Further mathematical connections between some equations of Dirichlet L-functions, some equations of D-Branes and the Rogers-Ramanujan continued fractions. III

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Abstract

In this research thesis, we have described some new mathematical connections between some equations of Dirichlet L-functions, some equations of D-Branes and Rogers-Ramanujan continued fractions.

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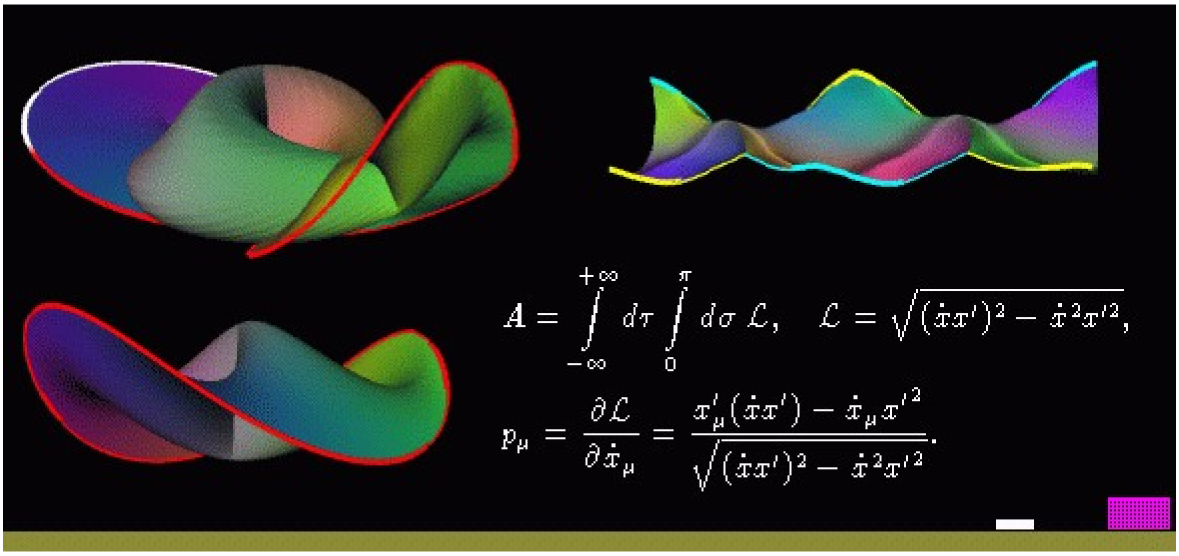

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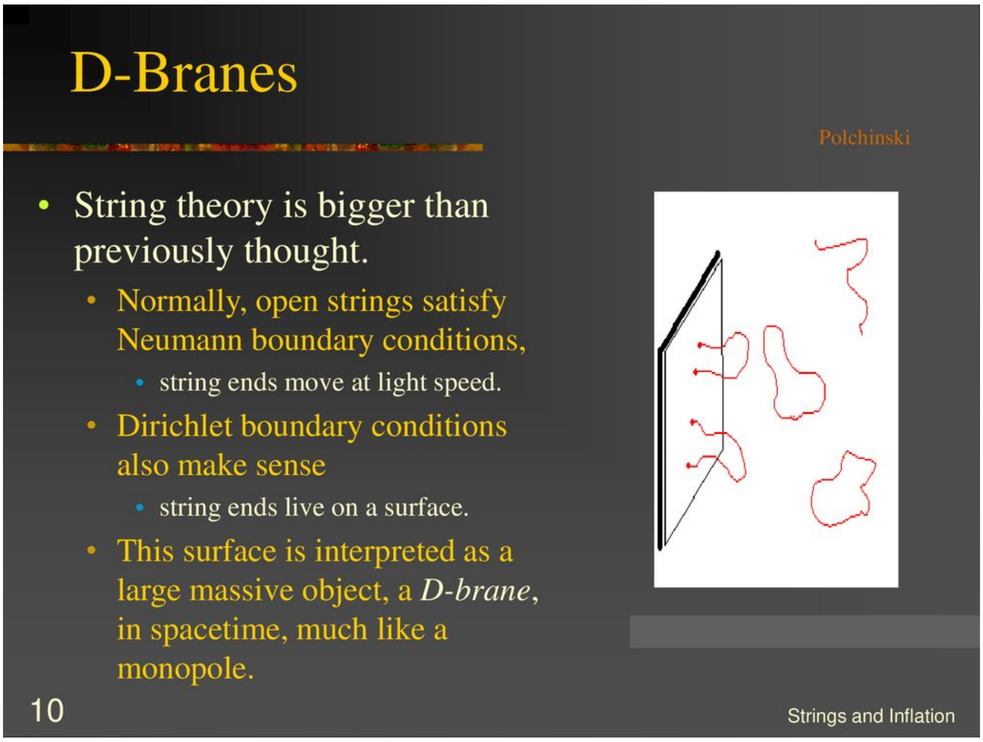

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Anatoly Alexeyevich Karatsuba (Russian Mathematician)

https://commons.wikimedia.org/wiki/File:AnatolyA.Karatsuba.jpg

Srinivasa Ramanujan (Indian Mathematician)

https://www.britannica.com/biography/Srinivasa-Ramanujan

From:

Quantization of the Riemann Zeta-Function and Cosmology
I.Ya. Aref'eva and I. V. Volovich

$$Z(\tau) == e^{i\vartheta(\tau)}\zeta\Big(\frac{1}{2}+i\tau\Big) \tag{42}$$

where

$$e^{i\vartheta(\tau)} = \pi^{-i\tau/2}\frac{\Gamma(\frac{1}{4}+\frac{i\tau}{2})}{|\Gamma(\frac{1}{4}+\frac{i\tau}{2})|}.$$

The function $Z(\tau)$ is called the Riemann-Siegel (or Hardy) function [9]. It is known that $Z(\tau)$ is real for real τ, $|Z(\tau)| = |\zeta(\tau)|$ and there is a bound

$$Z(\tau) = O(|\tau|^{\frac{1}{6}+\epsilon}), \quad \epsilon > 0. \tag{43}$$

and from:

A. A. Karatsuba, **On the zeros of a special type of function connected with Dirichlet series**, Izv. Akad. Nauk SSSR Ser. Mat., 1991, Volume 55, Issue 3, 483–514

$$I_1 = \int_t^{t+h} |Z(u)|\,du, \quad I_2 = \Big|\int_t^{t+h} Z(u)\,du\Big|.$$

for t = 2, t + h = 3, we obtain:

integrate [Pi^(-1/2)*(((gamma (1/4+1/2) / gamma (1/4+1/2))))*zeta (1+1/2)]x, [2, 3]

Definite integral:

$$\int_2^3 \frac{\left(\pi^{-1/2}\,\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)\zeta\left(1+\frac{1}{2}\right)\right)x}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\,dx = \frac{5\,\zeta\left(\frac{3}{2}\right)}{2\sqrt{\pi}} \approx 3.68469$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

We note that 3.68469 is very near to the following sum of values of Ramanujan continued fraction:

2,0663656771 + 0,5683000031 + 1,0018674362 = 3,6365331164

$I_1 + I_2 = 7.36937$

Indeed:

Input:

$$2\int_2^3\left(\pi^{-1/2}\times\frac{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\zeta\left(1+\frac{1}{2}\right)\right)x\,dx$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{5\,\zeta\left(\frac{3}{2}\right)}{\sqrt{\pi}}\approx 7.36937$$

From the first integral, we obtain:

((((((integrate [Pi^(-1/2) * (((gamma (1/4+1/2) / gamma (1/4+1/2)))) * zeta (1+1/2)]x, [2, 3]))))))^1/(e)

Input:

$$\sqrt[e]{\int_2^3\left(\pi^{-1/2}\times\frac{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\zeta\left(1+\frac{1}{2}\right)\right)x\,dx}$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\pi^{-1/(2e)}\sqrt[e]{\frac{5\,\zeta\left(\frac{3}{2}\right)}{2}}\approx 1.61572$$

1.61572

Computation result:

$$\sqrt[e]{\int_2^3 \frac{\left(\pi^{-1/2}\,\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)\zeta\left(1+\frac{1}{2}\right)\right)x}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\,dx} = \pi^{-1/(2e)}\sqrt[e]{\frac{5\,\zeta\left(\frac{3}{2}\right)}{2}}$$

From the above total integral

$$2\int_2^3 \left(\pi^{-1/2}\times\frac{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\zeta\left(1+\frac{1}{2}\right)\right)x\,dx$$

we obtain:

((((((2*integrate [Pi^(-1/2) * (((gamma (1/4+1/2) / gamma (1/4+1/2)))) * zeta (1+1/2)]x, [2, 3]))))))^1/4

Input:

$$\sqrt[4]{2\int_2^3 \left(\pi^{-1/2}\times\frac{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\zeta\left(1+\frac{1}{2}\right)\right)x\,dx}$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{\sqrt[4]{5\,\zeta\left(\frac{3}{2}\right)}}{\sqrt[8]{\pi}} \approx 1.64762$$

$1.64762 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Computation result:

$$\sqrt[4]{2\int_2^3 \frac{\left(\pi^{-1/2}\,\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)\zeta\left(1+\frac{1}{2}\right)\right)x}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\,dx} = \frac{\sqrt[4]{5\,\zeta\left(\frac{3}{2}\right)}}{\sqrt[8]{\pi}}$$

-29/10^3+((((((2*integrate [Pi^(-1/2) * (((gamma (1/4+1/2) / gamma (1/4+1/2)))) * zeta (1+1/2)]x, [2, 3]))))))^1/4

Input:

$$-\frac{29}{10^3}+\sqrt[4]{2\int_2^3\left(\pi^{-1/2}\times\frac{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\zeta\left(1+\frac{1}{2}\right)\right)x\,dx}$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{\sqrt[4]{5\,\zeta\left(\frac{3}{2}\right)}}{\sqrt[8]{\pi}}-\frac{29}{1000}\approx 1.61862$$

1.61862

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Computation result:

$$-\frac{29}{10^3}+\sqrt[4]{2\int_2^3\frac{\left(\pi^{-1/2}\,\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)\zeta\left(1+\frac{1}{2}\right)\right)x}{\Gamma\left(\frac{1}{4}+\frac{1}{2}\right)}\,dx}=\frac{\sqrt[4]{5\,\zeta\left(\frac{3}{2}\right)}}{\sqrt[8]{\pi}}-\frac{29}{1000}$$

Alternate form:

$$\frac{1000\sqrt[4]{5\,\zeta\left(\frac{3}{2}\right)}-29\sqrt[8]{\pi}}{1000\sqrt[8]{\pi}}$$

From: (pag.14-15)

We introduce three more functions

Введём ещё три функции:

$$\rho(s,\chi)=\varepsilon(\overline{\chi})\left(\frac{k}{\pi}\right)^{1/2-s}\frac{\Gamma\left(\frac{1-s}{2}+\frac{a}{2}\right)}{\Gamma\left(\frac{s}{2}+\frac{a}{2}\right)};\qquad(10)$$

$$e^{i\theta(t,\chi)}=\left\{\rho\left(\frac{1}{2}+it,\chi\right)\right\}^{-1/2};\qquad(11)$$

$$Z(t,\chi)=e^{i\theta(t,\chi)}L\left(\frac{1}{2}+it,\chi\right).\qquad(12)$$

Далее, из определения (11) функции $\theta(t, \chi)$ находим

$$e^{i\theta(t,\chi)} = \left\{\rho\left(\frac{1}{2} + it, \chi\right)\right\}^{-1/2} = \left\{\frac{i^a \sqrt{k}}{\tau(\bar{\chi})}\right\}^{-1/2} \left(\frac{k}{\pi}\right)^{it/2} \left\{\frac{\Gamma\left(\frac{1}{4} + \frac{a}{2} + \frac{it}{2}\right)}{\Gamma\left(\frac{1}{4} + \frac{a}{2} - \frac{it}{2}\right)}\right\}^{1/2}.$$

Поэтому

$$F(t, \chi) = e^{i\theta(t,\chi)} F_1(t, \chi) = \sqrt{\varepsilon(\chi)} \left(\frac{k}{\pi}\right)^{it/2} \left\{\frac{\Gamma\left(\frac{1}{4} + \frac{a}{2} + \frac{it}{2}\right)}{\Gamma\left(\frac{1}{4} + \frac{a}{2} - \frac{it}{2}\right)}\right\}^{1/2} \times$$

$$\times \sum_{\lambda \leq P_1} \frac{a(\lambda)}{\sqrt{\lambda}} \lambda^{-it} + \frac{i^{-a}}{\sqrt{\varepsilon(\chi)}} \left(\frac{k}{\pi}\right)^{it/2} \left\{\frac{\Gamma\left(\frac{1}{4} + \frac{a}{2} + \frac{it}{2}\right)}{\Gamma\left(\frac{1}{4} + \frac{a}{2} - \frac{it}{2}\right)}\right\}^{1/2} \times$$

$$\times \exp\left(i\left(\frac{\pi}{4} + t - 2t \log P_1\right)\right) \sum_{\lambda \leq P_1} \frac{\overline{a(\lambda)}}{\sqrt{\lambda}} \lambda^{it} + O(T^{-0,25} X \log^2 T).$$

Пользуясь формулой Стирлинга, легко найдём асимптотическую формулу для φ, где

$$e^{i\varphi} = \frac{\Gamma\left(\frac{1}{4} + \frac{a}{2} + \frac{it}{2}\right)}{\Gamma\left(\frac{1}{4} + \frac{a}{2} - \frac{it}{2}\right)}.$$

Действительно, логарифмируя обе части этого равенства, а затем применяя формулу Стирлинга (см., например, [12, с. 137]), получаем

$$i\varphi = \log\Gamma\left(\frac{1}{4}+\frac{a}{2}+\frac{it}{2}\right) - \log\Gamma\left(\frac{1}{4}+\frac{a}{2}-\frac{it}{2}\right) =$$

$$= \left(\frac{a}{2}-\frac{1}{4}+\frac{it}{2}\right)\log\left(\frac{1}{4}+\frac{a}{2}+\frac{it}{2}\right) - \left(\frac{1}{4}+\frac{a}{2}+\frac{it}{2}\right) +$$

$$+ \int_0^\infty \frac{\rho_1(u)\,du}{u+\frac{1}{4}+\frac{a}{2}+\frac{it}{2}} - \left(\frac{a}{2}-\frac{1}{4}-\frac{it}{2}\right)\log\left(\frac{1}{4}+\frac{a}{2}-\frac{it}{2}\right) +$$

$$+ \left(\frac{1}{4}+\frac{a}{2}-\frac{it}{2}\right) - \int_0^\infty \frac{\rho_1(u)\,du}{u+\frac{1}{4}+\frac{a}{2}-\frac{it}{2}} =$$

$$= \left(\frac{a}{2}-\frac{1}{4}\right)\log\frac{\frac{1}{4}+\frac{a}{2}+\frac{it}{2}}{\frac{1}{4}+\frac{a}{2}-\frac{it}{2}} + \frac{it}{2}\log\left(\left(\frac{1}{4}+\frac{a}{2}\right)^2+\frac{t^2}{4}\right) -$$

$$- it - it\int_0^\infty \frac{\rho_1(u)\,du}{\left(u+\frac{1}{4}+\frac{a}{2}\right)^2+\frac{t^2}{4}} =$$

$$= i\left(a-\frac{1}{2}\right)\left(\frac{\pi}{2}-\operatorname{arctg}\frac{1/2+a}{t}\right) + it\log\frac{t}{2} +$$

$$+ \frac{it}{2}\log\left(1+\left(\frac{a+1/2}{t}\right)^2\right) - it - it\int_0^\infty \frac{\rho_1(u)\,du}{\left(u+\frac{1}{4}+\frac{a}{2}\right)^2+\frac{t^2}{4}};$$

$$\rho_1(u) = 1/2 - \{u\}.$$

From:

$$= i\left(a-\frac{1}{2}\right)\left(\frac{\pi}{2}-\operatorname{arctg}\frac{1/2+a}{t}\right) + it\log\frac{t}{2} +$$

$$+ \frac{it}{2}\log\left(1+\left(\frac{a+1/2}{t}\right)^2\right) - it - it\int_0^\infty \frac{\rho_1(u)\,du}{\left(u+\frac{1}{4}+\frac{a}{2}\right)^2+\frac{t^2}{4}};$$

$$\rho_1(u) = 1/2 - \{u\}.$$

$$a \geqslant \sqrt{t},$$

$a = \sqrt{2}$, $t = 2$

i(sqrt(2)-1/2)(Pi/2-arctang(((((1/2+sqrt(2))/2)))+i*2*ln(1)+i*t/2 ln(1+((((sqrt(2)+1/2)/2)))^2)-i*t-i*t

((((i(sqrt(2)-1/2)(Pi/2-arctang(((((1/2+sqrt(2))/2)))+i*2*ln(1)+i*2/2 ln(1+((((sqrt(2)+1/2)/2)))^2)-i*2-i*2)))))

integrate [1/2*1/((1+1/4+sqrt(2)/2))^2+1]x

$$= i\left(a-\frac{1}{2}\right)\left(\frac{\pi}{2}-\operatorname{arctg}\frac{1/2+a}{t}\right)+it\log\frac{t}{2}+$$
$$+\frac{it}{2}\log\left(1+\left(\frac{a+1/2}{t}\right)^2\right)-it-it\int_0^\infty\frac{\rho_1(u)\,du}{\left(u+\frac{1}{4}+\frac{a}{2}\right)^2+\frac{t^2}{4}};$$
$$\rho_1(u)=1/2-\{u\}.$$

Input:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)+i\times 2\log(1)+i\times\frac{2}{2}\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)-i\times 2-i\times 2\right)$$

Exact Result:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)$$

(result in radians)

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

$$\int_0^\infty\frac{\rho_1(u)\,du}{\left(u+\frac{1}{4}+\frac{a}{2}\right)^2+\frac{t^2}{4}}$$

Indefinite integral:

$$\int\frac{x}{2\left(\left(1+\frac{1}{4}+\frac{\sqrt{2}}{2}\right)^2+1\right)}\,dx=\frac{x^2}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}+\text{constant}$$

(((((x^2/(4 (1 + (5/4 + 1/sqrt(2))^2)))))) * i (-1/2 + sqrt(2)) (π/2 + i tanh^(-1)(4 + 1/2 i (1/2 + sqrt(2)) - log(1 + 1/4 (1/2 + sqrt(2))^2)))))))

$$\frac{x^2}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} \left(i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)$$

$\log(x)$ is the natural logarithm

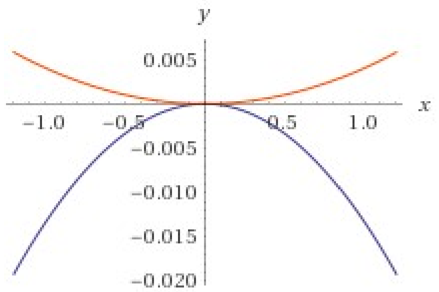
$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Exact result:

$$\frac{i\left(\sqrt{2}-\frac{1}{2}\right)x^2\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

Plot:



- **Alternate forms:**

$$\frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} i\left(\sqrt{2}-\frac{1}{2}\right)x^2\left(\frac{\pi}{2}+\frac{1}{2}\,i\left(\log\left(5+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)-\log\left(-3-\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)+\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)$$

- $$\frac{i\,(2\sqrt{2}-1)\,\pi\,x^2}{49+20\sqrt{2}}-\frac{2\,(2\sqrt{2}-1)\,x^2\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{49+20\sqrt{2}}$$

- $$x^2\left(-\frac{i\,\pi}{49+20\sqrt{2}}+\frac{2\,i\sqrt{2}\,\pi}{49+20\sqrt{2}}+\frac{2\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{49+20\sqrt{2}}-\frac{4\sqrt{2}\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{49+20\sqrt{2}}\right)$$

- **Expanded form:**

$$\frac{i\pi x^2}{4\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \frac{i\pi x^2}{16\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} -$$

$$\frac{x^2 \tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{2\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} +$$

$$\frac{x^2 \tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{8\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

- **Alternate form assuming x is real:**

$$\frac{x^2 \log\left(\frac{1}{4}\left(-\frac{1}{2}-\sqrt{2}\right)^2+\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)^2\right)}{8\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} -$$

$$\frac{x^2 \log\left(\frac{1}{4}\left(-\frac{1}{2}-\sqrt{2}\right)^2+\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)^2\right)}{32\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} -$$

$$\frac{x^2 \log\left(\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2+\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2\right)}{8\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} +$$

$$\frac{x^2 \log\left(\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2+\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2\right)}{32\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} +$$

$$i\left(\frac{x^2\tan^{-1}\left(\frac{-\frac{1}{2}-\sqrt{2}}{2\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)}\right)}{4\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \frac{x^2\tan^{-1}\left(\frac{-\frac{1}{2}-\sqrt{2}}{2\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)}\right)}{16\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \right.$$

$$\left.\frac{x^2\tan^{-1}\left(\frac{\frac{1}{2}+\sqrt{2}}{2\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}\right)}{4\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} + \frac{x^2\tan^{-1}\left(\frac{\frac{1}{2}+\sqrt{2}}{2\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}\right)}{16\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

- **Root:**

$x = 0$

- **Polynomial discriminant:**

$\Delta = 0$

- **Property as a function:**

  Parity

even

-

Derivative:

$$\frac{d}{dx}\left(\frac{x^2\left(i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\right)=$$

$$\frac{2i(2\sqrt{2}-1)x\left(\pi+2i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log\left(\frac{16}{25+4\sqrt{2}}\right)\right)\right)}{49+20\sqrt{2}}$$

- **Indefinite integral:**

$$\int\frac{x^2\left(i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\,dx=$$

$$\frac{i\pi x^3}{12\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}-\frac{i\pi x^3}{48\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}-\frac{x^3\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{6\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}+\frac{x^3\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{24\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}+\text{constant}$$

For x = 3

Input:

$$\frac{3^2}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\left(i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)$$

log(x) is the natural logarithm

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Exact result:

$$\frac{9i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

Decimal approximation:

-0.11981369355410630511317236782500010419207142141798911 8... +
0.03624411116460400117646231263179578879400430236639973 28... i

-0.119813693554106305 11+0.036244111164604i

Input interpretation:

$-0.11981369355410630511+0.036244111164604\,i$

i is the imaginary unit

Result:

$-0.119813693554106... + 0.036244111164604... i$

Polar coordinates:

$r = 0.125175703541819$ (radius), $\theta = 163.169228599584°$ (angle)

0.125175703541819

(((exp(0.125175703541819))))^4

Input interpretation:

$\exp^4(0.125175703541819)$

Result:

1.64988042265356...

1.649880422.... ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

18/10^3+4/10^3+(((exp(0.125175703541819))))^4

Input interpretation:

$\frac{18}{10^3} + \frac{4}{10^3} + \exp^4(0.125175703541819)$

Result:

1.67188042265356...

1.671880422....

We note that 1.671880422... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

-(29/10^3+2/10^3)+(((exp(0.125175703541819))))^4

Input interpretation:

$$-\left(\frac{29}{10^3}+\frac{2}{10^3}\right)+\exp^4(0.125175703541819)$$

Result:

1.61888042265356...

1.61888042265356...

1/(sqrt(exp(0.125175703541819)))

Input interpretation:

$$\frac{1}{\sqrt{\exp(0.125175703541819)}}$$

Result:

0.9393305373373498...

0.93933053733...

And:

(0.125175703541819)^1/48

Input interpretation:

$$\sqrt[48]{0.125175703541819}$$

Result:

0.95763130379980905...

0.95763130379...

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\dots}}}}\approx 0.9568666373$$

Or:

Input:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\times 2\log(1)+i\times\frac{2}{2}\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)-i\times 2-i\times 2\right)\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

i is the imaginary unit

Exact Result:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)$$

(result in radians)

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Decimal approximation:

−0.2572142775498841449176169323469793029486238404644223 40... + 0.0778083255102338300718229914518107325627586263006029525... i

(result in radians)

Alternate forms:

$$\frac{1}{4}i\left(2\sqrt{2}-1\right)\left(\pi+2i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log\left(\frac{16}{25+4\sqrt{2}}\right)\right)\right)$$

- $$\frac{1}{4}i\left(2\sqrt{2}-1\right)\left(\pi+2i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log(16)-\log\left(25+4\sqrt{2}\right)\right)\right)$$

- $$\frac{1}{2}i\left(\sqrt{2}-\frac{1}{2}\right)\left(\pi+2i\tanh^{-1}\left(\frac{1}{4}\left((16+i)+2i\sqrt{2}+16\log(2)-4\log\left(25+4\sqrt{2}\right)\right)\right)\right)$$

Alternative representations:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=i\left(\frac{\pi}{2}-\tan^{-1}\left(1,-4i+2i\log(1)+i\log\left(1+\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)^2\right)+\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=i\left(\frac{\pi}{2}-\tan^{-1}\left(-4i+2i\log(a)\log_a(1)+i\log(a)\log_a\left(1+\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)^2\right)+\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=i\left(\frac{\pi}{2}-\tan^{-1}\left(-4\,i+2\,i\log_e(1)+i\log_e\left(1+\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)^2\right)+\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=i\left(\frac{\pi}{2}-\frac{1}{2}\,i\left(\log\left(1-i\left(-4\,i+2\,i\log(1)+i\log\left(1+\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)^2\right)+\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)-\log\left(1+i\left(-4\,i+2\,i\log(1)+i\log\left(1+\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)^2\right)+\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)$$

$\tan^{-1}(x, y)$ is the inverse tangent function
$\log_b(x)$ is the base- b logarithm

Series representations:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)+\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(\frac{1}{2}+\sqrt{2}\right)^{2k}}{k}\right)\right)$$

- $$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=\frac{1}{4}\,i\left(-1+2\sqrt{2}\right)\left(\pi+2\,i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(\frac{1}{2}+\sqrt{2}\right)^{2k}}{k}\right)\right)$$

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$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$
$$\frac{1}{4}\,i\left(-1+2\sqrt{2}\right)\left(\pi+2\,i\tanh^{-1}(z_0)+i\sum_{k=1}^{\infty}\frac{\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log\left(\frac{16}{25+4\sqrt{2}}\right)-z_0\right)^k\left((1-z_0)^{-k}+(-1)^{1+k}(1+z_0)^{-k}\right)}{k}\right)$$

for ($z_0 \notin \mathbb{R}$ or (not ($1 \le z_0 < \infty$) and not ($-\infty < z_0 \le -1$)))

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$
$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\left(\tanh^{-1}(z_0)+\frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-z_0\right)^k\left((1-z_0)^{-k}+(-1)^{-1+k}(1+z_0)^{-k}\right)\right)\right)$$

for ($z_0 \notin \mathbb{R}$ or (not ($1 \le z_0 < \infty$) and not ($-\infty < z_0 \le -1$)))

$\mathbb{R}$ is the set of real numbers

Integral representations:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$
$$\frac{1}{8}\left(-1+2\sqrt{2}\right)\left(2\,i\,\pi+(-16-i)-2\,i\sqrt{2}+4\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\int_0^1\frac{1}{1+\frac{1}{16}t^2\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2}\,dt\right)$$

•

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$
$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\int_0^1\frac{1}{1-t^2\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}\,dt\right)$$

- $$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$
$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\frac{(16+i)+2i\sqrt{2}-4\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)}{16\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{16^s\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)\left((1-16i)+2\sqrt{2}+4i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^{-2s}}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$

$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}-\frac{(1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{16}k^2\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2}{1+2k}\right)}\right)=i\left(-\frac{1}{2}+\sqrt{2}\right)$$

$$\left(\frac{\pi}{2}-\frac{(1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)}{4\left(1+\frac{\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2}{16\left(3+\frac{\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2}{4\left(5+\frac{9\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2}{16\left(7+\frac{\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2}{9+\ldots}\right)}\right)}\right)}\right)}\right)$$

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$

$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\frac{i\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-k^2\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{1+2k}}\right)=$$

$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\cfrac{i\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{1+-\cfrac{\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{3-\cfrac{4\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{5-\cfrac{9\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{7-\cfrac{16\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{9+\ldots}}}}}\right)$$

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^2\right)\right)2-i\,2-i\,2\right)\right)=$$

$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)+\frac{\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^3}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\left(1+(-1)^{1+k}+k\right)^2\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{3+2k}}\right)\right)=$$

$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)+\frac{\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^3}{3+-\frac{9\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{5-\frac{4\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{7-\frac{25\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{9-\frac{16\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}{11+\ldots}}}}}\right)\right)$$

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{2}+\sqrt{2}\right)+i\,2\log(1)+\frac{1}{2}\left(i\log\left(1+\left(\frac{1}{2}\left(\sqrt{2}+\frac{1}{2}\right)\right)^{2}\right)\right)2-i\,2-i\,2\right)\right)=$$

$$i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\left(i\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)\right)\Big/\left(1-\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}}{(1+2k)\left(1-\frac{1}{2}\left(1+(-1)^{k}\right)\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}\right)}\right)\right)$$

$$=i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\left(i\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)\right)\Big/\left(1-\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}+\left(2\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}\right)\Big/\left(3+\left(2\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}\right)\Big/\left(5\left(1-\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}\right)+\frac{12\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}}{7+\frac{12\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}}{9\left(1-\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^{2}\right)\right)^{2}\right)+\ldots}}\right)\right)\right)\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

$\lfloor x \rfloor$ is the floor function integrate [1/2*1/(((((1+1/4+sqrt(2)/2))^2+1)))]x

Indefinite integral:

$$\int \frac{x}{2\left(\left(1+\frac{1}{4}+\frac{\sqrt{2}}{2}\right)^2+1\right)}\,dx = \frac{x^2}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} + \text{constant}$$

x^2/(4 (1 + (5/4 + 1/sqrt(2))^2)

Exact Result:

$$i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)$$

(result in radians)

i (-1/2 + sqrt(2)) (π/2 + i tanh^(-1)(4 + 1/2 i (1/2 + sqrt(2)) - log(1 + 1/4 (1/2 + sqrt(2))^2)))

x^2/(4 (1 + (5/4 + 1/sqrt(2))^2)) * i (-1/2 + sqrt(2)) (π/2 + i tanh^(-1)(4 + 1/2 i (1/2 + sqrt(2)) - log(1 + 1/4 (1/2 + sqrt(2))^2)))

Input:

$$\left(\frac{x^2}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\,i\right)\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)$$

$\log(x)$ is the natural logarithm

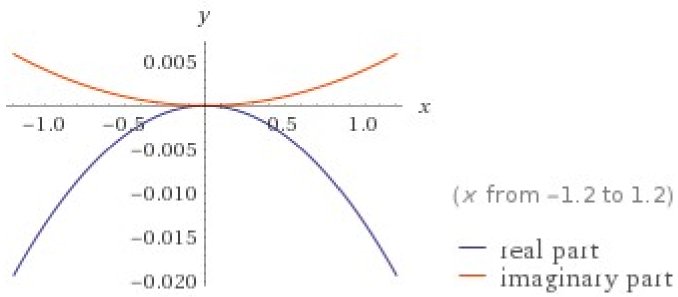
$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Exact result:

$$\frac{i\left(\sqrt{2}-\frac{1}{2}\right)x^2\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

Plot:



Alternate forms:

$$\frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} i\left(\sqrt{2}-\frac{1}{2}\right)x^2\left(\frac{\pi}{2}+\frac{1}{2}i\left(\log\left(5+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)-\log\left(-3-\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)\right)+\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)$$

•

$$\frac{i(2\sqrt{2}-1)\pi x^2}{49+20\sqrt{2}}-\frac{2(2\sqrt{2}-1)x^2\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{49+20\sqrt{2}}$$

•

$$x^2\left(-\frac{i\pi}{49+20\sqrt{2}}+\frac{2i\sqrt{2}\pi}{49+20\sqrt{2}}+\frac{2\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{49+20\sqrt{2}}-\frac{4\sqrt{2}\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{49+20\sqrt{2}}\right)$$

•

Expanded form:

$$\frac{i\pi x^2}{4\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}-\frac{i\pi x^2}{16\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}-\frac{x^2\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{2\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}+\frac{x^2\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{8\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

•

Alternate form assuming x is real:

$$\frac{x^2 \log\left(\frac{1}{4}\left(-\frac{1}{2}-\sqrt{2}\right)^2+\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)^2\right)}{8\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} -$$
$$\frac{x^2 \log\left(\frac{1}{4}\left(-\frac{1}{2}-\sqrt{2}\right)^2+\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)^2\right)}{32\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} -$$
$$\frac{x^2 \log\left(\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2+\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2\right)}{8\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} +$$
$$\frac{x^2 \log\left(\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2+\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2\right)}{32\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} +$$
$$i\left(\frac{x^2 \tan^{-1}\left(\frac{-\frac{1}{2}-\sqrt{2}}{2\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)}\right)}{4\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \frac{x^2 \tan^{-1}\left(\frac{-\frac{1}{2}-\sqrt{2}}{2\left(\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-3\right)}\right)}{16\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \right.$$
$$\left.\frac{x^2 \tan^{-1}\left(\frac{\frac{1}{2}+\sqrt{2}}{2\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}\right)}{4\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} + \frac{x^2 \tan^{-1}\left(\frac{\frac{1}{2}+\sqrt{2}}{2\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}\right)}{16\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Root:

$x = 0$

- **Polynomial discriminant:**

$\Delta = 0$

- **Property as a function:**

Parity

even

- **Derivative:**

$$\frac{d}{dx}\left(\frac{(x^2\, i)\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\, i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\right) =$$
$$\frac{2\, i\,(2\sqrt{2}-1)\, x\left(\pi+2\, i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log\left(\frac{16}{25+4\sqrt{2}}\right)\right)\right)}{49+20\sqrt{2}}$$

- **Indefinite integral:**

$$\int \frac{(x^2 i)\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\,dx =$$

$$\frac{i\pi x^3}{12\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \frac{i\pi x^3}{48\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} - \frac{x^3\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{6\sqrt{2}\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} + \frac{x^3\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)}{24\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} + \text{constant}$$

For x = 3, we obtain:

Input:

$$\left(\frac{3^2}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\,i\right)\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)$$

$\log(x)$ is the natural logarithm

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

i is the imaginary unit

Exact result:

$$\frac{9i\left(\sqrt{2}-\frac{1}{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

Decimal approximation:

$-0.1198136935541063051131723678250001041920714214179891 18... +$
$0.0362441111646040011764623126317957887940043023663997328...\,i$

Alternate forms:

- $$\frac{9i(2\sqrt{2}-1)\left(\pi+2i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log\left(\frac{16}{25+4\sqrt{2}}\right)\right)\right)}{49+20\sqrt{2}}$$
- $$\frac{9i(2\sqrt{2}-1)\left(\pi+2i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log(16)-\log(25+4\sqrt{2})\right)\right)}{49+20\sqrt{2}}$$
-

$$\frac{9\,i\,(2\sqrt{2}-1)\left(\pi+2\,i\tanh^{-1}\left(\frac{1}{4}\left((16+i)+2\,i\sqrt{2}+16\log(2)-4\log(25+4\sqrt{2})\right)\right)\right)}{49+20\sqrt{2}}$$

Alternative representations:

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$
$$\frac{9\,i\left(\frac{\pi}{2}+i\tanh^{-1}\left(4-\log_e\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

-

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$
$$\frac{9\,i\left(\frac{\pi}{2}+i\tanh^{-1}\left(4-\log(a)\log_a\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

-

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$
$$\frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}9\,i\left(\frac{\pi}{2}+\frac{1}{2}\,i\left(-\log\left(-3+\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)\right)+\right.\right.$$
$$\left.\left.\log\left(5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)\right)\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)$$

-

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$
$$\frac{9\,i\left(\frac{\pi}{2}+i\log\left(\frac{\sqrt{5-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)}}{\sqrt{-3+\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)}}\right)\right)\left(-\frac{1}{2}+\sqrt{2}\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

$\log_b(x)$ is the base- b logarithm

-

Series representations:

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2 i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} = \frac{9i(-1+2\sqrt{2})\left(\pi+2i\tanh^{-1}\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(\frac{1}{2}+\sqrt{2}\right)^{2k}}{k}\right)\right)}{49+20\sqrt{2}}$$

- $$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2 i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} = \frac{9i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)+\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(\frac{1}{2}+\sqrt{2}\right)^{2k}}{k}\right)\right)}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}$$

- $$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2 i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} = \frac{1}{49+20\sqrt{2}}9i\left(-1+2\sqrt{2}\right)\left(\pi+2i\tanh^{-1}(z_0)+ i\sum_{k=1}^{\infty}\frac{\left(\left(4+\frac{i}{4}\right)+\frac{i}{\sqrt{2}}+\log\left(\frac{16}{25+4\sqrt{2}}\right)-z_0\right)^k\left((1-z_0)^{-k}+(-1)^{1+k}(1+z_0)^{-k}\right)}{k}\right)$$

  for $(z_0 \notin \mathbb{R}$ or $($ not $(1 \leq z_0 < \infty)$ and not $(-\infty < z_0 \leq -1)))$

  $$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2 i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} = \frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}9i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\left(\tanh^{-1}(z_0)+\frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k}\left(4+\frac{1}{2}i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)-z_0\right)^k\left((1-z_0)^{-k}+(-1)^{-1+k}(1+z_0)^{-k}\right)\right)\right)$$

  for $(z_0 \notin \mathbb{R}$ or $($ not $(1 \leq z_0 < \infty)$ and not $(-\infty < z_0 \leq -1)))$

$\mathbb{R}$ is the set of real numbers

Integral representations:

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$

$$\frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}9\,i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\int_0^1\frac{1}{1-t^2\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2}\,dt\right)$$

- $$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$

$$-\frac{1}{8\left(49+20\sqrt{2}\right)\pi^{3/2}}\,9\left(-1+2\sqrt{2}\right)\left(-8\,i\,\pi^{5/2}+(1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\left(1+\frac{1}{16}\left((1-16\,i)+2\sqrt{2}+4\,i\log\left(\frac{1}{16}\left(25+4\sqrt{2}\right)\right)\right)^2\right)^{-s}ds\right)\text{ for }0<\gamma<\frac{1}{2}$$

- $$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}=$$

$$\frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\,9\,i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\frac{4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)}{4\,\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\left(1-\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2\right)^{-s}ds\right)\text{ for }0<\gamma<\frac{1}{2}$$

-

$$\frac{\left(\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+i\tanh^{-1}\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)\right)\right)3^2\,i}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)} =$$

$$\frac{1}{4\left(1+\left(\frac{5}{4}+\frac{1}{\sqrt{2}}\right)^2\right)}\,9\,i\left(-\frac{1}{2}+\sqrt{2}\right)\left(\frac{\pi}{2}+\frac{4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)}{4\pi}\right.$$
$$\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)\left(-\left(4+\frac{1}{2}\,i\left(\frac{1}{2}+\sqrt{2}\right)-\log\left(1+\frac{1}{4}\left(\frac{1}{2}+\sqrt{2}\right)^2\right)\right)^2\right)^{-s}}{\Gamma\left(\frac{3}{2}-s\right)}$$
$$\left.ds\right) \quad \text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

$-0.1198136935541063051131723678250001041920714214179891 18\ldots +$
$0.0362441111646040011764623126317957887940043023663997328\ldots\, i$

-0.1198136935541063+0.036244111164604001176i

Input interpretation:

$-0.1198136935541063 + 0.036244111164604001176\, i$

i is the imaginary unit

Result:

$-0.1198136935541063\ldots +$
$0.036244111164604 00\ldots\, i$

Polar coordinates:

$r = 0.1251757035418194$ (radius), $\theta = 163.16922859958368°$ (angle)

0.1251757035418194

Input interpretation:

$0.1251757035418194 \times 8$

Result:

1.0014056283345552

1.00140562833…

Input interpretation:

$\frac{1}{2}(0.1251757035418194\times 11 + 0.1251757035418194\times 4)$

Result:

0.9388177765636455

0.9388177765…

18/10^3+((((0.1251757035418194*11)+(0.1251757035418194*4)))) / 2

Input interpretation:

$$\frac{18}{10^3}+\frac{1}{2}(0.1251757035418194\times 11+0.1251757035418194\times 4)$$

Result:

0.9568177765636455

0.9568177765…

Note also that:

(1/0.9388177765 + 1.00140562833) * 1/2 = 2.06657506273… value practically equal to the result of the Ramanujan continued fraction 2.0663656771

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}}=1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cfrac{e^{-8\pi}}{1+\ldots}}}}\approx 1.0018674362$$

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

$$\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!} + \cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\dots}}}}} \approx 2.0663656771$$

$$\sqrt{\frac{e\pi}{2}} erfc\left(\frac{\sqrt{2}}{2}\right) \approx 0.6556795424$$

Now, we have that:

$$\varphi = \left(a - \frac{1}{2}\right)\frac{\pi}{2} + t \log \frac{t}{2} - t + O\left(\frac{1}{t}\right).$$

$$\varphi_1 = \frac{t}{2} \log \frac{k}{\pi} + \frac{\pi}{4}\left(a - \frac{1}{2}\right) + \frac{t}{2} \log \frac{t}{2} - \frac{t}{2} + O\left(\frac{1}{t}\right) =$$
$$= \frac{t}{2} \log \frac{kt}{2\pi} - \frac{t}{2} + \frac{\pi}{4}\left(a - \frac{1}{2}\right) + O\left(\frac{1}{t}\right),$$

$$\varphi_2 = -\frac{a\pi}{2} + \frac{t}{2} \log \frac{k}{\pi} + \frac{\pi}{4}\left(a - \frac{1}{2}\right) + \frac{t}{2} \log \frac{t}{2} - \frac{t}{2} + O\left(\frac{1}{t}\right) +$$
$$+ \frac{\pi}{4} + t - t \log \frac{kt}{2\pi} = -\frac{t}{2} \log \frac{kt}{2\pi} + \frac{t}{2} - \frac{\pi}{4}\left(a - \frac{1}{2}\right) + O\left(\frac{1}{t}\right).$$

Обозначая через φ_3 число

$$\varphi_3 = \frac{t}{2} \log \frac{kt}{2\pi} - \frac{t}{2} + \frac{\pi}{4}\left(a - \frac{1}{2}\right).$$

For T = 3, t =2 and a = √2, and

$k \geqslant 3$

k = 5

we obtain:

((sqrt(2)-1/2))*Pi/2+2ln1-2+1/2

Input:

$$\left(\sqrt{2}-\frac{1}{2}\right)\times\frac{\pi}{2}+2\log(1)-2+\frac{1}{2}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi-\frac{3}{2}$$

Decimal approximation:

-0.063956694318265186107720350789528871741981505155931343 70...

-0.063956694…

Property:

$-\frac{3}{2}+\frac{1}{2}\left(-\frac{1}{2}+\sqrt{2}\right)\pi$ is a transcendental number

Alternate forms:

$$\left(\frac{1}{\sqrt{2}}-\frac{1}{4}\right)\pi-\frac{3}{2}$$

$$-\frac{3}{2}-\frac{\pi}{4}+\frac{\pi}{\sqrt{2}}$$

- $\frac{1}{4}\left(-6-\pi+2\sqrt{2}\,\pi\right)$

Alternative representations:

$$\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=-\frac{3}{2}+2\log_e(1)+\frac{1}{2}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=-\frac{3}{2}+2\log(a)\log_a(1)+\frac{1}{2}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$

- $\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=-\frac{3}{2}+4\tanh^{-1}(0)+\frac{1}{2}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$

$\log_b(x)$ is the base- b logarithm

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Series representations:

- $$\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=-\frac{3}{2}-\frac{\pi}{4}+2\log(1)+\frac{1}{2}\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

- $$\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=-\frac{3}{2}-\frac{\pi}{4}+2\log(1)+\frac{1}{2}\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}$$

- $$\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=\frac{1}{4}\left(-6-\pi+16i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor+8\log(x)-8\sum_{k=1}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}}{k}+2\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

i is the imaginary unit

$\mathbb{R}$ is the set of real numbers

Integral representation:

$$\frac{1}{2}\left(\sqrt{2}-\frac{1}{2}\right)\pi+2\log(1)-2+\frac{1}{2}=-\frac{3}{2}-\frac{\pi}{4}+2\int_1^1\frac{1}{t}\,dt+\frac{\pi\sqrt{2}}{2}$$

1*((log(5*2)/(2Pi)))-1+Pi/4((sqrt(2)-1/2))+(1/2)

Input:

$$1\times\frac{\log(5\times 2)}{2\pi}-1+\frac{\pi}{4}\left(\sqrt{2}-\frac{1}{2}\right)+\frac{1}{2}$$

$\log(x)$ is the natural logarithm

Exact result:

$$-\frac{1}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{\log(10)}{2\pi}$$

Decimal approximation:

0.5844894522805812773825335384748629024047557834536790017 25...

0.58448945228…

Alternate forms:

$$\frac{1}{4}\left(-2+\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{\log(100)}{\pi}\right)$$

- $$-\frac{1}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+\frac{\log(10)}{2\pi}$$

- $$\frac{-4\pi-\pi^2+2\sqrt{2}\,\pi^2+4\log(10)}{8\pi}$$

Alternative representations:

- $$\frac{\log(5\times 2)}{2\pi}-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=-\frac{1}{2}+\frac{\log_e(10)}{2\pi}+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$\frac{\log(5\times 2)}{2\pi}-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=-\frac{1}{2}+\frac{\log(a)\log_a(10)}{2\pi}+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$\frac{\log(5\times 2)}{2\pi}-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=-\frac{1}{2}-\frac{\text{Li}_1(-9)}{2\pi}+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi + \frac{1}{2} = -\frac{1}{2} - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + \frac{\log(9)}{2\pi} - \frac{\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{9}\right)^k}{k}}{2\pi}$$

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi + \frac{1}{2} = -\frac{1}{2} - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + i\left\lfloor \frac{\arg(10-x)}{2\pi} \right\rfloor + \frac{\log(x)}{2\pi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k}}{2\pi} \quad \text{for } x < 0$$

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi + \frac{1}{2} = -\frac{1}{2} - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + i\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \frac{\log(z_0)}{2\pi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}}{2\pi}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi + \frac{1}{2} = -\frac{1}{2} - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + \frac{1}{2\pi}\int_1^{10} \frac{1}{t}\, dt$$

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi + \frac{1}{2} = -\frac{1}{2} - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} - \frac{i}{4\pi^2}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

-1*((log(5*2)/(2Pi)))+1-Pi/4((sqrt(2)-1/2))+(1/2)

Input:

$$-\frac{\log(5\times 2)}{2\pi}+1-\frac{\pi}{4}\left(\sqrt{2}-\frac{1}{2}\right)+\frac{1}{2}$$

log(x) is the natural logarithm

Exact result:

$$\frac{3}{2}-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi-\frac{\log(10)}{2\pi}$$

Decimal approximation:

0.415510547719418722617466461525137097595244216546320998274...

0.415510547719…

Alternate forms:

- $$\frac{1}{8}\left(12+\pi-2\sqrt{2}\,\pi-\frac{4\log(10)}{\pi}\right)$$
- $$\frac{3}{2}+\frac{\pi}{8}-\frac{\pi}{2\sqrt{2}}-\frac{\log(10)}{2\pi}$$
- $$\frac{1}{4}\left(6+\left(\frac{1}{2}-\sqrt{2}\right)\pi-\frac{2\log(10)}{\pi}\right)$$

Alternative representations:

$$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=\frac{3}{2}-\frac{\log_e(10)}{2\pi}-\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=\frac{3}{2}-\frac{\log(a)\log_a(10)}{2\pi}-\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$
- $$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=\frac{3}{2}+\frac{\mathrm{Li}_1(-9)}{2\pi}-\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

- $$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=\frac{3}{2}+\frac{\pi}{8}-\frac{\pi}{2\sqrt{2}}-\frac{\log(9)}{2\pi}+\frac{\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{9}\right)^k}{k}}{2\pi}$$

- $$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=$$
$$\frac{3}{2}+\frac{\pi}{8}-\frac{\pi}{2\sqrt{2}}-i\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor-\frac{\log(x)}{2\pi}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}}{2\pi}\quad\text{for } x<0$$

- $$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=$$
$$\frac{3}{2}+\frac{\pi}{8}-\frac{\pi}{2\sqrt{2}}-i\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\frac{\log(z_0)}{2\pi}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}}{2\pi}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations:

$$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=\frac{3}{2}+\frac{\pi}{8}-\frac{\pi}{2\sqrt{2}}-\frac{1}{2\pi}\int_1^{10}\frac{1}{t}\,dt$$

- $$-\frac{\log(5\times 2)}{2\pi}+1-\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{1}{2}=$$
$$\frac{3}{2}+\frac{\pi}{8}-\frac{\pi}{2\sqrt{2}}+\frac{i}{4\pi^2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

1*((log(5*2)/(2Pi)))-1+Pi/4((sqrt(2)-1/2))

Input:

$$1\times\frac{\log(5\times 2)}{2\pi}-1+\frac{\pi}{4}\left(\sqrt{2}-\frac{1}{2}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{\log(10)}{2\pi}$$

Decimal approximation:

0.084489452280581277382533538474862902404755783453679001725...

0.08448945228…

Alternate forms:

- $$\frac{1}{4}\left(-4+\left(\sqrt{2}-\frac{1}{2}\right)\pi+\frac{\log(100)}{\pi}\right)$$
- $$-1-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+\frac{\log(10)}{2\pi}$$
- $$-1+\frac{1}{8}\left(2\sqrt{2}-1\right)\pi+\frac{\log(10)}{2\pi}$$

Alternative representations:

$$\frac{\log(5\times 2)}{2\pi}-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-1+\frac{\log_e(10)}{2\pi}+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$\frac{\log(5\times 2)}{2\pi}-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-1+\frac{\log(a)\log_a(10)}{2\pi}+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$
- $$\frac{\log(5\times 2)}{2\pi}-1+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-1-\frac{\text{Li}_1(-9)}{2\pi}+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi = -1 - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + \frac{\log(9)}{2\pi} - \frac{\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{9}\right)^k}{k}}{2\pi}$$

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi = -1 - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + i\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor + \frac{\log(x)}{2\pi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k}}{2\pi} \quad \text{for } x < 0$$

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi = -1 - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + i\left\lfloor\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi}\right\rfloor + \frac{\log(z_0)}{2\pi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}}{2\pi}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi = -1 - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} + \frac{1}{2\pi}\int_1^{10} \frac{1}{t}\, dt$$

- $$\frac{\log(5\times 2)}{2\pi} - 1 + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi = -1 - \frac{\pi}{8} + \frac{\pi}{2\sqrt{2}} - \frac{i}{4\pi^2}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

From the following sum:

-0.063956694 + 0.58448945228 + 0.415510547719 + 0.08448945228

Input interpretation:

−0.063956694 + 0.58448945228 + 0.415510547719 + 0.08448945228

Result:

1.020532758279

1.020532758279

Note that from the following values of Rogers-Ramanujan continued fractions:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

1/2(((1/0.9568666373)+(1/0.9991104684)))

Input interpretation:

$$\frac{1}{2}\left(\frac{1}{0.9568666373}+\frac{1}{0.9991104684}\right)$$

Result:

1.022984019876352948168796053849360636152194103588348225836...

1.022984019….

The two results 1.020532758… and 1.022984019 are very closed

Now:

$$\varphi_0 = -t \log \frac{tk}{2\pi} + t + \frac{\pi}{4} .$$

$$\varphi_4 = \frac{t}{2} \log \frac{kT}{2\pi} - \frac{T}{2} + \frac{\pi}{4}\left(a - \frac{1}{2}\right),$$

For T = 3, t =2 and a = √2, and

$k \geqslant 3$

k = 5

-2ln(10/(2Pi))+2+Pi/4

Input:

$$-2 \log\left(\frac{10}{2\pi}\right) + 2 + \frac{\pi}{4}$$

$\log(x)$ is the natural logarithm

Exact result:

$$2 + \frac{\pi}{4} - 2 \log\left(\frac{5}{\pi}\right)$$

Decimal approximation:

1.855982110228047908700996882073617865292679267137364154445...

1.855982110228…

Alternate forms:

- $$\frac{1}{4}\left(8 + \pi + 8 \log\left(\frac{\pi}{5}\right)\right)$$
- $$\frac{1}{4}\left(8 + \pi - 8 \log\left(\frac{5}{\pi}\right)\right)$$
- $$2 + \frac{\pi}{4} - 2 \log(5) + 2 \log(\pi)$$

Alternative representations:

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} = 2-2\log_e\left(\frac{10}{2\pi}\right)+\frac{\pi}{4}$$

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} = 2-2\log(a)\log_a\left(\frac{10}{2\pi}\right)+\frac{\pi}{4}$$

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} = 2+2\,\mathrm{Li}_1\left(1-\frac{10}{2\pi}\right)+\frac{\pi}{4}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations:

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} = 2+\frac{\pi}{4}+2\sum_{k=1}^{\infty}\frac{\left(\frac{-5+\pi}{\pi}\right)^k}{k}$$

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} = 2+\frac{\pi}{4}-4\,i\,\pi\left\lfloor\frac{\arg\left(\frac{5}{\pi}-x\right)}{2\pi}\right\rfloor-2\log(x)+2\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{5}{\pi}-x\right)^k x^{-k}}{k}$$
  for $x<0$

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} =$$
  $$2+\frac{\pi}{4}-4\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-2\log(z_0)+2\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{5}{\pi}-z_0\right)^k z_0^{-k}}{k}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations:

$$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4} = 2+\frac{\pi}{4}-2\int_1^{\frac{5}{\pi}}\frac{1}{t}\,dt$$

- $$-2\log\left(\frac{10}{2\pi}\right)+2+\frac{\pi}{4}=2+\frac{\pi}{4}+\frac{i}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\frac{5}{\pi}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

1*ln(15/(2Pi))-3/2+Pi/4*(((sqrt(2)-1/2))

Input:

$$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{\pi}{4}\left(\sqrt{2}-\frac{1}{2}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi+\log\left(\frac{15}{2\pi}\right)$$

Decimal approximation:

0.088194787533731989381484921942713628579306212237734676162...

0.0881947875337…

Alternate forms:

- $$-\frac{3}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+\log\left(\frac{15}{2\pi}\right)$$

- $$-\frac{3}{2}+\frac{1}{8}\left(2\sqrt{2}-1\right)\pi+\log\left(\frac{15}{2\pi}\right)$$

- $$-\frac{3}{2}+\left(\frac{1}{2\sqrt{2}}-\frac{1}{8}\right)\pi-\log\left(\frac{2\pi}{15}\right)$$

Alternative representations:

- $$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-\frac{3}{2}+\log_e\left(\frac{15}{2\pi}\right)+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

-

$$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-\frac{3}{2}+\log(a)\log_a\left(\frac{15}{2\pi}\right)+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

- $$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-\frac{3}{2}-\text{Li}_1\left(1-\frac{15}{2\pi}\right)+\frac{1}{4}\pi\left(-\frac{1}{2}+\sqrt{2}\right)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

- $$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-\frac{3}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+\log\left(-1+\frac{15}{2\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{2\pi}{15-2\pi}\right)^k}{k}$$

- $$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-\frac{3}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+\log\left(-1+\frac{15}{2\pi}\right)-\sum_{k=1}^{\infty}\frac{(2\pi)^k\left(\frac{1}{-15+2\pi}\right)^k}{k}$$

- $$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=$$
$$-\frac{3}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+2i\pi\left\lfloor\frac{\arg\left(\frac{15}{2\pi}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{15}{2\pi}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations:

$$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi=-\frac{3}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}+\int_1^{\frac{15}{2\pi}}\frac{1}{t}\,dt$$

-

$$1\log\left(\frac{15}{2\pi}\right)-\frac{3}{2}+\frac{1}{4}\left(\sqrt{2}-\frac{1}{2}\right)\pi =$$
$$-\frac{3}{2}-\frac{\pi}{8}+\frac{\pi}{2\sqrt{2}}-\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{2\pi}{15-2\pi}\right)^s\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

The total sum of the various results is:

(-0.063956694 + 0.58448945228 + 0.415510547719 + 0.08448945228 + 1.855982110228 + 0.0881947875337)

Input interpretation:

$-0.063956694 + 0.58448945228 + 0.415510547719 + 0.08448945228 + 1.855982110228 + 0.0881947875337$

Result:

2.9647096560407

2.9647096560407

From the following two Rogers-Ramanujan continued fractions:

$$\sqrt{\frac{e\pi}{2}}=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!!}+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\dots}}}}}\approx 2.0663656771$$

$$4\int_0^{\infty}\frac{t\,dt}{e^{\sqrt{5}t}\cosh t}=\cfrac{1}{1+\cfrac{1^2}{1+\cfrac{1^2}{1+\cfrac{2^2}{1+\cfrac{2^2}{1+\cfrac{3^2}{1+\cfrac{3^2}{1+\dots}}}}}}}\approx 0.5683000031$$

$$2\int_0^\infty \frac{t^2 dt}{e^{\sqrt{3}t}\sinh t} = \cfrac{1}{1+\cfrac{1^3}{1+\cfrac{1^3}{3+\cfrac{2^3}{1+\cfrac{2^3}{5+\cfrac{3^3}{1+\cfrac{3^3}{7+\dots}}}}}}} \approx 0.5269391135$$

Now, from the following calculation, we obtain:

1/(0.5683000031+0.5269391135) + 2.0663656771

Input interpretation:

$$\frac{1}{0.5683000031+0.5269391135}+2.0663656771$$

Result:

2.979408303904952813534307983827903394297011177440263458902...

2.9794083039… result very near to the previous value 2.9647096560407

We have that:

((((1/(0.5683000031+0.5269391135) + 2.0663656771))))^1/((2+Pi)^0.5))

Input interpretation:

$$\sqrt[\sqrt{2+\pi}]{\frac{1}{0.5683000031+0.5269391135}+2.0663656771}$$

Result:

1.618442953...

1.618442953…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\, d\,s}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

-55/10^3+((((1/(0.5683000031+0.5269391135) + 2.0663656771))))^1/2

Input interpretation:

$$-\frac{55}{10^3} + \sqrt{\frac{1}{0.5683000031 + 0.5269391135} + 2.0663656771}$$

Result:

1.6710962615...

1.6710962615...

We note that 1.6710962615... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

(2/10^3-(55*2)/10^3)+((((1/(0.5683000031+0.5269391135) + 2.0663656771))))^1/2

Input interpretation:

$$\left(\frac{2}{10^3} - \frac{55 \times 2}{10^3}\right) + \sqrt{\frac{1}{0.5683000031 + 0.5269391135} + 2.0663656771}$$

Result:

1.6180962615...

1.6180962615...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have that:

$$\varphi_4 = \frac{t}{2} \log \frac{kT}{2\pi} - \frac{T}{2} + \frac{\pi}{4}\left(a - \frac{1}{2}\right),$$

For T = 3, t =2 and a = √2, and

$k \geqslant 3$

k = 5

We know that:

ln(15/(2Pi))-3/2+Pi/4*(((sqrt(2)-1/2))

$$\log\left(\frac{15}{2\pi}\right) - \frac{3}{2} + \frac{\pi}{4}\left(\sqrt{2} - \frac{1}{2}\right)$$

$$-\frac{3}{2} + \frac{1}{4}\left(\sqrt{2} - \frac{1}{2}\right)\pi + \log\left(\frac{15}{2\pi}\right)$$

0.088194787533731989381484921942713628579306212237734676162...

0.088194787…

$$Z(t, \chi) = e^{i\theta(t, \chi)} L\left(\frac{1}{2} + it, \chi\right)$$

$$\chi = \chi(n) = 1, \; L(s, \chi) = \zeta(s)$$

$$Z(t, \chi) = Z(t) = e^{i\theta(t)} \zeta\left(\frac{1}{2} + it\right)$$

$$a \geqslant \sqrt{t},$$

$$s = 0{,}5 + it$$

$$\rho(s, \chi) = \varepsilon(\bar{\chi}) \left(\frac{k}{\pi}\right)^{1/2 - s} \frac{\Gamma\left(\frac{1-s}{2} + \frac{a}{2}\right)}{\Gamma\left(\frac{s}{2} + \frac{a}{2}\right)};$$

$$e^{i\theta(t, \chi)} = \left\{\rho\left(\frac{1}{2} + it, \chi\right)\right\}^{-1/2};$$

$$\varepsilon(\chi) = \frac{i^a \sqrt{k}}{\tau(\chi)}.$$

$$|\tau(\chi)| = \sqrt{k}$$

$$\rho(s, \chi) = \varepsilon(\bar{\chi}) \left(\frac{k}{\pi}\right)^{1/2 - s} \frac{\Gamma\left(\frac{1-s}{2} + \frac{a}{2}\right)}{\Gamma\left(\frac{s}{2} + \frac{a}{2}\right)};$$

$$e^{i\theta(t, \chi)} = \left\{\rho\left(\frac{1}{2} + it, \chi\right)\right\}^{-1/2};$$

From:

$$\varepsilon(\chi) = \frac{i^a \sqrt{k}}{\tau(\chi)}.$$

i^(sqrt(2))*(sqrt(5)) / (sqrt(5))

$$i^{\sqrt{2}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$i^{\sqrt{2}}$$

$-0.6056998670788134288044363562502044409630387809861072875\ldots +$
$0.7956932015674808719286969020197189349039323597968150918\ldots i$

$i^{\sqrt{2}}$ is a transcendental number

Exact form

$r = 1$ (radius), $\theta \approx 127.279°$ (angle)

For T = 3, t =2 and a = √2, and

$$k \geqslant 3$$

k = 5

$$s = 0{,}5 + it$$

From:

$$\rho(s,\chi) = \varepsilon(\bar{\chi})\left(\frac{k}{\pi}\right)^{1/2-s} \frac{\Gamma\left(\frac{1-s}{2}+\frac{a}{2}\right)}{\Gamma\left(\frac{s}{2}+\frac{a}{2}\right)};$$

$$e^{i\theta(t,\chi)} = \left\{\rho\left(\frac{1}{2}+it,\chi\right)\right\}^{-1/2};$$

(3/Pi)^((1/2-(0.5+2i))) * gamma (((((1-(0.5+2i)*(1/2))+(sqrt(2))/2)))) / gamma (((((0.5+2i)/2+(((sqrt(2))/2))))))

Input:

$$\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)} \times \frac{\Gamma\left(\left(1-(0.5+2i)\times\frac{1}{2}\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}\times(0.5+2i)+\frac{\sqrt{2}}{2}\right)}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

1.04723... +
0.349266... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.10394$ (radius), $\theta = 18.4443°$ (angle)

1.10394 result very near to a multiple of the Cosmological Constant $1.1056 * 10^{-52}$

Alternative representations:

$$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)} = \frac{\left(\frac{1}{2}(-0.5-2i)+\frac{\sqrt{2}}{2}\right)!\left(\frac{3}{\pi}\right)^{0-2i}}{\left(-1+\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)!}$$

-

$$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)} = \frac{\Gamma\left(1+\frac{1}{2}(-0.5-2i)+\frac{\sqrt{2}}{2},0\right)\left(\frac{3}{\pi}\right)^{0-2i}}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2},0\right)}$$

-

$$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)}=\frac{(1)_{\frac{1}{2}(-0.5-2i)+\frac{\sqrt{2}}{2}}\left(\frac{3}{\pi}\right)^{0-2i}}{(1)_{-1+\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}}}$$

$n!$ is the factorial function

$\Gamma(a,\ x)$ is the incomplete gamma function

$(a)_n$ is the Pochhammer symbol (rising factorial)

Series representations:

$$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)}=\frac{9^{-i}\left(\frac{1}{\pi}\right)^{-2i}\sum_{k=0}^{\infty}\frac{\left(-0.25-i+0.5\sqrt{2}\right)^k\Gamma^{(k)}(1)}{k!}}{\sum_{k=0}^{\infty}\frac{2^{-k}\sqrt{2}^k\Gamma^{(k)}(0.25+i)}{k!}}$$

- $$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)}=\frac{9^{-i}\left(\frac{1}{\pi}\right)^{-2i}\sum_{k=0}^{\infty}\frac{\left(0.75-i+0.5\sqrt{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{\sum_{k=0}^{\infty}\frac{\left(0.25+i+0.5\sqrt{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}$$

  for $(z_0\notin\mathbb{Z}\text{ or }z_0>0)$

- $$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)}=\frac{9^{-i}\left(\frac{1}{\pi}\right)^{-2i}\Gamma\left(0.75-i+0.5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)}{\Gamma\left(0.25+i+0.5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)}$$

  for not $((z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$

$\mathbb{Z}$ is the set of integers

$\mathbb{R}$ is the set of real numbers

Integral representations:

$$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)} = \frac{9^{-i}\left(\frac{1}{\pi}\right)^{-2i}\int_0^1 \log^{-0.25-i+0.5\sqrt{2}}\left(\frac{1}{t}\right)dt}{\int_0^1 \log^{-0.75+i+0.5\sqrt{2}}\left(\frac{1}{t}\right)dt}$$

- $$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)} = \frac{9^{-i}\left(\frac{1}{\pi}\right)^{-2i}\int_0^\infty e^{-t}\,t^{-0.25-i+0.5\sqrt{2}}\,dt}{\int_0^\infty e^{-t}\,t^{-0.75+i+0.5\sqrt{2}}\,dt}$$

- $$\frac{\left(\frac{3}{\pi}\right)^{1/2-(0.5+2i)}\Gamma\left(\left(1-\frac{1}{2}(0.5+2i)\right)+\frac{\sqrt{2}}{2}\right)}{\Gamma\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)} =$$
$$3^{0-2i}\exp\left(\int_0^1\left(\frac{-1+x^{1+1/2(-0.5-2i)+\sqrt{2}/2}-(-1+x)\left(1+\frac{1}{2}(-0.5-2i)+\frac{\sqrt{2}}{2}\right)}{(-1+x)\log(x)} - \frac{-1+x^{1/2(0.5+2i)+\sqrt{2}/2}-(-1+x)\left(\frac{1}{2}(0.5+2i)+\frac{\sqrt{2}}{2}\right)}{(-1+x)\log(x)}\right)dx\right)\left(\frac{1}{\pi}\right)^{0-2i}$$

$\log(x)$ is the natural logarithm

(((1.10394(1/2+2i)))^(-1/2)

Input interpretation:

$$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}$$

i is the imaginary unit

Result:

0.522480... −
0.407940... i

Polar coordinates:

$r = 0.662873$ (radius), $\theta = -37.9819°$ (angle)

0.662873

(((((1.10394(1/2+2i)))^(-1/2)))) * zeta (1/2+2i)

Input interpretation:

$$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2} \zeta\left(\frac{1}{2}+2i\right)$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Result:

0.103043... −
0.342545... i

Polar coordinates:

$r = 0.357708$ (radius), $\theta = -73.2578°$ (angle)

0.357708

Alternative representations:

$$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2} \zeta\left(\frac{1}{2}+2i\right) = \frac{\zeta\left(\frac{1}{2}+2i, 1\right)}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}}$$

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2} \zeta\left(\frac{1}{2}+2i\right) = \frac{\zeta\left(\frac{1}{2}+2i, 1\right)}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}}$$

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2} \zeta\left(\frac{1}{2}+2i\right) = \frac{\zeta\left(\frac{1}{2}+2i, \frac{1}{2}\right)}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}\left(-1+2^{1/2+2i}\right)}$$

$\zeta(s, a)$ is the Hurwitz zeta function

$\zeta(s, a)$ is the generalized Riemann zeta function

Series representations:

$$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)=-\frac{1.34599\times 4^{i}\sum_{k=1}^{\infty}(-1)^k k^{-1/2-2i}}{\left(-1.41421+4^{i}\right)\sqrt{1+4i}}$$

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)=$$
$$\frac{0.95176\,\gamma}{\sqrt{\frac{1}{2}+2i}}+\frac{1.90352}{\sqrt{\frac{1}{2}+2i}\,(-1+4i)}+\frac{0.95176\sum_{k=1}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+2i\right)^k\gamma_k}{k!}}{\sqrt{\frac{1}{2}+2i}}$$

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)=\frac{0.168249\sqrt{1+4i}\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}(-1)^k(1+k)^{1/2-2i}\binom{n}{k}}{1+n}}{(-0.25+i)(0.25+i)}$$

$n!$ is the factorial function

γ_n is the n^{th} Stieltjes constant

γ is the Euler-Mascheroni constant

$\binom{n}{m}$ is the binomial coefficient

Integral representations:

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)=-\frac{2.69198\,(0.25+i)}{\sqrt{1+4i}}\int_0^{\infty}t^{-1/2+2i}\,\mathrm{frac}\left(\frac{1}{t}\right)dt$$

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)=\frac{1.34599\times 4^{i}}{\left(-1.41421+4^{i}\right)\sqrt{1+4i}\,\Gamma\left(\frac{1}{2}+2i\right)}\int_0^{\infty}\frac{t^{-1/2+2i}}{1+e^{t}}\,dt$$

- $$\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)=$$
$$-\frac{0.95176\times 4^{2i}}{\left(1.41421-4^{i}\right)\sqrt{1+4i}\,\Gamma\left(\frac{3}{2}+2i\right)}\int_0^{\infty}t^{1/2+2i}\,\mathrm{sech}^2(t)\,dt$$

sqrt[1/((((((((1.10394(1/2+2i)))^(-1/2)))) * zeta (1/2+2i))))]

Input interpretation:

$$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2\,i\right)\right)^{-1/2}\,\zeta\left(\frac{1}{2}+2\,i\right)}}$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Result:

$1.34181\ldots +$
$0.997564\ldots\, i$

Polar coordinates:

$r = 1.672$ (radius), $\theta = 36.6289°$ (angle)

1.672 result very near to the proton mass

All 2nd roots of 0.805308 + 2.67707 i:

- Polar form
  $1.672\, e^{0.639295\, i} \approx 1.3418 + 0.99756\, i$ (principal root)
- $1.672\, e^{-2.5023\, i} \approx -1.3418 - 0.99756\, i$

Alternative representations:

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2\,i\right)\right)^{-1/2}\,\zeta\left(\frac{1}{2}+2\,i\right)}} = \sqrt{\frac{1}{\frac{\zeta\left(\frac{1}{2}+2\,i,1\right)}{\sqrt{1.10394\left(\frac{1}{2}+2\,i\right)}}}}$$
-

$$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{\frac{1}{\frac{\zeta\left(\frac{1}{2}+2i,1\right)}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}}}}$$

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{\frac{1}{\frac{S_{-\frac{1}{2}+2i,1}{}^{(1)}}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}}}}$$

$\zeta(s, a)$ is the Hurwitz zeta function

$\zeta(s, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations:

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{-\frac{0.742947\times 4^{-i}\left(-1.41421+4^{i}\right)\sqrt{1+4i}}{\sum_{k=1}^{\infty}(-1)^k k^{-1/2-2i}}}$$

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{-1+\frac{0.742947\sqrt{1+4i}}{\zeta\left(\frac{1}{2}+2i\right)}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+\frac{0.742947\sqrt{1+4i}}{\zeta\left(\frac{1}{2}+2i\right)}\right)^{-k}$$

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{\frac{0.742947\sqrt{1+4i}}{\gamma+\frac{2}{-1+4i}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+2i\right)^k\gamma_k}{k!}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

γ_n is the n^{th} Stieltjes constant

γ is the Euler-Mascheroni constant

Integral representations:

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{-\frac{0.371473\sqrt{1+4i}}{0.25+i\int_0^\infty t^{-1/2+2i}\operatorname{frac}\left(\frac{1}{t}\right)dt}}$$

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{\frac{1.05069\times 4^{-2i}\left(-1.41421+4^{i}\right)\sqrt{1+4i}\,\Gamma\left(\frac{3}{2}+2i\right)}{\int_0^\infty t^{1/2+2i}\operatorname{sech}^2(t)\,dt}}$$

- $$\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}} = \sqrt{\frac{0.742947\times 4^{-i}\left(-1.41421+4^{i}\right)\sqrt{1+4i}\,\Gamma\left(\frac{1}{2}+2i\right)}{\int_0^\infty \frac{t^{-1/2+2i}}{1+e^t}\,dt}}$$

And:

1/ sqrt[1/((((((((1.10394(1/2+2i)))^(-1/2)))) * zeta (1/2+2i))))]

Input interpretation:

$$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}}$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Result:

0.479975... –
0.356837... i

Polar coordinates:

$r = 0.598087$ (radius), $\theta = -36.6289°$ (angle)

0.598087

Alternative representations:

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{\frac{1}{\zeta\left(\frac{1}{2}+2i,1\right)}}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}}}}$$

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{\frac{1}{\zeta\left(\frac{1}{2}+2i,1\right)}}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}}}}$$

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{\frac{1}{\zeta\left(\frac{1}{2}+2i,\frac{1}{2}\right)}}{\sqrt{1.10394\left(\frac{1}{2}+2i\right)}\left(-1+2^{1/2+2i}\right)}}}$$

$\zeta(s, a)$ is the Hurwitz zeta function

$\zeta(s, a)$ is the generalized Riemann zeta function

Series representations:

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{-1+\frac{0.742947\sqrt{1+4i}}{\zeta\left(\frac{1}{2}+2i\right)}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+\frac{0.742947\sqrt{1+4i}}{\zeta\left(\frac{1}{2}+2i\right)}\right)^{-k}}}$$

-

$$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{0.742947\sqrt{1+4i}}{\gamma+\frac{2}{-1+4i}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+2i\right)^k\gamma_k}{k!}}}}$$

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{0.742947\sqrt{1+4i}}{\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}+2i-s_0\right)^k\zeta^{(k)}(s_0)}{k!}}}} \quad \text{for } s_0 \neq 1$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

γ_n is the n^{th} Stieltjes constant

γ is the Euler-Mascheroni constant

Integral representations:

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{-\frac{0.371473\sqrt{1+4i}}{0.25+i\int_0^{\infty}t^{-1/2+2i}\operatorname{frac}\left(\frac{1}{t}\right)dt}}}$$

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{1.05069\times 4^{-2i}\left(-1.41421+4^{i}\right)\sqrt{1+4i}\,\Gamma\left(\frac{3}{2}+2i\right)}{\int_0^{\infty}t^{1/2+2i}\operatorname{sech}^2(t)\,dt}}}$$

- $$\frac{1}{\sqrt{\frac{1}{\left(1.10394\left(\frac{1}{2}+2i\right)\right)^{-1/2}\zeta\left(\frac{1}{2}+2i\right)}}} = \frac{1}{\sqrt{\frac{0.742947\times 4^{-i}\left(-1.41421+4^{i}\right)\sqrt{1+4i}\,\Gamma\left(\frac{1}{2}+2i\right)}{\int_0^{\infty}\frac{t^{-1/2+2i}}{1+e^t}\,dt}}}$$

From:

$$4\int_0^\infty \frac{t\,dt}{e^{\sqrt{5}t}\cosh t} = \cfrac{1}{1+\cfrac{1^2}{1+\cfrac{1^2}{1+\cfrac{2^2}{1+\cfrac{2^2}{1+\cfrac{3^2}{1+\cfrac{3^2}{1+\dots}}}}}}} \approx 0.5683000031$$

$$2\int_0^\infty \frac{t^2\,dt}{e^{\sqrt{3}t}\sinh t} = \cfrac{1}{1+\cfrac{1^3}{1+\cfrac{1^3}{3+\cfrac{2^3}{1+\cfrac{2^3}{5+\cfrac{3^3}{1+\cfrac{3^3}{7+\dots}}}}}}} \approx 0.5269391135$$

$$\sqrt{\frac{e\pi}{2}}\,erfc\left(\frac{\sqrt{2}}{2}\right) \approx 0.6556795424$$

We obtain:

(0,5269391135 + 0,5683000031 + 0,6556795424)/3 = 0,583639553

We note that the previous result 0.598087, is very near to 0.583639553

Cosmological constant

From:

Haramein, N. And Val Baker, A. (2019) **Resolving the Vacuum Catastrophe: A Generalized Holographic Approach.** Journal of High Energy Physics, Gravitation and Cosmology, 5, 412-424. https://doi.org/10.4236/jhepgc.2019.52023

Now we analyze the values of N. Haramein:

Using the current value of $H_o = 67.4 \pm 0.5 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ for Hubble's constant [14], gives the critical density at the present time as, $\rho_{crit} = 8.53 \times 10^{-30} \text{ g/cm}^3$ and thus $\rho_b = 0.049 \rho_{crit} = 4.18 \times 10^{-31} \text{ g/cm}^3$, $\rho_d = 0.268 \rho_{crit} = 2.29 \times 10^{-30} \text{ g/cm}^3$ and $\rho_\Lambda = 0.683 \rho_{crit} = 5.83 \times 10^{-30} \text{ g/cm}^3$. The vacuum energy density at the cosmological scale is thus of the order 10^{-30} g/cm^3.

However, quantum field theory determines the vacuum energy density by summing the energies $\hbar\omega/2$ over all oscillatory modes. See reference [1] for a more detailed overview. As quantum fluctuations predict infinite oscillatory modes [15] [16] this yields an infinite result unless renormalized at the Planck cutoff. In utilizing such a cutoff value, the vacuum energy density is found to be:

$$\rho_{vac} = \frac{c^5}{\hbar G^2} = \frac{m_\ell}{\ell^3} = 5.16 \times 10^{93} \text{ g/cm}^3 \tag{8}$$

$8{,}53*10^{-30} - 5{,}83*10^{-30} = 2.7*10^{-30}$; $4.18*10^{-31} - 2.29*10^{-30} = -1{,}872*10^{-30}$;

$2{,}7*10^{-30} - 1{,}872*10^{-30} = 0.828*10^{-30}$

From:

$$\rho(s, \chi) = \varepsilon(\bar{\chi}) \left(\frac{k}{\pi}\right)^{1/2-s} \frac{\Gamma\left(\frac{1-s}{2} + \frac{a}{2}\right)}{\Gamma\left(\frac{s}{2} + \frac{a}{2}\right)};$$

$= 1.10394$

Now:

$1.10394 - 0.828 = 0{,}27594$; and $\sqrt{0.27594} = 0{,}525299914334...$

This result is very near to the value of the following Rogers-Ramanujan continued fraction!

$$2\int_0^\infty \frac{t^2\,dt}{e^{\sqrt{3}t}\sinh t} = \cfrac{1}{1+\cfrac{1^3}{1+\cfrac{1^3}{3+\cfrac{2^3}{1+\cfrac{2^3}{5+\cfrac{3^3}{1+\cfrac{3^3}{7+\ldots}}}}}}} \approx 0.5269391135$$

$$= 0.5269391135$$

Note

Another fundamental Ramanujan formula, linked to the Rogers-Ramanujan continued fractions, is:

$$R(e^{-2\pi\sqrt{5}}) = \frac{\sqrt{5}}{1+\sqrt[5]{5^{3/4}\left(\frac{\sqrt{5}-1}{2}\right)^{5/2}-1}} - \frac{\sqrt{5}+1}{2},$$

We obtain:

*(((sqrt(5)))) / (((1+(5^0.75 * (1/golden ratio)^2.5 - 1)^1/5))) - golden ratio*

Input:

$$\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}} - \phi$$

ϕ is the golden ratio

Result:

0.060209406562451820435729675819574923372619370646311866424...

0.060209406562451 82...

Series representations:

$$\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi = -\phi+\frac{\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}$$

- $$\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi = -\phi+\frac{\sqrt{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}$$

- $$\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi = -\phi+\frac{\sqrt{z_0}\sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}$$

  for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

*1+(((sqrt(5)))) / (((1+(5^0.75 * (1/golden ratio)^2.5 - 1)^1/5))) - golden ratio*

Input:

$$1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi$$

ϕ is the golden ratio

Result:

1.060209406562451820435729675819574923372619370646311866424...

1.06020940656245…

Series representations:

$$1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi=1-\phi+\frac{\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}$$

$$1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi=1-\phi+\frac{\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}$$

- $$1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi=1-\phi+\frac{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}$$

  for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We have also that:

*1+1/((((1+(((sqrt(5)))) / (((1+(5^0.75 * (1/golden ratio)^2.5 - 1)^1/5))) - golden ratio))))^7*

Input:

$$1+\frac{1}{\left(1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi\right)^7}$$

ϕ is the golden ratio

Result:

1.664138150067678648861289799236134011783090897871287901274...

1.664138150067… is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ *= 1164,2696 i.e. 1,65578...*

Series representations:

$$1+\frac{1}{\left(1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi\right)^7}=1+\frac{1}{\left(1-\phi+\frac{\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}\right)^7}$$

- $$1+\frac{1}{\left(1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi\right)^7}=1+\frac{1}{\left(1-\phi+\frac{\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}\right)^7}$$

- $$1+\frac{1}{\left(1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi\right)^7}=1+\frac{1}{\left(1-\phi+\frac{\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\left(1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}\right)\sqrt{\pi}}\right)^7}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\text{Res}_{z=z_0} f$ is a complex residue

And:

*7/10^3+1+1/((((1+(((sqrt(5)))) / (((1+(5^0.75 * (1/golden ratio)^2.5 - 1)^1/5))) - golden ratio))))^7*

Where 7 is a Lucas number

Input:

$$\frac{7}{10^3}+1+\frac{1}{\left(1+\frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}}-\phi\right)^7}$$

ϕ is the golden ratio

Result:

1.671138150067678648861289799236134011783090897871287901274...

1.671138150067….

We note that 1.671138150067... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-2}$ *gm*

that is the holographic proton mass

Series representations:

$$\frac{7}{10^3} + 1 + \frac{1}{\left(1 + \frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}} - \phi\right)^7} = \frac{1007}{1000} + \frac{1}{\left(1 - \phi + \frac{\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}\right)^7}$$

- $$\frac{7}{10^3} + 1 + \frac{1}{\left(1 + \frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}} - \phi\right)^7} = \frac{1007}{1000} + \frac{1}{\left(1 - \phi + \frac{\sqrt{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}}\right)^7}$$

- $$\frac{7}{10^3} + 1 + \frac{1}{\left(1 + \frac{\sqrt{5}}{1+\sqrt[5]{5^{0.75}\left(\frac{1}{\phi}\right)^{2.5}-1}} - \phi\right)^7} = \frac{1007}{1000} + \frac{1}{\left(1 - \phi + \frac{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\left(1+\sqrt[5]{-1+3.3437\left(\frac{1}{\phi}\right)^{2.5}}\right)\sqrt{\pi}}\right)^7}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

We have that:

$$\sum_{a_1 < x \leqslant b_1} (kx + l)^{-0,5-it} = \int_{a_1}^{b_1} (kx + l)^{-0,5-it}\, dx + O\left(a^{-0,5}\right) =$$
$$= \frac{1}{0,5 - it}\left(b^{0,5-it} - a^{0,5-it}\right) + O\left(a^{-0,5}\right);$$

For T = 3, t =2 and a = √2, and

$k \geqslant 3$ b = 2

k = 5

$s = 0,5 + it$

1/(0.5-2i)*(((((2^(0.5-2i)-(sqrt(2))^(0.5-2i))))))+((((((sqrt(2)^(-0.5)))))))

Input:

$$\frac{1}{0.5 - 2i}\left(2^{0.5-2i} - \sqrt{2}^{0.5-2i}\right) + \frac{1}{\sqrt{2}^{0.5}}$$

i is the imaginary unit

Result:

1.06044... −
0.382553... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.12733$ (radius), $\theta = -19.837°$ (angle)

1.12733

1/2*[1/(0.5-2i)*(((((2^(0.5-2i)-(sqrt(2))^(0.5-2i))))))+((((((sqrt(2)^(-0.5)))))))]

Input:

$$\frac{1}{2}\left(\frac{1}{0.5-2i}\left(2^{0.5-2i}-\sqrt{2}^{0.5-2i}\right)+\frac{1}{\sqrt{2}^{0.5}}\right)$$

i is the imaginary unit

Result:

0.530218... −
0.191277... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 0.563664$ (radius), $\theta = -19.837°$ (angle)

0.563664

This result is very near to the value of the following Rogers-Ramanujan continued fraction:

$$4\int_0^\infty \frac{t\,dt}{e^{\sqrt{5}t}\cosh t} = \cfrac{1}{1+\cfrac{1^2}{1+\cfrac{1^2}{1+\cfrac{2^2}{1+\cfrac{2^2}{1+\cfrac{3^2}{1+\cfrac{3^2}{1+\dots}}}}}}} \approx 0.5683000031$$

$= 0.5683000031$

[1/(0.5-2i)*(((((2^(0.5-2i)-(sqrt(2))^(0.5-2i))))))+((((((sqrt(2)^(-0.5)))))))]^4

Input:

$$\left(\frac{1}{0.5-2i}\left(2^{0.5-2i}-\sqrt{2}^{0.5-2i}\right)+\frac{1}{\sqrt{2}^{0.5}}\right)^4$$

i is the imaginary unit

Result:

0.298546... −
1.58728... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.61511$ (radius), $\theta = -79.3479°$ (angle)

1.61511

-(47/10^3+7/10^3+3/10^3)i+[1/(0.5-2i)*(((((2^(0.5-2i)-(sqrt(2))^(0.5-2i))))))+((((((sqrt(2)^(-0.5))))))))]^4

Input:

$$-\left(\frac{47}{10^3}+\frac{7}{10^3}+\frac{3}{10^3}\right)i+\left(\frac{1}{0.5-2i}\left(2^{0.5-2i}-\sqrt{2}^{0.5-2i}\right)+\frac{1}{\sqrt{2}^{0.5}}\right)^4$$

i is the imaginary unit

Result:

0.298546... −
1.64428... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.67116$ (radius), $\theta = -79.7091°$ (angle)

1.67116

We note that 1.67116... is a result practically equal to the value of the formula:

$$m_{p\prime} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

$$\beta_1 = f^{(1)}(b_1) = -\frac{tk}{2\pi b};$$

-10/(4Pi) = -0.795774715 = β

b = 2; t = 2; k = 5

$\log Y \geqslant 4.$

ln(54.59815) = 4; Y = 55

$T \geqslant 10$

$$\int_0^{1/4} \frac{(\alpha+t)^\beta}{\theta_1^2+t^2}\,dt \leqslant \int_0^\alpha \frac{2\alpha^\beta dt}{\alpha^2} + \int_\alpha^{1/4} \frac{2t^\beta}{t^2}\,dt = O\left(\alpha^{-1+\beta}\right) = O\left((\log Y)^{1-\beta}\right),$$

(((ln(55))^(1+0.795774715)))

Input interpretation:

$\log^{1+0.795774715}(55)$

log(x) is the natural logarithm

Result:

12.0946298...

12.0946298…. result very near to the black hole entropy 12.1904

(18+2)/(((ln(55))^(1+0.795774715)))

Where 18 and 2 are Lucas numbers

Input interpretation:

$\frac{18+2}{\log^{1+0.795774715}(55)}$

log(x) is the natural logarithm

Result:

1.65362647...

1.65362647…. is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$ = 1164,2696 i.e. 1,65578...

Alternative representations:

$$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{20}{\log_e^{1.79577}(55)}$$

- $$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{20}{(\log(a)\log_a(55))^{1.79577}}$$

- $$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{20}{(-\text{Li}_1(-54))^{1.79577}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{20}{\left(\log(54) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{54}\right)^k}{k}\right)^{1.79577}}$$

- $$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{20}{\left(2 i \pi \left\lfloor \frac{\arg(55-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (55-x)^k x^{-k}}{k}\right)^{1.79577}} \quad \text{for } x < 0$$

- $$\frac{18+2}{\log^{1+0.795775}(55)} =$$
$$\frac{20}{\left(\log(z_0) + \left\lfloor \frac{\arg(55-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (55-z_0)^k z_0^{-k}}{k}\right)^{1.79577}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{20}{\left(\int_1^{55} \frac{1}{t}\, dt\right)^{1.79577}}$$

- $$\frac{18+2}{\log^{1+0.795775}(55)} = \frac{69.4404}{\left(\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{54^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^{1.79577}} \quad \text{for } -1 < \gamma < 0$$

18/10^3+20/(((ln(55))^(1+0.795774715)))

Where 18 is a Lucas number

Input interpretation:

$$\frac{18}{10^3} + \frac{20}{\log^{1+0.795774715}(55)}$$

$\log(x)$ is the natural logarithm

Result:

1.67162647...

1.67162647…

We note that 1.67162647... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

Alternative representations:

$$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{18}{10^3} + \frac{20}{\log_e^{1.79577}(55)}$$

- $$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{18}{10^3} + \frac{20}{(\log(a)\log_a(55))^{1.79577}}$$

- $$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{18}{10^3} + \frac{20}{(-\text{Li}_1(-54))^{1.79577}}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{9}{500} + \frac{20}{\left(\log(54) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{54}\right)^k}{k}\right)^{1.79577}}$$

- $$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{9}{500} + \frac{20}{\left(2 i \pi \left\lfloor \frac{\arg(55-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (55-x)^k x^{-k}}{k}\right)^{1.79577}} \quad \text{for } x < 0$$

- $$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{9}{500} + \frac{20}{\left(\log(z_0) + \left\lfloor \frac{\arg(55-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (55-z_0)^k z_0^{-k}}{k}\right)^{1.79577}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{9}{500} + \frac{20}{\left(\int_1^{55} \frac{1}{t}\, dt\right)^{1.79577}}$$

- $$\frac{18}{10^3} + \frac{20}{\log^{1+0.795775}(55)} = \frac{9}{500} + \frac{69.4404}{\left(\frac{1}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{54^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{1.79577}} \quad \text{for } -1 < \gamma < 0$$

From the following data:

$$T \leqslant t \leqslant T + H,\ H \leqslant \sqrt[3]{T},\ 1 \leqslant X \leqslant T^{0,01}$$

-10/(4Pi) = -0.795774715 = β

b = 2; t = 2; k = 5

$$\log Y \geqslant 4.$$

ln(54.59815) = 4; Y = 55

$$T \geqslant 10$$

$$1 \leqslant X \leqslant T^{0,01}$$

$T^{0.01}$ = 1,0232929922;   X = 1.0061571663

ε_2 = 0.008

$H = T^{27/82+\varepsilon_1}$ = 10^27/82+0.01 = 10^0,339268292 = 2.18407874

$$A = c_1 r\,(\log T) \times (\log X)^{-2\beta},\ \beta\varphi\,(K) = 1,\ c > 1,\ c_1 > 1$$

$r = [c \log\log T]$ c = 1.2 c_1 = 1.3   r = 1.00083893

$$h = A\,(\log T)^{-1} \qquad h_1 = hr^{-1}$$

1.2 * 1.00083893 (ln10) * (ln10)^(-2*-0.795774715) = A = 10.429063158

h = 10.429063158 (ln10)^-1 = 4.5292845809

h_1 = 4.5292845809*(1.00083893^-1) = 4.52548801324

$$I_{22} \ll h_1^{2r} H \left(\varepsilon_2 (\log T)(\log X)^{-2\beta} + T^{-\varepsilon_1}\right).$$

$$I_{22} \ll h_1^{2r} \int_T^{T+H+1} |F_2(t;\chi)|^2\, dt \ll h_1^{2r} \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H+1}\right)^2\right) \times$$

$$\times \left| \sum_{P^{1-\varepsilon_2}<\lambda\leqslant P} \frac{a(\lambda)}{\sqrt{\lambda}} \lambda^{-i(T+t)} \right|^2 dt \ll h_1^{2r} H \left(\sum_{P^{1-\varepsilon_2}<\lambda\leqslant P} \frac{|a(\lambda)|^2}{\lambda} + |W_3| \right),$$

$$I_{22} \ll h_1^{2r} H \left(\varepsilon_2 (\log T)(\log X)^{-2\beta} + T^{-\varepsilon_1}\right).$$

4.52548801324^2.00167786 * 2.18407874 * (((0.008 ln10*(ln1.0061571663)^(-2*-0.795774715)+10^(-0.01)))

Input interpretation:

$$4.52548801324^{2.00167786} \times 2.18407874 \left(0.008 \log(10) \log^{-2\times(-0.795774715)}(1.0061571663) + \frac{1}{10^{0.01}}\right)$$

log(x) is the natural logarithm

Result:

43.82296054118301490643493489826921207542917995895681642390...

43.8229605…

43.822960541183014906 4349…

Alternative representations:

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = 2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} + 0.008 \log_e(10) \log_e^{1.59155}(1.00615716630000)\right)$$

•

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2 (-1) 0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = 2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} + 0.008 \log(a) \log_a(10) (\log(a) \log_a(1.00615716630000))^{1.59155}\right)$$

- $$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2 (-1) 0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = 2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} - 0.008 \operatorname{Li}_1(-9) (-\operatorname{Li}_1(-0.00615716630000))^{1.59155}\right)$$

$\log_b(x)$ is the base- b logarithm

$\operatorname{Li}_n(x)$ is the polylogarithm function

Series representations:

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2 (-1) 0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = 0.717496 \left(61.0773 + i \pi \left\lfloor \frac{\arg(10-x)}{2\pi} \right\rfloor \left(2 i \pi \left\lfloor \frac{\arg(1.00615716630000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - x)^k x^{-k}}{k}\right)^{1.59155} + 0.5 \log(x) \left(2 i \pi \left\lfloor \frac{\arg(1.00615716630000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - x)^k x^{-k}}{k}\right)^{1.59155} - 0.5 \left(2 i \pi \left\lfloor \frac{\arg(1.00615716630000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - x)^k x^{-k}}{k}\right)^{1.59155} \sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k}\right) \text{ for } x < 0$$

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) =$$

$$0.717496 \left(61.0773 + i\pi \left\lfloor -\frac{-\pi + \arg\left(\frac{10}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor \left(2 i\pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1.00615716630000}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} + 0.5 \log(z_0) \left(2 i\pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1.00615716630000}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} - 0.5 \left(2 i\pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1.00615716630000}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k}\right)$$

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = 0.358748 \left(122.155 + \left\lfloor \frac{\arg(10 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) \left(\log(z_0) + \left\lfloor \frac{\arg(1.00615716630000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} + \log(z_0) \left(\log(z_0) + \left\lfloor \frac{\arg(1.00615716630000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} + \left\lfloor \frac{\arg(10 - z_0)}{2\pi} \right\rfloor \log(z_0) \left(\log(z_0) + \left\lfloor \frac{\arg(1.00615716630000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} - \left(\log(z_0) + \left\lfloor \frac{\arg(1.00615716630000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.00615716630000 - z_0)^k z_0^{-k}}{k}\right)^{1.59155} \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k}\right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representations:

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = 0.358748 \left(122.155 + \left(\int_1^{1.00615716630000} \frac{1}{t}\, dt\right)^{1.59155} \int_1^{10} \frac{1}{t}\, dt\right)$$

$$4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right) = \frac{1}{i\pi} 0.0595189 \left(736.282\, i\pi + \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds\right) \left(\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{5.090138623510\, s}\, \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^{1.59155}\right) \text{ for } -1 < \gamma < 0$$

-(11/10^3+3/10^3)+((((((47/ (((((4.52548801324^2.00167786 * 2.18407874 * (((0.008 ln10*(ln1.0061571663)^(-2*-0.795774715)+10^(-0.01))))))))))))))))^7

Where 3, 7, 11 and 47 are Lucas numbers

Input interpretation:

$$-\left(\frac{11}{10^3} + \frac{3}{10^3}\right) + \left(47 \Big/ \left(4.52548801324^{2.00167786} \times 2.18407874 \left(0.008 \log(10) \log^{-2\times(-0.795774715)}(1.0061571663) + \frac{1}{10^{0.01}}\right)\right)\right)^7$$

log(x) is the natural logarithm

Result:

1.618198630290829139047945511164247076891565226497548443472...

1.6181986302…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternative representations:

$$-\left(\frac{11}{10^3} + \frac{3}{10^3}\right) + \left(47 \Big/ \left(4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right)\right)\right)^7 = -\frac{14}{10^3} + \left(47 \Big/ \left(2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} + 0.008 \log_e(10) \log_e^{1.59155}(1.00615716630000)\right)\right)\right)^7$$

•

$$-\left(\frac{11}{10^3}+\frac{3}{10^3}\right)+\left(47\left/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7=$$
$$-\frac{14}{10^3}+\left(47\left/\left(2.18408\times 4.525488013240000^{2.00168}\left(\frac{1}{10^{0.01}}+0.008\log(a)\log_a(10)\left(\log(a)\log_a(1.00615716630000)\right)^{1.59155}\right)\right)\right)^7$$

•

$$-\left(\frac{11}{10^3}+\frac{3}{10^3}\right)+\left(47\left/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7=$$
$$-\frac{14}{10^3}+\left(47\left/\left(2.18408\times 4.525488013240000^{2.00168}\left(\frac{1}{10^{0.01}}-0.008\,\text{Li}_1(-9)\left(-\text{Li}_1(-0.00615716630000)\right)^{1.59155}\right)\right)\right)^7$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$-\left(\frac{11}{10^3}+\frac{3}{10^3}\right)+\left(47\left/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7=$$
$$-\frac{7}{500}+1.38928\left/\left(0.977237+0.008\left(2 i\pi\left\lfloor\frac{\arg(1.00615716630000-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-x)^k x^{-k}}{k}\right)^{1.59155}\left(2 i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)\right)^7\right.\quad \text{for } x<0$$

•

$$-\left(\frac{11}{10^3}+\frac{3}{10^3}\right)+\left(47\Big/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7 = -\frac{7}{500}+1.38928\Big/\left(0.977237+0.008\left(\log(z_0)+\left\lfloor\frac{\arg(1.00615716630000-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-z_0)^k z_0^{-k}}{k}\right)^{1.59155}\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)\right)^7$$

$$-\left(\frac{11}{10^3}+\frac{3}{10^3}\right)+\left(47\Big/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7 = -\frac{7}{500}+1.38928\Big/\left(0.977237+0.008\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1.00615716630000}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-z_0)^k z_0^{-k}}{k}\right)^{1.59155}\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{10}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)\right)^7$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

(29/10^3+7/10^3+3/10^3)+((((((47/ (((((4.52548801324^2.00167786 * 2.18407874 * (((0.008 ln10*(ln1.0061571663)^(-2*-0.795774715)+10^(-0.01))))))))))))))))^7

Input interpretation:

$$\left(\frac{29}{10^3}+\frac{7}{10^3}+\frac{3}{10^3}\right)+\left(47\Big/\left(4.52548801324^{2.00167786}\times 2.18407874\left(0.008\log(10)\log^{-2\times(-0.795774715)}(1.0061571663)+\frac{1}{10^{0.01}}\right)\right)\right)^7$$

$\log(x)$ is the natural logarithm

Result:

1.6711986302908291390479455111642470768915652264975484434 72...

1.6711986302…

We note that 1.6711986302... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

Alternative representations:

$$\left(\frac{29}{10^3} + \frac{7}{10^3} + \frac{3}{10^3}\right) + \left(47 \Big/ \left(4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right)\right)\right)^7 = \frac{39}{10^3} + \left(47 \Big/ \left(2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} + 0.008 \log_e(10) \log_e^{1.59155}(1.00615716630000)\right)\right)\right)^7$$

-

$$\left(\frac{29}{10^3} + \frac{7}{10^3} + \frac{3}{10^3}\right) + \left(47 \Big/ \left(4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right)\right)\right)^7 = \frac{39}{10^3} + \left(47 \Big/ \left(2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} + 0.008 \log(a) \log_a(10) (\log(a) \log_a(1.00615716630000))^{1.59155}\right)\right)\right)^7$$

-

$$\left(\frac{29}{10^3} + \frac{7}{10^3} + \frac{3}{10^3}\right) + \left(47 \Big/ \left(4.525488013240000^{2.00168} \times 2.18408 \left(0.008 \log(10) \log^{-2(-1)0.795775}(1.00615716630000) + \frac{1}{10^{0.01}}\right)\right)\right)^7 = \frac{39}{10^3} + \left(47 \Big/ \left(2.18408 \times 4.525488013240000^{2.00168} \left(\frac{1}{10^{0.01}} - 0.008 \operatorname{Li}_1(-9) (-\operatorname{Li}_1(-0.00615716630000))^{1.59155}\right)\right)\right)^7$$

$\log_b(x)$ is the base- b logarithm

$\operatorname{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\left(\frac{29}{10^3}+\frac{7}{10^3}+\frac{3}{10^3}\right)+\left(47\Big/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7 = \frac{39}{1000}+1.38928\Big/\left(0.977237+0.008\left(2i\pi\left\lfloor\frac{\arg(1.00615716630000-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-x)^k x^{-k}}{k}\right)^{1.59155}\left(2i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)\right)^7 \quad \text{for } x<0$$

-

$$\left(\frac{29}{10^3}+\frac{7}{10^3}+\frac{3}{10^3}\right)+\left(47\Big/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7 = \frac{39}{1000}+1.38928\Big/\left(0.977237+0.008\left(\log(z_0)+\left\lfloor\frac{\arg(1.00615716630000-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-z_0)^k z_0^{-k}}{k}\right)^{1.59155}\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)\right)^7$$

$$\left(\frac{29}{10^3}+\frac{7}{10^3}+\frac{3}{10^3}\right)+\left(47\Big/\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\right)^7 = \frac{39}{1000}+1.38928\Big/\left(0.977237+0.008\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1.00615716630000}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-z_0)^k z_0^{-k}}{k}\right)^{1.59155}\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{10}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)\right)^7$$

arg(z) is the complex argument

$\lfloor x\rfloor$ is the floor function *i* is the imaginary unit

We have also that:

(((((4.52548801324^2.00167786 * 2.18407874 * (((0.008 ln10*(ln1.0061571663)^(-2*-0.795774715)+10^(-0.01)))))))))^1/8

Input interpretation:

$$\left(4.52548801324^{2.00167786}\times 2.18407874\left(0.008\log(10)\log^{-2\times(-0.795774715)}(1.0061571663)+\frac{1}{10^{0.01}}\right)\right)\hat{}\,(1/8)$$

log(*x*) is the natural logarithm

Result:

1.604030840754374597643212058472401424296892908189773818280...

1.60403084…. result very near to the elementary charge

(47/10^3+18/10^3+2/10^3)+(((((4.52548801324^2.00167786 * 2.18407874 * (((0.008 ln10*(ln1.0061571663)^(-2*-0.795774715)+10^(-0.01)))))))))^1/8

Where 2, 18 and 47 are Lucas numbers

Input interpretation:

$$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.52548801324^{2.00167786}\times 2.18407874\left(0.008\log(10)\log^{-2\times(-0.795774715)}(1.0061571663)+\frac{1}{10^{0.01}}\right)\right)\hat{}\,(1/8)$$

log(*x*) is the natural logarithm

Result:

1.671030840754374597643212058472401424296892908189773818280...

1.67103084075…

We note that 1.67103084... is a result practically equal to the value of the formula:

$m_{p'} = 2\times\frac{\eta}{R}m_P = 1.6714213\times 10^{-24}$ gm

that is the holographic proton mass

Alternative representations:

$$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\wedge(1/8)=\frac{67}{10^3}+\left(2.18408\times 4.525488013240000^{2.00168}\left(\frac{1}{10^{0.01}}+0.008\log_e(10)\log_e^{1.59155}(1.00615716630000)\right)\right)\wedge(1/8)$$

- $$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\wedge(1/8)=\frac{67}{10^3}+\left(2.18408\times 4.525488013240000^{2.00168}\left(\frac{1}{10^{0.01}}+0.008\log(a)\log_a(10)(\log(a)\log_a(1.00615716630000))^{1.59155}\right)\right)\wedge(1/8)$$

- $$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\wedge(1/8)=\frac{67}{10^3}+\left(2.18408\times 4.525488013240000^{2.00168}\left(\frac{1}{10^{0.01}}-0.008\,\text{Li}_1(-9)(-\text{Li}_1(-0.00615716630000))^{1.59155}\right)\right)\wedge(1/8)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

-

Series representations:

$$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\hat{}\,(1/8)=$$

$$1.60865\left(0.0416497+\left(0.977237+0.008\left(2i\pi\left\lfloor\frac{\arg(1.00615716630000-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-x)^k x^{-k}}{k}\right)^{1.59155}\left(2i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)\right)\hat{}\,(1/8)\right)\quad\text{for } x<0$$

-

$$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\hat{}\,(1/8)=$$

$$1.60865\left(0.0416497+\left(0.977237+0.008\left(\log(z_0)+\left\lfloor\frac{\arg(1.00615716630000-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(1.00615716630000-z_0)^k z_0^{-k}}{k}\right)^{1.59155}\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)\right)\hat{}\,(1/8)\right)$$

$$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\text{^}(1/8)=$$
$$1.60865\left(0.0416497+\left(0.977237+0.008\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1.00615716630000}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(1.00615716630000-z_0)^k\,z_0^{-k}}{k}\right)^{1.59155}\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{10}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)\right)\text{^}(1/8)\right)$$

arg(z) is the complex argument

⌊x⌋ is the floor function

i is the imaginary unit

Integral representations:

$$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\text{^}(1/8)=$$
$$1.60865\left(0.0416497+\sqrt[8]{0.977237+0.008\left(\int_1^{1.00615716630000}\frac{1}{t}\,dt\right)^{1.59155}\int_1^{10}\frac{1}{t}\,dt}\right)$$

- $$\left(\frac{47}{10^3}+\frac{18}{10^3}+\frac{2}{10^3}\right)+\left(4.525488013240000^{2.00168}\times 2.18408\left(0.008\log(10)\log^{-2(-1)0.795775}(1.00615716630000)+\frac{1}{10^{0.01}}\right)\right)\text{^}(1/8)=$$
$$1.60865\left(0.0416497+\left(0.977237+\frac{1}{i\,\pi}\,0.00132726\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\left(\frac{1}{i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{5.090138623510\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{1.59155}\right)\text{^}(1/8)\right)\quad\text{for } -1<\gamma<0$$

Γ(x) is the gamma function

Now, we have that:

$$I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll$$

$$\ll H\left(\sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P\lambda^{-1})^{2r}} + |W_2| \right) \ll$$

$$\ll H\left(\left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} \sum_{\lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right),$$

$$I_{21} \ll H\left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T)(\log X)^{-2\beta} + \right.$$

$$\left. + \left(\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\varepsilon_1} \right\}.$$

Data

Let a be an arbitrary fixed number with $0 < a < 1$

-10/(4Pi) = -0.795774715 = β

b = 2; t = 2; k = 5

$\log Y \geqslant 4.$

ln(54.59815) = 4; Y = 55

$T \geqslant 10$

$1 \leqslant X \leqslant T^{0,01}$

$T^{0.01}$ = 1,0232929922; X = 1.0061571663; ε_1 = 0.01; ε_2 = 0.008

$H = T^{27/82+\varepsilon_1}$ = 10^27/82+0.01 = 10^0,339268292 = 2.18407874

$A = c_1 r (\log T) \times (\log X)^{-2\beta}, \ \beta\varphi(K) = 1, \ c > 1, \ c_1 > 1$

$r = [c \log \log T]$ c, c_1 = 1.2 r = 1.00083893

$h = A (\log T)^{-1} \quad h_1 = hr^{-1}$

1.2 * 1.00083893 (ln10) * (ln10)^(-2*-0.795774715) = A = 10.429063158

h = 10.429063158 (ln10)^-1 = 4.5292845809

h_1 = 4.5292845809*(1.00083893^-1) = 4.52548801324

Thence:

2.18407874*(((((4/(0.008 ln(10)))^(2.00167786) * ln(10) ln(1.0061571663)^ -(1.59154943) + (((0.008^-(2.00167786) ln(10)^-(2.00167786) + 0.008^-(1.00083893)* 4.52548801324^(1.00083893)*ln(10)^-(1.00083893)))*10^-(0.01)))))

Input interpretation:

$$\frac{\left(\frac{4}{0.008\log(10)}\right)^{2.00167786}\log(10)}{\log^{1.5915493}(1.0061571663)}$$

log(x) is the natural logarithm

Result:

$3.63142\ldots\times10^{8}$

Partial result

2.18407874*(3.63142*10^8)+(((0.008^-(2.00167786) ln(10)^-(2.00167786) + 0.008^-(1.00083893)* 4.52548801324^(1.00083893)*ln(10)^-(1.00083893)))*10^-(0.01))

Input interpretation:

$$2.18407874\times3.63142\times10^{8}+\left(\frac{1}{0.008^{2.00167786}\log^{2.00167786}(10)}+\frac{4.52548801324^{1.00083893}}{0.008^{1.00083893}\log^{1.00083893}(10)}\right)\times10^{-0.01}$$

log(x) is the natural logarithm

Result:

$7.93133862334800504570304142069014510208874797926497200\ldots\times10^{8}$

793133862.3348005 * 10^8 Total result

Input interpretation:

$7.931397650527512071340741120268742044987164054 73\times10^{8}$

Decimal form:

793139765.05275120713407411202687420449871640547 3

793139765.05275

We have that:

(793139765.05275)^1/43

Input interpretation:

$$\sqrt[43]{7.9313976505275\times 10^{8}}$$

Result:

1.610503309080049...

1.6105033.... result practically equal to the value of the following Ramanujan mock theta function:

$$\left(1+\frac{0.449329}{1-0.449329^2}+\frac{0.449329^4}{(1-0.449329^3)(1-0.449329^4)}\right)+\frac{0.449329^9}{(1-0.449329^4)(1-0.449329^5)(1-0.449329^6)}$$ = **1.61052934557...**

Now from the sum of ln of the I_1 and I_2 results, we obtain:

ln(7.9313386233480050457030414206901451020887479792 6 × 10^8) + ln(43.822960541183014906434 9)

Input interpretation:

$$\log(7.93133862334800504570304142069014510208874797926\times 10^{8})+\log(43.8229605411830149064349)$$

log(x) is the natural logarithm

Result:

24.27166046361472741203147...

24.271660.... result practically equal to the black hole entropy 24.2477 ln(7.93133862334800504570304142069014510208874797926 × 10^8) - ln(43.8229605411830149064349)

Input interpretation:

$\log(7.93133862334800504570304142069014510208874797926 \times 10^8) - \log(43.8229605411830149064349)$

log(x) is the natural logarithm

Result:

16.7113446770154427525458 1...

16.71134467.... result very near to the black hole entropy 16.8741

From:

1/10 (((ln(7.93133862 × 10^8) - ln(43.82296054))))

Where 10 is the number of dimensions in superstring theories

Input interpretation:

$\frac{1}{10}(\log(7.93133862 \times 10^8) - \log(43.82296054))$

log(x) is the natural logarithm

Result:

1.671134468...

1.671134468...

We note that 1.671134468... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

ln (((7.93133862 × 10^8+43.82296054)))

Input interpretation:

$\log(7.93133862 \times 10^8 + 43.82296054)$

log(x) is the natural logarithm

Result:

20.49150263...

20.49150263.... result very near to the black hole entropy 20.5520

(((ln (((7.93133862 × 10^8+43.82296054))))))^1/6

Input interpretation:

$$\sqrt[6]{\log(7.93133862\times 10^8 + 43.82296054)}$$

log(x) is the natural logarithm

Result:

1.6542290162...

1.6542290162.... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$ = 1164,2696 i.e. 1,65578...

18/10^3+(((ln (((7.93133862 × 10^8+43.82296054))))))^1/6

Input interpretation:

$$\frac{18}{10^3} + \sqrt[6]{\log(7.93133862\times 10^8 + 43.82296054)}$$

log(x) is the natural logarithm

Result:

1.6722290162...

1.6722290162.... result very near to the proton mass

-(34/10^3+2/10^3)+(((ln (((7.93133862 × 10^8+43.82296054))))))^1/6

Input interpretation:

$$-\left(\frac{34}{10^3} + \frac{2}{10^3}\right) + \sqrt[6]{\log(7.93133862\times 10^8 + 43.82296054)}$$

log(x) is the natural logarithm

Result:

1.6182290162...

1.6182290162...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

From the above result 24.271660, considered as an entropy, for the mathematical connections between *the zeros of a special type of function connected with Dirichlet series* of the paper of russian mathematician A. A. Karatsuba and the Boundary Conditions of D-Branes (D-Brane → Black Hole → Black Brane), we obtain the usual excellent approximation to the Golden Ratio:

Mass = 4.590431e-8

Radius = 6.816109e-35

Temperature = 2.673394e+30

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[[[[1/(((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(4.590431e-8)* sqrt[[-((((2.673394e+30 * 4*Pi*(6.816109e-35)^3-(6.816109e-35)^2))))) / ((6.67*10^-11))]]]]]

Input interpretation:

$$\sqrt{\left(1\Bigg/\left(\frac{4\times1.962364415\times10^{19}}{5\times0.0864055^2}\times\frac{1}{4.590431\times10^{-8}}\sqrt{-\frac{2.673394\times10^{30}\times4\pi\,(6.816109\times10^{-35})^3-(6.816109\times10^{-35})^2}{6.67\times10^{-11}}}\right)\right)}$$

Result:

1.618249177258575656358015315290507395476410230056539657615...

1.618249177…

And:

1/sqrt[[[[1/(((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(4.590431e-8)* sqrt[[-((((2.673394e+30 * 4*Pi*(6.816109e-35)^3-(6.816109e-35)^2))))) / ((6.67*10^-11))]]]]]

Input interpretation:

$$1/\left(\sqrt{\left(1/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{4.590431\times 10^{-8}}\sqrt{-\frac{2.673394\times 10^{30}\times 4\pi(6.816109\times 10^{-35})^3-(6.816109\times 10^{-35})^2}{6.67\times 10^{-11}}}\right)\right)}\right)$$

Result:

0.617951804983499587733002946944000494952031510296536692886...

0.61795180498…

From:

Scalar Perturbations of Nonlinear Charged Lifshitz Black Branes with Hyperscaling Violation - *P. A. González, Yerko Vásquez* - https://arxiv.org/abs/1509.00802v2

We have that:

Following the argument used in [57], adapted to Lifshitz geometries with hyperscaling violation, we can verify when the imaginary part of the quasinormal frequency ω is always negative. By using outgoing Eddington-Filkenstein coordinates $v = t + x$, metric (4) can be transformed to

$$ds^2 = r^{2\alpha}\left(r^{2z}f(r)dv^2 + 2r^{z-1}dvdr + r^2\sum_{i=1}^{D-2}dx_i^2\right). \tag{18}$$

Now, taking as ansatz

$$\varphi = e^{-i\omega t}e^{i\vec{\kappa}\cdot\vec{x}}\frac{\psi(r)}{r^n}, \tag{19}$$

with $n = \frac{(D-2)(1+\alpha)}{2}$, the Klein-Gordon equation yields

$$\frac{d}{dr}(r^{1+z}f(r)\psi'(r)) - 2i\omega\psi'(r) - V(r)\psi(r) = 0, \tag{20}$$

where

$$V(r) = nr^z f'(r) + n(n+z)r^{z-1}f(r) + \kappa^2 r^{z-3} + m^2 r^{z+2\alpha-1}. \tag{21}$$

Notice that $n > -\frac{z}{2}$ according to the inequalities (7). Then, multiplying equation (20) by ψ^* and performing integrations by parts, and using Dirichlet boundary condition for the scalar field at spatial infinity, one can obtain

$$\int_{r_h}^{\infty} dr\left(r^{1+z}f(r)\left|\frac{d\psi}{dr}\right|^2 + V(r)\left|\psi\right|^2\right) = -\frac{|\omega|^2\left|\psi(r=r_h)\right|^2}{Im(\omega)}, \tag{22}$$

thus, the stability of the black brane under scalar field perturbations is guaranteed for a strictly positive potential $V(r)$ outside the horizon, because in this case, equation (22) is satisfied only for $Im(\omega) < 0$. Notice that the potential (21) is positive for $n > 0$ or $\alpha > -1$, what guaranties the stability of the black brane solution. In this work we focus our attention to this case ($\alpha > -1$).

From:

Dirichlet Boundary State in Linear Dilaton Background

Miao Li - https://arxiv.org/abs/hep-th/9512042v3

Finally, the boundary state is given by

$$|B,p\rangle = \int [dx]|x,p\rangle \exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{-\frac{1}{2Q}X(\theta)}\right). \tag{14}$$

The following remarks on (14) are in order. The solution to the infinite set of differential equations (9) is by no means unique. However, we trust that the solution given by (14) is the appropriate generalization of the usual Dirichlet boundary state to the background of linear dilaton condensate, because not only the solution looks very elegant, but also it appears to return to the usual Dirichlet boundary state in the limit $Q \to 0$. In this limit, whenever $X(\theta) \neq 0$, the exponent $\int d\theta \exp(-X/(2Q)$ is large, so the integral in (14) tends to center at $X = 0$ which is the usual Dirichlet state. As a consistency check, take $p = -iQ$, then $\Phi = 1$. Integrating over x we obtain the Neumann boundary state discussed before. For a real Q, this is unphysical if we are interested in real momentum transfer.

As we have expected, the form of (14) tells us that a boundary operator which is to replace the "wave function" $\Phi(x)$ is needed in order to restore conformal invariance. What is a little surprising is that the coefficient $2iQ(p+iQ)$ is fixed for a given Q and p. If one attempts to replace x_m in $X(\theta)$ by $\alpha_m - \tilde{\alpha}_{-m}$, one obtains operator ϕ without the zero mode part. Again, integrating over the x's results in the Neumann boundary state, except that the zero mode part is that of Dirichlet. We conclude that the generalized Dirichlet boundary state in a linear dilaton background is obtained by applying a boundary operator

$$\exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{\frac{1}{2Q}\phi_{oc}}\right)$$

to the Neumann boundary state carrying momentum p. We note in passing that a similar interaction boundary term is studied in [10], where no background charge is introduced. The usual Dirichlet boundary state is achieved by letting the coupling constant of the

From:

A. A. Karatsuba, **On the zeros of a special type of function connected with Dirichlet series**, Izv. Akad. Nauk SSSR Ser. Mat., 1991, Volume 55, Issue 3, 483–514

We have calculated:

$$I_{21} \leqslant \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leqslant P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda)\, \lambda^{-i(T-t)} \right|^2 dt \leqslant$$

$$\leqslant H \Big(\sum_{\lambda \leqslant P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P \lambda^{-1})^{2r}} + |W_2| \Big) \leqslant$$

$$\leqslant H \Big(\Big(\frac{4}{\varepsilon_2 \log T} \Big)^{2r} \sum_{\lambda \leqslant P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \Big),$$

$$I_{21} \leqslant H \Big\{ \Big(\frac{4}{\varepsilon_2 \log T} \Big)^{2r} (\log T)\, (\log X)^{-2\beta} +$$

$$+ (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r})\, T^{-\varepsilon_1} \Big\}.$$

= 793139765.05275

$$I_{22} \leqslant h_1^{2r} \int_{T}^{T+H+1} |F_2(t;\chi)|^2\, dt \leqslant h_1^{2r} \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H+1}\right)^2\right) \times$$

$$\times \left| \sum_{P^{1-\varepsilon_2} < \lambda \leqslant P} \frac{a(\lambda)}{\sqrt{\lambda}} \lambda^{-i(T+t)} \right|^2 dt \leqslant h_1^{2r} H \Big(\sum_{P^{1-\varepsilon_2} < \lambda \leqslant P} \frac{|a(\lambda)|^2}{\lambda} + |W_3| \Big),$$

$$I_{22} \leqslant h_1^{2r} H\, (\varepsilon_2 (\log T)\, (\log X)^{-2\beta} + T^{-\varepsilon_1}).$$

= 43.8229605411830149064349…

We have the following possible mathematical connections and solutions:

$$\int_{r_h}^{\infty} dr \left(r^{1+z} f(r) \left|\frac{d\psi}{dr}\right|^2 + V(r)\,|\psi|^2 \right) = -\frac{|\omega|^2\,|\psi(r=r_h)|^2}{Im(\omega)}\,,$$

⇒

⇒ $$|B,p\rangle = \int [dx]|x,p\rangle \exp\left(2iQ(p+iQ)\oint \frac{d0}{2\pi} e^{-\frac{1}{2Q}X(\theta)}\right)$$

⇒ $$\exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{\frac{1}{2Q}\phi_{oc}}\right)$$

$$I_{21} \leqslant \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right)\left|\sum_{\lambda\leq P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda)\,\lambda^{-i(T+t)}\right|^2 dt \leqslant$$

$$\leqslant H\left(\sum_{\lambda\leq P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda}\cdot\frac{4^r}{(\log P\lambda^{-1})^{2r}} + |W_2|\right) \leqslant$$

$$\leqslant H\left(\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} \sum_{\lambda\leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2|\right),$$

$$I_{21} \leqslant H\left\{\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} (\log T)\,(\log X)^{-2\beta} + \right.$$

$$\left. + \left(\varepsilon_2^{-2r}\,(\log T)^{-2r} + \varepsilon_2^{-r} h_1^r\,(\log T)^{-r}\right) T^{-\varepsilon_1}\right\}.$$

= 793139765.05275

$$\int_{r_h}^{\infty} dr \left(r^{1+z} f(r) \left|\frac{d\psi}{dr}\right|^2 + V(r)\,|\psi|^2 \right) = -\frac{|\omega|^2\,|\psi(r=r_h)|^2}{Im(\omega)}\,,$$

⇒

⇒ $$|B,p\rangle - \int [dx]|x,p\rangle \exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{-\frac{1}{2Q}X(\theta)}\right)$$

⇒ $$\exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{\frac{1}{2Q}\phi_{oc}}\right)$$

$$I_{22} \ll h_1^{2r} \int_T^{T+H+1} |F_2(t;\chi)|^2 \, dt \ll h_1^{2r} \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H+1}\right)^2\right) \times$$

$$\times \left| \sum_{P^{1-\varepsilon_2}<\lambda\leqslant P} \frac{a(\lambda)}{\sqrt{\lambda}} \lambda^{-i(T+t)} \right|^2 dt \ll h_1^{2r} H \left(\sum_{P^{1-\varepsilon_2}<\lambda\leqslant P} \frac{|a(\lambda)|^2}{\lambda} + |W_3| \right),$$

$$I_{22} \ll h_1^{2r} H \left(\varepsilon_2 (\log T)(\log X)^{-2\beta} + T^{-\varepsilon_1}\right).$$

= 43.82296054118301490643490…

Note that:

$793139765.05 / (43.82296)^5 = 1/3*(793139765,05 / 161625001,567) =$

= 1,63576129385…

And:

ln(7.93133862334800504570304142069014510208874797926 × 10^8) + ln(43.82296054118301490643490)

24.271660463614727412031 47...

24.271660....

Thence:

$$\ln\left(\int_{r_h}^{\infty} dr \left(r^{1+z} f(r) \left| \frac{d\psi}{dr} \right|^2 + V(r)\,|\psi|^2 \right) = -\frac{|\omega|^2 \, |\psi(r=r_h)|^2}{Im(\omega)}, \right) \Rightarrow$$

$$\Rightarrow \ln\left(\begin{array}{c} I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H\left(\sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P\lambda^{-1})^{2r}} + |W_2| \right) \ll \\ \ll H\left(\left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} \sum_{\lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right), \\ I_{21} \ll H\left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T)(\log X)^{-2\beta} + \right. \\ \left. + \left(\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\varepsilon_1} \right\}. \end{array}\right) +$$

$$+ \ln\left(\begin{array}{c} I_{22} \ll h_1^{2r} \int_T^{T+H+1} |F_2(t;\chi)|^2 dt \ll h_1^{2r} \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H+1}\right)^2\right) \times \\ \times \left| \sum_{P^{1-\varepsilon_2} < \lambda \leq P} \frac{a(\lambda)}{\sqrt{\lambda}} \lambda^{-i(T+t)} \right|^2 dt \ll h_1^{2r} H \left(\sum_{P^{1-\varepsilon_2} < \lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_3| \right), \\ I_{22} \ll h_1^{2r} H \left(\varepsilon_2 (\log T)(\log X)^{-2\beta} + T^{-\varepsilon_1} \right). \end{array}\right)$$

= ln 793139765.05275 + ln 43.822960541183014906 4349 =

= 24.271660....

We have also the following mathematical connection:

$$|B,p\rangle = \int [dx]|x,p\rangle \exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{-\frac{1}{2Q}X(\theta)}\right)$$

$$\Rightarrow \exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{\frac{1}{2Q}\phi_{oc}}\right)$$

$$\ln\left(\begin{array}{c} |B,p\rangle = \int [dx]|x,p\rangle \exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{-\frac{1}{2Q}X(\theta)}\right) \\ \Rightarrow \exp\left(2iQ(p+iQ)\oint \frac{d\theta}{2\pi} e^{\frac{1}{2Q}\phi_{oc}}\right) \end{array}\right) \Rightarrow$$

$$\Rightarrow \ln\left(\begin{array}{c} I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right)\left|\sum_{\lambda\le P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda)\,\lambda^{-i(T+t)}\right|^2 dt \ll \\ \ll H\left(\sum_{\lambda\le P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda}\cdot\frac{4^r}{(\log P\lambda^{-1})^{2r}} + |W_2|\right) \ll \\ \ll H\left(\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} \sum_{\lambda\le P} \frac{|a(\lambda)|^2}{\lambda} + |W_2|\right), \\ I_{21} \ll H\left\{\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} (\log T)(\log X)^{-2\beta} + \right. \\ \left. + \left(\varepsilon_2^{-2r}(\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}\right) T^{-\varepsilon_1}\right\}. \end{array}\right) +$$

$$+ \ln\left(\begin{array}{c} I_{22} \ll h_1^{2r} \int_T^{T+H+1} |F_2(t;\chi)|^2\, dt \ll h_1^{2r} \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H+1}\right)^2\right) \times \\ \times \left|\sum_{P^{1-\varepsilon_2}<\lambda\le P} \frac{a(\lambda)}{\sqrt{\lambda}} \lambda^{-i(T+t)}\right|^2 dt \ll h_1^{2r} H\left(\sum_{P^{1-\varepsilon_2}<\lambda\le P} \frac{|a(\lambda)|^2}{\lambda} + |W_3|\right), \\ I_{22} \ll h_1^{2r} H\left(\varepsilon_2 (\log T)(\log X)^{-2\beta} + T^{-\varepsilon_1}\right). \end{array}\right)$$

$= \ln 793139765.05275 + \ln 43.8229605411830149064349 =$

$= 24.271660....$

And:

$$\frac{1}{3}\times\left(\begin{aligned}
&I_{21}\ll\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{t}{H}\right)^2\right)\Big|\sum_{\lambda\le P^{1-\varepsilon_2}}\frac{a(\lambda)}{\sqrt{\lambda}}B(\lambda)\lambda^{-i(T+t)}\Big|^2dt\ll\\
&\ll H\Big(\sum_{\lambda\le P^{1-\varepsilon_2}}\frac{|a(\lambda)|^2}{\lambda}\cdot\frac{4^r}{(\log P\lambda^{-1})^{2r}}+|W_2|\Big)\ll\\
&\ll H\Big(\Big(\frac{4}{\varepsilon_2\log T}\Big)^{2r}\sum_{\lambda\le P}\frac{|a(\lambda)|^2}{\lambda}+|W_2|\Big),\\
&I_{21}\ll H\Big\{\Big(\frac{4}{\varepsilon_2\log T}\Big)^{2r}(\log T)(\log X)^{-2\beta}+\\
&+(\varepsilon_2^{-2r}(\log T)^{-2r}+\varepsilon_2^{-r}h_1^r(\log T)^{-r})T^{-\varepsilon_1}\Big\}.
\end{aligned}\right)/$$

$$\left(\begin{aligned}
&I_{22}\ll h_1^{2r}\int_T^{T+H+1}|F_2(t;\chi)|^2dt\ll h_1^{2r}\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{t}{H+1}\right)^2\right)\times\\
&\times\Big|\sum_{P^{1-\varepsilon_2}<\lambda\le P}\frac{a(\lambda)}{\sqrt{\lambda}}\lambda^{-i(T+t)}\Big|^2dt\ll h_1^{2r}H\Big(\sum_{P^{1-\varepsilon_2}<\lambda\le P}\frac{|a(\lambda)|^2}{\lambda}+|W_3|\Big),\\
&I_{22}\ll h_1^{2r}H(\varepsilon_2(\log T)(\log X)^{-2\beta}+T^{-\varepsilon_1}).
\end{aligned}\right)^5=$$

= 1,63576129385…

From:

Изв. Акад. Наук СССР
Сер. Матем. Том 55 (1991), № 3

Math. USSR Izvestiya
Vol. 38 (1992), No. 3

ON THE ZEROS OF A SPECIAL TYPE OF FUNCTION CONNECTED WITH DIRICHLET SERIES

UDC 511

A. A. KARATSUBA

To find a lower bound for the integral

$$J = \int_{T-h}^{T+H} |F(t)|^a \, dt = \left(\frac{|r(n_1)|}{\sqrt{n_1}}\right)^a \int_{T+h}^{T+H} \left| \sum_{n=n_1}^{\infty} A(n) \left(\frac{n}{n_1}\right)^{-0.5-it} \right|^a dt$$

we apply a theorem of Gabriel (see [17], or else [2] or [18]) in the following formulation:

If

$$J(\sigma, \lambda) = \left(\int_0^{H-h} |f(\sigma + it)|^{1/\lambda} \, dt \right)^{\lambda}, \qquad \lambda > 0,$$

$$f(\sigma + it) = \sum_{n=n_1}^{\infty} A(n) \left(\frac{n}{n_1}\right)^{-\sigma - i(T+t)},$$

then

$$J(\sigma, p\lambda + q\mu) \le J^p(\alpha, \lambda) J^q(\beta, \mu), \tag{70}$$

where $\alpha < \sigma < \beta$, $p = (\beta - \sigma)/(\beta - \alpha)$, *and* $q = (\sigma - \alpha)/(\beta - \alpha)$.

In this inequality we set $\alpha = 1/2$, $\lambda = 1/a$, $\beta = 2$, $\mu = 1/(2-a)$, $p = a/2$, $q = (2-a)/2$, and $\sigma = 2 - 3a/4$. Then $p\lambda + q\mu = 1$, and

$$\begin{aligned} J(\sigma, p\lambda + q\mu) &= J\left(2 - \frac{3a}{4}, 1\right) = \int_0^{H-h} \left| \sum_{n=n_1}^{\infty} A(n) \left(\frac{n}{n_1}\right)^{-2-3a/4-i(T+t)} \right| dt \\ &\ge \left| H - h + \sum_{n=n_1+1}^{\infty} A(n) \left(\frac{n}{n_1}\right)^{-2+3a/4-iT} \int_0^{H-h} \left(\frac{n}{n_1}\right)^{-it} dt \right| \\ &\ge H - h + O(1) = H + O(1). \end{aligned} \tag{71}$$

Furthermore,

$$\begin{aligned} J(\beta, \mu) &= J\left(2, \frac{1}{2-a}\right) = \left(\int_0^{H-h} \left| \sum_{n=n_1}^{\infty} A(n) \left(\frac{n}{n_1}\right)^{-2-i(T+t)} \right|^{2-a} dt \right)^{1/(2-a)} \\ &\le \left(\int_0^{H-h} \left(n_1^2 \sum_{n=n_1}^{\infty} |A(n)| n^{-2} \right)^{2-a} dt \right)^{1/(2-a)} \le c_2 H^{1/(2-a)}. \end{aligned} \tag{72}$$

From (70)–(72) we find that

$$H + O(1) \le J^{a/2}(\tfrac{1}{2}, \tfrac{1}{a}) (c_2 H^{1/(2-a)})^{(2-a)/2},$$

$$J^{a/2}(\tfrac{1}{2}, \tfrac{1}{a}) \ge c_3 H^{1/2}, \qquad J(\tfrac{1}{2}, \tfrac{1}{a}) \ge c_3^{2/a} H^{1/a},$$

$$\left(\int_{T+h}^{T+H} \left| \sum_{n=n_1}^{\infty} A(n) \left(\frac{n}{n_1}\right)^{-0.5-it} \right|^a dt \right)^{1/a} \ge c_3^{2/a} H^{1/a},$$

$$J = \int_{T+h}^{T+H} |F(t)|^a \, dt \ge \left(\frac{|r(n_1)|}{\sqrt{n_1}}\right)^a c_3^2 H.$$

Together with (69), this gives us the necessary lower bound for I_3:

$$I_3 \ge c_4 h_1'^{a} H.$$

Data

Let a be an arbitrary fixed number with $0 < a < 1$

a = 0.5

-10/(4Pi) = -0.795774715 = β

b = 2; t = 2; k = 5

$\log Y \geqslant 4.$

ln(54.59815) = 4; Y = 55

$T \geqslant 10$

$1 \leqslant X \leqslant T^{0,01}$

$T^{0.01}$ = 1,0232929922;   X = 1.0061571663; ε_1 = 0.01; ε_2 = 0.008

$H = T^{27/82+\varepsilon_1}$ = 10^27/82+0.01 = 10^0,339268292 = 2.18407874

$A = c_1 r (\log T) \times (\log X)^{-2\beta}, \beta\varphi(K) - 1, c > 1, c_1 > 1$

$r = [c \log\log T]$ c; c_1 = 1.2   r = 1.00083893

$h = A (\log T)^{-1}$ $h_1 = hr^{-1}$

c_2 = 1.4; c_3 = 1.8

1.2 * 1.00083893 (ln10) * (ln10)^(-2*-0.795774715) = A = 10.429063158

h = 10.429063158 (ln10)^-1 = 4.5292845809

h_1 = 4.5292845809*(1.00083893^-1) = 4.52548801324

Now, we have that:

$$J(\beta\,,\,\mu) = J\left(2\,,\,\frac{1}{2-a}\right) = \left(\int_0^{H-h}\left|\sum_{n=n_1}^{\infty} A(n)\left(\frac{n}{n_1}\right)^{-2-i(T+t)}\right|^{2-a} dt\right)^{1/(2-a)}$$

$$\leq \left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a} dt\right)^{1/(2-a)} \leq c_2 H^{1/(2-a)}\,.$$

$$\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a} dt\right)^{1/(2-a)} \leq c_2 H^{1/(2-a)}$$

1.4 * 2.18407874^(1/1.5)

Input interpretation:

$1.4\,\sqrt[1.5]{2.18407874}$

Result:

2.356714080790795072048499031194903127847222607435132551476...

2.35671408…

(((1.4 * 2.18407874^(1/1.5))))^0.58

Input interpretation:

$\left(1.4\,\sqrt[1.5]{2.18407874}\right)^{0.58}$

Result:

1.644136989168840918689193014566036107697791035968397241051...

1.644136989…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

And:

29/10^3-2/10^3+(((1.4 * 2.18407874^(1/1.5))))^0.58

Where 29 and 2 are Lucas numbers

Input interpretation:

$$\frac{29}{10^3} - \frac{2}{10^3} + \left(1.4^{\sqrt[1.5]{2.18407874}}\right)^{0.58}$$

Result:

1.671136989168840918689193014566036107697791035968397241051...

1.6711369891…

We note that 1.6711369891... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

Now:

$$\left(\int_{T+h}^{T+H} \left| \sum_{n=n_1}^{\infty} A(n) \left(\frac{n}{n_1} \right)^{-0.5-it} \right|^a dt \right)^{1/a} \geq c_3^{2/a} H^{1/a},$$

1.8^4 * 2.18407874^2

Input interpretation:

$1.8^4 \times 2.18407874^2$

Result:

50.07565091659782182976

50.0756509…

(((1.8^4 * 2.18407874^2)))^1/8

Input interpretation:

$\sqrt[8]{1.8^4 \times 2.18407874^2}$

Result:

1.630997612868126335500879433124520367385457562517823362263...

1.6309976…

-(11/10^3+2/10^3)+(((1.8^4 * 2.18407874^2)))^1/8

Input interpretation:

$$-\left(\frac{11}{10^3}+\frac{2}{10^3}\right)+\sqrt[8]{1.8^4\times 2.18407874^2}$$

Result:

1.617997612868126335500879433124520367385457562517823362263...

1.6179976128…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

From the difference from the two results, we obtain:

(50.0756509 - 2.35671408)^1/8

Input interpretation:

$$\sqrt[8]{50.0756509-2.35671408}$$

Result:

1.621199066038709110829927193890188767558372311697960574347...

1.621199066...

(47+3)/10^3 + (50.0756509 - 2.35671408)^1/8

Input interpretation:

$$\frac{47+3}{10^3}+\sqrt[8]{50.0756509-2.35671408}$$

Result:

1.671199066...

1.671199066...

We note that 1.671199066... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

-3/10^3+(50.0756509 - 2.35671408)^1/8

Input interpretation:

$$-\frac{3}{10^3} + \sqrt[8]{50.0756509 - 2.35671408}$$

Result:

1.618199066038709110829927193890188767558372311697960574347...

1.618199066...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

We have, in conclusion:

$$\sqrt[0.58]{\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)} \leq c_2 H^{1/(2-a)}} =$$

= 1.644136989…

$$\sqrt[8]{\left(\int_{T+h}^{T+H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a} \geq c_3^{2/a}H^{1/a},} =$$

= 1.6309976…

And:

$$\left(\frac{29}{10^3}-\frac{2}{10^3}+\sqrt[0.58]{\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)}\leq c_2H^{1/(2-a)}}\right)=$$

= 1.6711369891…

$$\left(-\left(\frac{11}{10^3}+\frac{2}{10^3}\right)+\sqrt[8]{\left(\int_{T+h}^{T+H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a}\geq c_3^{2/a}H^{1/a},}\right)=$$

= 1.6179976128…

From the difference of the following formulas:

$$\left(\int_{T+h}^{T+H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a}\geq c_3^{2/a}H^{1/a},$$

$$\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)}\leq c_2H^{1/(2-a)}$$

we obtain:

$$\left(\sqrt[8]{\left(\int_{T+h}^{T+H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a}\geq c_3^{2/a}H^{1/a},\ -\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)}\leq c_2H^{1/(2-a)}}\right)$$

= 1.621199066…

$$\left(-\frac{3}{10^3}+\sqrt[8]{\left(\int_{T+h}^{T-H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a}\geq c_3^{2/a}H^{1/a},\ -\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)}\leq c_2H^{1/(2-a)}}\right)$$

= 1.618199066...

$$\left(\frac{47+3}{10^3}+\sqrt[8]{\left(\int_{T+h}^{T+H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a}\geq c_3^{2/a}H^{1/a},\ -\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)}\leq c_2H^{1/(2-a)}}\right)$$

= 1.671199066…

$$\int_{r_h}^{\infty}dr\left(r^{1+z}f(r)\left|\frac{d\psi}{dr}\right|^2+V(r)\,|\psi|^2\right)=-\frac{|\omega|^2\,|\psi(r=r_h)|^2}{Im(\omega)}$$

$$\left(\int_{r_h}^{\infty}dr\left(r^{1+z}f(r)\left|\frac{d\psi}{dr}\right|^2+V(r)\,|\psi|^2\right)=-\frac{|\omega|^2\,|\psi(r=r_h)|^2}{Im(\omega)}\right)\Rightarrow$$

$\Rightarrow$

$$\left(\frac{47+3}{10^3}+\sqrt[8]{\left(\int_{T+h}^{T+H}\left|\sum_{n=n_1}^{\infty}A(n)\left(\frac{n}{n_1}\right)^{-0.5-it}\right|^a dt\right)^{1/a}\geq c_3^{2/a}H^{1/a},\ -\left(\int_0^{H-h}\left(n_1^2\sum_{n=n_1}^{\infty}|A(n)|n^{-2}\right)^{2-a}dt\right)^{1/(2-a)}\leq c_2H^{1/(2-a)}}\right)$$

= 1.671199066….

This result is practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass

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