3}{7}\right)\Gamma\left(\frac{5}{7}\right)\Gamma\left(\frac{6}{7}\right)} \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{1}{4320} \left(\frac{1}{12}\right)^4 + \frac{1}{38880} \left(\frac{1}{12}\right)^6\right)$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{e^{7466257/116095057920} \Gamma\left(\frac{1}{7}\right)\Gamma\left(\frac{2}{7}\right)\Gamma\left(\frac{4}{7}\right)}{\Gamma\left(\frac{3}{7}\right)\Gamma\left(\frac{5}{7}\right)\Gamma\left(\frac{6}{7}\right)}$$

Decimal approximation:

11.01790143176059304669967621461421319734457152179828477266...

11.01790143176059...

Alternate form:

$$\frac{45 e^{7466257/116095057920} \frac{1}{7}! \times \frac{2}{7}! \times \frac{4}{7}!}{4 \times \frac{3}{7}! \times \frac{5}{7}! \times \frac{6}{7}!}$$

$n!$ is the factorial function

Alternative representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{7}\right)\Gamma\left(\frac{2}{7}\right)\Gamma\left(\frac{4}{7}\right)}{\Gamma\left(\frac{3}{7}\right)\Gamma\left(\frac{5}{7}\right)\Gamma\left(\frac{6}{7}\right)} = \frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(-1 + \frac{1}{7}\right)! \left(-1 + \frac{2}{7}\right)! \left(-1 + \frac{4}{7}\right)!}{\left(-1 + \frac{3}{7}\right)! \left(-1 + \frac{5}{7}\right)! \left(-1 + \frac{6}{7}\right)!}$$

•

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \Gamma\left(\frac{1}{7}\right)\Gamma\left(\frac{2}{7}\right)\Gamma\left(\frac{4}{7}\right)}{\Gamma\left(\frac{3}{7}\right)\Gamma\left(\frac{5}{7}\right)\Gamma\left(\frac{6}{7}\right)} = \frac{G\left(1 + \frac{1}{7}\right) G\left(1 + \frac{2}{7}\right) G\left(1 + \frac{4}{7}\right) \exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right)}{\frac{G\left(\frac{1}{7}\right)G\left(\frac{2}{7}\right)G\left(\frac{4}{7}\right)\left(G\left(1+\frac{3}{7}\right)G\left(1+\frac{5}{7}\right)G\left(1+\frac{6}{7}\right)\right)}{G\left(\frac{3}{7}\right)G\left(\frac{5}{7}\right)G\left(\frac{6}{7}\right)}}$$

•

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(\Gamma\left(\frac{1}{7}\right) \Gamma\left(\frac{2}{7}\right) \Gamma\left(\frac{4}{7}\right)\right)}{\Gamma\left(\frac{3}{7}\right) \Gamma\left(\frac{5}{7}\right) \Gamma\left(\frac{6}{7}\right)} =$$

$$\left(\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) e^{-\log G(1/7) + \log G(1+1/7)} \right.$$

$$\left. e^{-\log G(2/7) + \log G(1+2/7)} e^{-\log G(4/7) + \log G(1+4/7)} \right) /$$

$$\left(e^{-\log G(3/7) + \log G(1+3/7)} e^{-\log G(5/7) + \log G(1+5/7)} e^{-\log G(6/7) + \log G(1+6/7)} \right)$$

$G(z)$ is the Barnes G-function

$\log G(z)$ gives the logarithm of the Barnes G-function

Integral representations:

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(\Gamma\left(\frac{1}{7}\right) \Gamma\left(\frac{2}{7}\right) \Gamma\left(\frac{4}{7}\right)\right)}{\Gamma\left(\frac{3}{7}\right) \Gamma\left(\frac{5}{7}\right) \Gamma\left(\frac{6}{7}\right)} =$$

$$\exp\left(\frac{7466257}{116095057920} + \gamma + \int_0^1 \frac{\sqrt[7]{x} + x^{2/7} - x^{3/7} + x^{4/7} - x^{5/7} - x^{6/7} + \log(x)}{(-1+x)\log(x)} dx\right)$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(\Gamma\left(\frac{1}{7}\right) \Gamma\left(\frac{2}{7}\right) \Gamma\left(\frac{4}{7}\right)\right)}{\Gamma\left(\frac{3}{7}\right) \Gamma\left(\frac{5}{7}\right) \Gamma\left(\frac{6}{7}\right)} =$$

$$\exp\left(\frac{7466257}{116095057920} + \int_0^1 \frac{(-1+x^{2/7})^2 (1+x^{2/7})}{(1+\sqrt[7]{x} + x^{2/7} + x^{3/7} + x^{4/7} + x^{5/7} + x^{6/7}) \log(x)} dx\right)$$

$$\frac{\exp\left(\frac{1}{108} \left(\frac{1}{12}\right)^2 + \frac{\left(\frac{1}{12}\right)^4}{4320} + \frac{\left(\frac{1}{12}\right)^6}{38880}\right) \left(\Gamma\left(\frac{1}{7}\right) \Gamma\left(\frac{2}{7}\right) \Gamma\left(\frac{4}{7}\right)\right)}{\Gamma\left(\frac{3}{7}\right) \Gamma\left(\frac{5}{7}\right) \Gamma\left(\frac{6}{7}\right)} =$$

$$\frac{e^{7466257/116095057920} \left(\int_0^\infty \frac{e^{-t}}{t^{6/7}} dt\right) \left(\int_0^\infty \frac{e^{-t}}{t^{5/7}} dt\right) \int_0^\infty \frac{e^{-t}}{t^{3/7}} dt}{\left(\int_0^\infty \frac{e^{-t}}{t^{4/7}} dt\right) \left(\int_0^\infty \frac{e^{-t}}{t^{2/7}} dt\right) \int_0^\infty \frac{e^{-t}}{\sqrt[7]{t}} dt}$$

$\log(x)$ is the natural logarithm

From these results:

11.01790143176059; 2.958865402388967; 4.931553822283411;

1.978491495476199 we obtain:

$$(11.01790143176059 * 2.958865402388967 * 4.931553822283411 * 1.978491495476199)^{\ln((1/\sqrt{2})+(2\pi))}$$

Input interpretation:

$$(11.01790143176059 \times 2.958865402388967 \times 4.931553822283411 \times 1.978491495476199)^{\log(1/\sqrt{2} + 2\pi)}$$

$\log(x)$ is the natural logarithm

Result:

73493.5020934615...

[73493.502093...](#)

Alternative representations:

$$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020^{\log_e(2\pi + 1/\sqrt{2})}$$

- $$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = \left(2\pi + \frac{1}{\sqrt{2}}\right)^{\log(318.0841710029814020)}$$

- $$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020^{\log(a) \log_a(2\pi + 1/\sqrt{2})}$$

$\log_b(x)$ is the base- b logarithm

Series representations:

$$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020 \log(-1+2\pi+1/\sqrt{2}) - \sum_{k=1}^{\infty} \left((-1)^k \left(-1+2\pi + \frac{1}{\sqrt{2}} \right)^{-k} \right) / k$$

$$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020 \left[\frac{2i\pi \left[\arg\left(2\pi - x + \frac{1}{\sqrt{2}}\right)\right]}{(2\pi)} + \log(x) - \sum_{k=1}^{\infty} \left((-1)^k x^{-k} \left(2\pi - x + \frac{1}{\sqrt{2}} \right)^k \right) \right] / k$$

for $x < 0$

$$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020 \left[\frac{\log(z_0) + \left[\arg\left(2\pi + \frac{1}{\sqrt{2}} - z_0\right)\right]}{(2\pi)} \right] \left(\log(1/z_0) + \log(z_0) \right) - \sum_{k=1}^{\infty} \left((-1)^k \left(2\pi + \frac{1}{\sqrt{2}} - z_0 \right)^k z_0^{-k} \right) / k$$

$\arg(z)$ is the complex argument

$[x]$ is the floor function

i is the imaginary unit

Integral representations:

$$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020 \int_1^{2\pi+1/\sqrt{2}} \frac{1}{t} dt$$

$$(11.017901431760590000 \times 2.9588654023889670000 \times 4.9315538222834110000 \times 1.9784914954761990000)^{\log(1/\sqrt{2} + 2\pi)} = 318.0841710029814020 \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\Gamma(-s)^2 \Gamma(1+s) \left(-1+2\pi + \frac{1}{\sqrt{2}} \right)^{-s} \right) / \Gamma(1-s) ds$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

We have the following mathematical connections:

$$\begin{aligned}
& \left((11.01790143176059 \times 2.958865402388967 \times \right. \\
& \quad \left. 4.931553822283411 \times 1.978491495476199)^{\log(1/\sqrt{2} + 2\pi)} \right) \Rightarrow \\
& \Rightarrow -3927 + 2 \left(\sqrt[13]{ N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS} + \right. \\
& \quad \left. \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS} \right) = \\
& \quad -3927 + 2 \sqrt[13]{ 2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59} } \\
& = 73490.8437525 \dots \Rightarrow
\end{aligned}$$

(the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$)

$$\begin{aligned}
& \Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow \\
& \Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) = \\
& = 73491.78832548118710549159572042220548025195726563413398700 \dots \\
& = 73491.7883254 \dots \Rightarrow
\end{aligned}$$

(the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane)

$$\begin{aligned}
& \left(I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(-\left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leq p1-\epsilon_1} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right. \\
& \quad \left. \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\epsilon_1} \right\} \right) \\
& / (26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662 \dots
\end{aligned}$$

(the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series)

we have that:

$$1 / (((((1 + (1 / 11.01790143176059 * 1 / 2.958865402388967 * 1 / 4.931553822283411 * 1 / 1.978491495476199))))))$$

Input interpretation:

$$\frac{1}{1 + \frac{1}{11.01790143176059} \times \frac{1}{2.958865402388967} \times \frac{1}{4.931553822283411} \times \frac{1}{1.978491495476199}}$$

Result:

0.996866030687587268696958593552342615071405890058620209870...

0.9968660306875...

From the sum of the above results, we obtain:

$$(11.01790143176059 + 2.958865402388967 + 4.931553822283411 + 1.978491495476199)$$

Input interpretation:

$$11.01790143176059 + 2.958865402388967 + 4.931553822283411 + 1.978491495476199$$

Result:

20.886812151909167

20.886812... result very near to the Fibonacci number 21

From the difference, we obtain:

$$(11.01790143176059 - 2.958865402388967 - 4.931553822283411 - 1.978491495476199)$$

Input interpretation:

$$11.01790143176059 - 2.958865402388967 - 4.931553822283411 - 1.978491495476199$$

Result:

1.148990711612013

1.148990711... result very near to the value of the following Ramanujan mock theta function:

$$\left(1 + \frac{0.449329^2}{1 + 0.449329} + \frac{0.449329^6}{(1 + 0.449329) + (1 + 0.449329^2)}\right) + \frac{0.449329^{12}}{(1 + 0.449329)(1 + 0.449329^2)(1 + 0.449329^3)} =$$

$$= 1.142443242201380904097917635488946328383797361320962332093...$$

f(q) = 1.1424432422...

furthermore, we obtain:

$$1 + 1 / (11.01790143176059 - 2.958865402388967 - 4.931553822283411 - 1.978491495476199)^{\pi}$$

Input interpretation:

$$1 + 1 / (11.01790143176059 - 2.958865402388967 - 4.931553822283411 - 1.978491495476199)^{\pi}$$

Result:

$$1.6464129829975...$$

$$1.64641298... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

Alternative representations:

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + \frac{1}{1.148990711612013000^{180^\circ}}$$

•

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + \frac{1}{1.148990711612013000^{-i \log(-1)}}$$

•

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + \frac{1}{1.148990711612013000^{\cos^{-1}(-1)}}$$

log(x) is the natural logarithm

i is the imaginary unit

$\cos^{-1}(x)$ is the inverse cosine function

Series representations:

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + 1.148990711612013000 e^{-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

•

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + 1.320179655370680025 e^{-0.277767829890712717 \times \sum_{k=1}^{\infty} 2^k / \binom{2k}{k}}$$

•

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + 1.148990711612013000 e^{-\sum_{k=0}^{\infty} (2^{-k} (-6+50k)) / \binom{3k}{k}}$$

$\binom{n}{m}$ is the binomial coefficient

Integral representations:

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + e^{-0.277767829890712717 \int_0^{\infty} 1/(1+t^2) dt}$$

•

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + e^{-0.555535659781425434 \int_0^1 \sqrt{1-t^2} dt}$$

•

$$1 + 1 / (11.017901431760590000 - 2.9588654023889670000 - 4.9315538222834110000 - 1.9784914954761990000)^{\pi} = 1 + e^{-0.277767829890712717 \int_0^{\infty} \sin(t)/t dt}$$

$$(11.01790143176059 + 2.958865402388967 + 4.931553822283411 + 1.978491495476199)^{1/6}$$

Input interpretation:

$$(11.01790143176059 + 2.958865402388967 + 4.931553822283411 + 1.978491495476199)^{(1/6)}$$

Result:

$$1.6595054899737257\dots$$

1.6595054899... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

$$1/10^{27} * (((13/10^3 + (11.01790143176059 + 2.958865402388967 + 4.931553822283411 + 1.978491495476199)^{1/6})))$$

Input interpretation:

$$\frac{1}{10^{27}} \left(\frac{13}{10^3} + (11.01790143176059 + 2.958865402388967 + 4.931553822283411 + 1.978491495476199)^{(1/6)} \right)$$

Result:

$$1.6725054899737257\dots \times 10^{-27}$$

1.6725054899... * 10⁻²⁷ result practically equal to the proton mass

*This ... is... magic: "The number 24 appearing in Ramanujan's function is also the origin of the miraculous cancellations occurring in String Theory ... each of the 24 modes in the Ramanujan function corresponds to a physical vibration of a string. Whenever the string executes its complex motions in space-time by splitting and recombining, a large number of highly complex mathematical identities must be satisfied. These are precisely the mathematical identities discovered by Ramanujan ... the string vibrates in ten dimensions because it requires generalized Ramanujan functions in order to remain self-consistent. ~ Michio Kaku, in **Hyperspace : A Scientific Odyssey Through Parallel Universes, Time Warps, and the Tenth Dimension (1995) Ch.7 Superstrings!***

From Wikipedia:

Ramanujan tau function

The **Ramanujan tau function**, studied by Ramanujan (1916), is the function $\tau : \mathbf{N} \rightarrow \mathbf{Z}$ defined by the following identity:

$$\sum_{n \geq 1} \tau(n)q^n = q \prod_{n \geq 1} (1 - q^n)^{24} = \eta(z)^{24} = \Delta(z),$$

where $q = \exp(2\pi iz)$ with $\Im z > 0$ and η is the Dedekind eta function and the function $\Delta(z)$ is a holomorphic cusp form of weight 12 and level 1, known as the discriminant modular form. It appears in connection to an "error term" involved in counting the number of ways of expressing an integer as a sum of 24 squares. A formula due to Ian G. Macdonald was given in Dyson (1972).

Values

The first few values of the tau function are given in the following table (sequence A000594 in the OEIS):

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\tau(n)$	1	-24	252	-1472	4830	-6048	-16744	84480	-113643	-115920	534612	-370944	-577738	401856	1217160	987136

Rogers–Ramanujan continued fraction

The **Rogers–Ramanujan continued fraction** is a continued fraction discovered by Rogers (1894) and independently by Srinivasa Ramanujan, and closely related to the Rogers–Ramanujan identities. It can be evaluated explicitly for a broad class of values of its argument.

Given the functions $G(q)$ and $H(q)$ appearing in the Rogers–Ramanujan identities,

$$\begin{aligned} G(q) &= \sum_{n=0}^{\infty} \frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)} = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q; q)_n} = \frac{1}{(q; q^5)_{\infty} (q^4; q^5)_{\infty}} \\ &= \prod_{n=1}^{\infty} \frac{1}{(1 - q^{5n-1})(1 - q^{5n-4})} \\ &= \sqrt[60]{qj} {}_2F_1 \left(-\frac{1}{60}, \frac{19}{60}; \frac{4}{5}; \frac{1728}{j} \right) \\ &= \sqrt[60]{q(j-1728)} {}_2F_1 \left(-\frac{1}{60}, \frac{29}{60}; \frac{4}{5}; -\frac{1728}{j-1728} \right) \\ &= 1 + q + q^2 + q^3 + 2q^4 + 2q^5 + 3q^6 + \cdots \end{aligned}$$

and,

$$\begin{aligned}
 H(q) &= \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)} = \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q; q)_n} = \frac{1}{(q^2; q^5)_{\infty} (q^3; q^5)_{\infty}} \\
 &= \prod_{n=1}^{\infty} \frac{1}{(1-q^{5n-2})(1-q^{5n-3})} \\
 &= \frac{1}{\sqrt[60]{q^{11} j^{11}}} {}_2F_1\left(\frac{11}{60}, \frac{31}{60}; \frac{6}{5}; \frac{1728}{j}\right) \\
 &= \frac{1}{\sqrt[60]{q^{11} (j-1728)^{11}}} {}_2F_1\left(\frac{11}{60}, \frac{41}{60}; \frac{6}{5}; -\frac{1728}{j-1728}\right) \\
 &= 1 + q^2 + q^3 + q^4 + q^5 + 2q^6 + 2q^7 + \cdots
 \end{aligned}$$

OEIS: A003114 and OEIS: A003106, respectively, where $(a; q)_{\infty}$ denotes the infinite q -Pochhammer symbol, j is the j -function, and ${}_2F_1$ is the hypergeometric function, then the Rogers–Ramanujan continued fraction is,

$$\begin{aligned}
 R(q) &= \frac{q^{\frac{11}{60}} H(q)}{q^{-\frac{1}{60}} G(q)} = q^{\frac{1}{5}} \prod_{n=1}^{\infty} \frac{(1-q^{5n-1})(1-q^{5n-4})}{(1-q^{5n-2})(1-q^{5n-3})} \\
 &= \frac{q^{1/5}}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \ddots}}}}
 \end{aligned}$$

If $q = e^{2\pi i \tau}$, then $q^{-\frac{1}{60}} G(q)$ and $q^{\frac{11}{60}} H(q)$, as well as their quotient $R(q)$, are modular functions of τ . Since they have integral coefficients, the theory of complex multiplication implies that their values for τ an imaginary quadratic irrational are algebraic numbers that can be evaluated explicitly.

Examples

$$R(e^{-2\pi}) = \frac{e^{-\frac{2\pi}{5}}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}} = \sqrt{\frac{5 + \sqrt{5}}{2}} - \phi$$

$$R(e^{-2\sqrt{5}\pi}) = \frac{e^{-\frac{2\pi}{\sqrt{5}}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}} = \frac{\sqrt{5}}{1 + (5^{3/4}(\phi - 1)^{5/2} - 1)^{1/5}} - \phi$$

where $\phi = \frac{1 + \sqrt{5}}{2}$ is the golden ratio.

and these four Rogers – Ramanujan continued fraction, that link e , ϕ and π :

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\phi\sqrt{5}} - \phi} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$\frac{e^{\frac{2\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}} - \phi \approx 1.0000007913$$

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\phi-1)\sqrt{5}} - \phi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \varphi + 1 \approx 0.9991104684$$

Always to the genius of Ramanujan we owe the following stupendous formula:

$$\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!} + \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \dots}}}}} \approx 2.0663656771$$

Neither the infinite series nor the continuous fraction can be expressed (at least as far as we know) through e or π , but their sum, incredibly yes! The continued fraction is worth:

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2}}{2}\right) \approx 0.6556795424$$

(<http://www.bitman.name/math/article/102/109/>)

From Wikipedia:

Relation to j-function

Among the many formulas of the j -function, one is,

$$j(\tau) = \frac{(x^2 + 10x + 5)^3}{x}$$

where

$$x = \left[\frac{\sqrt{5} \eta(5\tau)}{\eta(\tau)} \right]^6$$

Eliminating the eta quotient, one can then express $j(\tau)$ in terms of $r = R(q)$ as,

$$j(\tau) = -\frac{(r^{20} - 228r^{15} + 494r^{10} + 228r^5 + 1)^3}{r^5(r^{10} + 11r^5 - 1)^5}$$

$$j(\tau) - 1728 = -\frac{(r^{30} + 522r^{25} - 10005r^{20} - 10005r^{10} - 522r^5 + 1)^2}{r^5(r^{10} + 11r^5 - 1)^5}$$

where the numerator and denominator are polynomial invariants of the icosahedron. Using the modular equation between $R(q)$ and $R(q^5)$, one finds that,

$$j(5\tau) = -\frac{(r^{20} + 12r^{15} + 14r^{10} - 12r^5 + 1)^3}{r^{25}(r^{10} + 11r^5 - 1)}$$

let $z = r^5 - \frac{1}{r^5}$, then $j(5\tau) = -\frac{(z^2 + 12z + 16)^3}{z + 11}$

where

$$z_\infty = -\left[\frac{\sqrt{5} \eta(25\tau)}{\eta(5\tau)} \right]^6 - 11, \quad z_0 = -\left[\frac{\eta(\tau)}{\eta(5\tau)} \right]^6 - 11, \quad z_1 = \left[\frac{\eta(\frac{5\tau+2}{5})}{\eta(5\tau)} \right]^6 - 11,$$

$$z_2 = -\left[\frac{\eta(\frac{5\tau+4}{5})}{\eta(5\tau)} \right]^6 - 11, \quad z_3 = \left[\frac{\eta(\frac{5\tau+6}{5})}{\eta(5\tau)} \right]^6 - 11, \quad z_4 = -\left[\frac{\eta(\frac{5\tau+8}{5})}{\eta(5\tau)} \right]^6 - 11$$

which in fact is the j -invariant of the elliptic curve,

$$y^2 + (1 + r^5)xy + r^5y = x^3 + r^5x^2$$

parameterized by the non-cusp points of the modular curve $X_1(5)$.

Now, from:

$$j(\tau) = -\frac{(r^{20} - 228r^{15} + 494r^{10} + 228r^5 + 1)^3}{r^5(r^{10} + 11r^5 - 1)^5}$$

for $r = 1$, we obtain:

$$-(1-228+494+228+1)^3 / (1+11-1)^5$$

Input:

$$\frac{(1 - 228 + 494 + 228 + 1)^3}{(1 + 11 - 1)^5}$$

Exact result:

$$-\frac{122023936}{161051}$$

Decimal approximation:

$$-757.672637860056752208927606782944533098211125668266573942\dots$$

-757.67263786.... result very near to the rest mass of Charged rho meson 775.11 with minus sign

Mixed fraction:

$$-757\frac{108329}{161051}$$

Repeating decimal:

(period 29 282)

Continued fraction:

$$-757 + \frac{1}{-1 + \frac{1}{-2 + \frac{1}{-18 + \frac{1}{-3 + \frac{1}{-1 + \frac{1}{-1 + \frac{1}{-1 + \frac{1}{-3 + \frac{1}{-1 + \frac{1}{-27 + \frac{1}{-2}}}}}}}}}}}}$$

From:

$$j(\tau) - 1728 = -\frac{(r^{30} + 522r^{25} - 10005r^{20} - 10005r^{10} - 522r^5 + 1)^2}{r^5(r^{10} + 11r^5 - 1)^5}$$

For $r=1$, we obtain:

$$-(1+522-10005-10005-522+1)^2 / (1+11-1)^5$$

Input:

$$-\frac{(1 + 522 - 10\,005 - 10\,005 - 522 + 1)^2}{(1 + 11 - 1)^5}$$

Exact result:

$$-\frac{400\,320\,064}{161\,051}$$

Decimal approximation:

$$-2485.67263786005675220892760678294453309821112566826657394\dots$$

-2485.67263786..... result very near to the rest mass of charmed Xi baryon 2470.88 with minus sign

Mixed fraction:

$$-2485\frac{108329}{161051}$$

Repeating decimal:

(period 29 282)

Continued fraction:

$$-2485 + \frac{1}{-1 + \frac{1}{-2 + \frac{1}{-18 + \frac{1}{-3 + \frac{1}{-1 + \frac{1}{-1 + \frac{1}{-1 + \frac{1}{-3 + \frac{1}{-1 + \frac{1}{-27 + \frac{1}{-2}}}}}}}}}}}}$$

Thence:

$$j(\tau) - 1728 = -\frac{(r^{30} + 522r^{25} - 10005r^{20} - 10005r^{10} - 522r^5 + 1)^2}{r^5(r^{10} + 11r^5 - 1)^5}$$

$$\begin{aligned} & -757.672637860056752208927606782944533098211125668266573942 - 1728 = \\ & = -2485.67263786005675220892760678294453309821112566826657394; \end{aligned}$$

and:

$$1728 = 2485.67263786005675220892760678294453309821112566826657394 - 757.672637860056752208927606782944533098211125668266573942$$

Input interpretation:

$$1728 = 2485.67263786005675220892760678294453309821112566826657394 - 757.672637860056752208927606782944533098211125668266573942$$

Result:

True

$$1728 = 2485.67263786 - 757.67263786$$

Now, from:

$$j(5\tau) = -\frac{(r^{20} + 12r^{15} + 14r^{10} - 12r^5 + 1)^3}{r^{25}(r^{10} + 11r^5 - 1)}$$

For r = 1 we obtain:

$$-(1+12+14-12+1)^3 / (1+11-1)$$

Input:

$$-\frac{(1 + 12 + 14 - 12 + 1)^3}{1 + 11 - 1}$$

Exact result:

$$-\frac{4096}{11}$$

Decimal approximation:

-124.12...

-124.12121... result very near to the Higgs boson mass with minus sign.

From:

$$j(\tau) = \frac{(x^2 + 10x + 5)^3}{x}$$

We obtain:

$$-757.67263786 x = (x^2 + 10x + 5)^3$$

Input interpretation:

$$-757.67263786 x = (x^2 + 10 x + 5)^3$$

Result:

$$-757.67263786 x = (x^2 + 10 x + 5)^3$$

Alternate forms:

$$-757.67263786 x = (x(x + 10) + 5)^3$$

$$-x^6 - 30 x^5 - 315 x^4 - 1300 x^3 - 1575 x^2 - 1507.67263786 x - 125 = 0$$

Expanded form:

$$-757.67263786 x = x^6 + 30 x^5 + 315 x^4 + 1300 x^3 + 1575 x^2 + 750 x + 125$$

Real solutions:

$$x \approx -11.3636364$$

•
$$x \approx -0.09090909091$$

Complex solutions:

$$x = -8.701942 - 2.106217 i$$

$$x = -8.701942 + 2.106217 i$$

$$x = -0.57078550 - 1.08797339 i$$

$$x = -0.57078550 + 1.08797339 i$$

From:

$$y^2 + (1 + r^5)xy + r^5y = x^3 + r^5x^2$$

For $x = -11.3636364$, we obtain:

$$(-11.3636364)^3 + (-11.3636364)^2$$

Input interpretation:

$$(-11.3636364)^3 + (-11.3636364)^2$$

Result:

$$-1338.279502366641666596544$$

•

Repeating decimal:

$$-1338.279502366641666596544$$

-1338.27950236.... result very near to the rest mass of scalar meson $f_0(1370)$ with minus sign

For $x = 8.95321$, we obtain:

$$(8.95321)^3 + (8.95321)^2$$

Input interpretation:

$$8.95321^3 + 8.95321^2$$

Result:

$$797.849008077261161$$

797.8490008..... result very near to the rest mass of Omega meson 782.65

For $x = 1.22861$, we obtain:

$$(1.22861)^3 + (1.22861)^2$$

Input interpretation:

$$1.22861^3 + 1.22861^2$$

Result:

$$3.364047865863381$$

$$3.364047865863381$$

For $x = -0.09090909091$, we obtain:

$$(-0.09090909091)^3 + (-0.09090909091)^2$$

Input interpretation:

$$(-0.09090909091)^3 + (-0.09090909091)^2$$

Result:

$$0.007513148009158527422990833959429$$

$$0.007513148009158527422990833959429$$

From the two previous equation, we obtain:

$$[-(-(1+522-10005-10005-522+1)^2 / (1+11-1)^5 + (1-228+494+228+1)^3 / (1+11-1)^5)]^{1/15}$$

Input:

$$\sqrt[15]{-\left(-\frac{(1+522-10005-10005-522+1)^2}{(1+11-1)^5} + \frac{(1-228+494+228+1)^3}{(1+11-1)^5}\right)}$$

Result:

$$2^{2/5} \sqrt[5]{3}$$

Decimal approximation:

$$1.643751829517225762308497936230979517383492589945475200411...$$

$$1.6437518295172.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

From the previous solution $(-11.3636364)^3 + (-11.3636364)^2$ we obtain:

$$((-1/ ((((-11.3636364)^3 + (-11.3636364)^2))))))^{1/512}$$

Input interpretation:

$$\sqrt[512]{-\frac{1}{(-11.3636364)^3 + (-11.3636364)^2}}$$

Result:

0.98603757111...

0.98603757111...

From the previous solution $(-0.09090909091)^3 + (-0.09090909091)^2$ we obtain:

$$1/(((((-0.09090909091)^3 + (-0.09090909091)^2))))$$

Input interpretation:

$$\frac{1}{(-0.09090909091)^3 + (-0.09090909091)^2}$$

Result:

133.0999999974711000000374010999995050011000061600010999263...

133.0999999..... result very near to the Pion meson 134.9766

On some equations concerning the Fermi and Yukawa theory meson

From:

Are Mesons Elementary Particles?

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(Received August 24, 1949)

We have the following equations:

Using (10), (11), and (15), and carrying out the integration one finds:

$$R = i \left(\frac{2\pi\hbar^3 c^3}{\Omega\mu c^2} \right)^{\frac{1}{2}} (5.3\gamma_1\gamma_2\gamma_3 + 0.11\gamma_1\gamma_2\gamma_3\gamma_4). \quad (18)$$

This expression can be compared with the conventional interaction of a pseudoscalar meson with nucleons in the Yukawa theory.⁶ There are two essentially independent coupling constants: f , the so-called pseudoscalar interaction, and g , the pseudovector interaction. The nucleon-meson interaction Hamiltonian is:

$$i \int N^* \left\{ f\gamma_1\gamma_2\gamma_3\phi + \sum_{\nu} \frac{\hbar}{\mu c} g\gamma_1\gamma_2\gamma_3\gamma_{\nu} \frac{\partial\phi}{\partial x_{\nu}} \right\} P d^3\mathbf{r} \quad (19)$$

where ϕ is the pseudoscalar meson field.

The corresponding matrix element for the production of a meson at rest is

$$i \frac{\hbar c}{(2\Omega\mu c^2)^{\frac{1}{2}}} \int N^* (f\gamma_1\gamma_2\gamma_3 + g\gamma_1\gamma_2\gamma_3\gamma_4) P d^3\mathbf{r}.$$

Comparison with (18) gives

$$f = (4\pi\hbar c)^{\frac{1}{2}} \times 5.3, \quad g = (4\pi\hbar c)^{\frac{1}{2}} \times 0.11. \quad (20)$$

It has been proved by Case⁷ that the terms f and g produce up to the second approximation nuclear forces of the same type. Indeed, their joint contribution is the same as would be obtained by putting $f=0$ and substituting g by

$$g' = g + f(\mu/2M). \quad (21)$$

We find, therefore,

$$g' = (4\pi\hbar c)^{\frac{1}{2}} \times 0.52$$

yielding for $g'^2/4\pi\hbar c$, that is for the analog of the fine structure constant, the value 0.27, which appears quite reasonable.

Naturally the similarity between the present point of view and the Yukawa theory can be carried on only up to a limited extent. The similarity breaks down on the one hand because of the finite size of the meson which introduces naturally a cut-off at short distances. On the other hand it breaks down for phenomena in which sufficiently high energies are involved to break up the meson.

We have that from (20):

$$5.3 * \text{sqrt}(\text{(((4 * \text{Pi} * 1.054 * 10^{(-34)} * 3 * 10^8))))})$$

Input interpretation:

$$5.3 \sqrt{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}$$

Result:

$$3.34089... \times 10^{-12}$$

$$3.34089... * 10^{-12}$$

And:

$$0.11 * \text{sqrt}(\text{(((4 * \text{Pi} * 1.054 * 10^{(-34)} * 3 * 10^8))))})$$

Input interpretation:

$$0.11 \sqrt{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}$$

Result:

$$6.93392... \times 10^{-14}$$

$$6.93392... * 10^{-14}$$

Note that:

$$\text{(((1 / (((3.34089 \times 10^{-12}) * 1 / (6.93392 * 10^{-14}))))))})^{1/256}$$

Input interpretation:

$$\sqrt[256]{\frac{1}{3.34089 \times 10^{-12} \times \frac{1}{\frac{6.93392}{10^{14}}}}}$$

Result:

$$0.98497733...$$

$$0.98497733...$$

From eq. (21)

$$6.93392 * 10^{(-14)} + \text{((((5.3 * \text{sqrt}(\text{(((4 * \text{Pi} * 1.054 * 10^{(-34)} * 3 * 10^8)))))))))$$

Input interpretation:

$$6.93392 \times 10^{-14} + 5.3 \sqrt{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}$$

Result:

$$3.41023... \times 10^{-12}$$

$$3.41023... * 10^{-12}$$

We have that:

$$0.52 * \text{sqrt}(\text{(((4 * \pi * 1.054 * 10^{(-34)} * 3 * 10^8))))})$$

Input interpretation:

$$0.52 \sqrt{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}$$

Result:

$$3.27785... \times 10^{-13}$$

$$3.27785... * 10^{-13}$$

Note that:

$$1 / \text{((((3.41023 * 10^{-12}) * 1 / (3.27785 * 10^{-13}))))}^{1/256}$$

Input interpretation:

$$\frac{1}{\sqrt[256]{\frac{3.41023}{10^{12}} \times \frac{1}{\frac{3.27785}{10^{13}}}}}$$

Result:

$$0.99089260...$$

$$0.9908926...$$

Now:

$$\text{(((3.27785 * 10^{(-13)}))}^2 / \text{((((4 * \pi * 1.054 * 10^{(-34)} * 3 * 10^8))))})$$

Input interpretation:

$$\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}$$

Result:

0.270400...

0.2704...

$$10^3 + \left(\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8} \right)^2$$

Input interpretation:

$$10^3 + \left(10^2 \times \frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8} \right)^2$$

Result:

1731.16...

1731.16

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

$$\left[10^3 + \left(\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8} \right)^2 \right]^{1/15}$$

Input interpretation:

$$\sqrt[15]{10^3 + \left(10^2 \times \frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8} \right)^2}$$

Result:

1.643952...

$$1.643952... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$\sqrt{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}$$

Input interpretation:

$$\sqrt{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}$$

Result:

0.520000...

0.52

$$1 + (-\ln \sqrt{\left(\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}\right)^{1/3}})$$

Input interpretation:

$$1 - \log \left(\sqrt[3]{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}} \right)$$

log(x) is the natural logarithm

Result:

1.65393...

1.6539267... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

$$1/\left(\frac{0.0047}{\left(\frac{1}{\left(\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}\right)^{1/3}}\right)^{1/3}}\right)$$

$$0.0047 = 0.0052 + 0.0011 - 0.0027 + 0.0011$$

Input interpretation:

$$0.0047 \times \frac{1}{\sqrt[3]{\frac{1}{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}}}}$$

Result:

137.585...

137.585... result very near to the rest mass of Pion meson 139.57

$$2 \times \frac{1}{\left(\frac{0.52 + 0.27}{\left(\frac{1}{\left(\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}\right)^{1/3}}\right)^{1/3}}\right)}$$

Input interpretation:

$$2 \times \frac{1}{(0.52 + 0.27) \sqrt[3]{\frac{1}{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}}}}$$

Result:

1.63709...

$$1.63709\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$1/3.16 * 0.52 * 1 / (((((((((((((0.79((((((((((1/(((((((((((3.27785 * 10^{(-13)}))^2 / (((4 * \pi * 1.054 * 10^{(-34)} * 3 * 10^8)))))))))))))))))^1/3))))))))))))))$$

Input interpretation:

$$\frac{1}{3.16} \times 0.52 \times \frac{1}{0.79 \sqrt[3]{\frac{1}{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}}}}$$

Result:

0.134697...

0.134697...

$$10^3 \frac{1}{3.16} * 0.52 * 1 / (((((((((((((0.52 + 0.27)((((((((((1/(((((((((((3.27785 * 10^{(-13)}))^2 / (((4 * \pi * 1.054 * 10^{(-34)} * 3 * 10^8)))))))))))))))))^1/3))))))))))))))$$

Input interpretation:

$$10^3 \times \frac{1}{3.16} \times 0.52 \times \frac{1}{(0.52 + 0.27) \sqrt[3]{\frac{1}{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}}}}$$

Result:

134.697...

Or:

Input interpretation:

$$10^3 \times \frac{1}{\sqrt{10}} \times 0.52 \times \frac{1}{(0.52 + 0.27) \sqrt[3]{\frac{1}{\frac{(3.27785 \times 10^{-13})^2}{4\pi \times 1.054 \times 10^{-34} \times 3 \times 10^8}}}}}$$

Result:

134.600...

134.600 and 134.697... very near to the rest mass of Pion meson 134.9766

In conclusion:

$$1 / (((1/10 * 0.27^2)))$$

Input:

$$\frac{1}{\frac{1}{10} \times 0.27^2}$$

Result:

137.1742112482853223593964334705075445816186556927297668038...

137.1742... very near to the reciprocal of fine-structure constant

Now, we have that:

$$64 * \ln \left(\frac{0.27 + 0.52}{\sqrt{4 * \pi * 1.054571 * 10^{-34} * 3 * 10^8}} \right)$$

Input interpretation:

$$64 \log \left(\frac{0.27 + 0.52}{\sqrt{4 \pi \times 1.054571 \times 10^{-34} \times 3 \times 10^8}} \right)$$

log(x) is the natural logarithm

Result:

1782.816...

1782.819... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

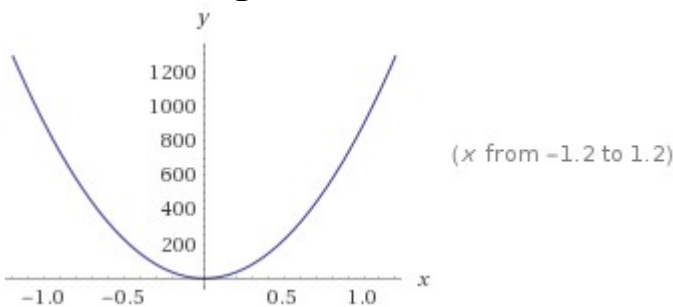
Integrate [64 * ln sqrt((((((0.27+0.52)/(4*Pi*1.054571*10^(-34)*3*10^8)))))))]x

Indefinite integral:

$$\int \left(64 \log \left(\sqrt{\frac{0.27 + 0.52}{4 \pi \cdot 1.054571 \times 10^{-34} \cdot 3 \times 10^8}} \right) \right) x \, dx = 895.18 x^2 + \text{constant}$$

log(x) is the natural logarithm

Plot of the integral:



Alternate form assuming x is real:

$$895.18 x^2 + 0 + \text{constant}$$

For $x^2 = 1.08094974^5$

$895.18 * 1.08094974^5$

Input interpretation:

$895.18 \times 1.08094974^5$

Result:

1321.106647043284321978706216428783110467783232

1321.1066... result very near to the rest mass of Xi-baryon 1321.71

Now, from:

The lowest eigenvalue must be $E = \mu c^2$, the rest energy of the meson. This condition determines⁴ the depth V_0 of the potential (6). Assuming the ratio 6.46 between the proton and meson masses one finds:⁴

$$V_0 = 26.4 \text{ Mc}^2 = 24.6 \text{ Bev.} \quad (9)$$

The corresponding normalized solution in a large volume Ω is:

$$r > r_0 = \frac{\hbar}{Mc} \begin{cases} f_1 = -\frac{0.236}{(r_0^3 \Omega)^{\frac{1}{2}}} \frac{1}{u} e^{-u} \\ f_2 = f_3 = -\frac{0.218}{(r_0^3 \Omega)^{\frac{1}{2}}} e^{-u} \left[\frac{1}{u} + \frac{1}{u^2} \right] \\ f_4 = \frac{0.202}{(r_0^3 \Omega)^{\frac{1}{2}}} \frac{1}{u} e^{-u}, \end{cases} \quad (10)$$

$$r < r_0 \begin{cases} f_1 = -\frac{0.0136 \sin v}{(r_0^3 \Omega)^{\frac{1}{2}}} \frac{1}{v} \\ f_2 = f_3 = \frac{0.370}{(r_0^3 \Omega)^{\frac{1}{2}}} \left[\frac{\cos v}{v} - \frac{\sin v}{v^2} \right] \\ f_4 = -\frac{0.0147 \sin v}{(r_0^3 \Omega)^{\frac{1}{2}}} \frac{1}{v}, \end{cases} \quad (11)$$

where

$$u = rc/\hbar [M^2 - (\mu^2/4)]^{\frac{1}{2}}, \quad v = 2.03(r/r_0).$$

We have that $0.236+0.218+0.202 = 0.656$

And:

$$\left(\left(\left(\left(\left(\left(0.236+0.218+0.202\right)/\sqrt{\left(4*\pi*1.054571*10^{\left(-34\right)}*3*10^8\right)}\right)\right)\right)\right)\right)\right)$$

Input interpretation:

$$\frac{0.236 + 0.218 + 0.202}{\sqrt{4\pi \times 1.054571 \times 10^{-34} \times 3 \times 10^8}}$$

Result:

$$1.04040... \times 10^{12}$$

$$1.04040... * 10^{12}$$

$$64*\ln\left(\left(\left(\left(\left(\left(0.202+0.236+0.218\right)/\sqrt{\left(4*\pi*1.054571*10^{\left(-34\right)}*3*10^8\right)}\right)\right)\right)\right)\right)\right)$$

Input interpretation:

$$64 \log \left(\frac{0.202 + 0.236 + 0.218}{\sqrt{4\pi \times 1.054571 \times 10^{-34} \times 3 \times 10^8}} \right)$$

$\log(x)$ is the natural logarithm

Result:

$$1770.920...$$

1770.920... result in the range of the mass of candidate “glueball” $f_0(1710)$ and the hypothetical mass of Gluino (“glueball” = 1760 ± 15 MeV; gluino = 1785.16 GeV).

$$64*\ln\left(\left(\left(\left(\left(\left(0.0136+0.0147+0.370\right)/\sqrt{\left(4*\pi*1.054571*10^{\left(-34\right)}*3*10^8\right)}\right)\right)\right)\right)\right)\right)$$

Input interpretation:

$$64 \log \left(\frac{0.0136 + 0.0147 + 0.37}{\sqrt{4\pi \times 1.054571 \times 10^{-34} \times 3 \times 10^8}} \right)$$

$\log(x)$ is the natural logarithm

Result:

$$1738.987...$$

1738.987... result in the range of the mass of candidate “glueball” $f_0(1710)$

$$\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(64 \cdot \ln\left(\frac{0.0136+0.0147+0.370}{\sqrt{4 \cdot \pi \cdot 1.054571 \cdot 10^{-34} \cdot 3 \cdot 10^8}}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{64 \log\left(\frac{0.0136 + 0.0147 + 0.37}{\sqrt{4 \pi \times 1.054571 \times 10^{-34} \times 3 \times 10^8}}\right)}$$

log(x) is the natural logarithm

Result:

1.64444652...

$$1.64444652... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$1.0061571663\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(64 \cdot \ln\left(\frac{0.0136+0.0147+0.370}{\sqrt{4 \cdot \pi \cdot 1.054571 \cdot 10^{-34} \cdot 3 \cdot 10^8}}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$1.0061571663 \sqrt[15]{64 \log\left(\frac{0.0136 + 0.0147 + 0.37}{\sqrt{4 \pi \times 1.054571 \times 10^{-34} \times 3 \times 10^8}}\right)}$$

log(x) is the natural logarithm

Result:

1.65457165...

1.654571... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$0.202+0.236+0.218+0.0136+0.0147+0.370$$

$$(0.202+0.236+0.218+0.0136+0.0147+0.370)^9$$

Input:

$$(0.202 + 0.236 + 0.218 + 0.0136 + 0.0147 + 0.37)^9$$

Result:

1.609451369129162146466137412028231343

1.6094513... result that is a golden number, very near to the electric charge of positron.

And:

$1/6(((((((((\exp(0.202+0.236+0.218+0.0136+0.0147+0.370))))))^{1/2} + ((((\exp(0.202+0.236+0.218+0.0136+0.0147+0.370))))^{1/3}))))))^{1/2}$

Input:

$$\frac{1}{6} \left(\sqrt{\exp(0.202 + 0.236 + 0.218 + 0.0136 + 0.0147 + 0.37)} + \sqrt[3]{\exp(0.202 + 0.236 + 0.218 + 0.0136 + 0.0147 + 0.37)} \right)^2$$

Result:

1.617412098130703317191873174547237097456086883303230956391...

1.617412.....

This result is a very good approximation to the value of the golden ratio 1,618033988749...

From:

On the Interaction of Elementary Particles

H. Yukawa

(Received 1935)

or by (16)

$$\frac{4\pi gg'}{\lambda^2} \int \int \int \tilde{\nu}(\vec{r}) u(\vec{r}) \sum_{k,l} \tilde{\psi}(\vec{r}) \delta_{kl} \tilde{\phi}'_l(\vec{r}) dV, \quad (19)$$

which is the same as the expression (21) of Fermi, corresponding to the emission of a neutrino and an electron of positive energy states $\phi'_k(\vec{r})$ and $\psi_k(\vec{r})$, except that the factor $\frac{4\pi gg'}{\lambda^2}$ is substituted for Fermi's g .

Thus the result is the same as that of Fermi's theory, in this approximation, if we take

$$\frac{4\pi gg'}{\lambda^2} = 4 \times 10^{-50} \text{cm}^3 \cdot \text{erg},$$

from which the constant g' can be determined. Taking, for example, $\lambda = 5 \times 10^{12}$ and $g = 2 \times 10^{-9}$, we obtain $g' \cong 4 \times 10^{-17}$, which is about 10^{-8} times as small as g .

This means that the interaction between the neutrino and the electron is much smaller than between the neutron and the proton so that the neutrino will be far more penetrating than the neutron and consequently more difficult to observe. The difference of g and g' may be due to the difference of masses of heavy and light particles.

We have that:

$$(((4 * \text{Pi} * (2 * 10^{-9}) (4 * 10^{-17})))) / (4 * 10^{-50})$$

Input interpretation:

$$\frac{4\pi \times 2 \times 10^{-9} \times 4 \times 10^{-17}}{4 \times 10^{-50}}$$

Result:

8 000 000 000 000 000 000 000 000 π

Decimal approximation:

2.5132741228718345907701147066236023073577355195000846... $\times 10^{25}$

2.51327... $\times 10^{25}$

Property:

8 000 000 000 000 000 000 000 000 π is a transcendental number

$$\text{sqrt}(\text{(((((((4*\text{Pi}*(2*10^{\wedge}-9)(4*10^{\wedge}-17)))))) / (4*10^{\wedge}-50))))))$$

Input interpretation:

$$\sqrt{\frac{4\pi \times 2 \times 10^{-9} \times 4 \times 10^{-17}}{4 \times 10^{-50}}}$$

Result:

2 000 000 000 000 $\sqrt{2\pi}$

Decimal approximation:

5.0132565492620010048315305696220905060139734812198766... $\times 10^{12}$

5.013256... $\times 10^{12}$

Property:

2 000 000 000 000 $\sqrt{2\pi}$ is a transcendental number

$$\text{sqrt}(21) \ln(\text{(((((((\text{sqrt}(\text{(((((((4*\text{Pi}*(2*10^{\wedge}-9)(4*10^{\wedge}-17)))))) / (4*10^{\wedge}-50)))))))))$$

Input interpretation:

$$\sqrt{21} \log\left(\sqrt{\frac{4\pi \times 2 \times 10^{-9} \times 4 \times 10^{-17}}{4 \times 10^{-50}}}\right)$$

$\log(x)$ is the natural logarithm

Result:

$\sqrt{21} \log(2\,000\,000\,000\,000 \sqrt{2\pi})$

Decimal approximation:

134.0087506027490326030379012717895284087398176598106120082...

134.00875... result very near to the rest mass of Pion meson 134.9766

$$-13 + 13 \cdot \sqrt{21} \ln \left(\frac{\sqrt{4\pi \cdot 2 \times 10^{-9} \cdot 4 \times 10^{-17}}}{4 \times 10^{-50}} \right)$$

Input interpretation:

$$-13 + 13 \sqrt{21} \log \left(\sqrt{\frac{4 \pi \times 2 \times 10^{-9} \times 4 \times 10^{-17}}{4 \times 10^{-50}}} \right)$$

log(x) is the natural logarithm

Result:

$$13 \sqrt{21} \log(200000000000 \sqrt{2\pi}) - 13$$

Decimal approximation:

1729.113757835737423839492716533263869313617629577537956107...

1729.11375... result in the range of the mass of candidate "glueball" $f_0(1710)$

$$\left(\frac{-13 + 13 \cdot \sqrt{21} \ln \left(\frac{\sqrt{4\pi \cdot 2 \times 10^{-9} \cdot 4 \times 10^{-17}}}{4 \times 10^{-50}} \right)}{15} \right)^{15}$$

Input interpretation:

$$\sqrt[15]{-13 + 13 \sqrt{21} \log \left(\sqrt{\frac{4 \pi \times 2 \times 10^{-9} \times 4 \times 10^{-17}}{4 \times 10^{-50}}} \right)}$$

log(x) is the natural logarithm

Result:

$$\sqrt[15]{13 \sqrt{21} \log(200000000000 \sqrt{2\pi}) - 13}$$

Decimal approximation:

1.643822438739917290895309483013173301886407117030932783615...

$$1.643822... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$34 + 5 + 13 \cdot \sqrt{21} \ln \left(\frac{\sqrt{4\pi \cdot 2 \times 10^{-9} \cdot 4 \times 10^{-17}}}{4 \times 10^{-50}} \right)$$

Input interpretation:

$$34 + 5 + 13 \sqrt{21} \log \left(\sqrt{\frac{4 \pi \times 2 \times 10^{-9} \times 4 \times 10^{-17}}{4 \times 10^{-50}}} \right)$$

log(x) is the natural logarithm

Result:

$$39 + 13 \sqrt{21} \log(200000000000 \sqrt{2\pi})$$

Decimal approximation:

1781.113757835737423839492716533263869313617629577537956107...

1781.1137.... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

From paper II

The summation in (21) can be replaced by integration and we obtain after some calculations

$$\begin{aligned}
 K(\vec{r}) &= -\frac{2g^4}{\pi \hbar c} \cdot \frac{1}{r^2} \int_0^\infty \frac{k \sin kr \cos kr \cdot dk}{(k^2 + \kappa^2)^{\frac{3}{2}}} \\
 &= -\frac{g^4}{\hbar c} \frac{iH_0^{(1)}(2i\kappa r)}{r} = K(r)
 \end{aligned}
 \tag{22}$$

where $H_0^{(1)}$ is the Hankel function of zero order. This gives ordinary attractive force between two neutrons with the potential $K(r)$. The range of force in this case is only half of that between unlike particles, as $K(r)$ has the asymptotic form

$$K(r) = \frac{g^4}{\hbar c} \frac{e^{-2\kappa r}}{\sqrt{\pi \kappa r^3}} \left\{ 1 + O\left(\frac{1}{\kappa r}\right) \right\}
 \tag{23}$$

for large r . Exactly the same result can be obtained for the case of two protons.

Relative magnitude of the forces between like and unlike particles is given by

$$\frac{|K(r)|}{|J(r)|} = \frac{g^2}{\hbar c} e^{\kappa r} iH_0^{(1)}(2i\kappa r)
 \tag{24}$$

which varies with κr as follows⁽⁹⁾, if we omit the constant factor $g^2/\hbar c$.

κr	0.05	0.1	0.25	0.5	1.0	1.5
$e^{\kappa r} \cdot iH_0^{(1)}(2i\kappa r)$	1.62	1.23	0.76	0.44	0.20	0.1

From the eqs. (9), (20) and (21) of Fermi's paper, we obtain:

$$2.46 \cdot 10^{10} / ((((((0.52 + 0.11))) / 5.3))) \cdot 10^3$$

Input interpretation:

$$\frac{2.46 \times 10^{10}}{\frac{0.52 + 0.11}{5.3} \times 10^3}$$

Result:

$$2.06952380952380952380952380952380952380952380952380952... \times 10^8$$

$$2.0695238... * 10^8$$

We obtain, from (24):

$$2.0695238 * 10^8 / ((((((2 * 10^{-9})^2 / (((1.054571817 * 10^{-34}) * (3 * 10^8))))))))))$$

Where $g = 2 * 10^{-9}$ (see paper 1 Yukawa)

Input interpretation:

$$\frac{2.0695238 \times 10^8}{\frac{(2 \times 10^{-9})^2}{1.054571817 \times 10^{-34} \times 3 \times 10^8}}$$

Result:

$$1.63684610556805845$$

1.6368461.... a result that is a good approximation, i.e. very near to the value 1.62

Thence, from (24), we obtain:

$$(((((((2 * 10^{-9})^2 / (((1.054571817 * 10^{-34}) * (3 * 10^8)))))) * 1.6368461$$

Input interpretation:

$$\frac{(2 \times 10^{-9})^2}{(1.054571817 \times 10^{-34} \times 3 \times 10^8) \times 1.6368461}$$

Result:

$$7.72422165470324266781912014437129568046557363370486145... \times 10^7$$

$$7.7242216547... * 10^7 \approx 77242216.5$$

We have that:

$$\text{sqrt}(((2.0695238 * 10^8) / (7.7242216547 * 10^7)))$$

Input interpretation:

$$\sqrt{\frac{2.0695238 \times 10^8}{7.7242216547 \times 10^7}}$$

Result:

1.6368461...

$$1.6368461... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

And:

$$1/(((2.0695238 * 10^8) / (7.7242216547 * 10^7)))^{1/64}$$

Input interpretation:

$$\frac{1}{\sqrt[64]{\frac{2.0695238 \times 10^8}{7.7242216547 \times 10^7}}}$$

Result:

0.9847188573...

0.9847188573...

And:

$$24 * 4 \ln \left(\frac{(2 * 10^{-9})^2}{(1.054571817 * 10^{-34} * 3 * 10^8) * 1.6368461} \right)$$

Input interpretation:

$$24 \times 4 \log \left(\frac{(2 \times 10^{-9})^2}{(1.054571817 \times 10^{-34} \times 3 \times 10^8) \times 1.6368461} \right)$$

log(x) is the natural logarithm

Result:

1743.59584...

1743.595... result in the range of the mass of candidate “glueball” $f_0(1710)$

$$\left(\frac{(24 * 4 \ln \left(\frac{(2 * 10^{-9})^2}{(1.054571817 * 10^{-34} * 3 * 10^8) * 1.6368461} \right))^{1/15}} \right)$$

Input interpretation:

$$\sqrt[15]{24 \times 4 \log \left(\frac{(2 \times 10^{-9})^2}{(1.054571817 \times 10^{-34} \times 3 \times 10^8) \times 1.6368461} \right)}$$

log(x) is the natural logarithm

Result:

1.644736719...

$$1.64473... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$1.0061571663^4 * 24 * 4 \ln \left(\frac{(2 * 10^{-9})^2}{(1.054571817 * 10^{-34})(3 * 10^8)) * 1.6368461} \right)$$

Input interpretation:

$$1.0061571663^4 \times 24 \times 4 \log \left(\frac{(2 \times 10^{-9})^2}{(1.054571817 \times 10^{-34} \times 3 \times 10^8) \times 1.6368461} \right)$$

log(x) is the natural logarithm

Result:

1786.93652...

1786.936... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

From the result of formula, we also obtain:

$$\left(\int_0^{\frac{1}{54\pi}} (7.7242216547 \times 10^7)x \, dx - 21 \right)$$

Input interpretation:

$$\int_0^{\frac{1}{54\pi}} 7.7242216547 \times 10^7 x \, dx - 21$$

Result:

1320.9534993

1320.953... result very near to the rest mass of Xi baryon 1321.71

$$e + \frac{1}{10} \left(\int_0^{\frac{1}{54\pi}} (7.7242216547 \times 10^7)x \, dx - 21 \right)$$

Input interpretation:

$$e + \frac{1}{10} \left(\int_0^{\frac{1}{54\pi}} 7.7242216547 \times 10^7 x \, dx - 21 \right)$$

Result:

134.81363175

134.813... result very near to the rest mass of Pion meson 134.9766

$$(-21^2 - 144 - 21 - 8) + \left(\int_0^{\frac{1}{54\pi}} (7.7242216547 \times 10^7)x \, dx - 21 \right)$$

Input interpretation:

$$(-21^2 - 144 - 21 - 8) + \int_0^{\frac{1}{54\pi}} 7.7242216547 \times 10^7 x \, dx$$

Result:

727.9534993

727.953...

result practically equal to the famous Ramanujan cube formula $9^3 - 1^3 = 728$

Note that, we have also:

$$(-21^2 - 89 - 21 - 8) + (((((((integrate (7.7242216547 \times 10^7)x, [0, 1/(54\pi)]))))))$$

Input interpretation:

$$(-21^2 - 89 - 21 - 8) + \int_0^{\frac{1}{54\pi}} 7.7242216547 \times 10^7 x dx$$

Result:

782.9534993

782.953... result very near to the rest mass of Omega meson 782.65

From (24) for the value 0.76, we obtain:

$$((((((2 \times 10^{-9})^2 / (((((1.054571817 \times 10^{-34})(3 \times 10^8))) * 0.76)))))))$$

Input interpretation:

$$\frac{(2 \times 10^{-9})^2}{(1.054571817 \times 10^{-34} \times 3 \times 10^8) \times 0.76}$$

Result:1.66360027513638808133859504128231477454170005086745144... $\times 10^8$ 1.6636... $\times 10^8$

Note that:

$$1 / (((1.6636 \times 10^8) / (7.7242216547 \times 10^7)))^{1/64}$$

Input interpretation:

$$\frac{1}{\sqrt[64]{\frac{1.6636 \times 10^8}{7.7242216547 \times 10^7}}}$$

Result:

0.9880839...

0.9880839...

And:

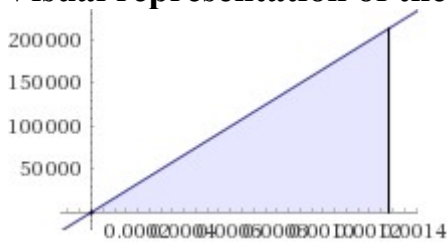
integrate $(1.66360027513638808133859504128231477454170005086745144 \times 10^8)$
 $x, [0, 1/(248\pi)]$

Definite integral:

$$\int_0^{\frac{1}{248\pi}} 1.66360027513638808133859504128231477454170005086745144 \times 10^8 x \, dx = 137.03$$

137.03, very near to the inverse of fine-structure constant 137.0359...

Visual representation of the integral:



From paper III Yukawa, we have that:

The corresponding probability, when the heavy quantum is moving with the velocity v and energy E , is reduced to

$$w = w_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{2g_1'^2 + g_2'^2}{6\hbar c} \cdot \frac{m_r c^2}{\hbar} \cdot \frac{m_r c^2}{E} \quad (67)$$

owing to the change of time scale under Lorentz transformation. The mean life time τ and the mean free path λ of the heavy quantum with the energy E can be defined by the relations

$$\tau = \frac{1}{w}, \quad \lambda = v\tau \quad (68)$$

If we take $g_1' = g_2' = g' = 4 \times 10^{-17}$, a value which was determined from the probability of β^+ -disintegration in § 4, I, and $m_r = 100 m$,⁽¹⁾ we obtain

$$w = 2 \times 10^8 \frac{m c^2}{E} \quad (69)$$

We obtain:

$$\frac{(2 \cdot (4 \cdot 10^{-17})^2 + (4 \cdot 10^{-17})^2)}{((6 \cdot 1.054571 \cdot 10^{-34} \cdot 3 \cdot 10^8))} \cdot \frac{(100 \cdot (3 \cdot 10^8)^2)}{((1.054571 \cdot 10^{-34}) \cdot E)}$$

Input interpretation:

$$\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}}$$

Result:

2.1580405107899897527223221742908299793179295820195144... × 10⁴⁵
 2.15804051... * 10⁴⁵

We have also:

$$17 \ln \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}} \right)$$

Input interpretation:

$$17 \log \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}} \right)$$

log(x) is the natural logarithm

Result:

1774.55401...
 1774.55401... result in the range of the mass of candidate “glueball” f₀(1710) and the hypothetical mass of Gluino (“glueball” = 1760 ± 15 MeV; gluino = 1785.16 GeV).

We note that:

$$\left(\frac{17 \ln \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}} \right)}{15} \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{17 \log \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}} \right)}$$

log(x) is the natural logarithm

Result:

1.646667629...
 1.646667... ≈ ζ(2) = π²/6 = 1.6449

And:

$$2\sqrt[6]{6^{15} \left(\frac{17 \ln \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \right)}{1.054571 \times 10^{-34}} \right)^{15}}^{1/15}$$

Input interpretation:

$$2 \sqrt[6]{6^{15} \sqrt[15]{17 \log \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}} \right)}}$$

$\log(x)$ is the natural logarithm

Result:

6.286495296...

6.286495296... $\approx 2\pi$

We have also that:

$$27 + [10^3 \sqrt[6]{6^{15} \left(\frac{17 \ln \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \right)}{1.054571 \times 10^{-34}} \right)^{15}}^{1/15}]$$

Input interpretation:

$$27 + 10^3 \sqrt[6]{6^{15} \sqrt[15]{17 \log \left(\frac{2(4 \times 10^{-17})^2 + (4 \times 10^{-17})^2}{6 \times 1.054571 \times 10^{-34} \times 3 \times 10^8} \times \frac{100(3 \times 10^8)^2}{1.054571 \times 10^{-34}} \right)}}$$

$\log(x)$ is the natural logarithm

Result:

1673.667629...

1673.667... result very near to the rest mass of Omega baryon 1672.45

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