THE COLLATZ CONJECTURE
Order and harmony in the sequence numbers

by

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Abstract: I propose a numerical table that demonstrates visually that the sequences formed with Collatz's algorithm always reach 1.

Keywords: Collatz Conjecture, 3n+1 problem, number theory.

Introduction: The conjecture was raised by the mathematician Lothar Collatz in 1937. It is also known as other names: The 3n + 1 problem, the Ulam conjecture, the Kakutani problem, the conjecture of Thwaites, the Hasse algorithm or the Syracuse problem.

The conjecture reads as follows:

1 - Any natural number is chosen, n.
2 - If it is even divide by 2, (n / 2).
3 - If odd, multiply by 3 and add 1 to the result, (3n + 1).

The process is repeated with each result and a sequence is obtained that always ends in 1 and always It is so, whatever the initial number. The unknown is why it happens and if it happens with all natural numbers.

Overview: For the Collatz conjecture, I identify two types of odd numbers: Those of the form 4n + 3 and those of the 4n + 1 form.
Applying (3m + 1) / 2 to those of the form 4n + 3, an odd number is found that is greater than the previous one and the sequence is ascending.

Applying the same operation to those of the 4n + 1 form, it is an even number that requires more divisions by 2, so the odd number they arrive at is always smaller than the previous one and the sequence is descending.

A sequence of Collatz will be more or less descending and more or less long as obtained more or less odd numbers of the form 4n + 1.

If in the set of odd numbers there are the same number of both, do they have the same probability of going out in the Collatz sequences? Can you predict how many will there be and at what moment the numbers of the form 4n + 1 will appear and the sequence will descend?

I also identify two types of even numbers: Those of the form 4n + 2 and those of the form 4n + 4. Those of the form 4n + 2 admit a single division n / 2 and the odd number that results is greater than the previous one and the sequence is ascending.

Those of the form 4n + 4 admit two or more divisions n / 2 and the odd number they arrive at is smaller than the previous one and the sequence is descending.

A sequence of Collatz will be more or less descending and more or less long as more or less even numbers of the form 4n + 4 are obtained.

If in the set of even numbers there are the same number of both, do they have the same probability of going out in the Collatz sequences? Can you predict how many there will be and at what time the numbers of the 4n + 4 form will appear and the sequence will descend?

The answers to these two questions may be found in two tables, one for odd numbers and one for even numbers, which I explain below.

TABLE OF THE ODD NUMBERS OF THE SEQUENCES

On a table with k columns, I write in the first row the numbers 2k-1.
In the successive rows I apply (3m + 1) / 2 to the odd numbers, leaving the even number as the last number of each column.
In red they are the odd numbers of the form 4n + 3 and those of green are the odd numbers of the form 4n + 1. Even numbers have been left without color.

The value of n of the last number of the columns corresponds to the number of odd numbers and the number of applications (3m + 1) / 2 in them.

Example: In column k (48) there are 5 odd numbers and the odd 95 needs 5 steps to reach the even number 728.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
n																																																				
0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	. . .
1	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	
2		8		17		26		35		44		53		62		71		80		89		98		107		116		125		134		143		152		161		170		179		188		197		206		215		224		
3				26				53				80				107				134				161				188				215				242				269				296				323				
4								80								161								242								323								404								485				
5																242																485																728				
6																																728																				
7																																																				

k	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	. . .
n																																																				
0	103	105	107	109	111	113	115	117	119	121	123	125	127	129	131	133	135	137	139	141	143	145	147	149	151	153	155	157	159	161	163	165	167	169	171	173	175	177	179	181	183	185	187	189	191	193	195	197	199	201	203	. . .
1	155	158	161	164	167	170	173	176	179	182	185	188	191	194	197	200	203	206	209	212	215	218	221	224	227	230	233	236	239	242	245	248	251	254	257	260	263	266	269	272	275	278	281	284	287	290	293	296	299	302	305	. . .
2	233		242		251		260		269		278		287		296		305		314		323		332		341		350		359		368		377		386		395		404		413		422		431		440		449		458	
3	350				377				404				431				458				485				512				539				566				593				620				647				674			
4					566								647								728								809								890								971							
5													971																1214																1457							
6													1457																																2186							
7													2186																																							

This formula defines the number in the table: a (n) = 2k (3/2) ^ n-1

In each column: a (n + 1) = a (n) + k (3/2) ^ n

The amount of numbers in each column is always the same, depending on the value of k:

		n		valor de k												cantidad de números por columna
2k-1		0		1	3	5	7	9	11	13	15	17	. . .			0 impares rojos, 1 número impar verde y un número par
2(2k-1)		1		2	6	10	14	18	22	26	30	34	. . .			1 número impar rojo, 1 número impar verde y un número par
4(2k-1)		2		4	12	20	28	36	44	52	60	68	. . .			2 números impares rojos, 1 número impar verde y un número par
8(2k-1)		3		8	24	40	56	72	88	104	120	136	. . .			3 números impares rojos, 1 número impar verde y un número par
16(2k-1)		4		16	48	80	112	144	176	208	240	272	. . .			4 números impares rojos, 1 número impar verde y un número par
32(2k-1)		5		32	96	160	224	288	352	416	480	544	. . .			5 números impares rojos, 1 número impar verde y un número par
64(2k-1)		6		64	192	320	448	576	704	832	960	1088	. . .			6 números impares rojos, 1 número impar verde y un número par
. . .		. . .		. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .			. . .

In each of the columns k = 2 ^ n * (2k-1) there are n odd red numbers, one last green odd number and the even number that closes the column.

Example: in column k = 2 ^ 50 there are 50 red odd numbers, the first is number 2251799813685247 and the 50th is 957197316922470118360331 and the last odd of the column, which is green, is number 1435795975383705177540497. The even number that closes The column is 2153693963075557766310746. In Annex 1, the sequence development started with this first column number.

They will also have the same amount of numbers each of the columns k = (2k-1) * 2 ^ 50.

A sequence obtained with the Collatz conjecture algorithm is formed by an indeterminate number of columns in the table or cycles.

A sequence started with the first odd number of any column k = (2k-1) * 2 ^ 1000000 would have in that column, 1000000 odd red, one last green odd and an even number closing the column. There will be the same odd number in all columns, for every value of (2k-1).

If the sequence were started with a power number of 2 close to infinity, the first column of the table or cycle of that sequence would have that same amount of odd numbers.

The table is infinite but all its columns or cycles are bounded geometric progressions.

Example of a sequence starting with the number 279:

279, 838, 419, 1258, 629, 1888, 944, 472, 236, 118, 59, 178, 89, 268, 134, 67, 202, 101, 304, 152,
k(140) k(30) k(34)

76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.
k(10) k(6) k(7) k(3) k(1)

The columns involved in the sequence:

140	30	34	10	6	7	3	1
279	59	67	19	11	13	5	1
419	89	101	29	17	20	8	2
629	134	152	44	26			
944							

The sequence obtained starting with the number 27 and the 18 columns or cycles that form it:

k	14	16	61	46	52	88	223	84	142	160	456	289	217	163	31	12	3	1
n																		
0	27	31	121	91	103	175	445	167	283	319	911	577	433	325	61	23	5	1
1	41	47	182	137	155	263	668	251	425	479	1367	866	650	488	92	35	8	2
2	62	71		206	233	395		377	638	719	2051					53		
3		107			350	593		566		1079	3077					80		
4		161				890				1619	4616							
5		242								2429								
6										3644								
7																		
8																		
9																		

The same columns without the odd ones of the form 4n + 3:

k	14	16	61	46	52	88	223	84	142	160	456	289	217	163	31	12	3	1
n																		
0			121				445					577	433	325	61		5	1
1	41		182	137			668		425			866	650	488	92		8	2
2	62			206	233			377	638							53		
3					350	593		566			3077					80		
4		161				890					4616							
5		242								2429								
6										3644								
7																		
8																		

The odd green ones followed by the even number at the end of each column k (even) are also in the columns k (odd).

K	**21**	**81**	**61**	**69**	**117**	**297**	**223**	**189**	**213**	**1215**	**1539**	**289**	**217**	**163**	**31**	**27**	**3**	**1**
	41	161	121	137	233	593	445	377	425	2429	3077	577	433	325	61	53	5	1
	62	242	182	206	350	890	668	566	638	3644	4616	866	650	488	92	80	8	2

Equivalence of k even and **K** odd : k(3/2)^n=**K** Example: k160*(3/2)^5=**K**1215

At each repetition of the algorithm operation (3m + 1) / 2, applied to the first odd number of the column, the sequence “jumps” from one column to another, leaving it at a single step to reach the even number that can be divided at least 2^2. This will always occur in any sequence until the green odd is in row n (0).

An example of the evolution of the columns of the table in which “loses” an odd number in each jump, k * 3/2:

32*3/2=48, 48*3/2=72, 72*3/2=108, 108*3/2=162 y 162*3/2=243 32*(3/2)^5=243

k = 32, **K** = 243

n \ k	32	48	72	108	162	243
0	63	95	143	215	323	485
1	95	143	215	323	485	728
2	143	215	323	485	728	
3	215	323	485	728		
4	323	485	728			
5	485	728				
6	728					
7						

Column k (32) contains the odd numbers and an even number of a Collatz sequence from number 63 to number 728, occupying rows n (0) through n (6).
The same numbers are in columns k (32), k (48), k (72), k (108), k (162) and k (243), occupying row n (0), because in a sequence there may be the numbers of any of these columns and not necessarily all of them.

The cycle that follows the previous one is that of the column of the number 91, because 728/2 ^ 3 = 91

n \ k	46	69
0	91	137
1	137	206
2	206	
3		
4		
5		
6		
7		

The next cycle: 206/2 = 103

n \ k	52	78	117
0	103	155	233
1	155	233	350
2	233	350	
3	350		
4			
5			
6			
7			

It is followed by eleven more cycles until the number 5 results, which heads the last column before reaching 1.

Odd numbers of the form 4n + 3 always arrive at an odd number of the form 4n + 1.

THE EVEN NUMBERS OF SEQUENCES IN A TABLE

On a table with k columns, we write in the first row the numbers 2k.
In the successive rows, the even numbers are divided by 2, leaving the odd number as the last number in each column.

In red they are even numbers that divide them by 2 produce another even number and green ones are even numbers that divide by 2 produce an odd number.
Odd numbers have been left without color.

The value of n of the last number of the columns corresponds to the number of even numbers and the number of divisions between 2 in them.

Example: In column k (48) there are 5 even numbers and the pair 96 needs 5 steps to reach the odd number 3.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	...
n																																																				
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	...
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	...
2		1		2		3		4		5		6		7		8		9		10		11		12		13		14		15		16		17		18		19		20		21		22		23		24		25		...
3				1				2				3				4				5				6				7				8				9				10				11				12				...
4								1								2								3								4								5								6				...
5																1																2																3				...
6																																1																				...
7																																																				

	k	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	...
n																																																					
0		104	106	108	110	112	114	116	118	120	122	124	126	128	130	132	134	136	138	140	142	144	146	148	150	152	154	156	158	160	162	164	166	168	170	172	174	176	178	180	182	184	186	188	190	192	194	196	198	200	202	204	...
1		52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	...
2		26		27		28		29		30		31		32		33		34		35		36		37		38		39		40		41		42		43		44		45		46		47		48		49		50		51	...
3		13				14				15				16				17				18				19				20				21				22				23				24				25			...
4						7								8								9								10								11								12							...
5														4																5																6							...
6														2																																3							...
7														1																																							...
8																																																					

The amount of numbers in each column is always the same, depending on the value of k:

		n		valor de k												cantidad de números por columna
2k-1		0		1	3	5	7	9	11	13	15	17	. . .			0 pares rojos, 1 número par verde y un número impar
2(2k-1)		1		2	6	10	14	18	22	26	30	34	. . .			1 número par rojo, 1 número par verde y un número impar
4(2k-1)		2		4	12	20	28	36	44	52	60	68	. . .			2 números pares rojos, 1 número par verde y un número impar
8(2k-1)		3		8	24	40	56	72	88	104	120	136	. . .			3 números pares rojos, 1 número par verde y un número impar
16(2k-1)		4		16	48	80	112	144	176	208	240	272	. . .			4 números pares rojos, 1 número par verde y un número impar
32(2k-1)		5		32	96	160	224	288	352	416	480	544	. . .			5 números pares rojos, 1 número par verde y un número impar
64(2k-1)		6		64	192	320	448	576	704	832	960	1088	. . .			6 números pares rojos, 1 número par verde y un número impar
. . .		. . .		. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .			. . .

In each of the columns k = 2 ^ n * (2k-1) there are n red even numbers, one last green pair and the odd number that closes the column.

Examples: In column k = 2 ^ 50 there are 50 red even numbers, the first one is number 2251799813685248 and the 50th is 4 and the last pair of the column, which is green, is number 2. The odd number that closes the column is 1.

They would also have the same amount of numbers each of the columns k = (2k-1) * 2 ^ 50.

Having both tables the same amount of numbers in their columns with the same value of k and the same value of n and the same amount of red numbers, green numbers and numbers without color, could a single table be formed to develop the sequence path from Collatz ?.

Comparison of the two tables:

Odd numbers:

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
n																																																				
0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	. . .
1	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	
2		8		17		26		35		44		53		62		71		80		89		98		107		116		125		134		143		152		161		170		179		188		197		206		215		224		
3				26				53				80				107				134				161				188				215				242				269				296				323				
4								80								161								242								323								404								485				
5																242																485																728				
6																																728																				
7																																																				

Even numbers:

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
n																																																				
0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	. . .
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
2		1		2		3		4		5		6		7		8		9		10		11		12		13		14		15		16		17		18		19		20		21		22		23		24		25		. . .
3				1				2				3				4				5				6				7				8				9				10				11				12				. . .
4								1								2								3								4								5								6				. . .
5																1																2																3				. . .
6																																1																				. . .
7																																																				

With the columns of the table of even numbers, we can form the column with the same value of k from the table of odd numbers.

Example: column k (4)

From even to odd:	From odd to even:
1*8-1=7	(7+1)/1=8
3*4-1=11	(11+1)/3=4
9*2-1=17	(17+1)/9=2
27*1-1=26	(26+1)/27=1
3^n*even-1=odd	**(odd+1)/3^n=even**

A TRIANGLE WITH THE COLUMNS OF THE TABLE

A triangle T (n):

A number n is written at the top vertex and the triangle is formed as follows:
For the columns, multiply n by 2 and add 1 to the result, 2n + 1. Repeat with the results obtained.
For the rows, multiply n by 3, add 1 to the result and divide by 2, (3n + 1) / 2. Repeat until you reach an even number.
A triangle T (n) for each value of (n) = 0, 4, 6, 10, 12, 16, 18, 22, 24,. . . . and we can write on them all odd numbers.

The triangle T (0):

0											
1	2										
3	5	8									
7	11	17	26								
15	23	35	53	80							
31	47	71	107	161	242						
63	95	143	215	323	485	728					
127	191	287	431	647	971	1457	2186				
255	383	575	863	1295	1943	2915	4373	6560			
511	767	1151	1727	2591	3887	5831	8747	13121	19682		
1023	1535	2303	3455	5183	7775	11663	17495	26243	39365	59048	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

Each row k of the triangle is column k of the table and the value of k is any number of the triangle plus 1.

0										
1	2									
3	5	8								
7	11	17	26							
15	23	35	53	80						
31	47	71	107	161	242					
63	95	143	215	323	485	728				
127	191	287	431	647	971	1457	2186			
255	383	575	863	1295	1943	2915	4373	6560		
511	767	1151	1727	2591	3887	5831	8747	13121	19682	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

Examples:

Column k (8) of the table has the numbers 15, 23, 35, 53 and 80.
Column k (12) of the table has the numbers 23, 35, 53 and 80.
Column k (18) of the table has the numbers 35, 53 and 80.
Column k (27) of the table has the numbers 53 and 80.

8	12	18	27
15	23	35	53
23	35	53	80
35	53	80	
53	80		
80			

The triangle T(4):

4											
9	14										
19	29	44									
39	59	89	134								
79	119	179	269	404							
159	239	359	539	809	1214						
319	479	719	1079	1619	2429	3644					
639	959	1439	2159	3239	4859	7289	10934				
1279	1919	2879	4319	6479	9719	14579	21869	32804			
2559	3839	5759	8639	12959	19439	29159	43739	65609	98414		
5119	7679	11519	17279	25919	38879	58319	87479	131219	196829	295244	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

The triangle T(6):

6											
13	20										
27	41	62									
55	83	125	188								
111	167	251	377	566							
223	335	503	755	1133	1700						
447	671	1007	1511	2267	3401	5102					
895	1343	2015	3023	4535	6803	10205	15308				
1791	2687	4031	6047	9071	13607	20411	30617	45926			
3583	5375	8063	12095	18143	27215	40823	61235	91853	137780		
7167	10751	16127	24191	36287	54431	81647	122471	183707	275561	413342	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

The triangle T (n) that contains the first odd number of the Collatz sequence is what will be the beginning or first cycle of it.

In the columns of the table and in the rows of the triangles, the numbers:
a (n + 1) = (a (n) * 3 + 1) / 2.

MULTIPLY BY 3 AND DIVIDE BETWEEN 2

If in triangle T (0) I add 1 to their numbers, another triangle T (1) results:

1											
2	3										
4	6	9									
8	12	18	27								
16	24	36	54	81							
32	48	72	108	162	243						
64	96	144	216	324	486	729					
128	192	288	432	648	972	1458	2187				
256	384	576	864	1296	1944	2916	4374	6561			
512	768	1152	1728	2592	3888	5832	8748	13122	19683		
1024	1536	2304	3456	5184	7776	11664	17496	26244	39366	59049	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

In each column of this triangle, the even numbers of the sequences and the odd numbers they reach after being subjected to n divisions by 2, which coincide with the columns in the table of even numbers.

A triangle T (n) for each value of (n) = 1, 5, 7, 11, 13, 17, 19, 23, 25,. . . . (6n-1, 6n + 1) and we can write on them all even numbers.

In the rows of the triangle, each number: a (n + 1) = a (n) * 3/2. All have as prime factors 3 and 2.

Multiplying by 3 and dividing by 2 is the script with which a sequence obtained with the Collatz algorithm is developed and this keeps the odd ones in the sequence in the same row.

Example: 8 * 3 = 24, 24/2 = 12, 12 * 3 = 36, 36/2 = 18, 18 * 3 = 54, 54/2 = 27. (numbers 7, 11, 17, 26 of the sequence).

To develop a sequence of Collatz in this triangle and to see its entire route, from its beginning until it reaches 1, we form a table with the triangle as a base.
Starting with each last number of the rows of the triangle, we apply these two operations: To the odd we add 1 and to the even we divide it by 2.
We write each result above the previous one, in the same column, until we reach 1.
In this extension of the triangle, the numbers are in red.

														1	1	. . .
												1	1	2	2	. . .
											1	2	2	3	4	. . .
									1	1	2	3	4	5	7	. . .
							1	1	2	2	3	5	7	10	14	. . .
						1	2	2	3	4	6	9	13	19	28	. . .
				1	1	2	3	4	5	8	11	17	25	37	55	. . .
		1	1	2	2	3	5	7	10	15	22	33	49	73	110	. . .
	1	2	2	3	4	6	9	13	20	29	44	65	98	146	219	. . .
1	2	3	4	6	8	12	18	26	39	58	87	130	195	292	438	. . .
2	3	5	7	11	16	23	35	52	77	116	173	260	390	584	876	. . .
4	6	9	14	21	31	46	69	103	154	231	346	519	779	1168	1752	. . .
8	12	18	27	41	61	92	137	206	308	462	692	1038	1557	2336	3504	. . .
16	24	36	54	81	122	183	274	411	616	923	1384	2076	3114	4671	7007	. . .
32	48	72	108	162	243	365	547	821	1231	1846	2768	4152	6228	9342	14013	. . .
64	96	144	216	324	486	729	1094	1641	2461	3691	5536	8304	12456	18684	28026	. . .
128	192	288	432	648	972	1458	2187	3281	4921	7382	11072	16608	24912	37367	56051	. . .
256	384	576	864	1296	1944	2916	4374	6561	9842	14763	22144	33216	49823	74734	112101	. . .
512	768	1152	1728	2592	3888	5832	8748	13122	19683	29525	44287	66431	99646	149468	224202	. . .
1024	1536	2304	3456	5184	7776	11664	17496	26244	39366	59049	88574	132861	199291	298936	448404	. . .
2048	3072	4608	6912	10368	15552	23328	34992	52488	78732	118098	177147	265721	398581	597872	896807	. . .
4096	6144	9216	13824	20736	31104	46656	69984	104976	157464	236196	354294	531441	797162	1195743	1793614	. . .
8192	12288	18432	27648	41472	62208	93312	139968	209952	314928	472392	708588	1062882	1594323	2391485	3587227	. . .
16384	24576	36864	55296	82944	124416	186624	279936	419904	629856	944784	1417176	2125764	3188646	4782969	7174454	. . .
32768	49152	73728	110592	165888	248832	373248	559872	839808	1259712	1889568	2834352	4251528	6377292	9565938	14348907	. . .
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

In this table we can visualize any sequence of Collatz, whose first odd number of it, is in the triangle. If it is not, it can be displayed in the table whose base or triangle contains that number. All sequences started with the odd numbers of a triangle row will have the same trajectory.

Like the triangles, the tables obtained from them, are unique for each T(n) and the representation of the sequences will be made in the table formed from the triangle that contains the first odd number of the same.

For example, to visualize the sequence started with the number 508, whose first odd number is 127, we will represent it in the table of triangle T(1), because in row 2^8 there is the number 127.

																. . .
														1	1	. . .
												1	1	2	2	. . .
											1	2	2	3	4	. . .
									1	1	2	3	4	5	7	. . .
							1	1	2	2	3	5	7	10	14	. . .
						1	2	2	3	4	6	9	13	19	28	. . .
				1	1	2	3	4	5	8	11	17	25	37	55	. . .
		1	1	2	2	3	5	7	10	15	22	33	49	73	110	. . .
	1	2	2	3	4	6	9	13	20	29	44	65	98	146	219	. . .
1	2	3	4	6	8	12	18	26	39	58	87	130	195	292	438	. . .
2	3	5	7	11	16	23	35	52	77	116	173	260	390	584	876	. . .
4	6	9	14	21	31	46	69	103	154	231	346	519	779	1168	1752	. . .
8	12	18	27	41	61	92	137	206	308	462	692	1038	1557	2336	3504	. . .
16	24	36	54	81	122	183	274	411	616	923	1384	2076	3114	4671	7007	. . .
32	48	72	108	162	243	365	547	821	1231	1846	2768	4152	6228	9342	14013	. . .
64	96	144	216	324	486	729	1094	1641	2461	3691	5536	8304	12456	18684	28026	. . .
128	192	288	432	648	972	1458	2187	3281	4921	7382	11072	16608	24912	37367	56051	. . .
256	384	576	864	1296	1944	2916	4374	6561	9842	14763	22144	33216	49823	74734	112101	. . .
512	768	1152	1728	2592	3888	5832	8748	13122	19683	29525	44287	66431	99646	149468	224202	. . .
1024	1536	2304	3456	5184	7776	11664	17496	26244	39366	59049	88574	132861	199291	298936	448404	. . .
2048	3072	4608	6912	10368	15552	23328	34992	52488	78732	118098	177147	265721	398581	597872	896807	. . .
4096	6144	9216	13824	20736	31104	46656	69984	104976	157464	236196	354294	531441	797162	1195743	1793614	. . .
8192	12288	18432	27648	41472	62208	93312	139968	209952	314928	472392	708588	1062882	1594323	2391485	3587227	. . .
16384	24576	36864	55296	82944	124416	186624	279936	419904	629856	944784	1417176	2125764	3188646	4782969	7174454	. . .
32768	49152	73728	110592	165888	248832	373248	559872	839808	1259712	1889568	2834352	4251528	6377292	9565938	14348907	. . .
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

The sequence obtained with the Collatz algorithm, started with the number 508:

508, 254, 127, 382, 191, 574, 287, 862, 431, 1294, 647, 1942, 971, 2914, 1457, 4372, 2186, 1093, 3280, 1640, 820, 410, 205, 616, 308, 154, 77, 232, 116, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

The black numbers are in the base triangle and the red ones are in the extension of this one.

The odd ones of the triangle: 127, 191, 287, 431, 647, 971, 1457 are in the same row, that is, none of them moves away from the 1, descending to lower rows.

The same happens with the odd ones of the red zone, none descends to lower rows and only when changing the column it descends to the lower row, but goes back up with the division between 2.

Each column change occurs when the odd number is multiplied by 3. The more odd the sequence has, the more column changes there will be and the longer it will reach 1.

Example: In the following table the odd 19 descends a row to the number 57 when it is multiplied by 3, but goes back up to 29 when the number 58 is divided by 2.

The development of a Collatz sequence started with the number 39:

												. . .
										1	1	. . .
								1	1	2	2	. . .
							1	2	2	3	4	. . .
					1	1	2	3	4	5	7	. . .
			1	1	2	2	3	5	7	10	14	. . .
		1	2	2	3	4	6	9	13	19	28	. . .
1	1	2	3	4	5	8	11	17	25	37	55	. . .
2	2	3	5	7	10	15	22	33	49	73	109	. . .
3	4	6	9	13	19	29	43	65	97	145	217	. . .
5	8	12	17	26	38	57	86	129	193	289	433	. . .
10	15	23	34	51	76	114	171	257	385	577	865	. . .
20	30	45	68	102	152	228	342	513	769	1154	1730	. . .
40	60	90	135	203	304	456	684	1026	1538	2307	3460	. . .
80	120	180	270	405	608	912	1367	2051	3076	4614	6920	. . .
160	240	360	540	810	1215	1823	2734	4101	6151	9227	13840	. . .
320	480	720	1080	1620	2430	3645	5468	8202	12302	18453	27680	. . .
640	960	1440	2160	3240	4860	7290	10935	16403	24604	36906	55359	. . .
1280	1920	2880	4320	6480	9720	14580	21870	32805	49208	73812	110717	. . .
2560	3840	5760	8640	12960	19440	29160	43740	65610	98415	147623	221434	. . .
5120	7680	11520	17280	25920	38880	58320	87480	131220	196830	295245	442868	. . .
10240	15360	23040	34560	51840	77760	116640	174960	262440	393660	590490	885735	. . .
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

The sequence obtained with the Collatz algorithm, started with the number 39:

39, 118, 59, 178, 89, 268, 134, 67, 202, 101, 304, 152, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

The sequence runs through the table developed from triangle T (5), because the first odd number in the sequence, 39, is in this triangle. The sequences started with the numbers 59, 89 and 134 will have the same route.

For triangle extension numbers, (in red), if 1 is subtracted from the odd number instead of adding 1, the table is equally valid:

Impar-1

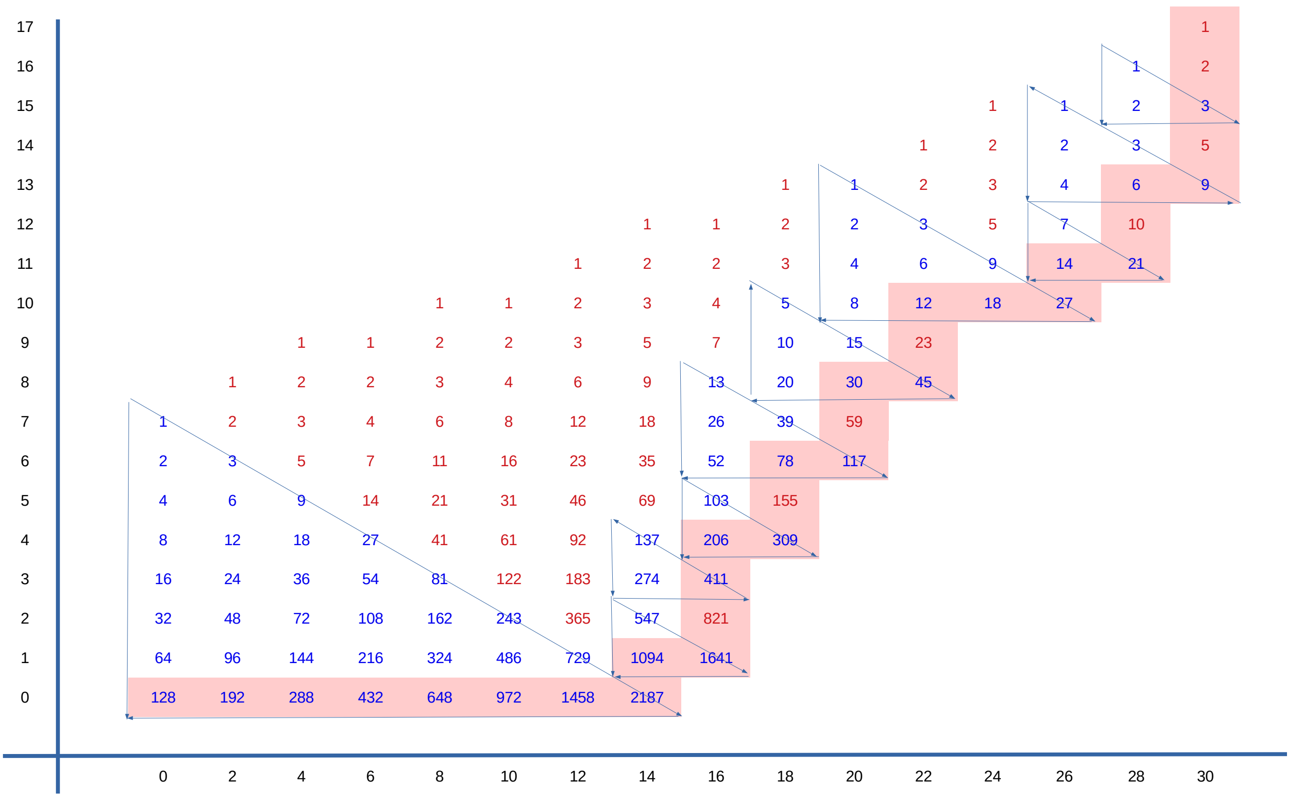
	0	1	2	3	4	5	6	7
11								1
10							1	2
9					1	1	2	4
8				1	2	3	5	8
7		1	1	2	4	6	10	16
6	1	2	3	5	8	13	20	
5	3	4	7	11	17	26	40	
4	6	9	14	22	34	52		
3	12	19	29	44				
2	25	38	58	88				
1	51	77	116					
0		154	232					

Impar+1

	0	1	2	3	4	5	6	7
11							1	1
10					1	1	2	2
9				1	2	2	3	4
8		1	1	2	3	4	5	8
7	1	2	2	3	5	7	10	16
6	2	3	4	6	9	13	20	
5	4	5	8	11	17	26	40	
4	7	10	15	22	34	52		
3	13	20	29	44				
2	26	39	58	88				
1	51	77	116					
0		154	232					

Next, the sequence table started with the number 127, considering as sequences cycles the columns of the odd numbers table and the rows of the triangles, since both are formed by the same numbers.

The sequence table started with the number 127 and the triangles involved or cycles:



The sequence has gone through 30 horizontal steps or iterations of odd numbers and 16 vertical steps or iterations of even numbers, divisions by 2.

Sequence started with the number 27 and the triangles or cycles that form it:

1										
2	3									
4	6	9								
8	12	18	27							
16	24	36	54	81						
32	48	72	108	162	243					
64	96	144	216	324	486	729				
128	192	288	432	648	972	1458	2187			
256	384	576	864	1296	1944	2916	4374	6561		
512	768	1152	1728	2592	3888	5832	8748	13122	19683	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .

127,191, 287, 431, 647, 971, 1457, 2186

547										
1094	1641									
2188	3282	4923								
4376	6564	9846	14769							
8752	13128	19692	29538	44307						
17504	26256	39384	59076	88614	132921					
. . .	. . .	. . .	. . .	. . .	. . .	. . .				

1093, 1640

103										
206	309									
412	618	927								
824	1236	1854	2781							
1648	2472	3708	5562	8343						
3296	4944	7416	11124	16686	25029					
...	...	...	...	...	...	...				

205, 308

13										
26	39									
52	78	117								
104	156	234	351							
208	312	468	702	1053						
416	624	936	1404	2106	3159					
...	...	...	...	...	...	...				

77, 116

5										
10	15									
20	30	45								
40	60	90	135							
80	120	180	270	405						
160	240	360	540	810	1215					
...	...	...	...	...	...	...				

29, 44

1										
2	3									
4	6	9								
8	12	18	27							
16	24	36	54	81						
32	48	72	108	162	243					
...	...	...	...	...	...	...				

11, 17, 26

7										
14	21									
28	42	63								
56	84	126	189							
112	168	252	378	567						
224	336	504	756	1134	1701					
...	...	...	...	...	...	...				

13, 20

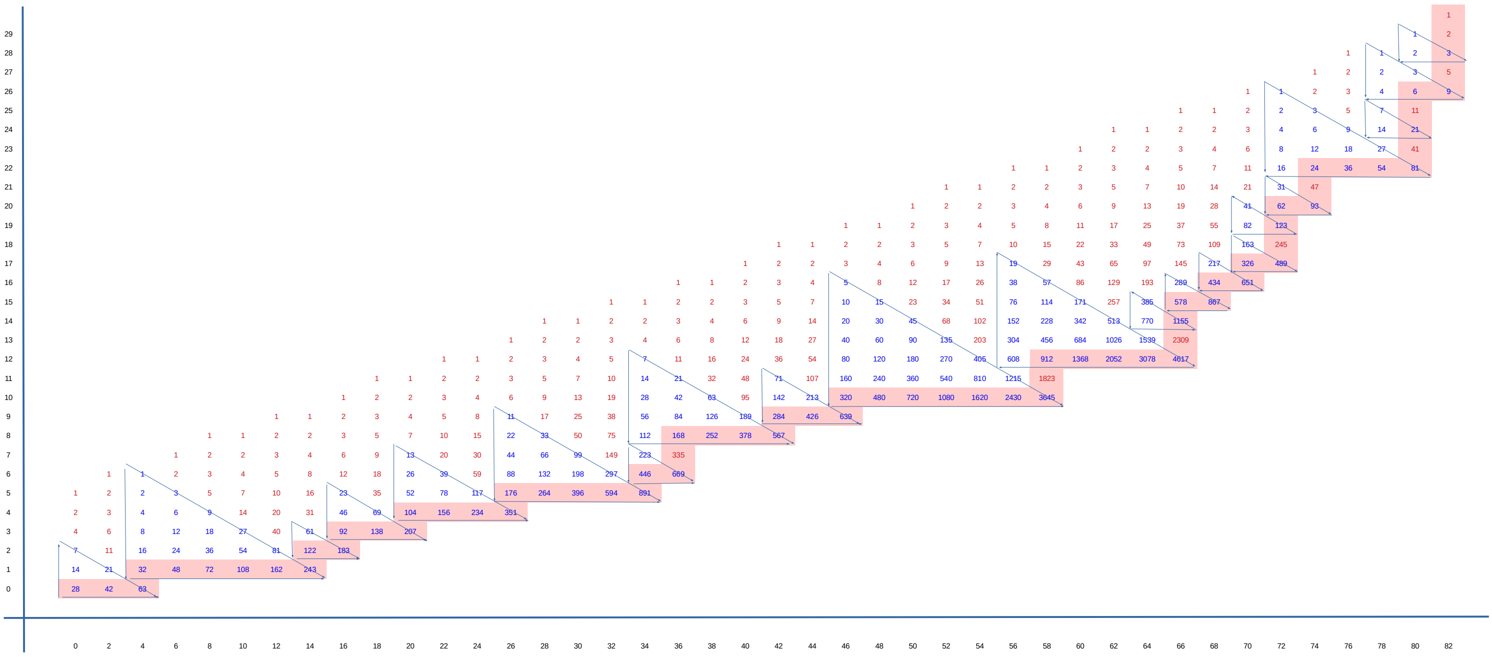
1										
2	3									
4	6	9								
8	12	18	27							
16	24	36	54	81						
32	48	72	108	162	243					
...	...	...	...	...	...	...				

5, 8

The sequence started with the number 27 in the 18 columns of the table or cycles:

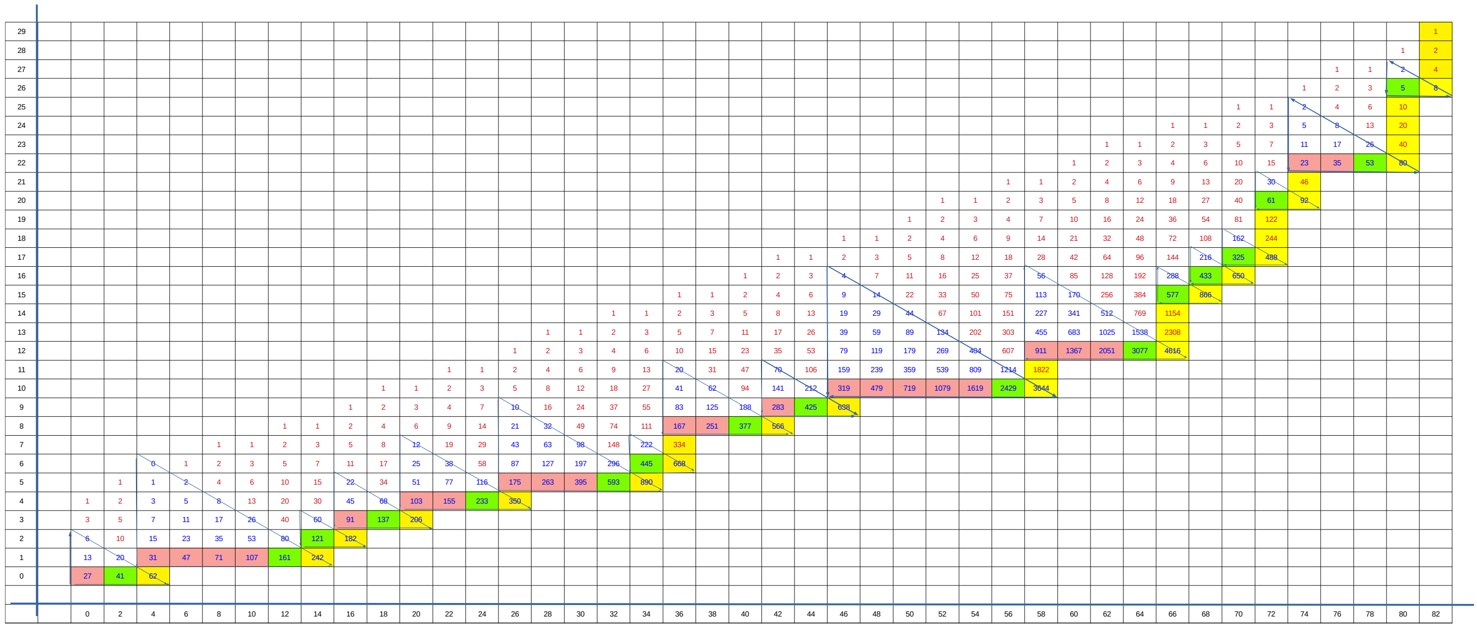
k	14	16	61	46	52	88	223	84	142	160	456	289	217	163	31	12	3	1
n																		
0	27	31	121	91	103	175	445	167	283	319	911	577	433	325	61	23	5	1
1	41	47	182	137	155	263	668	251	425	479	1367	866	650	488	92	35	8	2
	62	71		206	233	395		377	638	719	2051					53		
3		107			350	593		566		1079	3077					80		
4		161				890				1619	4616							
5		242								2429								
6										3644								
7																		
8																		
9																		

The same sequence in the table of triangles:



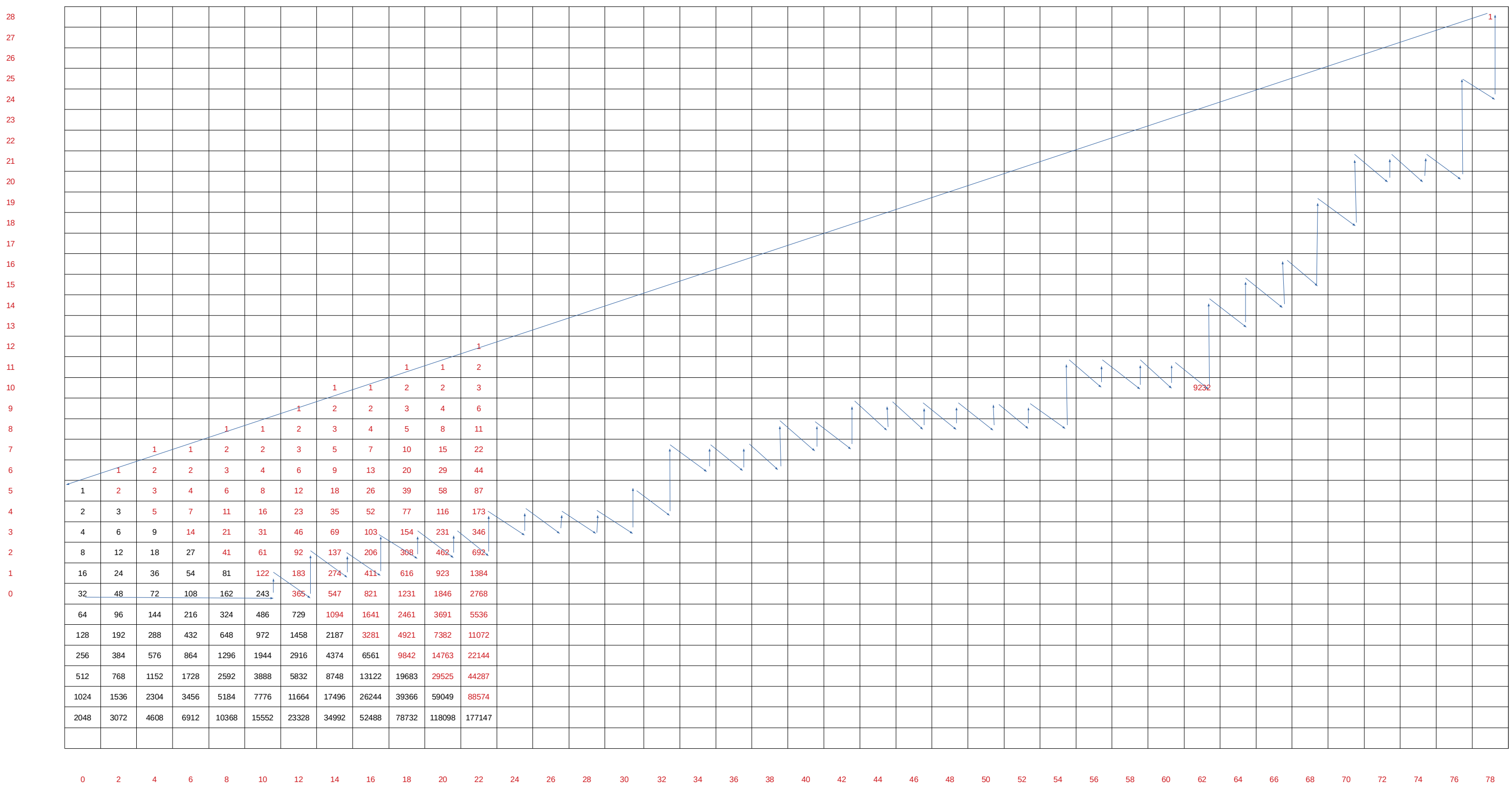
The sequence started with the number 27 has 82 horizontal iterations of the odd numbers and 29 vertical iterations of the even numbers.

We also see the development of the sequence with the columns of the odd number table, such as rows of triangles:



The rows of the triangles that form the cycles of the sequence, with the odd ones of the 4n + 3 form in red, the odd ones of the 4n + 1 form of green and the even numbers that join the cycles in yellow.

In the following table, a sequence started with the number 31:



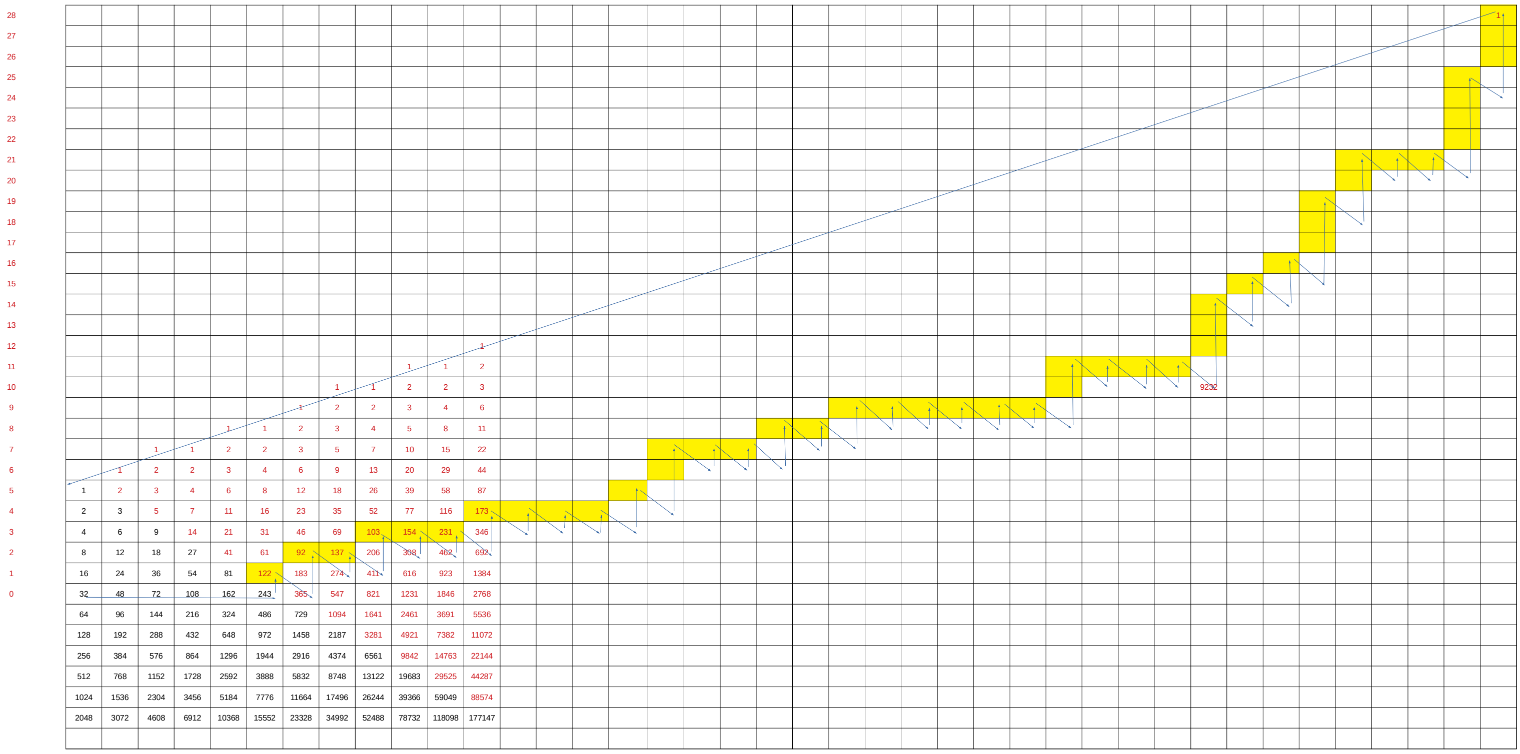
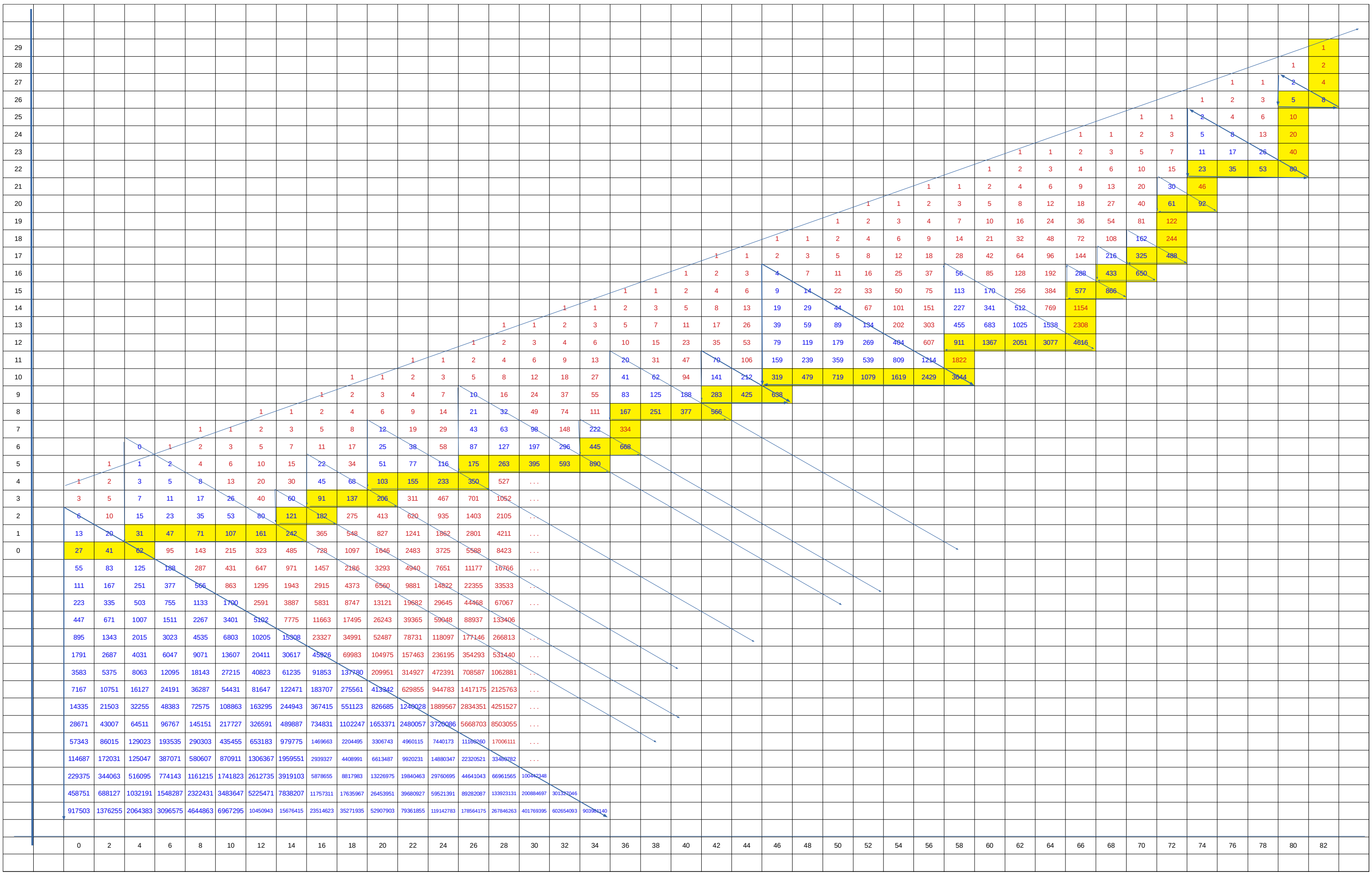


Table and the triangles of the sequence started with the number 27. You could complete all the numbers of each triangle, but they are not necessary to see the development of the sequence.



Applying (3m + 1) / 2 to an odd number causes the sequence to move to the next column of the table, but in the same row and ascends to higher rows when at the end of each cycle, the even number is divided by 2. In this table, ascending means approaching number 1.

In the table, each cycle or triangle is above the previous one and the sequence never descends to a lower row. Neither will it go parallel to the line of numbers 1 nor will it be separated, because the cycles or columns with the largest number of odd numbers (longer rows) are very scarce:
In the first 10,000 columns, the one with the largest amount of odd numbers is column k(8192), with 14 numbers. It is followed by column k(4096) with 13 numbers, two columns have 12 numbers, five columns have 11 numbers, ten columns with 10 numbers, twenty columns with 9 numbers, etc.
In the first 50,000, the column with the most numbers k(32768) has 16.
In the first 300,000, the column with the most numbers k(262144) has 19.
In the first 1,000,000, the column with the most numbers k(524288) has 20.

All the rows of the triangles or columns in the table are finite and their last number is even, so it will inevitably end in a number 1.

It is a visual demonstration that, although the numbers suffer increasing and decreasing oscillations, all sequences will ascend to number 1.

THE AMOUNT OF ODD NUMBERS in each column of the odd table and the amount of even numbers in the even table: 1, 2, 1, 3, 1, 2, 1, 4, 1, 2,. . . matches the number of digits 1 to the right of the last 0 in the binary representation of the odd numbers and the number of digits 0 to the right of the last 1 in the binary representation of the even numbers. Example:

1	3	5	7	9	11	13	15	17	19	21
1	11	101	111	1001	1011	1101	1111	10001	10011	10101
2	4	6	8	10	12	14	16	18	20	22
10	100	110	1000	1010	1100	1110	10000	10010	10100	10110
1	2	1	3	1	2	1	4	1	2	1

The last row corresponds to the amount of numbers in the columns of the tables and forms a fractal sequence with infinite sequences of the natural numbers, as follows: The numbers 1 are in the values of odd k and the following terms are in 2k.

Example: The terms of the first sequence are in k(1), k(2), k(4), k(8), k(16),. . .
The terms of the second sequence are in k(3), k(6), k(12), k(24), k(48),. . .
The terms of the third sequence are in k(5), k(10), k(20), k(40), k(80),. . .

	k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
n																																																					
0		1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	. . .
1		2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	
2			8		17		26		35		44		53		62		71		80		89		98		107		116		125		134		143		152		161		170		179		188		197		206		215		224		
3					26				53				80				107				134				161				188				215				242				269				296				323				
4									80								161								242								323								404								485				
5																	242																485																728				
6																																	728																				
7																																																					

Impares	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	6	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	. . .

"Impares" is the amount of those numbers in every column.
It is also the value of n of the even number at the end of each column.
It is also the number of steps or iterations that the first number of the column needs to reach the even number, applying the Collatz algorithm.
It is also the number of digits 1 on the right after 0, of odd numbers written in binary. Example: Column k (24) has 4 even numbers and the number 47 in binary is 101111.
This is a fractal sequence and forms infinite sequences of natural numbers, as follows: The numbers 1 are in the values of odd k and the following terms are in 2k.
Example: The terms of the first sequence are in k(1), k(2), k(4), k(8), k(16),. . .
The terms of the second sequence are in k(3), k(6), k(12), k(24), k(48),. . .
The terms of the third sequence are in k(5), k(10), k(20), k(40), k(80),. . .

	k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
n																																																					
0		2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	. . .
1		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	. . .
2			1		2		3		4		5		6		7		8		9		10		11		12		13		14		15		16		17		18		19		20		21		22		23		24		25		. . .
3					1				2				3				4				5				6				7				8				9				10				11				12				. . .
4									1								2								3								4								5								6				. . .
5																	1																2																3				. . .
6																																	1																				. . .
7																																																					

Pares	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	6	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	. . .

"Pares" is the amount of those numbers in every column.
It is also the value of n of the odd number at the end of each column.
It is also the number of steps or iterations that the first number of the column needs to reach the odd number, applying the Collatz algorithm.
It is also the number of digits 0 on the right after 1, of the even numbers written in binary. Example: Column k (24) has 4 even numbers and the number 48 in binary is 110000.
This is a fractal sequence and forms infinite sequences of natural numbers, as in the odd numbers table.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41
	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62
		8		17		26		35		44		53		62		71		80		89	
				26				53				80				107				134	
								80								161					
																242					

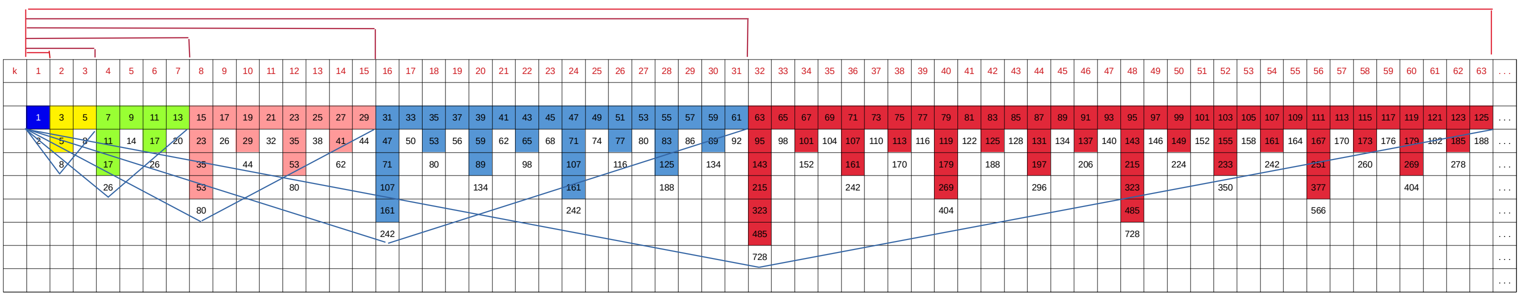
k	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83
	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125
	98		107		116		125		134		143		152		161		170		179		188
			161				188				215				242				269		
			242								323								404		
											485										
											728										

k	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	. . .
	85	87	89	91	93	95	97	99	101	103	105	107	109	111	113	115	117	119	121	123	125	. . .
	128	131	134	137	140	143	146	149	152	155	158	161	164	167	170	173	176	179	182	185	188	. . .
		197		206		215		224		233		242		251		260		269		278		. . .
		296				323				350				377				404				. . .
						485								566								. . .
						728																. . .
																						. . .
																						. . .

In the first column there is only 1 odd number.
In the first 3 columns, there are 4 odd numbers and one that has 2 odd numbers.
In the first 7 columns, there are 11 odd numbers and one that has 3 odd numbers.
In the first 15 columns, there are 26 odd numbers and one that has 4 odd numbers.
In the first 31 columns, there are 57 odd numbers and one that has 5 odd numbers.
In the first 63 columns, there are 120 odd numbers and one that has 6 odd numbers.
. . .

In the first k (2 ^ n-1), there are 2 ^ n-n-1 odd numbers and the column k (2 ^ (n-1)) has n odd numbers and is the one with the largest amount.

k											
0	0										
1	1	2									
2	3	5	8								
3	7	11	17	26							
4	15	23	35	53	80						
5	31	47	71	107	161	242					
6	63	95	143	215	323	485	728				
7	127	191	287	431	647	971	1457	2186			
8	255	383	575	863	1295	1943	2915	4373	6560		
9	511	767	1151	1727	2591	3887	5831	8747	13121	19682	
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .



In each “triangle” T (n) of the fractal there are 2 * T (n) + n odd numbers and the even number is 3 ^ n-1.

Example:

T(1) 2*T(0)+1= 1 odd number.
T(2) 2*T(1)+2= 4 odd numbers and the even 8.
T(3) 2*T(2)+3= 11 odd numbers and the even 26.
T(4) 2*T(3)+4= 26 odd numbers and the even 80.
T(5) 2*T(4)+5= 57 odd numbers and the even 242.
T(6) 2*T(5)+6= 120 odd numbers and the even 728.
T(7) . . .

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61
	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92
		8		17		26		35		44		53		62		71		80		89		98		107		116		125		134	
				26				53				80				107				134				161				188			
								80								161								242							
																242															
	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1
	1	3		7				15								31															
	1																														
	4																														
	11																														
	26																														
	57																														
	120																														

k	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	. . .
	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	103	105	107	109	111	113	115	117	119	121	123	125	. . .
	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	155	158	161	164	167	170	173	176	179	182	185	188	. . .
	143		152		161		170		179		188		197		206		215		224		233		242		251		260		269		278		. . .
	215				242				269				296				323				350				377				404				. . .
	323								404								485								566								. . .
	485																728																. . .
	728																																. . .
																																	. . .
	6	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	. . .
	63																																. . .

1		1																															
2		2	1																														
4		3	1	2	1																												
8		4	1	2	1	3	1	2	1																								
16		5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1																
32		6	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1
64		7	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1

1																																
2																																
4																																
8																																
16																																
32																																
64	6	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1

APPENDIX 1

Sequence started with the number 2251799813685247, which is the first number in column k (2 ^ 50) of the table.

In this column are the first 50 odd ones of the sequence, which are of the form 4n-1 and occupy the even steps from 0 to 98. The odd 51º, which is of the form 4n-3 in step 100 and the pair 52º which is the last number of the column, in step 102.

step: value

```
 0: 2251799813685247
 1: 6755399441055742
 2: 3377699720527871
 3: 10133099161583614
 4: 5066549580791807
 5: 15199648742375422
 6: 7599824371187711
 7: 22799473113563134
 8: 11399736556781567
 9: 34199209670344702
10: 17099604835172351
11: 51298814505517054
12: 25649407252758527
13: 76948221758275582
14: 38474110879137791
15: 115422332637413374
16: 57711166318706687
17: 173133498956120062
18: 86566749478060031
19: 259700248434180094
20: 129850124217090047
21: 389550372651270142
22: 194775186325635071
23: 584325558976905214
24: 292162779488452607
25: 876488338465357822
26: 438244169232678911
27: 1314732507698036734
28: 657366253849018367
29: 1972098761547055102
30: 986049380773527551
31: 2958148142320582654
32: 1479074071160291327
33: 4437222213480873982
34: 2218611106740436991
35: 6655833320221310974
36: 3327916660110655487
37: 9983749980331966462
38: 4991874990165983231
39: 14975624970497949694
40: 7487812485248974847
41: 22463437455746924542
42: 11231718727873462271
43: 33695156183620386814
44: 16847578091810193407
45: 50542734275430580222
46: 25271367137715290111
47: 75814101413145870334
48: 37907050706572935167
49: 113721152119718805502
50: 56860576059859402751
```

51: 170581728179578208254
52: 85290864089789104127
53: 255872592269367312382
54: 127936296134683656191
55: 383808888404050968574
56: 191904444202025484287
57: 575713332606076452862
58: 287856666303038226431
59: 863569998909114679294
60: 431784999454557339647
61: 1295354998363672018942
62: 647677499181836009471
63: 1943032497545508028414
64: 971516248772754014207
65: 2914548746318262042622
66: 1457274373159131021311
67: 4371823119477393063934
68: 2185911559738696531967
69: 6557734679216089595902
70: 3278867339608044797951
71: 9836602018824134393854
72: 4918301009412067196927
73: 14754903028236201590782
74: 7377451514118100795391
75: 22132354542354302386174
76: 11066177271177151193087
77: 33198531813531453579262
78: 16599265906765726789631
79: 49797797720297180368894
80: 24898898860148590184447
81: 74696696580445770553342
82: 37348348290222885276671
83: 112045044870668655830014
84: 56022522435334327915007
85: 168067567306002983745022
86: 84033783653001491872511
87: 252101350959004475617534
88: 126050675479502237808767
89: 378152026438506713426302
90: 189076013219253356713151
91: 567228039657760070139454
92: 283614019828880035069727
93: 850842059486640105209182
94: 425421029743320052604591
95: 1276263089229960157813774
96: 638131544614980078906887
97: 1914394633844940236720662
98: 957197316922470118360331
99: 2871591950767410355080994
100: 1435795975383705177540497
101: 4307387926151115532621492
102: 2153693963075557766310746
103: 1076846981537778883155373
104: 3230540944613336649466120
105: 1615270472306668324733060
106: 807635236153334162366530
107: 403817618076667081183265
108: 1211452854230001243549796
109: 605726427115000621774898
110: 302863213557500310887449
111: 908589640672500932662348
112: 454294820336250466331174
113: 227147410168125233165587
114: 681442230504375699496762

115: 340721115252187849748381
116: 1022163345756563549245144
117: 511081672878281774622572
118: 255540836439140887311286
119: 127770418219570443655643
120: 383311254658711330966930
121: 191655627329355665483465
122: 574966881988066996450396
123: 287483440994033498225198
124: 143741720497016749112599
125: 431225161491050247337798
126: 215612580745525123668899
127: 646837742236575371006698
128: 323418871118287685503349
129: 970256613354863056510048
130: 485128306677431528255024
131: 242564153338715764127512
132: 121282076669357882063756
133: 60641038334678941031878
134: 30320519167339470515939
135: 90961557502018411547818
136: 45480778751009205773909
137: 136442336253027617321728
138: 68221168126513808660864
139: 34110584063256904330432
140: 17055292031628452165216
141: 8527646015814226082608
142: 4263823007907113041304
143: 2131911503953556520652
144: 1065955751976778260326
145: 532977875988389130163
146: 1598933627965167390490
147: 799466813982583695245
148: 2398400441947751085736
149: 1199200220973875542868
150: 599600110486937771434
151: 299800055243468885717
152: 899400165730406657152
153: 449700082865203328576
154: 224850041432601664288
155: 112425020716300832144
156: 56212510358150416072
157: 28106255179075208036
158: 14053127589537604018
159: 7026563794768802009
160: 21079691384306406028
161: 10539845692153203014
162: 5269922846076601507
163: 15809768538229804522
164: 7904884269114902261
165: 23714652807344706784
166: 11857326403672353392
167: 5928663201836176696
168: 2964331600918088348
169: 1482165800459044174
170: 741082900229522087
171: 2223248700688566262
172: 1111624350344283131
173: 3334873051032849394
174: 1667436525516424697
175: 5002309576549274092
176: 2501154788274637046
177: 1250577394137318523
178: 3751732182411955570

179: 1875866091205977785
180: 5627598273617933356
181: 2813799136808966678
182: 1406899568404483339
183: 4220698705213450018
184: 2110349352606725009
185: 6331048057820175028
186: 3165524028910087514
187: 1582762014455043757
188: 4748286043365131272
189: 2374143021682565636
190: 1187071510841282818
191: 593535755420641409
192: 1780607266261924228
193: 890303633130962114
194: 445151816565481057
195: 1335455449696443172
196: 667727724848221586
197: 333863862424110793
198: 1001591587272332380
199: 500795793636166190
200: 250397896818083095
201: 751193690454249286
202: 375596845227124643
203: 1126790535681373930
204: 563395267840686965
205: 1690185803522060896
206: 845092901761030448
207: 422546450880515224
208: 211273225440257612
209: 105636612720128806
210: 52818306360064403
211: 158454919080193210
212: 79227459540096605
213: 237682378620289816
214: 118841189310144908
215: 59420594655072454
216: 29710297327536227
217: 89130891982608682
218: 44565445991304341
219: 133696337973913024
220: 66848168986956512
221: 33424084493478256
222: 16712042246739128
223: 8356021123369564
224: 4178010561684782
225: 2089005280842391
226: 6267015842527174
227: 3133507921263587
228: 9400523763790762
229: 4700261881895381
230: 14100785645686144
231: 7050392822843072
232: 3525196411421536
233: 1762598205710768
234: 881299102855384
235: 440649551427692
236: 220324775713846
237: 110162387856923
238: 330487163570770
239: 165243581785385
240: 495730745356156
241: 247865372678078
242: 123932686339039

243: 371798059017118
244: 185899029508559
245: 557697088525678
246: 278848544262839
247: 836545632788518
248: 418272816394259
249: 1254818449182778
250: 627409224591389
251: 1882227673774168
252: 941113836887084
253: 470556918443542
254: 235278459221771
255: 705835377665314
256: 352917688832657
257: 1058753066497972
258: 529376533248986
259: 264688266624493
260: 794064799873480
261: 397032399936740
262: 198516199968370
263: 99258099984185
264: 297774299952556
265: 148887149976278
266: 74443574988139
267: 223330724964418
268: 111665362482209
269: 334996087446628
270: 167498043723314
271: 83749021861657
272: 251247065584972
273: 125623532792486
274: 62811766396243
275: 188435299188730
276: 94217649594365
277: 282652948783096
278: 141326474391548
279: 70663237195774
280: 35331618597887
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282: 52997427896831
283: 158992283690494
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285: 238488425535742
286: 119244212767871
287: 357732638303614
288: 178866319151807
289: 536598957455422
290: 268299478727711
291: 804898436183134
292: 402449218091567
293: 1207347654274702
294: 603673827137351
295: 1811021481412054
296: 905510740706027
297: 2716532222118082
298: 1358266111059041
299: 4074798333177124
300: 2037399166588562
301: 1018699583294281
302: 3056098749882844
303: 1528049374941422
304: 764024687470711
305: 2292074062412134
306: 1146037031206067

307: 3438111093618202
308: 1719055546809101
309: 5157166640427304
310: 2578583320213652
311: 1289291660106826
312: 644645830053413
313: 1933937490160240
314: 966968745080120
315: 483484372540060
316: 241742186270030
317: 120871093135015
318: 362613279405046
319: 181306639702523
320: 543919919107570
321: 271959959553785
322: 815879878661356
323: 407939939330678
324: 203969969665339
325: 611909908996018
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