Title
Conjectures about modulated Maxwell signals and, or, Ranada solutions.

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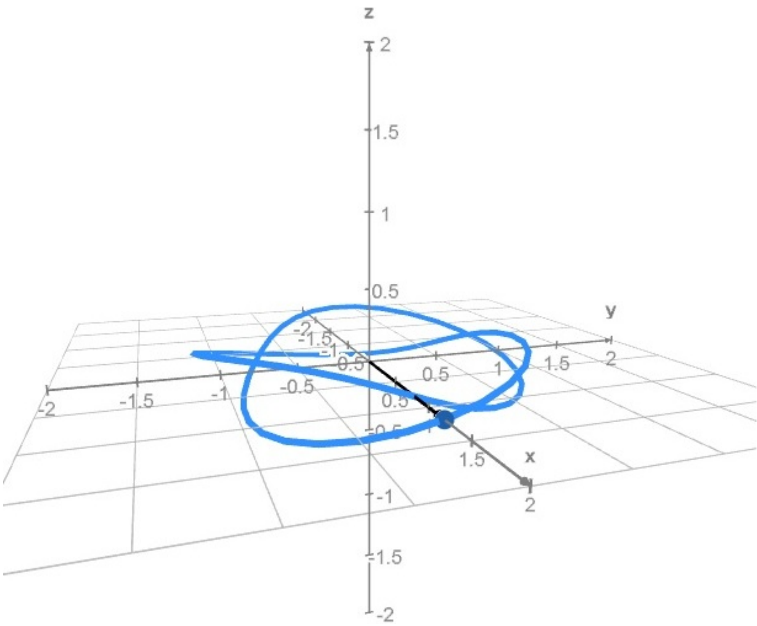
Abstract
I present some methods to generate electromagnetic fields wich, in my opinion, have a good chance to represent linked and knotted fields and, maybe, the electron.



1-Introduction

"Linked" and "knotted" solutions of Maxwell's equations are currently the subject of many studies and in particular have attracted the interest of David Hestenes as possible toroidal solutions for the electron .
See for example [1] and all the citations contained therein.
As is known, Hestenes is a historic proponent of a spatially extended solution for electron, possibly an electromagnetic solution .
However, it is not very easy or mathematically easy to visualize a toroidal solution.
I propose here an easy approach, which is based on the creation of analytic functions (or left monogenic functions) through the product operation.
For the exposition I make extensive use of the GA Geometric Algebra.
For the notations in use I refer to my paper "manuscripts of the last century" [2] .
Throughout all the present document, unless otherwise advised, I use the words "electromagnetic field", "analytic function", "left monogenic function" as synonyms.

2 - A trick?

In fact, there are already papers that use Geometric Algebra, for example , [3] SJ van Enk, "The covariant description of electric and magnetic field lines of null fields: application to Hopf-Ranada solutions"
Here it is said , with "v" vector product or (ab)v = ab-ba ……. (eg formula 93 of the text)

“We can describe a set of solutions to the free Maxwell equations, the Hopf-Rañada linked and knotted fields, in a compact geometric way by using Geometric Algebra. In particular, we represent the solutions in covariant form as F=$(\partial\alpha\partial^*\beta)_v$ with α and β real scalar potentials ”.

Actually in the text an asterisk * does not appear but an 'overbar' that denotes the so-called 'Clifford conjugate', described this way:
“Another involution, of great use in descriptions of relativity, is the Clifford conjugate, which reverses the directions of vectors and pseudovectors but leaves the scalar part the same . We denote the Clifford conjugate by a bar. "

In my opinion, some clues are derived from the paper.
First clue.
Alpha and beta are "real scalar potentials" so they have certain properties that can be useful, such as commutativity with the delta.
Second clue.
A vector product or an exterior product appears (ab)v = ab-ba.
That is : let's imagine an F field expressed as a vector product ab-ba. This I think can make you that taking the operation ∂^*, due to the difference the result is zero. So the clues are
real scalar potentials
and then
field as a vector product ab-ba so taking the ∂^*, the result is zero.
These are just vague clues, insights .
But perhaps we can think of a simpler situation.

If alpha and beta are harmonics and even real scalars , then $F = \partial\alpha\partial\beta$ is analytic i.e. is an electromagnetic field (?).

Although it is obvious in 2D , I wanted to verify it by developing the calculations in full.
If alpha and beta are real scalar harmonics , then $F = \partial\alpha\partial\beta$ is analytic: the demonstration involves 16 terms (16 addends) and is long and boring . The fact is that I did it , with patience, and it works.

Then all this opens up a series of questions I can't answer at the moment .
In particular: is the proof also valid in 4D?
If the answer is yes, therefore also in 4D the generation of analytic functions can take place in (at least) two ways. The ways would be the following:
1 - If alpha is harmonic , then $F = \partial\alpha$ is analytic.
2 - If alpha and beta are real scalar harmonics , then $F = \partial\alpha\partial\beta$ is analytic.
In fact, the second way is possible, and I can give a brief demonstration of it but only with the condition that $\partial\alpha$ commutes with the Dirac operator ∂^*.
The proof, which is valid both in 2D and in 4D, is the following .

With the aforementioned hypotheses, namely α, β real scalar harmonics
(1)
$$(\partial^* \partial \alpha) = 0$$
$$(\partial^* \partial \beta) = 0$$
and $\partial\alpha$ that commutes with the Dirac operator ∂^*, I apply ∂^* to the product $F = \partial\alpha\partial\beta$ with the product derivation rule (derivative of the first times the second, plus the first times the derivative of the second.). It is
(2)
$$\partial^* F = \partial^*(\partial\alpha\partial\beta) = (\partial^*\partial\alpha)\partial\beta + \partial^*\partial\alpha\partial\beta$$
In the last term $\partial^*\partial\alpha\partial\beta$ the operator ∂^* is to be understood to apply to $\partial\beta$ with $\partial\alpha$ being a constant. Thus a difficulty arises: in general ∂^* does not commute with $\partial\alpha$, due to the numbers (commutative and / or non-commutative) of Clifford's algebra. However, if they commute, one can write:
(3)
$$\partial^*\partial\alpha\partial\beta = \partial\alpha(\partial^*\partial\beta)$$
Thus replacing (3) in (2) and since they α, β satisfy the Laplace equation (1), it turns out
(4)
$$\partial^* F = \partial^*(\partial\alpha\partial\beta) = (\partial^*\partial\alpha)\partial\beta + \partial\alpha(\partial^*\partial\beta) = 0$$
QED.

Of these alpha and beta (real) there are several , see for example [4] , "Solid harmonics".
"In mathematics, solid harmonics are defined as solutions of the Laplace equation in spherical polar coordinates".
But more simply, are there explicit examples of real harmonics?
Example x ^ 2-y ^ 2, being a real part of z ^ 2. And therefore, in general, and also in 4D, the real part of an (any) analytic function.
Some clarifications are necessary.

1First of all, we did the proof with ∂^* that commutes with $\partial\alpha$ and α real. But with "real" we can also mean with α containing the index Tji. It in fact does commute with all the symbols of the Clifford algebra. We can still talk about "real" in a broad sense. This means that when we introduce $\partial\alpha$ (which is in effect an electromagnetic field) with the potential α as we have said, it happens that if α it also contains the index Tji we are sure we can also represent a magnetic field .

2The second clarification is that, for the proof, we do not hypothesized anything about β. Therefore the latter potential can be a potential A containing any symbol, just harmonic So the second analytic function that appears in the product, let's say , $F2 = \partial A$, is a Maxwell field in a broad sense.
Then we can say that if alpha is a real harmonic and ∂^* commutes with $\partial\alpha$, then it is analytic also the function
(5)
$$F = \partial\alpha F2$$
3Third, last but not least, strictly speaking, when we introduce ∂α (which is actually an electromagnetic field as I have noticed) with the potential α as we have said, nothing is necessary about α. Just be harmonic to have $F1 = \partial\alpha$ as electromagnetic field. So we may also say that if $F1, F2$ are analitic and ∂^* commutes with $F1$, then $F = F1F2$ is analytic.
Among these ways I prefer the second one.

3 - Extension

The initial proposition can thus be reformulated

If alpha is real harmonic and ∂^* commutes with $\partial\alpha$, then it is analytic also the function

(6) $$F = \partial\alpha F2$$

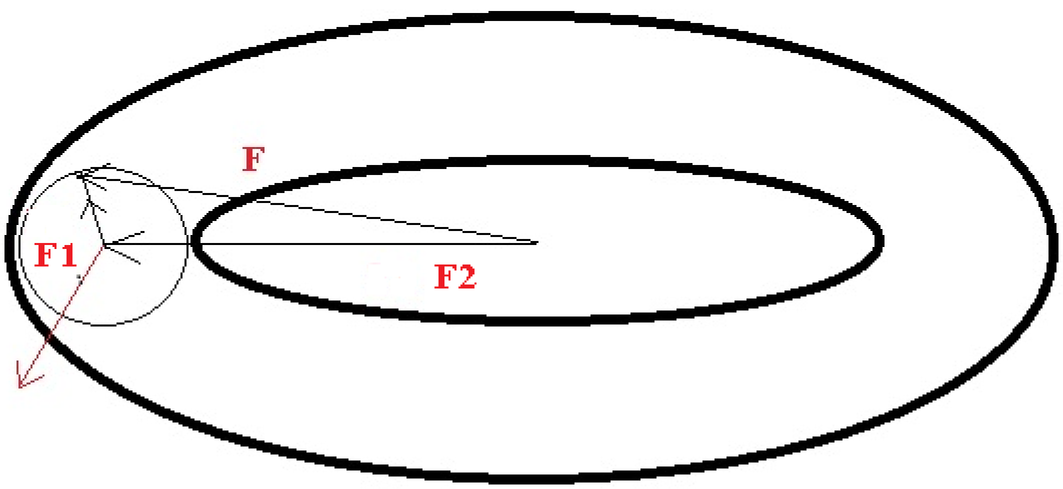
with $F2$ generic analytic function.

We can see this as the condition for forming a product to make fields that is still a field. Or: the condition to form a product between "left monogenics" that is still a "left monogenic", see literature. In fact in general the product $F = F1F2$ between two F1 and F2 (analytic) fields is no longer analytic.
Instead in this case we have a product between two analytic functions that is still an analytic function. In order for this to be possible we do not have many choices, we have only one possibility: that the first analytic function $F1 = \partial\alpha$ commutes with the ∂^*.
With these clarifications, and combining the fact that toroidal solutions have already been demonstrated elsewhere,

can we think of a product $F = F1F2$ as a modulation?

A modulation, for example, of a TE that I indicate with F2, drawing its electric field, modulated by a second field F1 , equipped with a charge, too.

To answer this question let us look at the parametric equations of a helix wrapped around a torus.
Let he equations for the vector indicated as F in the figure:
(7)
x = (R + r * cos (n * t)) * cos (t)
y = (R + r * cos (n * t)) * sin (t)
z = r * sin (n * t)



In these equations I can understand that the two different 'omega' that characterize the two rotations of the vector F are explicitly indicated . Here they are 1 or n, or rather they are in the ratio 1: n.

Question:
is it possible that this is an electric field? Is it possible to construct the corresponding field H? or is it possible that this is the $1\ i\ j\ T$ part of an analytic function?

I could reason like this:
let's admit that these are really the components of an electric field E , let's write them down and check if they know satisfy ... ... what? If this is an electric field, forming part of an electromagnetic field F, it must satisfy the Maxwell equations (with his own H).
But also, I would say, the 2nd order equation $\partial\partial^* F = 0$, to which then (scalar operator) must necessarily satisfy each of the components.
I write that these from (7) are the components of an E field , including a possible field component E_τ. (which I can also indicate with

$$E_\tau = -H_\tau$$

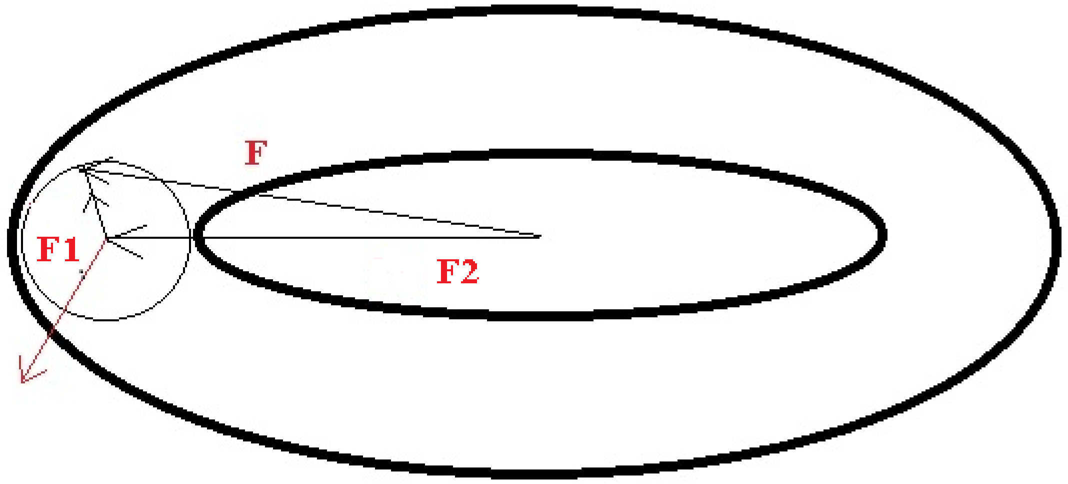
to be consistent with my other writings). So:
(8)

$$\boldsymbol{E_x = (R + rcosn\omega t)cos\omega t}$$
$$\boldsymbol{E_y = (R + rcosn\omega t)sin\omega t}$$
$$\boldsymbol{E_z = rsinn\omega t}$$
$$\boldsymbol{E_\tau = -H_\tau} = ??$$

The fourth field component I indicated with $E_\tau = -H_\tau$ could represent (I imagine) a charge (<u>Much </u>interesting. See later in paragraph 4 below).
Going forward in the conjectures you can do some examples or suggestive in my opinion ,
and however, not to be boring, I continue in two Appendices to which I refer.

At this point, I would like to make some considerations as an expert on radar, electromagnetism and signal theory or, if we want to put it more politically correct, as a "lover of the subject" .
I summarize.
Let the equations for the vector F in the figure (eventually an "electric field") these ones:
(13)

$$E_x = (R + r\cos n\omega t)\cos\omega t$$
$$E_y = (R + r\cos n\omega t)\sin\omega t$$
$$E_z = r\sin n\omega t$$



These are also the parametric equations of a "toroidal helix" which with $\omega t = 0 - 2\pi$ makes n turns on the torus, that is, at each complete $0 - 2\pi$ circumferential rotation it is wound n times on the torus itself.
This is a fact.
It is not possible to say that this is really an electric field.
Relying on pure intuition, however, we can try to link this vector to the (hypothetical) electric field vector of a (hypothetical) toroidal solution of Maxwell's equations.
Several observations concur to consider this hypothesis plausible.

Observation No. 1.
The equations of the helix, as well as the physics of the phenomenon, are reminiscent of a modulation and precisely an Amplitude Modulation (AM). It can be well highlighted by rewriting, for example, the:
(14)

$$E_x = (R + rcosn\omega t)cos\omega t$$

in the form of a modulated signal
(15)

$$E_x = \left(1 + \frac{r}{R}cosn\omega t\right)Rcos\omega t$$

This is the classical form of amplitude modulation with
$\frac{r}{R}$ = "Modulation index";
$cosn\omega t$ = “Amplitude modulation signal”;
$Rcos\omega t$ ="Sinusoidal carrier wave ".
See for example [5] "Sinusoidal Amplitude Modulation (AM)".

It is instructive to study the *modulation* of one sinusoid by another. In this section, we will look at sinusoidal *Amplitude Modulation (AM)*. The general AM formula is given by

$$x_\alpha(t) = [1 + \alpha \cdot a_m(t)] \cdot A_c \sin(\omega_c t + \phi_c),$$

where (A_c, ω_c, ϕ_c) are parameters of the sinusoidal *carrier* wave, $\alpha \in [0,1]$ is called the *modulation index* (or *AM index*), and $a_m(t) \in [-1,1]$ is the *amplitude modulation* signal.

In the Ref. here cited they also note that "the modulated signal can be written as the sum of the modulated carrier wave, the product of the carrier wave and the modulating wave". That is to say (15) is also written:
(16)

$$E_x = Rcos\omega t + \frac{r}{R}cosn\omega t(Rcos\omega t)$$

Nothing assures us that this may actually be an amplitude modulated electric field, but nothing forbids us to think of an electrictromagnetic field (analytic function) amplitude modulated, if it were possible to write it in this form.

Observation No. 2.
Using (6) we can form an analytic function in the equivalent forms (15) or (16) of modulated signal by writing:
(17)

$$F = \left(1 + \frac{r}{R}\partial\alpha\right)F2$$

or
(18)

$$F = F2 + \frac{r}{R}\partial\alpha F2$$

We could call it "Maxwell's modulated signal”, which may even include charges and currents.
The possibility comes from being able to write an analytic function as a product of analytic functions, according to the rule (6).
To do this in practice, and also to be as adherent as possible to the pictorial ideas exposed here, we may take as $F2$ a suitable TE in circular waveguide, at the cutoff frequency. Or in a circular cavity resonator. This could assure us that 1) is certainly an analytic function and 2) certainly there is an electric field that rotates as desired.
Anyway, let $F2$ the "carrier", ie the unmodulated signal.
As for the modulation index $\frac{r}{R}$ there are no problems.
As modulating wave $\partial\alpha$ we need a potential alpha that is real& harmonic, or more precisely, as we have seen, that contains only index 1 and index Tji. Also for this there are no problems, because we can take it as a real part of an arbitrary auxiliary analytic function

$$\alpha = ReF3 = \frac{1}{2}(F3 + F3^*)$$

As for $F3$ we can take some spherical or cylindrical coordinates solution, choosen from the solutions presented in [6] .

In doing so it is my opinion that we will be able to find:

1) toroidal solutions of Maxwell's equations;
2) the solutions of the electron.

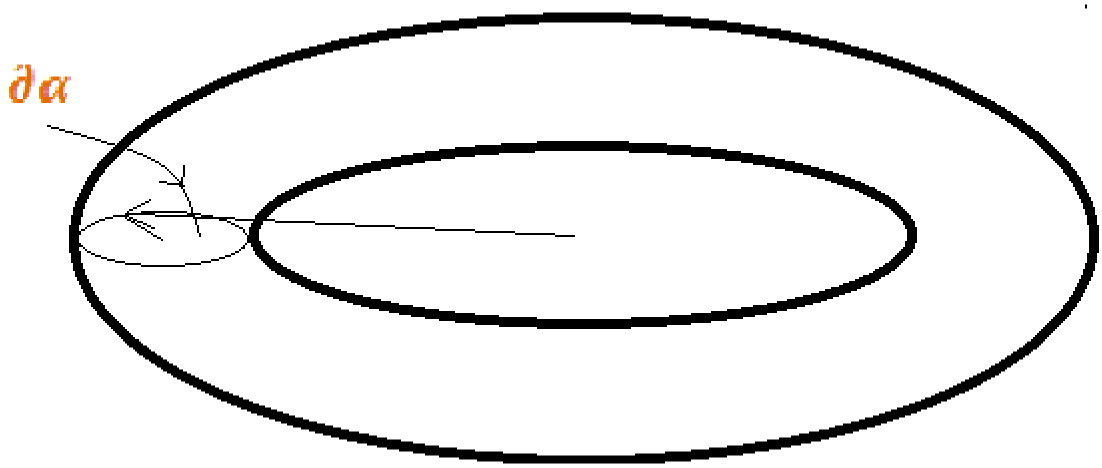
Point 2) has yet to be demonstrated.
As for point 1) , looking for toroidal solutions, I would say that it should be more easily accessible. With some warning.
I give an example: let’s suppose that the entire modulated signal F must be rotary, and for example rotate stably with law $exp(i\omega_0 t)$.
In this case it is necessary to consider that this multiplier will act correctly on F only for the parts that so to speak " lie in the rotation plane ". The right rotation formula will therefore be a formula with the product to the right and to the left, and with half the angle as an argument of the exponential.
I make no attempt here to guess this formula, I just note the problem.
However I want to point out an interesting possibility, which is connected, probably not by chance, to the various toroidal models of the electron that are present in the literature.
What I do in the next paragraph.

4-A possible interpretation

I refer to an electromagnetic field F as in the formulas (17) or (18).
First of all it is important to note that:
(note well)
as for "electromagnetic field" I mean a solution of the Maxwell equations in the broad sense, gauge-less, therefore to 8 components, so in particular with the possible presence of a E_τ component that gives rise to electric charge, eg ref. [2].
Given the above, it is already mentioned the difficulties that may arise in mathematically representing a rotating signal with this formula.
But let us return to consider the parametric equations of a toroidal helix (7) or the equations (13) of a potentially conceivable "electric field" having that form.
Looking more closely at these equations one notices that only the first two components

$$E_x = (R + rcosn\omega t)cos\omega t$$
$$E_y = (R + rcosn\omega t)sin\omega t$$

they take the form of a modulated signal.
There will be no difficulty in writing these components in the form (17) or (18) of analitic signal.
For this purpose will suffice that both $F2$ and the modulating wave $\partial\alpha$ lie on the x, y plane, and so the entire signal will be stably rotating with law $exp(i\omega t)$ without any difficulty.



It would remain to be only alone the third component

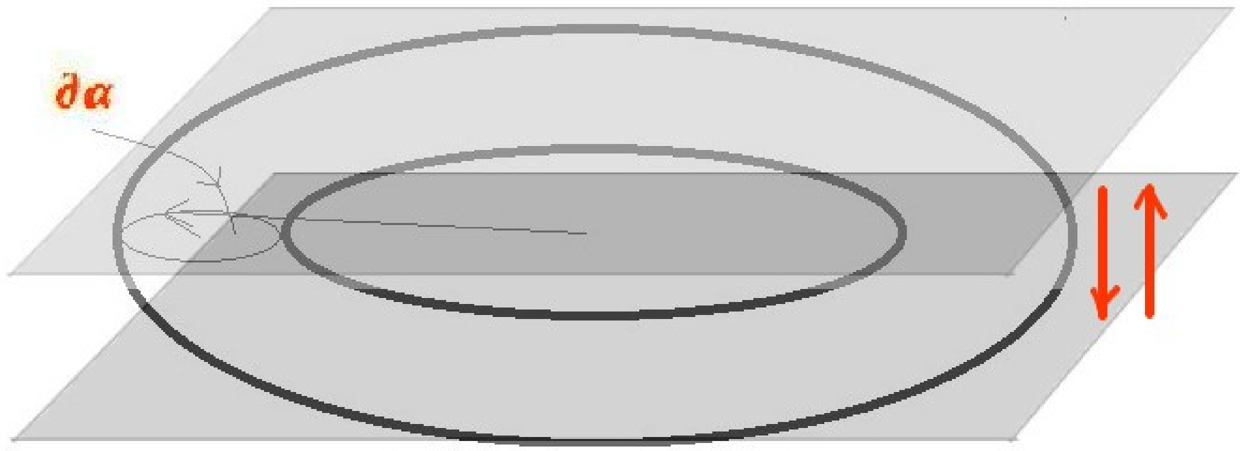
$$E_z = rsinn\omega t$$

How to proceed? I am proceeding in the hypothesis that the vector describing the toroidal helix is also interpretable as an electric field vector. Vector electric field forming part of a Maxwell field.
That is interpretable as $1\ i\ j\ T$ part of an analytic function.
So we have just seen that this would certainly be possible (it is certainly possible) for the E_x, E_y components.
It remains alone the single component E_z.

At this point a magic is possible.
It would not have been possible (it is not possible) if we consider Maxwell's equations in empty space in the strict sense, ie with 6 components. Instead it is possible if we consider them as Maxwell equations in the broad sense, then to 8 components.
E_z on its own it would not have been able to be an electric field component that was part of an analytic function . If instead E_z appears together with a fourth component $E_\tau = -H_\tau$,, the set of the two components can become a scalar wave that satisfies Maxwell's equations. A standing scalar wave, sum of a forward scalar wave (see for example [6]) and a reverse scalar wave. We can think that through a standing scalar electromagnetic wave we can add E_z, E_τ writing everything in the form
(19)

$$E_x = (R + rcosn\omega t)cos\omega t$$
$$E_y = (R + rcosn\omega t)sin\omega t$$
$$E_z = rsinn\omega t$$
$$E_\tau = -H_\tau = rcosn\omega t$$

Now it is interesting and perhaps not entirely irrelevant to note that this not only completes the analyticity but makes an electric charge appear.
In this case all the spatial part E_x, E_y, E_z describes a toroidal helix, and the $E_\tau = -H_\tau$ component becomes a charge which oscillates back and forth on a vertical dimension equal to the helix diameter.
The helix in (19) is wound n times at each revolution.



As you can see, unecessary to point out that the charge pattern suggested by (19) has little to do with the various models of literature, such as Consa, Hestenes, etc. These rather suggest a circulating ring, or toroidal helix, ie rotating and not pulsating up and down like (19).
I make some considerations in this regard in the following paragraph 5.

5-Considerations

I have seen how we can write the exact equations of an electromagnetic field in which the electric field vector E has a spatial part E_x, E_y, E_z that describes a toroidal helix, and the $E_\tau = -H_\tau$ component becomes a charge that oscillates back and forth on a vertical dimension equal to the diameter of the helix.
I mean, I don't have
written,
but I assumed that we could succeed in writing.
Proceeding then according to these same guidelines, we can hypothesize that we can write equations containing a rotating and non-pulsating charge , according to various models for example according to [7] , [8] .
But who knows what is right? Because this is an unexplored universe.
Example.
With $n = \frac{1}{2}$ il the electric field
(20)

$$E_x = \left(R + rcos\frac{1}{2}\omega t\right) cos\omega t$$
$$E_y = \left(R + rcos\frac{1}{2}\omega t\right) sin\omega t$$
$$E_z = rsin\frac{1}{2}\omega t$$
$$E_\tau = -H_\tau = rcos\frac{1}{2}\omega t$$

it assumes a "Moebius" pattern reminiscent of what I described semiquantitively in [9] .
What is right?
All these for the moment are only ideas, which suggest a possible line of research.

6 - Discussion

In the previous paragraphs I intended not only to suggest that an electromagnetic field (analytic function) having the form of a toroidal helix can be constructed, but also that it can be interpreted as a modulated signal.
As I wrote in the title, "modulated Maxwell signal".
In particular at the end of paragraph 4 above, we have seen that to complete the analyticity of a comprehensive analytic signal F we can, or perhaps better to say we must , also make use of the electric field components $E_\tau = -H_\tau$ which mean electrc charge.
In the case of (19) the whole spatial part E_x, E_y, E_z describes a toroidal helix, and the component $E_\tau = -H_\tau$ becomes a charge .
It doesn't seem to be a coincidence.
A portion of the signal that is a scalar field is born / needed. An electric charge.
This seemed suggestive to me as it is not wanted.
Besides being suggestive, I repeat it does not seem to be a coincidence. That is: the charge was not introduced "by force", but it appeared necessary.
It only depended on 'toroidal helix' condition and 'analyticity' condition.
It also suggests / confirms the validity of the Maxwell equations in the broad sense, therefore with 8 components, therefore in particular with electric charge.

I would now like to make some purely qualitative considerations here.
They concern the fact that in such a modulated signal - electromagnetic field (*) having the form of a toroidal helix - both of these things occur (they would occur) in a natural way:
1-quantization conditions;
2-relationships between all the geometric and physical parameters involved (**).
I proceed.
I refer to the formulas (19) or even (17), (18), with all the clarifications already mentioned.
Then I call $F2$ "carrier wave" and $\partial\alpha$ "modulating wave" (***).
Let $F2$ "carrier wave" characterized by amplitude and pulsation R, ω_R.
Let $\partial\alpha$ "modulating wave" characterized by amplitude and pulsation r, ω_r.
The modulated signal F is thus characterized by R, ω_R, r, ω_r.
At this point it is essential to note that all these parameters have a double interpretation:
geometric
and physics.

(*) I remind you again that here and throughout the text by " electromagnetic field" I mean it obeys to Maxwell's equations in the broad sense, therefore to 8 components, therefore in particular with electric charge.
(**) Everything is already present in [10] , "Knots in electromagnetism", M. Arrayás, D. Bouwmeester, JL Trueba. Except that I think this could be an alternative, more 'friendly' approach to manage and visualize.
(***) I keep these names, although usually the modulating wave has a frequency that is enormously lower than that of the carrier, whereas here the opposite happens or can happen.

6.1-Geometric interpretation

Geometrically speaking, the parameters determine the width and shape of a toroidal helix. However, if they ω_R, ω_r are arbitrary, particularly if their ratio is an irrational number, the helix never closes. Instead it closes if they ω_R, ω_r are quantized to integers. In other words, the helix closes after a certain number of turns, and is stable.

So geometrically, the closure gives rise to a quantization (or, if you want, quantization conditions determine the closure).

6.2-Physical interpretation .

We can do the reasoning in three steps.

First step:

I form an electromagnetic field, for example with the hypotheses already specified in paragraph 3 that is α real harmonic and $F2$ an arbitrary electromagnetic field.

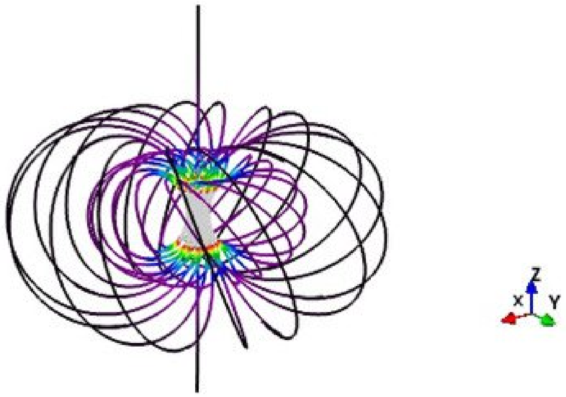
There is no problem, in the sense that the resulting function F is an electromagnetic field, in general also with charges and currents (except that it obeys physical conditions, such as not having infinite energy if integrated throughout the whole space).

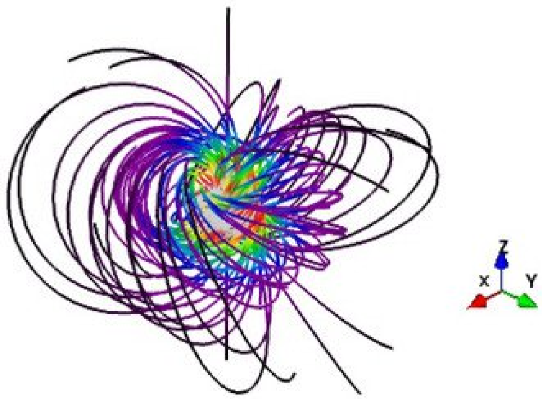
There is only one problem:

that this can be a "evolving" field, in the sense that it is never stable, is never periodically the same as itself, never obeys closed conditions, it is a "living" field, it does not represent any particular "entity" or elementary particle.

The figure (from [11]) serves to represent the situation. In the figure (below), taken from a completely different context and with a completely different meaning, the lines that are not closed represent the field.

If we suppose that we tried to represent a toroidal helix, we can say that part of the structure of the torus seems to still be present, but the field never closes.

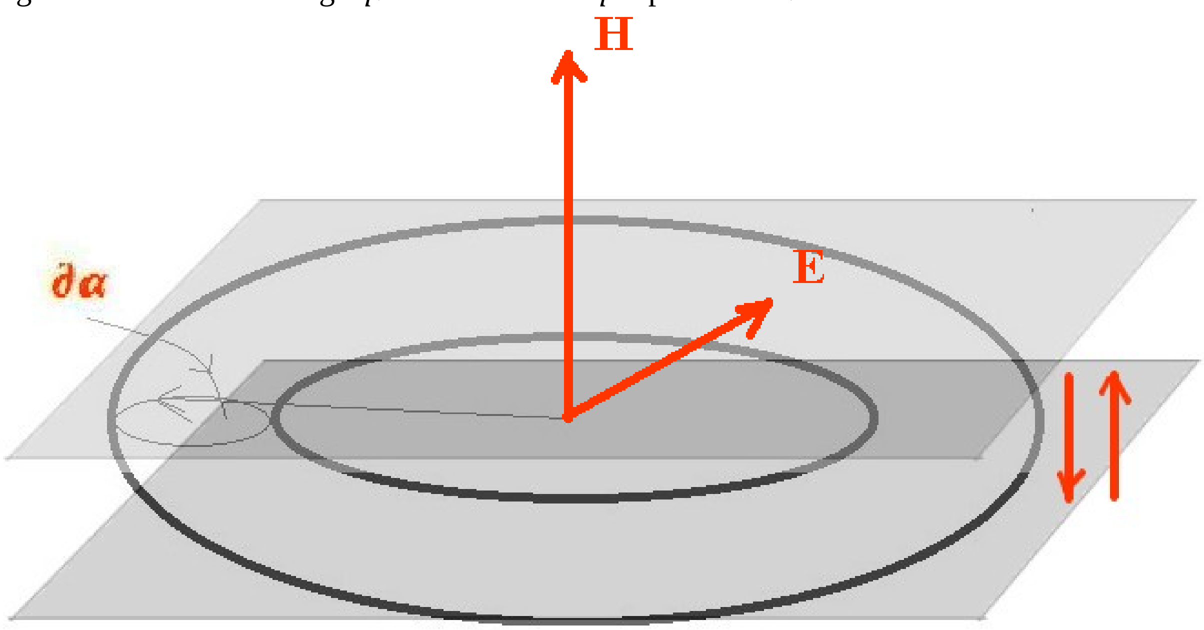




Second step:
while remaining in the field of analytic functions, I choose quantization conditions as additional conditions.
For example, trivially, if in the resulting function F the two pulsations ω_R, ω_r are arbitrary, in particular if their ratio is an irrational number, the field, eventually a helix, never closes.
If instead they ω_R, ω_r are quantized to integer numbers, the helix closes after a certain number of turns, and is stable. That is: if we want the helix to close after a certain number of turns, and it is stable, ω_R, ω_r must be quantized to integer numbers.
Once again the figure serves to represent the situation. In the figure (above), taken from a completely different context and with a completely different meaning, the closed lines represent the field.
If we suppose we tried to represent a toroidal helix, we can say that the structure of the torus is exactly present, the field "closes".
But there's more.

Third step:
We have seen that, given the analytic conditions, quantization conditions determine the closure (or, if you want, the closure gives rise to a quantization).
But in addition the field obeys Maxwell's equations.
What does this mean?
It means that (I jump directly to the conclusion) all the parameters R, ω_R, r, ω_r of the modulated signal F are so made;
a) - they are quantized, b) - they have a physical meaning, c) - they are interlaced.
That they have a physical meaning is obvious, that is an electromagnetic field so all its parameters have physical meaning. Such will be for example R because, for example, let $F2$ an electromagnetic field like that of a TE in a circular guide, at the cutoff frequency. This presents an electric field E that rotates as desired, and a perpendicular field H. Their amplitude goes with R.

Let the modulating wave be chosen with amplitude r.. The same r appears in the component that originates the electric charge q. So the value of q depends on r.



But, as I said, that analytic field F obeys Maxwell's equations.
Since it is an electromagnetic field in equilibrium, all the quantities will be subject to the formulas provided by the theory of electromagnetism, electric formulas. So it is likely that it H depends precisely on q.
If it H depends precisely on q, it follows that it R depends precisely on r, and this due to electric formulas.

This means that in that analytic function the modulation index $\frac{r}{R}$ will have a very specific value.
Similar things will also happen to all the other quantities involved.
So summarizing
quantized
having a physical meaning
and interlaced with each other by electric formulas.
More about the modulation index $\frac{r}{R}$:
the very specific value of the modulation index $\frac{r}{R}$ is without dimensions.
So it is a number.
More:
in that kind of field, eqs. (19), it does not depends on R, nor on r. As a matter of fact, the analicity equation $\partial^* F = 0$ is linear, so equations (19) must hold for AR and Ar. As indeed they do.
So the modulation index $\frac{Ar}{AR} = \frac{r}{R}$ is the same.
It is a number, but a constant, too.
It is suggestive to observe that, in some models of the electron found in the literature, the modulation index $\frac{r}{R}$ of a toroidal helix is assumed to be equal to the fine structure constant $alpha$, or $2alpha$, or so.

7 - Plane knots

As plane knot I mean here the projection of a given 3D knot onto a 2D plane.
That said, I intend to show a connection between the modulated Maxwell signals and the knots. And to do this I refer to the 2D plane.We defined in (17) a modulated Maxwell signal this way:
(17)

$$F = \left(1 + \frac{r}{R}\partial\alpha\right)F2$$

In the $z = (x + iy) = \rho e^{i\varphi}$ plane let's choose, following the instructions, the following functions: F2 as analytic, namely:
(28)

$$F2 = z = \rho e^{i\varphi}$$

and alpha as the real part of an analytic function, namely:
(29)

$$\alpha = \mathcal{R}e\left(\frac{2}{n+1}z^{n+1}\right) = \frac{2}{n+1}\cos\,(n+1)\varphi$$

Inserting the polar operator ∂
(30)

$$\partial = e^{-i\varphi}\left(\frac{\partial}{\partial\rho} - i\frac{1}{\rho}\frac{\partial}{\partial\varphi}\right)$$

we obtain from (29), (30), after a lengthy calculation:
(31)

$$\partial\alpha = z^n$$

and substituting (28), (31) in (17):
(32)

$$F = \left(1 + \frac{r}{R}\partial\alpha\right)F2 = \left(1 + \frac{r}{R}z^n\right)z$$

The modulated signal F is analytic , as indeed it must be.
To see the connection with knots, let's take the real part
(33)

$$\left(1 + \frac{r}{R}\mathcal{R}ez^n\right)z = \left(1 + \frac{r}{R}\rho^n\cos n\varphi\right)\rho e^{i\varphi}$$

Plotting this for $\rho = 1$ i.e.
(34)

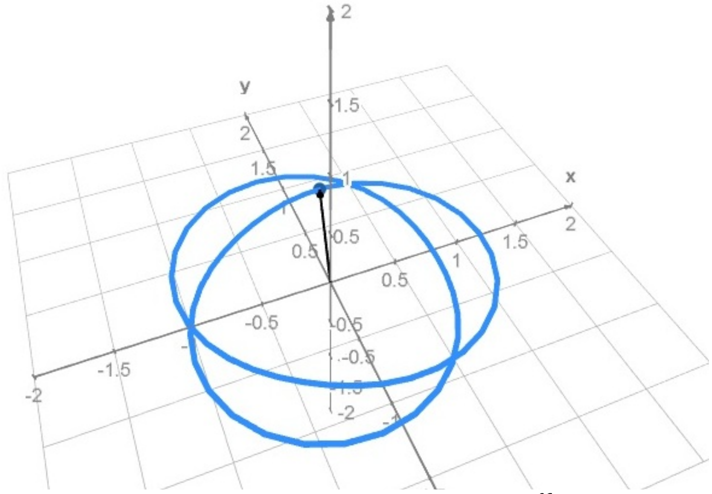
$$\left[\left(1 + \frac{r}{R}\mathcal{R}ez^n\right)z\right]_{\rho=1} = \left(1 + \frac{r}{R}\cos n\varphi\right)e^{i\varphi}$$

we get what in the language of electrical engineering we could call "a complex phasor $e^{i\varphi}$ modulated by the real signal $\cos n\varphi$ ". Which obviously becomes generalized to any value p, q.
(35)

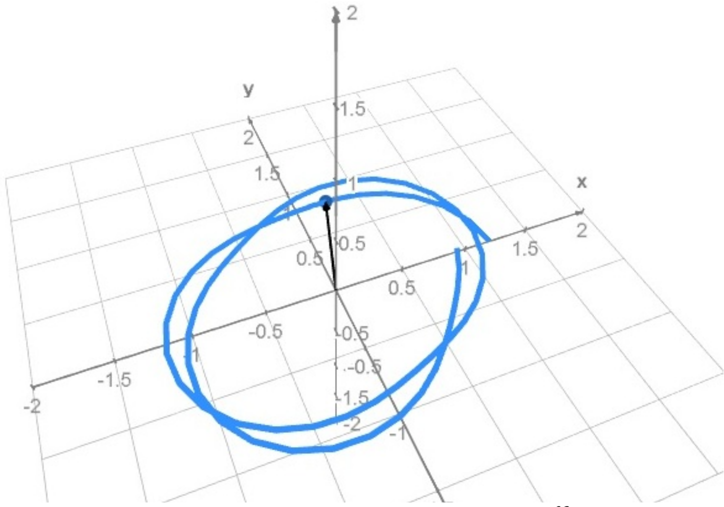
$$\left(1 + \frac{r}{R}\cos p\varphi\right)e^{iq\varphi}$$

That is in the language of the nodes the plane projection of any node T(p,q).
The following images show some example.

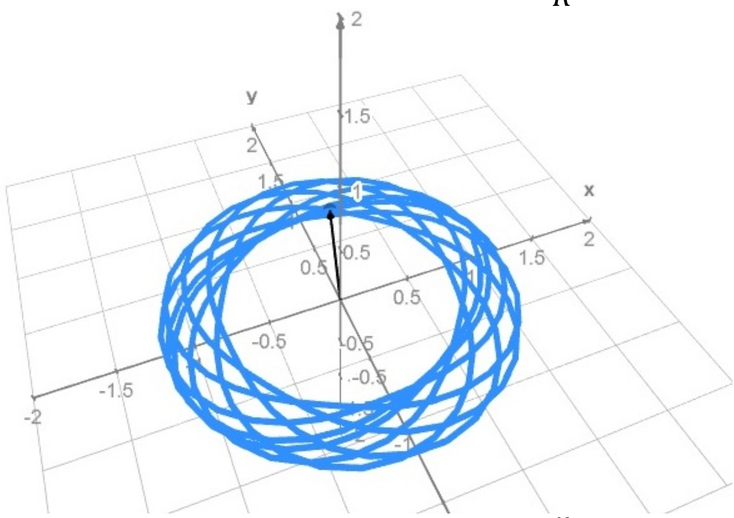
Plane plots of (35) for various (p,q).



$(p,q) = (3,2), modulation\ index \frac{r}{R} = 0.2, \varphi = 0 - 2\pi$



$(p,q) = (3.7,2), modulation\ index \frac{r}{R} = 0.2, \varphi = 0 - 2\pi$



$(p,q) = (3.7,2), modulation\ index \frac{r}{R} = 0.2, \varphi = 0 - 8\pi$

8 - Conclusions

A method is presented to generate strange or complicated solutions of Maxwell's equations, possibly toroidal solutions and in any case, more generally , solutions that are in effect "modulated Maxwell signals". These solutions, quite unusual, I think they comprise or extend the solutions presented by Ranada and that have recently been the subject of numerous studies. However, the method presented here is more easily interpreted by electromagnetic or electronic engineers.

8-References

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Appendix 1 .

From [12] , Trefoil torus .
"Start with the parametrization of the torus surface:
(9)
x=(a+bcosu)cosv,
y=(a+bcosu)sinv,
z=bsinu.
On that surface we get a curve that sits inside the trefoil. You get a parametrization of that curve $\vec{\gamma}$ =(x,y,z) by setting u=3s,v=2s and letting s range over the interval [0,2π]. This way you get a curve that wraps around the hole of the donut twice, and around the tube thrice".

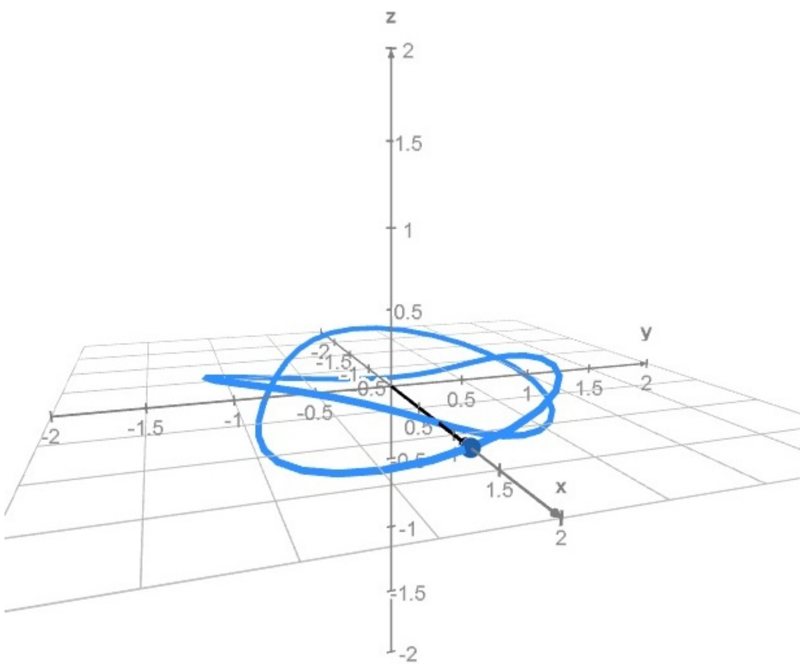
By comparison with (8)

$$E_x = (R + r cos n\omega t) cos\omega t$$
$$E_y = (R + r cos n\omega t) sin\omega t$$
$$E_z = r sin n\omega t$$
$$E_\tau = -H_\tau = ??$$

it turns out:
(10)

$$\boldsymbol{E_x = \left(1 + \frac{r}{R} cos3\omega t\right) cos2\omega t}$$
$$\boldsymbol{E_y = \left(1 + \frac{r}{R} cos3\omega t\right) sin2\omega t}$$
$$\boldsymbol{E_z = \frac{r}{R} sin3\omega t}$$
$$\boldsymbol{E_\tau = -H_\tau = ??}$$

In figure, (r/R)=0.2 and omega*t = [0,2π].



Appendix 2

With reference to [13], the formula of a Moebius ring on a torus is
(11)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(s)+(t)*\left(\cos\left(\frac{s}{2}\right)\right)*(\cos(s)) \\ \sin(s)+(t)*\left(\cos\left(\frac{s}{2}\right)\right)*(\sin(s)) \\ (t)*\left(\sin\left(\frac{s}{2}\right)\right) \end{pmatrix}$$

where s ranges from 0 to 2π and t ranges typically from -0.4 to 0.4

Always by comparison with (8)

$$E_x = (R + r\cos n\omega t)\cos\omega t$$
$$E_y = (R + r\cos n\omega t)\sin\omega t$$
$$E_z = r\sin n\omega t$$
$$E_\tau = -H_\tau = ??$$

it turns out:
(12)

$$E_x = \left(1 + \frac{r}{R}\cos 0.5\omega t\right)\cos\omega t$$
$$E_y = \left(1 + \frac{r}{R}\cos 0.5\omega t\right)\sin\omega t$$
$$E_z = \frac{r}{R}\sin 0.5\omega t$$
$$E_\tau = -H_\tau = ??$$

That is n = 1/2, of course, I would say.
In figure, (r/R)=0.2 and omega*t = [0,2π].