On the value of the function $\exp(ax)/f(a)$ at $a = 0$ for $f(a) = 0$

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Abstract: In this short note, we will consider the value of the function $\exp(ax)/f(a)$ at $a = 0$ for $f(a) = 0$. This case appears for the construction of the special solution of some differential operator $f(D)$ for the polynomial case of D with constant coefficients. We would like to show the power of the new method of the division by zero calculus, simply and typically.

Key Words: Division by zero calculus, construction of special solutions, ordinary differential equation.

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1 Introduction

In this short note, we will consider the value of the function $\exp(ax)/f(a)$ at $a = 0$ for $f(a) = 0$. This case appears for the construction of the special solution of some differential operator $f(D)$ for the polynomial case of D with constant coefficients. We would like to show the power of the new method of the division by zero calculus, simply and typically.

2 Division by zero calculus

For the statement of the conclusion, we will recall the division by zero calculus.

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n(z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z-a)^n,$$

we **define** the division by zero calculus by the identity

$$f(a) = C_0.$$

For many basic properties and applications of the division by zero calculus, see [7] and the references.

3 Conclusion

From the definition of the division by zero calculus, directly, we obtain the theorem, simply

Theorem: *For the function*

$$\frac{\exp(ax)}{f(a)}, \quad f(a) = 0$$

if $f(z)$ is analytic around $z = 0$ and $f'(a) = f''(a) = ... = f^{(m)}(a) = 0$ and $f^{(m+1)}(a) \neq 0$, by the division by zero calculus, we obtain the identity

$$\frac{x^{m+1}\exp(ax)}{f^{(m+1)}(a)}.$$

When $f(D)$ is an (polynomial) ordinary differential operator with $D = d/dx$ and with constant coefficients, in the ordinary differential equation

$$f(D)y = \exp(ax),$$

if $f'(a) = f''(a) = ... = f^{(m)}(a) = 0$ and $f^{(m+1)}(a) \neq 0$, then it gives a special solution.

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