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The binding energy and the total energy of a macroscopic body in the relativistic uniform model

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The total energy, binding energy, energy of fields, pressure energy and the potential energy of the system consisting of particles and four fields is precisely calculated in the relativistic uniform model. These energies are compared with the kinetic energy of particles. The relations between the coefficients of the acceleration field and the pressure field independent of the system's properties are found, which can be expressed in terms of each other and in terms of the gravitational constant and the vacuum permittivity. A noticeable difference is shown between the obtained results and the relations for simple systems in classical mechanics, in which the acceleration field and pressure field are not taken into account or the pressure is considered to be a simple scalar quantity. The conclusion is substantiated that as increasingly massive relativistic uniform systems are formed, the average density of these systems decreases as compared to the average density of the particles or bodies making up these systems. In this case the inertial mass of the massive system is less than the total inertial mass of the system's parts.
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1. Introduction

By definition, the relativistic energy of the system includes all forms of energy and should be written in a covariant form. In some cases, a constant term in the form of the rest energy of the system's particles must be subtracted from the relativistic energy. Thus we obtain the total energy of the system, which is usually used in classical mechanics when simple problems are solved that do not require the relativistic approach. As a rule, the total energy is divided into two main parts – the kinetic energy and the potential energy.

In large macroscopic systems several fields can be acting simultaneously, each of them changes the energy of the particles and can also have its own potential energy. This significantly complicates the expressions for the relativistic and total energies and often leads to the fact that different field theories give the expressions for the energy that are different in form. For example, in the general theory of relativity, the gravitational field energy is calculated not directly, but indirectly, with the help of the stress-energy pseudotensor and the spacetime metric obtained previously [1]. This means that the knowledge of the system metric is necessary to determine the energy even in case of an extremely weak field. But even with the known metric, there is some ambiguity in determination of the relativistic energy and inertial mass of the system and its individual parts [2], [3].

In contrast to this, in the covariant theory of gravitation there is a stress-energy tensor of the gravitational field, and the metric of any system in the weak field limit turns gradually into the constant metric of Minkowski space, where the gravitational energy no longer depends on the type of the system metric [4]. Similarly, under earthly conditions the spacetime metric is almost never used to calculate the energy of the bodies' electromagnetic field.

With this in mind, in the framework of the covariant theory of gravitation and the relativistic uniform model we will further determine the total energy and the binding energy of a macroscopic body, which is in equilibrium without rotation, and compare the obtained expressions with the results for simple systems in classical mechanics. We will also calculate the individual energy components, including the fields' energy, the pressure energy and the potential energy of the system. Our approach uses the field theory in the covariant notation, so that the results obtained for the flat spacetime can be easily adjusted for the curved spacetime with the corresponding metric, if necessary.

The thermodynamic properties of the considered physical system of particles and fields have previously been studied by Chernikov using the methods of the relativistic kinetic theory of gases and statistical physics [5]. For the case of a self-gravitating system with charged particles there is a special name – the Vlasov plasma. Vlasov used the additional idea of a self-consistent field [6] and pointed to the constraints of the Boltzmann's model with pair collisions, which did not take into account the action of the electromagnetic and gravitational fields at a distance. However, our approach is based not on the kinetic theory, but on the vector field theory, and instead of the general theory of relativity we use the covariant theory of gravitation. In addition, we describe the interaction of particles by means of the fields with the proper four-potentials, and this applies both to the pressure field and to the particles' acceleration field. Thus, obtaining the average values of physical quantities is not associated either with the distribution functions, the phase space or the Liouville's theorem, but with averaging the physical quantities directly in the equations, arising from the principle of least action.

Despite the fact that the approximation of the constant invariant mass density in the presence of sufficiently strong gravitational fields is a certain constraint, this approach is justified, as it gives an accurate description and can be applied to a number of macroscopic systems, such as the observable Universe, cosmic gas clouds and even neutron stars. In addition, the suggested approach can also be adjusted for the case of non-uniform mass density, as it was done in [7] for white dwarfs and the Sun, which is the main-sequence star.

2. Relativistic and kinetic energy

In [8] within the framework of the covariant theory of gravitation the formula was found to calculate the relativistic energy of a physical system of particles and fields associated with these particles. At the same time, the electromagnetic and gravitational fields, the acceleration field and the pressure field were taken into account, and the role of the stress-energy tensor of the matter was played by the stress-energy tensor of the acceleration field. A similar formula for the energy was presented in the general field concept for a macroscopic system in [9, 10].

With the help of this formula in [4] the energy of the equilibrium system with continuously distributed randomly moving matter was calculated, taking into account the energy of the fields, in the explicit form with an accuracy up to the terms that do not contain the square of the speed of light in the denominators. Taking into account the corrections, made in [11], the energy is equal to:

$$E_r = Mc^2 \approx mc^2\gamma_c^2 - \frac{7\eta m^2\gamma_c^2}{10a} - \frac{3Gm^2\gamma_c^2}{5a} + \frac{3q^2\gamma_c^2}{20\pi\varepsilon_0 a} + m\wp_c\gamma_c - \frac{2\sigma m^2\gamma_c^2}{5a}. \qquad (1)$$

Expression for the energy (1) differs noticeably from the energy accepted in the classical mechanics. The main difference arises from the acceleration field and the pressure field, which are considered as unique and independent vector fields that have stress-energy tensors determined in a covariant way.

In (1) it is assumed that the system has a spherical form with the radius a; the mass m and the charge q of the system are obtained by multiplying the mass density ρ_0 and the charge density ρ_{0q} by the volume of the fixed sphere, while ρ_0 and ρ_{0q} are the invariant densities of the particles that make up the sphere, measured in the reference frames associated with the particles. Within the relativistic uniform model these densities are assumed to be constants. Due to the random motion of matter, the global vector potentials and the corresponding solenoidal vectors of the system's fields are considered equal to zero, so that the contribution of the solenoidal vectors into the energy (1) is neglected.

At the center of the sphere the velocity of the particles is v_c and the Lorentz factor of the particles equals $\gamma_c = \frac{1}{\sqrt{1-v_c^2/c^2}}$, where c is the speed of light. Each field is characterized by its own coefficient: η is the acceleration field coefficient, G is the gravitational constant, ε_0 is the vacuum permittivity, σ is the pressure field constant. The quantity $\wp_c$ in (1) represents the scalar potential of the pressure field at the center of the sphere, and the mass M is the relativistic invariant inertial mass of the system.

The kinetic energy of motion of this system's particles was found in [12] by three methods – by the virial theorem, by subtracting the rest energy of the particles from the relativistic energy of their motion, and using the generalized three-momenta of the system's particles. All these methods give the same result:

$$E_k \approx mc^2\gamma_c^2 - \frac{3\eta m^2\gamma_c^2}{5a} - mc^2\gamma_c + \frac{3\eta m^2\gamma_c}{10a} \approx \frac{mv_c^2\gamma_c}{2} - \frac{3\eta m^2\gamma_c}{10a} \approx \frac{27\eta m^2\gamma_c}{20\sqrt{14}a}. \qquad (2)$$

Expressions (1) and (2) are the initial point for determining the components of the total energy and the binding energy of the system under consideration. The numerical coefficient $\frac{27}{20\sqrt{14}}$ in (2) is the consequence of the solution of the quadratic equation for the velocity v_c at the center of the sphere and is a property of the relativistic uniform system. As a result, similar numerical coefficients can be found in some other energy components.

3. The total energy components

According to [13], the Lorentz factor of the particles, moving randomly inside the sphere, depends on the current radius r:

$$\gamma' = \frac{c\gamma_c}{r\sqrt{4\pi\eta\rho_0}}\sin\left(\frac{r}{c}\sqrt{4\pi\eta\rho_0}\right) \approx \gamma_c - \frac{2\pi\eta\rho_0 r^2\gamma_c}{3c^2}. \qquad (3)$$

The Lorentz factor is maximal at the center of the sphere, where the particles are moving at the highest velocity. We use the reference frame K' that is associated with the center of the sphere, so that all the results will refer to the sphere, which is stationary relative to the observer. If necessary, the energies and momenta of individual fields and the entire system can be converted into the laboratory reference frame K by means of the corresponding Lorentz transformations at a known velocity of the sphere's motion in K.

Formula (3) was obtained by solving the wave equation for the scalar potential ϑ of the acceleration field in the same way as the potentials of the electromagnetic or gravitational fields are calculated inside the sphere. In this case in the Minkowski space the acceleration field potential becomes proportional to the Lorentz factor: $\vartheta = \gamma' c^2$.

The physical reason for the radial dependence in (3) is the need to maintain a balance of the acting forces. Thus, the gravitational force tends to compress the matter, and the pressure of the moving particles resists such compression. Indeed, the gravitational field strength inside the uniform system of a spherical form is proportional to the current radius and is directed towards the center of the sphere. The volumetric density of the gravitational force is proportional to the field strength and the mass density of the matter. As the observation point moves from the surface to the center along the radius, the total thickness and the mass of the observed spherical layer increase, as well as the total gravitational force from this layer towards the center. The gravitational pressure is balanced by the dynamic pressure of the particles, which according to the kinetic theory is two-thirds of the kinetic energy of the particles per unit volume. Consequently, the closer to the center, the greater is the pressure of particles in the matter.

Gravitation also forms some spherical boundary of the system with the radius a so that the particles do not on average go beyond this radius. If N charged particles were closed inside a sphere with a rigid boundary, then in the equilibrium state and in the absence of mass forces we could expect the uniform mass density, the same velocity and the constant Lorentz factor of the particles at each point of the system, which would depend only on the temperature. However, here is no rigid boundary in our model, and the proper gravitational and electromagnetic forces penetrating into any point of the system act as the mass forces. Thus, it turns out that the velocity of particles and their Lorentz factor are maximal at the center of the system under consideration and decrease with increasing current radius.

We will take into account now that as a consequence of the relativistic effect of length contraction the moving particles in the special theory of relativity must be considered as if they have a reduced volume and increased density. Indeed, ρ_0 is the mass density in the reference frames associated with the particles, γ' is the Lorentz factor of the moving particles, and $\rho_b = \rho_0 \gamma'$ gives the mass density of these particles from the viewpoint of an observer, who is stationary relative to the sphere.

For the motion of particles there should be some voids between them. Both the average accelerations and the average velocities of the particles inside the sphere are functions of the current radius. Dividing the particles' velocities by their acceleration, we can find the dependence of the average period of the oscillatory motion of particles on the radius. Finally, multiplying the velocity by the average period of motion, we can obtain an estimate of the size of the voids between the particles.

In order to calculate the volume of the sphere, it is necessary to sum up the volumes of all the typical particles moving inside the sphere, as well as the volumes of the voids between them. Suppose now that the sizes of the typical particles are much larger than the voids between the particles, and the volume of the voids is substantially less than the total volume of the particles. In this case, we can use the approximation of continuous medium, so that the unit of the mass of matter inside the sphere will be given by the approximate expression $dm \approx \rho_0 \gamma' dV$, where dV is the volume element of the fixed sphere.

The question whether it is acceptable to increase the sizes of typical particles up to the limit necessary for using the approximation of continuous medium can be answered as follows. In the gravitational field the acceleration of particles depends neither on the mass nor on the density of particles, which follows from the equivalence principle in the general theory of relativity and from the equation of motion in the covariant theory of gravitation. For the electric forces the acceleration is proportional to the ratio of densities $\frac{\rho_{0q}}{\rho_0}$ and does not depend on the mass of particles. The same applies to the motion velocity, Lorentz factor, kinetic energy and other quantities, which are determined not by the mass of particles, but by their density ρ_0. Thus, with a given density, we can choose the mass and, consequently, the sizes of typical particles within the limits we need.

The standard formula to calculate the kinetic energy all of the N particles of the system has the form:

$$E_k = \sum_{i=1}^{N} (\gamma_i - 1)\, m_i c^2 .$$

The mass m_i and the Lorentz factor γ_i of the particles were substituted in [12] with $dm = \rho_0 \gamma' dV$ and γ', respectively, and the sum for the particles was substituted with the integral over the sphere volume. This led to relation (2) for the kinetic energy of the particles, which also contains the expression for the total rest energy of the sphere's particles from the viewpoint of the observer associated with the sphere:

$$W_b = m_b c^2 = \rho_0 c^2 \int \gamma' \, r^2 \sin\theta \, dr \, d\theta \, d\phi \approx mc^2 \gamma_c - \frac{3\eta m^2 \gamma_c}{10a} . \qquad (4)$$

The average rest mass of one particle is obtained by dividing the total rest mass m_b by the number N of particles in the system.

In order to obtain the total energy of the system, we need to subtract the energy W_b from the relativistic energy (1):

$$E_t = E_r - W_b .$$

On the other hand, according to (2):

$$E_k \approx mc^2\gamma_c^2 - \frac{3\eta m^2\gamma_c^2}{5a} - W_b .$$

After eliminating W_b from these equations, taking into account (1) and collecting the similar terms, we obtain the expression for the total energy:

$$E_t = E_k - \frac{3G\,m^2\gamma_c^2}{5a} + \frac{3q^2\gamma_c^2}{20\pi\varepsilon_0 a} + m\wp_c\gamma_c - \frac{2\sigma m^2\gamma_c^2}{5a} - \frac{\eta\, m^2\gamma_c^2}{10a} . \qquad (5)$$

The total energy should consist of the kinetic and potential energies, $E_t = E_k + W_p$, therefore the potential energy of the system will be as follows:

$$W_p = -\frac{3G\,m^2\gamma_c^2}{5a} + \frac{3q^2\gamma_c^2}{20\pi\varepsilon_0 a} + m\wp_c\gamma_c - \frac{2\sigma m^2\gamma_c^2}{5a} - \frac{\eta\, m^2\gamma_c^2}{10a} . \qquad (6)$$

As we can see, the potential energy (6) contains the energy of the particles in the gravitational and electromagnetic fields and in the pressure field, as well as the energy of these fields themselves, with addition of the acceleration field energy.

To simplify expression (6) we will use the definition of the scalar potential of the pressure field in [14], according to which the potential at the center of the sphere is $\wp_c = \frac{p_{0c}\gamma_c}{\rho_0}$, which means that it is defined by the pressure p_{0c} in the reference frame K_p of the moving particle. On the other hand, the scalar potential of the pressure field from the viewpoint of the observer in K' equals $\wp_c = \frac{p_c}{\rho_c}$, where p_c is the pressure, $\rho_c = \rho_0\gamma_c$ is the mass density at the center of the sphere for this observer. Consequently, $p_c = p_{0c}\gamma_c^2$, and the pressure in the system due to the particles' motion increases more rapidly, in proportion to the square of the Lorentz factor, as compared to the mass density, which increases in proportion to the Lorentz factor.

If the radiation pressure is not taken into account, for the pressure in the reference frame K_p of the particle and for the pressure at the center of the sphere in the reference frame K' we can write the following:

$$p_{0c} = \frac{\rho_0 kT_{0c}}{\mu m_u}, \qquad\qquad p_c = \frac{\rho_c\gamma_c kT_{0c}}{\mu m_u} = \frac{\rho_c kT_c}{\mu m_u},$$

where μ is the parameter, which represents the number of nucleons per unit of relativistic ionized gas, m_u is the atomic mass unit, k is the Boltzmann constant, and the temperature at the center of the sphere T_{0c} in the reference frame K_p during transition from K_p into K' is transformed to the temperature $T_c = \gamma_c T_{0c}$.

Taking this into consideration, the scalar potential $\wp_c = \frac{p_c}{\rho_c}$ can be expressed in terms of the temperature T_c at the center of the sphere:

$$\wp_c = \frac{kT_c}{\mu m_u}. \qquad (7)$$

In [12] the square of the particles' velocity at the center of the sphere was estimated:

$$v_c^2 \approx \frac{3\eta m}{5a}\left(1+\frac{9}{2\sqrt{14}}\right). \qquad (8)$$

In derivation of (8) the value of the acceleration field coefficient η was not recorded, as well as the mass of the system's particles. Real bodies can contain several types of particles with different masses at the same time, such as atoms, ions, electrons, and individual nucleons. It is convenient to assume that the coefficient η refers to the particles with the effective mass $\bar{m} = \mu m_u$. In this case we can write for the mass density ρ at an arbitrary point of the sphere the following:

$$\rho \approx n_n m_u = n_p \bar{m} = n_p \mu m_u,$$

where n_n is the concentration of nucleons, n_p is the concentration of particles with the effective mass $\bar{m}$, contributing to the pressure, so that $p = \frac{\rho kT}{\mu m_u} = n_p kT$.

We will multiply relation (8) for v_c^2 by $\frac{\bar{m}}{2}$ and equate it to $\frac{3kT_c}{2}$, which gives the equality between the kinetic energy of one particle with the mass $\bar{m}$ and the kinetic temperature at the center of the sphere:

$$\frac{\eta m \bar{m}}{5a}\left(1+\frac{9}{2\sqrt{14}}\right) = kT_c.$$

Let us substitute the left-hand side of this equation in (7) instead of kT_c and take into account the definition $\bar{m} = \mu m_u$:

$$\wp_c = \frac{\eta m}{5a}\left(1+\frac{9}{2\sqrt{14}}\right). \qquad (9)$$

According to [13], for the scalar potential of the pressure field inside the sphere the following relation holds:

$$\wp = \wp_c - \frac{\sigma c^2 \gamma_c}{\eta} + \frac{\sigma c^3 \gamma_c}{r\eta\sqrt{4\pi\eta\rho_0}} \sin\left(\frac{r}{c}\sqrt{4\pi\eta\rho_0}\right) \approx \wp_c - \frac{2\pi\sigma\rho_0 r^2 \gamma_c}{3}. \qquad (10)$$

If we pass from the Lorentz factors to the squares of velocities in (3), we will obtain:

$$v'^2 \approx v_c^2 - \frac{4\pi\eta\rho_0 r^2 \gamma_c}{3}.$$

By multiplying this equation by $\frac{\bar{m}}{3k}$ we pass on to the relation for the temperatures inside the sphere:

$$T \approx T_c - \frac{4\pi \eta \bar{m} \rho_0 r^2 \gamma_c}{9k}.$$

Let us go ahead and multiply the last equation by $\frac{k}{\mu m_u}$. Taking into account the definition $\bar{m} = \mu m_u$ and (7) we have:

$$\wp \approx \wp_c - \frac{4\pi \eta \rho_0 r^2 \gamma_c}{9}.$$

From comparison of this relation with (10) we arrive at the fact that

$$\sigma = \frac{2\eta}{3}. \tag{11}$$

We will now use the relation between the field coefficients, which was obtained in [7] using the equation of motion:

$$\eta + \sigma = G - \frac{\rho_{0q}^2}{4\pi \varepsilon_0 \rho_0^2} = G - \frac{q^2}{4\pi \varepsilon_0 m^2}. \tag{12}$$

Combination of (11) and (12) gives the following for the coefficients of the pressure field and the acceleration field:

$$\eta = \frac{3}{5}\left(G - \frac{\rho_{0q}^2}{4\pi \varepsilon_0 \rho_0^2} \right), \qquad \sigma = \frac{2}{5}\left(G - \frac{\rho_{0q}^2}{4\pi \varepsilon_0 \rho_0^2} \right). \tag{13}$$

Let us now substitute (11) into (9):

$$\wp_c = \frac{3\sigma m}{10a}\left(1 + \frac{9}{2\sqrt{14}} \right). \tag{14}$$

In view of (14), we will sum up all the terms with the pressure in (6):

$$W_{pr} = m \wp_c \gamma_c - \frac{2\sigma m^2 \gamma_c^2}{5a} \approx \frac{\sigma m^2 \gamma_c}{10a}\left(\frac{27}{2\sqrt{14}} - 1 \right). \tag{15}$$

In view of (15), for the potential energy (6) we obtain:

$$W_p \approx -\frac{3G m^2 \gamma_c^2}{5a} + \frac{3q^2 \gamma_c^2}{20\pi \varepsilon_0 a} + \frac{\sigma m^2 \gamma_c}{10a}\left(\frac{27}{2\sqrt{14}} - 1 \right) - \frac{\eta m^2 \gamma_c^2}{10a}. \tag{16}$$

Relation (12) can be used to eliminate the coefficient η in (16) and to express the potential energy of the system in terms of the corresponding energies of the gravitational and electromagnetic fields and the pressure field:

$$W_p \approx -\frac{7G\,m^2\gamma_c^2}{10a} + \frac{7q^2\gamma_c^2}{40\pi\varepsilon_0 a} + \frac{27\sigma m^2\gamma_c}{20\sqrt{14}a}. \tag{17}$$

We will now express all the terms in (16) in terms of the acceleration field coefficient η using (12) and (11):

$$W_p \approx -\frac{\eta m^2\gamma_c}{30a}\left(35 - \frac{27}{\sqrt{14}}\right). \tag{18}$$

Hence we see that the potential energy of a relativistic uniform system that is in equilibrium condition is always negative.

Substituting (2) into (18) gives the following:

$$W_p \approx -\frac{2}{3}\left(\frac{35\sqrt{14}}{27} - 1\right)E_k. \tag{19}$$

Accordingly, the total energy is equal to:

$$E_t = E_k + W_p \approx -\frac{5}{3}\left(\frac{14\sqrt{14}}{27} - 1\right)E_k. \tag{20}$$

Despite the fact that in (19) the absolute value of the potential energy is much greater than the kinetic energy, the virial theorem is satisfied in the system under consideration. It was shown in [12], where the nonzero virial of the system and the forces acting on the particles were explicitly calculated. In particular, the average energy, associated with these forces, in view of (2) is equal to:

$$-\left\langle \sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i \right\rangle \approx \frac{3\eta m^2\gamma_c^2}{5a} - \frac{3\eta^2 m^3\gamma_c^2}{14a^2c^2} \approx \frac{4\sqrt{14}}{9}E_k.$$

We can also transform the pressure energy (15), using (11), and compare it with the kinetic energy (2):

$$W_{pr} \approx \frac{2}{3}\left(1 - \frac{2\sqrt{14}}{27}\right)E_k. \tag{21}$$

The entire energy, associated with the pressure, appears almost 2 times less than the kinetic energy of the particles' motion.

Most cosmic bodies are neutral, we can neglect their electromagnetic fields and can assume that $\rho_{0q} = 0$. In this case, according to (13) $\eta = \frac{3G}{5}$, and the potential energy (18) is equal to:

$$W_p \approx -\frac{Gm^2\gamma_c}{50a}\left(35 - \frac{27}{\sqrt{14}}\right) \approx -\frac{0.56Gm^2\gamma_c}{a}.$$

In the classical uniform model the body's matter is compressed by the gravitational forces, which are opposed by the internal pressure force, while the pressure is considered to be a scalar quantity. It is assumed that the main contribution into the potential energy is made by the gravitational energy, which equals the value

$$U_g \approx -\frac{0.6Gm^2}{a}.$$

We see that the potential energy W_p in the relativistic uniform model is very close in value to the gravitational energy U_g. This explains why in the classical model for estimating the potential energy of the system it is sufficient to calculate only the total gravitational energy of the system and there is no need to take into account neither the pressure field energy nor the acceleration field energy.

4. The binding energy

By definition, the binding energy of the physical system is obtained by subtracting the relativistic energy E_r (1) from the total rest energy of the particles W_b (4):

$$\Delta E = W_b - E_r. \tag{22}$$

Taking into account the definition of the total energy $E_t = E_r - W_b$, in view of (20), we find for the binding energy the following:

$$\Delta E = -E_t = -E_k - W_p \approx \frac{5}{3}\left(\frac{14\sqrt{14}}{27} - 1\right) E_k \approx 1.57 E_k. \tag{23}$$

For comparison, in simple systems, where there are only potential forces in the absence of pressure, due to the virial theorem $\Delta E \approx E_k$.

In our physical system of closely interacting particles and fields in addition to the electromagnetic field, we also take into account the contributions from the gravitational field, the acceleration field and the pressure field. In such a system, according to (19), the potential energy of the fields W_p is negative and it is much greater in its absolute value than the kinetic energy E_k in comparison to simple systems. This leads to the increased binding energy, which is required to separate the system's particles from each other and to scatter them to infinity. Thus it is expected that during the formation of a bound relativistic uniform system with charged particles under the action of the gravitational field, taking into account the contributions from the acceleration field and the pressure field, the system must emit the energy, which is equal to the binding energy $\Delta E \approx 1.57 E_k$.

Among all the energies it's most convenient to calculate the energies of the gravitational and electromagnetic fields, which go beyond the limits of the system to infinity. The sum of the energies of these external fields, taking into account (12) and (11), is equal to:

$$E_{og} + E_{oe} \approx -\frac{Gm^2\gamma_c^2}{2a} + \frac{q^2\gamma_c^2}{8\pi\varepsilon_0 a} = -\frac{\eta m^2\gamma_c^2}{2a} - \frac{\sigma m^2\gamma_c^2}{2a} = -\frac{5\eta m^2\gamma_c^2}{6a}. \tag{24}$$

From (4) the relation follows:

$$m\gamma_c \approx m_b + \frac{3\eta m^2\gamma_c}{10ac^2} \approx m_b + \frac{3\eta m_b^2}{10ac^2\gamma_c}.$$

Similarly, in [4] the charge q was associated with the charge q_b of the sphere, which was found by the observer in K':

$$q\gamma_c \approx q_b + \frac{3\eta m_b q_b}{10ac^2\gamma_c}.$$

Applying this to (24), we find:

$$E_{og} + E_{oe} = -\frac{Gm_b^2}{2a} + \frac{q_b^2}{8\pi\varepsilon_0 a}. \tag{25}$$

In (25) the energy of the gravitational and electromagnetic fields outside the sphere is expressed in terms of the total rest mass m_b of the particles inside the sphere and the total charge of the particles q_b. From (25) we see that the mass m_b is actually equal to the gravitational mass m_g, which is responsible for the gravitation outside the body.

Comparing (24) with the kinetic energy (2) and the binding energy (23) gives the following:

$$E_{og} + E_{oe} \approx -\frac{50\sqrt{14}\,\gamma_c}{81} E_k \approx -\frac{10\sqrt{14}\,\gamma_c}{\left(14\sqrt{14}-27\right)} \Delta E. \tag{26}$$

With the help of (26) we can easily estimate the binding energy ΔE, if we know the mass, charge and radius of the system, using which in (24) and (25) we can calculate the sum of the energies $E_{og} + E_{oe}$ outside the system. Although cosmic bodies with the same masses and sizes can differ in their state of matter, the binding energy of these bodies in (23), according to (26), will be the same. Indeed, the phase transformations of matter, arising from the energy transfer inside the system with the constant radius and mass, should not influence the energy of the external fields and the total binding energy of the system.

5. Estimation of the energy of fields

In this section we will consider the question about what contribution into the relativistic energy and the total energy is made by the energy E_f, associated with the system's fields. The energy E_f is calculated with the help of the volume integrals of the fields' tensor invariants, for which it is necessary to know the strengths and solenoidal vectors of the fields. As part of the relativistic energy of the system (1), the energy E_f according to [4] equals:

$$E_f = \frac{3Gm^2\gamma_c^2}{5a} - \frac{3q^2\gamma_c^2}{20\pi\varepsilon_0 a} - \frac{\sigma m^2\gamma_c^2}{10a} - \frac{\eta m^2\gamma_c^2}{10a}.$$

In this expression we will take into account (12), (11), (24), (2), (26) and (23):

$$E_f = -E_{og} - E_{oe} \approx \frac{50\sqrt{14}\,\gamma_c}{81} E_k \approx \frac{10\sqrt{14}\,\gamma_c}{\left(14\sqrt{14}-27\right)} \Delta E \approx -\frac{10\sqrt{14}\,\gamma_c}{\left(14\sqrt{14}-27\right)} E_t. \tag{27}$$

According to (27), the energy E_f of all the four fields is approximately $2.31\gamma_c$ times greater than the kinetic energy of the particles E_k and $1.47\gamma_c$ times greater than the binding energy ΔE, which is equal in its absolute value to the total energy of the system E_t. In addition, in (27) $E_f = -E_{og} - E_{oe}$, that is, the energy E_f is up to a sign equal to the sum of the energies of the gravitational and electromagnetic fields outside the sphere. Hence it follows that the sum of the energies of all the fields inside the sphere is equal to zero.

We will now calculate the sum of the particles' energies E_{gep} in the gravitational and electromagnetic fields and in the pressure field. According to (1) we have the following:

$$E_{gep} = -\frac{6G\,m^2\gamma_c^2}{5a} + \frac{3q^2\gamma_c^2}{10\pi\varepsilon_0 a} + m\wp_c\gamma_c - \frac{3\sigma m^2\gamma_c^2}{10a}.$$

Using (12), (14), (11), (2) and (24) we find:

$$E_{gep} \approx -\frac{2\eta\,m^2\gamma_c}{a}\left(1-\frac{9}{20\sqrt{14}}\right) \approx -\frac{2}{3}\left(\frac{20\sqrt{14}}{9}-1\right)E_k \approx \frac{12}{5\gamma_c}\left(1-\frac{9}{20\sqrt{14}}\right)(E_{og}+E_{oe}).$$

Comparison with (27) gives us the following:

$$E_{gep} \approx -\frac{12}{5\gamma_c}\left(1-\frac{9}{20\sqrt{14}}\right)E_f, \tag{28}$$

that is, the energy E_f, associated with the fields, in its absolute value is over 2 times less than the sum of the particles' energies E_{gep} in the gravitational and electromagnetic fields and in the pressure field. Note that all the conclusions are made in the weak field approximation, while the Lorentz factor γ_c of the particles at the center of the sphere does not differ significantly from unity.

As it was shown in [12], although the global vector potentials of the fields inside the sphere with the particles are equal to zero, there are also proper vector potentials of the fields inside the particles due to their motion. These vector potentials are part of the generalized momentum, with the help of which we can estimate the kinetic energy of the system's particles. Because of the proper vector potentials, during the particles' motion the corresponding solenoidal fields emerge, which make an additional contribution into the energy of the particles' fields E_f.

What can be the value of this contribution? Due to the motion, the energy of the fields becomes dependent on the velocity, however the total energy of the fields inside the body vanishes. If we assume that the same is true for each particle filling the sphere in the model under consideration, then the proper scalar and vector potentials of the fields inside the particles do not result in the total energy of the fields. At the same time the global scalar potentials of the fields, that are the scalar superposition of the external scalar potentials of individual particles, give the corresponding field strengths, with the help of which we can calculate the energy of the fields E_f inside and outside the sphere.

In contrast to this, the global vector potentials of the fields are the vector superposition of the external vector potentials of individual particles, and they are equal to zero due to the randomness of motion of the set of system's particles. This implies the equality of all the global solenoidal vectors to zero, and thus they do not contribute to the energy of the fields.

6. Conclusion

Our analysis of the energy for the system of a spherical form shows that in real massive bodies there are noticeable deviations of the total, kinetic and potential energies from the expressions for the energies of simple systems, interacting only by means of gravitational and electromagnetic forces at a distance. This is due to the additional contributions from the acceleration field and the pressure field.

Taking into account (23) and (19-20), the binding energy and the total energy can be expressed in terms of the kinetic energy of the particles or the potential energy of the system:

$$\Delta E = -E_t \approx \frac{5}{3}\left(\frac{14\sqrt{14}}{27}-1\right)E_k \approx -\frac{5\left(14\sqrt{14}-27\right)}{70\sqrt{14}-54}W_p\,, \qquad W_p \approx -\frac{2}{3}\left(\frac{35\sqrt{14}}{27}-1\right)E_k \approx -2.57E_k\,, \tag{29}$$

Relations (29) that we obtained can be compared to the standard expression for simple systems and the corresponding virial theorem in classical mechanics:

$$\Delta E = -E_t = -E_k - W_{pc} = -0.5W_{pc}\,, \qquad W_{pc} = -2E_k\,, \tag{30}$$

where the classical potential energy is expressed as the sum over all the particles in terms of the radius vectors of the particles and the forces acting on the particles from the potential fields:

$$W_{pc} = \left\langle \sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i \right\rangle.$$

In contrast to (30), in [12] we obtained the following expression for the virial theorem:

$$E_k \approx -\frac{9}{4\sqrt{14}}\left\langle \sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i \right\rangle. \tag{31}$$

From (29) and (31) it follows that taking into account the acceleration field and the pressure field in the system under consideration leads to a change from the classical value -0.5 to approximately -0.61 of the binding energy's share relative to the potential energy, and to a corresponding change in the value from -0.5 to approximately -0.6 of the kinetic energy's share relative to the energy associated with the action of the potential forces. In addition, (29) and (31) imply the inequality of the potential energy and the energy associated with the forces acting on the particles, so that $W_p \neq \left\langle \sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i \right\rangle$.

The binding energy of the system, according to the (26), can be expressed in terms of the sum of energies of the gravitational and electromagnetic fields outside the body:

$$\Delta E \approx -\frac{1}{5\gamma_c}\left(7-\frac{27}{2\sqrt{14}}\right)(E_{og}+E_{oe})\,. \tag{32}$$

The unique relation between the total, kinetic and potential energies and the binding energy in the system under consideration can be obtained due to the fact that we use the effective mass of the particles $\bar{m} = \mu m_u$, which is associated with the system's state of matter by means of the parameter μ (the number of nucleons per one particle of relativistic gas). This leads to relations between the coefficients of the acceleration field and the pressure field in (11) and to relations between these coefficients and the gravitational constant and the vacuum permittivity in (12-13), which allows us to compare the values of the energies.

In the last section we estimated the contribution of the energy E_f, which is made by the fields into the relativistic energy E_r, and we compared it with the energy E_{gep} in the gravitational and electromagnetic fields, as well as in the pressure field in relation (28). Using (2) and (4) we obtain the relation:

$$mc^2\gamma_c^2 - \frac{3\eta m^2\gamma_c^2}{5a} \approx W_b + E_k\,.$$

Taking this into account, from (1) and (26-28) for the relativistic energy of the system we find the following:

$$E_r = Mc^2 \approx mc^2\gamma_c^2 - \frac{3\eta m^2\gamma_c^2}{5a} + E_{gep} + E_f \approx W_b + E_k - \frac{1}{5}\left(7 - \frac{27}{5\sqrt{14}}\right)E_f \approx$$
$$\approx W_b + \frac{1}{5\gamma_c}\left(7 - \frac{27}{2\sqrt{14}}\right)(E_{og} + E_{oe}). \tag{33}$$

Expression (33) for the relativistic energy E_r, in view of (32), corresponds to expression (22) for the binding energy of the system.

Since W_b is the rest energy of the systems' particles according to (4), and the fields' energy $E_{og} + E_{oe}$ outside the sphere is typically negative due to the prevailing contribution of the gravitational energy over the electromagnetic energy, we can see that the inertial mass M of the system turns out to be less than the total invariant mass of the particles m_b, which is part of the equation $W_b = m_b c^2$. We should also note that the mass m_b is exactly equal to the gravitational mass m_g, according to [4].

After substituting (25) into (33) we arrive at the relation, which is only slightly different from relation (31) in [4]:

$$Mc^2 \approx m_b c^2 - \frac{1}{10\gamma_c}\left(7 - \frac{27}{2\sqrt{14}}\right)\left(\frac{Gm_b^2}{a} - \frac{q_b^2}{4\pi\varepsilon_0 a}\right). \tag{34}$$

Hence it follows that the introduction of the charge q_b into the system typically increases the inertial mass M, at least this holds exactly at the constant mass m_b. And conversely, since $m_b = m_g$, decrease in the gravitational mass m_g of the system is possible with increasing of the charge q_b. To see this, it suffices to solve (34) as a quadratic equation for m_b, and to fix the inertial mass M while q_b is changing:

$$m_b = m_g \approx M + \frac{1}{10\gamma_c}\left(7 - \frac{27}{2\sqrt{14}}\right)\frac{GM^2}{ac^2} - \frac{1}{10\gamma_c}\left(7 - \frac{27}{2\sqrt{14}}\right)\frac{q_b^2}{4\pi\varepsilon_0 ac^2}.$$

In the classical uniform system of a spherical shape with stationary particles the total gravitational energy summed up with the total electric energy is equal to the following: $E_g + E_e = -\frac{3Gm_b^2}{5a} + \frac{3q_b^2}{20\pi\varepsilon_0 a}$. Consequently, (34) can be written as follows:

$$Mc^2 \approx m_b c^2 + \frac{1}{6\gamma_c}\left(7 - \frac{27}{2\sqrt{14}}\right)\left(E_g + E_e\right).$$

As we can see, the inertial mass M of the relativistic uniform system differs from the rest mass of the particles m_b by approximately half of the total mass-energy of the gravitational and electric fields of the classical uniform system, whereas the presence of the electric field increases the mass M in contrast to the action of the gravitational field.

Now let us imagine that for an external observer the sphere with the particles has the invariant inertial mass M, the volume V and the corresponding mass density ρ_s: $M = \rho_s V$. Let us substitute (4) into (33) and take into account (24) and the relation $m = \rho_0 V$. This gives the following:

$$E_r = Mc^2 = \rho_s V c^2 \approx \rho_0 V c^2 \gamma_c \left[1 - \left(\frac{22}{15} - \frac{9}{4\sqrt{14}} \right) \frac{\eta m}{ac^2} \right]. \qquad (35)$$

In [12] we found expression (8) for the squared velocities of the particles at the center of the sphere, with the help of which we can estimate the Lorentz factor:

$$\gamma_c = \frac{1}{\sqrt{1 - v_c^2/c^2}} \approx 1 + \frac{v_c^2}{2c^2} + \frac{3v_c^4}{8c^4} \approx 1 + \frac{3\eta m}{10ac^2} \left(1 + \frac{9}{2\sqrt{14}} \right) + \frac{27\eta^2 m^2}{200a^2 c^4} \left(1 + \frac{9}{2\sqrt{14}} \right)^2 .$$

Substituting the expression for γ_c into (35), after reduction by Vc^2 we find:

$$\rho_s \approx \rho_0 \gamma_c \left[1 - \left(\frac{22}{15} - \frac{9}{4\sqrt{14}} \right) \frac{\eta m}{ac^2} \right] \approx \rho_0 \left[1 - \left(\frac{7}{6} - \frac{18}{5\sqrt{14}} \right) \frac{\eta m}{ac^2} \right]. \qquad (36)$$

Hence it follows that $\rho_s < \rho_0$, i.e., as more increasingly massive relativistic uniform systems are formed, the average density of these systems decreases as compared to the average density of the particles and bodies that make up these systems.

Let us assume that this conclusion holds true for neutron stars, mostly consisting of nucleons only with a small admixture of atomic nuclei and a certain number of electrons, which give a small contribution into the total mass. We will assume that the mass density ρ_0 in (36) represents the density of the nucleons' matter, and ρ_s is the average density of a neutron star, and we will use (13) for the case of an uncharged star with zero charge density ρ_{0q}. This gives the following:

$$\rho_s \approx \rho_0 \left[1 - \frac{3}{5} \left(\frac{7}{6} - \frac{18}{5\sqrt{14}} \right) \frac{Gm}{ac^2} \right]. \qquad (37)$$

Substituting here instead of m the mass of a typical star of 1.35 solar masses, taking as the nucleon density ρ_0 the proton density of 6×10^{17} kg/m^3 with the proton radius of 8.73×10^{-16} m according to [15], and using the estimate of the star density in the form $\rho_s = \frac{3m}{4\pi a^3}$, we find the corresponding radius of the neutron star: $a \approx 10.3$ km. This radius supposes tight packing of neutrons in the matter of the star and by the order of magnitude is in reasonable agreement with the observational data. However, it should be noted that in the star there are gaps between the nucleons. Therefore, in (37) for ρ_0 we should substitute not the mass density of the proton, but a smaller quantity. This leads to decreasing of ρ_s, so that the radius $a \approx 10.3$ km places the lower limit on the radius of the neutron star.

In the theory of infinite nesting of matter nucleons are similar in their properties to neutron stars, and for these objects the ratio of the central density to the average density is approximately 1.5 according to [7, 15]. Thus, in the first approximation nucleons and neutron stars are close enough in their properties to relativistic uniform systems. In addition it should be noted that these objects consist of particles, for which it is necessary at least to take into account the energy of the proper spin rotation and the energy of strong interaction. Consequently, our analysis with respect to such relativistic objects needs clarification, starting with introduction of additional terms into the Lagrangian and finishing with taking into account the metric in the equations, arising from the principle of least action. These calculations have not yet been made, but we can rely on the equation derived by Tolman, Oppenheimer, and Volkoff in the framework of the general theory of relativity [16].

From (37) and (33) it follows that as a certain large relativistic uniform system is formed from a number of small relativistic uniform systems, the average mass density of the system decreases as compared to the average density of its parts, while the inertial (invariant) mass of the large system is less than the sum of inertial (invariant) masses of the system's parts. At the same time, the relativistic energy density and the binding energy density decrease in the transition to increasingly more massive objects. From a qualitative point of view the density decrease can be explained by the presence of gaps between the individual parts of the system with reduced mass density. From a quantitative point of view the decrease in the average density of the system can be derived from the contributions of the particles' energy in the potentials of the system's proper fields with addition from the fields' energy, found through the strengths and solenoidal vectors of the fields.

In this regard, we note that the strengths and solenoidal vectors of the fields are the temporal and spatial rates of change of the field potentials, as they are calculated using the partial derivatives of the scalar and vector potentials. Therefore, in case of the known dependencies of the field potentials on time and coordinates, the full description of the system can be easily achieved and the main dependencies, including the equation of motion and the stress-energy tensor [14], can be easily found. In contrast to this, if only the strengths and solenoidal vector of the fields are given, in order to determine the potentials we need to perform integration and to take into account the initial conditions. And such integration, as is known, is much more difficult than differentiation. An additional advantage of the use of potentials in the field physics is the fact that they are calculated in the standard way using the corresponding wave equations [4], [13].

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