Variable Chaplygin gas in Kaluza-Klein framework

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We investigate cosmological features of the variable Chaplygin gas (VCG) describing a unified dark matter-energy scenario in a universe governed by the five dimensional (5D) Kaluza-Klein (KK) gravity. In such a proposal, the VCG evolves as from the dust-like phase to the phantom or the quintessence phases. It is concluded that the background evolution for the KK type VCG definition is equivalent to that for the dark energy interacting with the dark matter. Next, after performing neo-classical tests, we calculated the proper, luminosity and angular diameter distances. Additionally, we construct a connection between the VCG in the KK universe and a homogenous minimally coupled scalar field by introducing its self-interacting potential and also we confirm the stability of the KK type VCG model by making use of thermodynamics. Moreover, we use data from Type Ia Supernova (SN Ia), observational $H(z)$ dataset (OHD) and Planck-2015 results to place constraints on the model parameters. Subsequently, according to the best-fit values of the model parameters we analyze our results numerically.

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I. INTRODUCTION

Cosmological and astronomical data such as the type Ia supernovae (SNe-Ia)[1, 2], cosmic microwave background (CMB)[3], large scale structure (LSS)[4], Wilkinson Microwave Anisotropy Probe (WMAP)[5–7], Sloan Digital Sky Survey (SDSS)[8] and the Planck 2015[9] imply that the current universe experiences a speedy expansion stage. The reason of such a phase transition may be explained by introducing dark energy definitions. The cosmological constant is one of the primordial proposals of the dark energy, so there are other theoretical dark energy candidates such as phantom[10–13] and quintessence[14–19] models. The cosmological constant is the simplest definition of the dark energy but the model faces some serious problematic issues[20, 21] such as the fine tuning (unconventional small value) and cosmic coincidence problems (why the dark matter and the dark energy are of the same order today although the universe is in a speedy expansion phase?). Due to the dynamical dark energy is a useful alternative idea to lighten or even remove these problematic issues, the cosmological constant model has been transformed into a dynamical form in several methods. In order to implement the dynamized version of the cosmological constant dark energy definition, we can describe an interaction between the dark matter and the dark energy[22–27]. There are also some alternative significant dark energy models such as the Chaplygin gas (CG)[28, 29] and the Polytropic gas (PG)[31] which emerged from the string theory of gravity[32]. The noteworthy CG model has been developed also to the generalized[33–35], modified[36, 37], variable[38–42] and the extended[43] forms. A charming characteristic of the (original, generalized, modified, variable or extended) CG models is their capability of a unified definition of the dark matter-energy dominated era. The corresponding energy density interpolates between an early time decelerated expansion phase and a late time accelerated expansion phase. Cosmological indications relying on the dynamics of CG models have been widely investigated in literature[28, 29, 33–46].

On the other hand, an alternative idea to discuss the inflation and speedy expansion phases of our universe is to study with modified theories of gravity such as $f(R)$-gravity, $f(T)$-theory, $f(G)$-gravity and extra dimensional ideas. The KK theory is one of the most important extra dimensional theories which came forward as an attempt to couple electromagnetism and gravity[47, 48]. According to the idea behind the KK gravity, the Universe may have five dimensions (5Ds). Further investigations classified the 5D KK theory into two different forms such as the compact and non-compact ones[49–51]. In the compact KK theory, the extra fifth dimension is length-like. However, the non-compact KK theory includes a mass like fifth dimension and it is an outcome of the Campbell theorem in which we cannot describe any matter into 5D manifold by hand[21, 49–51]. Additionally, the original KK theory turned out to a be base for other extra dimensional ideas in different perspectives like brane models[52], string theory[53] and super gravity[54] (see review of the KK and other extra dimensional unified theories[55]). That's why it would be very interesting to construct the CG model in the KK universe.

The original CG[28, 29] description is given by an

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equation-of-state (EoS)

$$p = -B\rho^{-1}. \tag{1}$$

The VCG which is a phenomenologically extended model of the original CG model is characterized by an EoS

$$p = -\frac{Ba^{-n}}{\rho}, \tag{2}$$

where the time-dependent function $a(t)$ implies the cosmic scale factor given in the spacetime metric. In this study, we focus on the VCG model which can be reduced to the original CG proposal under a limiting condition.

We organize this paper as following: in the next section, we reconstruct the VCG model in the compact KK theory and investigate some of its cosmological implications. In the third section, we discuss neo-classical tests. In the fourth section, the relationship between a homogeneous minimally coupled scalar field and the KK type VCG is reevaluated by establishing its self-interacting potential. In the fifth section, we discuss the generalized second law of thermodynamics (GSL) in order to confirm stability of the KK type VCG proposal. The sixth section is devoted to the numerical analyzes in order to fit the model by using the recent observational data. In the seventh section of the study, we perform some additional numerical analyzes. Finally, in the eighth section, we give our closing remarks. All numerical calculations and analyzes had been performed by using Wolfram's MATHEMATICA sofware[30].

II. KK FORM OF THE VCG

The KK universe is represented by[56]

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) + (1-kr^2)dx_5^2], \tag{3}$$

where k is known as the curvature parameter for the flat ($k=0$), closed ($k=-1$) and open ($k=+1$) universe types. It is known that the recent astronomical observations sighted by SNe-Ia[2], WMAP[5], SDSS[8], Planck-2015[9] and X-ray[57] strongly suggest a spatially flat type universe.

We assume that the KK universe is dominated by perfect fluid which is described by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - g_{\mu\nu}p, \tag{4}$$

with $\mu, \nu = 0, 1, 2, 3, 5$. Here, ρ and p are, respectively, energy density and pressure of the VCG which is a combination of the dark energy and dark matter and u_μ is the five-velocity vector satisfying $u^\mu u_\mu = 1$.

Einstein's field equations are written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left[(\rho+p)u_\mu u_\nu - g_{\mu\nu}p\right] \tag{5}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, R and G show the Ricci tensor, metric tensor, Ricci scalar and the gravitational constant, respectively. Making use of equations (3) and (5) with $k=0$, it follows that

$$H^2 = \frac{4\pi G}{3}\rho, \tag{6}$$

$$2H^2 + \dot{H} = -\frac{8\pi G}{3}p, \tag{7}$$

where $H = \frac{\dot{a}}{a}$ is the cosmic Hubble expansion parameter and the over-dot implies derivative with respect to t. The continuity equation, i.e. $T^{\mu\nu}_{;\nu} = 0$, yields

$$\frac{d\rho}{d\ln a} + 4(1+\omega)\rho = 0, \tag{8}$$

with the fact that the EoS parameter of the VCG is given as $\omega = \frac{p}{\rho}$.

In a general form, the CG is described by the following expression

$$p = \sum_{i=1}^{\infty} A_i\rho^i - \frac{Ba^{-n}}{\rho^\alpha}, \tag{9}$$

where $0 < \alpha \leq 1$ and i, A_i, B and n denote free constants, so it can be seen that we have $(i+3)$ free parameters in the general model. In order to solve the equation (8) analytically, we consider the first order case and assume a significant set of free parameters, i.e. $A_1 = 0$ and $\alpha = 1$, defining the VCG[38–42]:

$$p = -\frac{Ba^{-n}}{\rho}. \tag{10}$$

Note that $n=0$ case defines the original CG (OCG) proposal interpolating between the de Sitter universe and a dust dominated one. Inserting equation (10) into the energy conservation relation (8), we calculate that the energy density of the VCG evolves as

$$\rho = a^{-4}\sqrt{8\int Ba^{7-n}da + \xi}, \tag{11}$$

where ξ implies an integration constant. Thus, one finds

$$\rho = \left[\frac{8}{8-n}\frac{B}{a^n} + \frac{\xi}{a^8}\right]^{\frac{1}{2}}. \tag{12}$$

From the equation (6), it is seen that the acceleration case $\ddot{a} > 0$ is equivalent to

$$\left[4 - \frac{8}{8-n}\right]a^{8-n} > \frac{\xi}{B}, \tag{13}$$

which requires $n < 6$. Hence, the present value of the VCG energy density is written as

$$\rho_0 = \left[\xi + \frac{8B}{8-n}\right]^{\frac{1}{2}} \tag{14}$$

when $a = a_0 = 1$. On the other hand, it is known[43] that the energy density of the universe (ρ) is related with the cosmological density (ρ_c) by $\rho = 1.31\rho_c$ which means we must take $\rho_0 = 1.31$ when $a_0 = 1$. Consequently, the integration constant ξ can be written as

$$\xi = 1.72 - \frac{8B}{8-n}. \tag{15}$$

Next, defining

$$A = \frac{\xi}{\xi + \frac{8B}{8-n}} \tag{16}$$

transforms the energy density into the following form

$$\rho = \rho_0 \sqrt{A a^{-8} + (1-A) a^{-n}}. \tag{17}$$

Compared to the OCG description, in the VCG model, the Universe tends to be filled with phantom dark energy ($n < 0$)[10] or quintessence one ($n > 0$)[58, 59] with the following EoS parameter

$$\omega = -1 + \frac{n}{8}. \tag{18}$$

On the right hand side of the equation (17), the second term is initially negligible and from this point of view one can rewrite the relation (17) approximately as $\rho \sim a^{-4}$, which corresponds to a dust-like matter dominated KK universe.

Writing relations in terms of the cosmic red shift parameter z helps us to use astrophysical data in order to identify free parameters in a theoretical model. The scale factor and red shift parameter are related to each other by

$$z = \frac{1}{a} - 1, \tag{19}$$

and we chose $a_0 = 1$ for convenience. Hence, we can write,

$$\rho = \rho_0 \sqrt{A(1+z)^8 + (1-A)(1+z)^n} \tag{20}$$

Additionally, after defining $E(z) \equiv H_0^{-1} H(z)$ where H_0 represents the current value of the cosmic Hubble parameter, we find

$$H(z) = H_0 \rho_0^{\frac{1}{4}} \left[A(1+z)^8 + (1-A)(1+z)^n \right]^{\frac{1}{4}}. \tag{21}$$

So that there are two free parameters in the KK type VCG model, B and n. According to the Planck 2015 results, the current value of the Hubble parameter is $H_0 = 67.8^{+0.9}_{-0.9}$km/s/Mpc.

Moreover, one can conclude that the background evolution for the VCG proposal is equivalent to that for an interaction between the dark energy and the dark matter[61, 62] with the EoS parameter $\omega = -1 + \frac{n}{8}$. Considering the scaling case for the dark energy-matter density, i.e. $\rho = \rho_m + \rho_e$ with $\rho_e \propto \rho_m a^{8-n}$, for the KK universe one can get

$$\rho_m + \rho_e = \rho_{cr} \sqrt{(1-\Omega_m^0)(1+z)^n + \Omega_m^0 (1+z)^8}, \tag{22}$$

where ρ_{cr} is a constant and it denotes the critical density while Ω_m^0 describes the matter density parameter. Comparing the equation (22) with (20), it can be seen that the parameter A is interpreted as an effective matter density. Now, we focus on $\omega_m = 0$ and $\omega = -1 + \frac{n}{8}$ relations. Thus, if the coupled case is defined by

$$\dot{\rho}_m + 4H\rho_m = \Gamma, \tag{23}$$

$$\dot{\rho}_e + \frac{n}{2}\rho_e = -\Gamma, \tag{24}$$

then the interaction is characterized by[63, 64]

$$\Gamma = 4H\left(\frac{n}{8} - 1\right) \frac{\rho_m}{1 + \frac{\rho_m}{\rho_e}}, \tag{25}$$

which describes the interaction between the dark contents. Negative Γ values describe energy transition from the dark matter era to the dark energy one, and positive values define the vice versa case. Such interaction case has recently been observed in the *Abell Cluster A586*[65, 66] and plays a significant role as a self-conserved dark component[67–69], however the importance of this event has not been identified clearly[70].

III. NEO-CLASSICAL TESTS

As a first step, we start with the proper distance $d(z)$. It is important to construct a causality relation between source and observer at any time. After assuming a source which is at position $\tilde{r}$ at time $\tilde{t}$ sending signal to an observer at distance r at time t, the corresponding proper distance between the observer and the source is defined by[71]

$$d(z) = \int_a^1 \frac{da}{a\dot{a}} \tag{26}$$

with $a_0 = 1$. For the KK type VCG model, the above relation yields

$$d(z) = \frac{4(1+z)f}{(4-n)H_0\rho_0^{\frac{1}{4}}} \left[\frac{1 + \frac{A(1+z)^{8-n}}{1-A}}{A(1+z)^8} + (1-A)(1+z)^n \right]^{\frac{1}{4}} \tag{27}$$

where

$$f = {}_2F_1\left[\frac{1}{4}, \frac{n-4}{4(n-8)}, 1 + \frac{n-4}{4(n-8)}; \frac{A(1+z)^{8-n}}{A-1}\right]. \tag{28}$$

Here, ${}_2F_1$ is the Kummer Confluent Hypergeometric function of the second kind which is written as

$$
\begin{aligned}
{}_2F_1[a,b,c;x] &= 1+\frac{ab}{x}+\frac{a(a+1)b(b+1)}{c(c+1)}\frac{x^2}{2!}+...\\
&= \sum_{i=1}^{\infty}\frac{a_ib_i}{c_i}\frac{x^i}{i!} \qquad (29)
\end{aligned}
$$

with $c\neq 0,-1,-2,...$ and $|x|<1$. It is seen that the conditions $c\neq 0$, $c\neq -1$ and $c\neq -2$ give $n\neq 7.2$, $n\neq 7.5$ and $n\neq 7.84$, respectively, which satisfy $n<6$.

In theoretical physics, the luminosity distance d_L helps to discuss the distribution of light to an observer who is at a distance from the source. In other words, the luminosity distance is given to describe the amount of light sent to a distant observer. It is defined as

$$d_L=\sqrt{\frac{L}{4\pi l}}, \qquad (30)$$

where L and l represent the total energy emitted by a source per unit time and apparent luminosity of an object, respectively[71]. Consequently, it can be arrived[71] at $d_L=(1+z)d(z)$. Making use of the result (27), one can calculate the corresponding luminosity distance

$$d_L=\frac{4(1+z)^2f}{(4-n)H_0\rho_0^{\frac{1}{4}}}\left[\frac{1+\frac{A(1+z)^{8-n}}{1-A}}{A(1+z)^8}+(1-A)(1+z)^n\right]^{\frac{1}{4}}. \qquad (31)$$

It is easy to conclude that $d(z)$ and d_L depend upon the cosmic redshift parameter and are also increasing functions of z.

Now, let's focus on the angular diameter distance d_A which describes a measure showing how large an object appears to be. For a light source at proper distance, it is written as[71]

$$d_A=\frac{d(z)}{(1+z)}=\frac{d_L}{(1+z)^2}. \qquad (32)$$

Therefore, it leads to

$$d_A=\frac{4H_0^{-1}f}{(4-n)\rho_0^{\frac{1}{4}}}\left[\frac{1+\frac{A(1+z)^{8-n}}{1-A}}{A(1+z)^8}+(1-A)(1+z)^n\right]^{\frac{1}{4}}. \qquad (33)$$

One can also conclude that the angular diameter distance d_A is the decreasing function of the red shift parameter. We see that the luminosity and angular diameter distances have different dependence on z. While the angular diameter is a decreasing function of z, the luminosity distance is directly proportional to the cosmic red shift parameter. Hence, it means that an object appears smaller as its distance increases. This is consistent with the recent observational data[1–9, 57].

IV. VCG AS A SCALAR FIELD

We can define the VCG proposal by introducing a scalar field ϕ having a self-interacting potential $V(\phi)$. The corresponding Lagrangian is written as

$$\pounds=\frac{\dot{\phi}^2}{2}-V(\phi). \qquad (34)$$

Making use of the following transformation relations, both the energy density and pressure of the VCG can be rewritten in terms of the scalar field function ϕ:

$$\rho_\phi=\frac{\dot{\phi}^2}{2}+V(\phi)=\rho, \qquad (35)$$

$$p_\phi=\frac{\dot{\phi}^2}{2}-V(\phi)=-\frac{Ba^{-n}}{\rho}. \qquad (36)$$

Hence, the kinetic energy and corresponding potential of the scalar field ϕ are defined as

$$\dot{\phi}^2=\rho_\phi+p_\phi=(1+\omega_\phi)\rho_\phi, \qquad (37)$$

$$V(\phi)=\frac{1}{2}(\rho_\phi-p_\phi)=\frac{1}{2}(1-\omega_\phi)\rho_\phi, \qquad (38)$$

where $\omega_\phi=\frac{p_\phi}{\rho_\phi}$. Since $\dot{\phi}=-(1+z)H\phi'$, we get

$$\phi'=-\sqrt{\frac{n}{8}}\frac{\rho_0^{\frac{1}{4}}}{H_0}(1+z)^{-1}. \qquad (39)$$

The equation (39) can be integrated easily and it gives

$$\phi-\phi_0=-\sqrt{\frac{n}{8}}\frac{\rho_0^{\frac{1}{4}}}{H_0}\ln(1+z). \qquad (40)$$

After assuming $\phi_0=0$, we find

$$z=-1+e^{-\sigma^2\phi}, \qquad (41)$$

where

$$\sigma^2=\sqrt{\frac{n}{8}}\frac{\rho_0^{\frac{1}{4}}}{H_0}. \qquad (42)$$

Next, the corresponding self-interacting potential is found as

$$V(\phi)=\left(1-\frac{n}{16}\right)\rho_0\sqrt{Ae^{-8\sigma^2\phi}+(1-A)e^{-n\sigma^2\phi}}. \qquad (43)$$

V. COSMOLOGY VIA THERMODYNAMICS

It is significant to discuss thermodynamical features of the model which may lead to the investigation of the generalized second law of thermodynamics (GSL). For this purpose, we assume the flat KK universe as a thermodynamical system which is bounded to the apparent horizon surface. Note that, in the flat geometry, the dynamical apparent horizon coincides with the Hubble horizon[72–74]:

$$R_h = \lim_{k\to 0}\left[H^2 + \frac{k}{a^2}\right]^{\frac{1}{2}} = \frac{1}{H}. \tag{44}$$

In addition to this, the temperature and the horizon entropy are defined by the following relations, respectively,

$$T = \frac{1}{2\pi R_h} = \frac{H}{2\pi}, \tag{45}$$

and

$$S_h = \frac{A}{4G} = \frac{\pi^2}{2G}\frac{1}{H^3}, \tag{46}$$

where $A = 2\pi^2 R_h^3$ is the surface area of 4-sphere.

Next, the internal entropy is defined by the Gibbs' expression[75]

$$TdS_I = pdV + dE_I, \tag{47}$$

where S_I, $E_I = \rho V$ and $V = \frac{1}{2}\pi^2 R_h^4$ are internal entropy, internal energy and the extra-dimensional volume of the system, respectively. Taking the time derivative of equation (47) and using equation (8), it can be found that

$$T\dot{S}_I = (\rho + p)(\dot{V} - 4HV), \tag{48}$$

and substituting relations (6) and (7) into this equation yields

$$T\dot{S}_I = \frac{3\pi}{4G}\frac{\dot{H}}{H^5}(\dot{H} - H^2). \tag{49}$$

Additionally, considering equations (45) and (46), the evolution of horizon entropy in the flat KK universe is found as

$$T\dot{S}_h = -\frac{3\pi}{4G}\frac{\dot{H}}{H^3}. \tag{50}$$

Now, one can discuss the GSL due to different contributions of the CG and horizon. Adding equations (48) and (49), we can get the GSL in the KK universe dominated by CG as

$$T\dot{S}_{tot} = \frac{3\pi}{4G}\frac{\dot{H}}{H^5}(\dot{H} - 2H^2), \tag{51}$$

where

$$S_{tot} = S_I + S_h. \tag{52}$$

In order to examine the validity of the GSL, i.e. $T\dot{S}_{tot}(t) \geq 0$, we should discuss the equation (51) graphically. Due to $\frac{d}{dt} = -(1+z)H\frac{d}{dz}$, the GSL transforms into another form, i.e. $TS'_{tot}(z) \leq 0$. Thus, we find

$$TS'_{tot} = -\frac{3\pi}{4G}\frac{H'}{H^4}\left[(1+z)H' + 2H\right]. \tag{53}$$

Therefore, substituting the relation (21) into the above result gives

$$TS'_{tot} = -\frac{3\pi H_0^{-2}\rho_0^{-\frac{1}{2}}}{64G} g_1^{-\frac{3}{2}}\left[(1+z)\frac{g_2}{g_1} + 8\right] \tag{54}$$

where

$$g_1 = A(1+z)^8 + (1-A)(1+z)^n, \tag{55}$$

$$g_2 = 8A(1+z)^7 + n(1-A)(1+z)^{n-1}. \tag{56}$$

We discuss this result graphically in the next section to check the validity of the GSL.

VI. FITTING THE MODEL PARAMETERS

We can fit the model according to the recent observational measurements. In this section, we use SN Ia dataset, OHD and Planck 2015 results in order to fix the auxiliary parameters B and n.

A. SN Ia data with Planck-2015 results

Now, we focus on the observational SN Ia dataset including information about the luminosity distance d_L. For the SN Ia measurements, χ^2 function is written as[76]

$$\chi^2_{SN} = \sum_i^{580}\frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i)]^2}{\sigma_i^2}, \tag{57}$$

where μ_{obs} and μ_{theo} represent the observational and theoretical forms of the distance modulus, respectively. Additionally, σ_i is the uncertainty in the distance modulus. The theoretical distance modulus is written as

$$\mu_{theo} = 5\log_{10} d_L(z_i) + \mu_0, \tag{58}$$

with

$$\mu_0 = 42.38 - 5\log_{10} h. \tag{59}$$

where h is the then-favored dimensionless Hubble parameter. In order to minimize the χ^2_{SN} function with respect to μ_0 for 580 recent data points of SN Ia[77], we write[78]

$$\tilde{\chi}^2_{SN} = P - \frac{Q^2}{R}, \tag{60}$$

where

$$P = \sum_i^{580}\frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i;\mu_0 = 0)]^2}{\sigma_i^2}, \tag{61}$$

$$Q = \sum_i^{580}\frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i;\mu_0 = 0)]}{\sigma_i^2}, \tag{62}$$

$$R = \sum_{i}^{580} \frac{1}{\sigma_i^2}. \tag{63}$$

The best-fit values of B and n in addition to χ^2_{min} (the minimum value of χ^2_{SN}) are presented in TABLE I.

TABLE I: χ^2_{min} and the best-fit values of the model parameters obtained by using the SN Ia dataset and Planck-2015 results in the 1σ confidence region.

Parameter	χ^2_{min}	B	n
Min. values	140.461	1.9	-1.15

B. OHD with Planck-2015 results

Now, considering some observable $H(z)$ data[79–90] given in TABLE II where DGA and RBAO means the Differential Galactic Age and the Radial Baryonic Acoustic Oscillation, respectively, we further investigate the validity of the constraints on the free parameters.

TABLE II: The recent observable $H(z)$ dataset.

z	$H(z)$	σ	Method, Reference
0.0708	69.00	$\mp$19.68	DGA, [80]
0.1200	68.60	$\mp$26.20	DGA, [80]
0.1700	83.00	$\mp$8.000	DGA, [81]
0.1990	75.00	$\mp$5.000	DGA, [82]
0.2400	79.69	$\mp$2.650	RBAO, [83]
0.2800	88.80	$\mp$36.60	DGA, [80]
0.3500	84.40	$\mp$7.000	RBAO, [84]
0.3802	83.00	$\mp$13.50	DGA, [82]
0.4000	95.00	$\mp$17.00	DGA, [81]
0.4247	87.10	$\mp$11.20	DGA, [85]
0.4300	86.45	$\mp$3.680	RBAO, [83]
0.4497	92.80	$\mp$12.90	DGA, [85]
0.4783	80.90	$\mp$9.000	DGA, [85]
0.4800	97.00	$\mp$62.00	DGA, [86, 87]
0.5700	92.40	$\mp$4.500	RBAO, [88]
0.5930	104.0	$\mp$13.00	DGA, [82]
0.6800	92.00	$\mp$8.000	DGA, [82]
0.7300	97.30	$\mp$7.000	RBAO, [89]
0.7810	105.0	$\mp$12.00	DGA, [82]
0.8750	125.0	$\mp$17.50	DGA, [82]
0.9000	117.0	$\mp$23.00	DGA, [81]
1.3000	168.0	$\mp$17.00	DGA, [90]
1.4300	177.0	$\mp$18.00	DGA, [90]

FIG. 1 shows the evolution of the Hubble parameter as a function of the cosmic red shift parameter in the 1σ confidence region. Note that the dots given in FIG. 1 indicate the recent observable values. The best-fit values of B and n are given in TABLE III.

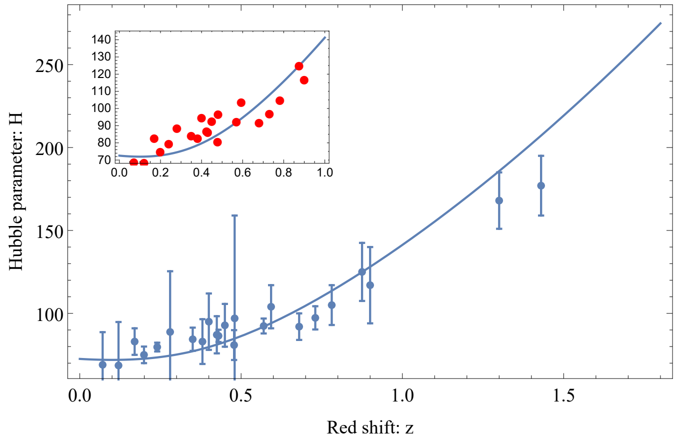

FIG. 1: The variation of Hubble parameter H against z.

TABLE III: The best-fit values of the auxiliary parameters B and n obtained by considering OHD and Planck-2105 results in the 1σ confidence region.

Parameter	B	n
Min. values	1.85	-1.1

VII. ADDITIONAL DISCUSSIONS

In the previous sections, we obtain some constraints and best-fit values for the free parameter of the model. Also, we have defined that

$$\xi = 1.72 - \frac{8B}{8-n}, \qquad A = \frac{\xi}{\xi + \frac{8B}{8-n}}. \tag{64}$$

Additionally, the recent observational data[9, 43] imply that $\rho_0 = 1.31$, $H_0 = 67.8 \pm 0.9$ km s^{-1} Mpc^{-1}.

In FIG. 2, we have depicted the evolution of d_L and d_A given in equations (31) and (33), respectively, as a function of the cosmic red shift parameter z. We show that the results we obtained are in consistence with the data presented in Refs[1–9, 57].

Next, we plot the relation $TS'_{tot} \sim z$ in FIG. 3. From this figure, one can see that the GSL is valid in the VCG dominated KK universe at all times.

VIII. CLOSING REMARKS

It is known that[91] the actions describing VCG and GCG are connected with the BornInfeld type lagrangian density[42] and its generalized definition[29], respectively. The CG proposals emerges from the dynamics of a extended d-brane in a $(d+1, 1)$ dimensional universe[42]. In this work, we mainly considered the VCG model in the compact KK framework which gives interesting and original conclusions.

We have written the corresponding energy density as a function of the cosmic red shift parameter and obtained

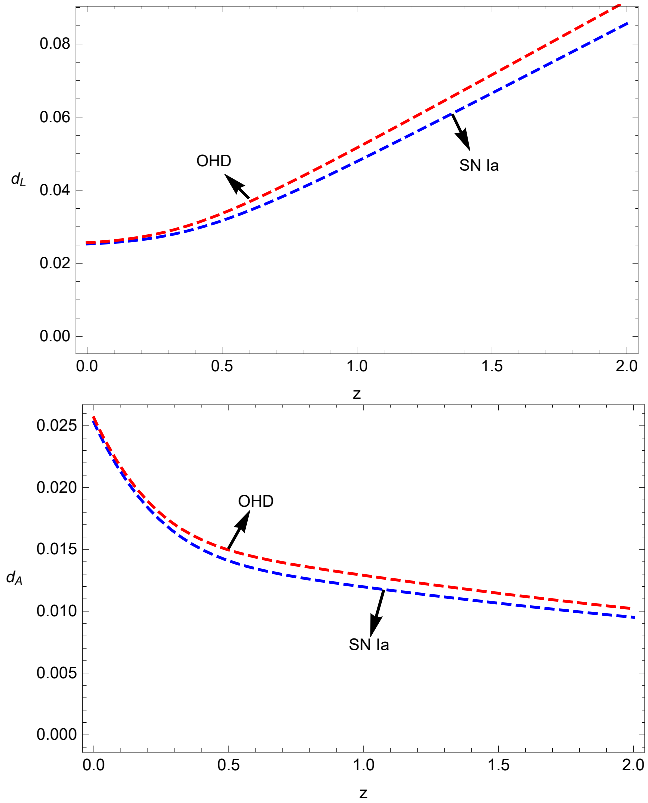

FIG. 2: Numerical analysis of the luminosity distance d_L and angular diameter distance d_A.

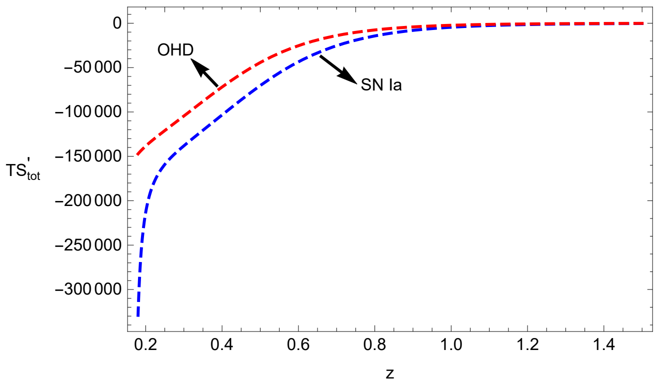

FIG. 3: The GSL $TS'_{tot} \leq 0$ in terms of the red shift parameter z. Here, we assume $8\pi G = 1$ for the sake of simplicity.

an analytical relation for the Hubble parameter. Next, we showed that there was no critical point in this type CG model. We also confirmed stability of the KK type VCG description using thermodynamics quantities. We can express our conclusions in the following way:

- In contrast to the OCG, we have seen that KK type VCG description gives more consistent results with experimental data,
- The parameter A is interpreted as an effective matter density,
- The background evolution of the KK type VCG description matches with an interaction between the dark energy and the dark matter (note that, in literature, the CG candidate is known as a unified form of the dark energy and the dark matter),
- The VCG proposal is equivalent to a model including a scalar field and self-interacting potential.

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