Electromagnetic Force and Special Relativity

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Introduction

Einstein claimed that "If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by transformation of the field to a system of co-ordinates at rest relatively to the electrical charge."[1]

- **A few thought experiments have been suggested to corroborate this claim[2,6]**
- **One of these thought experiments is produced by Feynman [3]**
- **Feynman's case and analysis is also adopted by Purcell and Morin [4]**
- **The example is generally being reproduced and taught in various universities [5]**
- **However, the results do not agree with Einstein's claim that the two methods produce the same answer for electromagnetic force acting on the charge**
- **This article presents and reviews Feynman's analysis and calculation of electromagnetic forces in the two inertial reference frames as described by Einstein**
- **It shows that Feynman's analysis is just a deliberate attempt to produce similar, though eventually not equal, results**
- **Correct analysis also does not support Einstein's claim**

Background

- **What is the electromagnetic force caused by a current-carrying wire on a passing electric charge**
- **The calculation can be done in the lab frame, S, in which the wire is fixed to the lab desk in front of the observer and the electric charge moves with a constant speed of u**
- **This case has been experimented**

- **The second calculation can be done by an observer in the frame of external electric charge, S', in which the charge is stationary and the wire is moving**
- **There is no report of an experiment for this case. Thus, scientists rely on thought experiment in their investigations in S' frame**

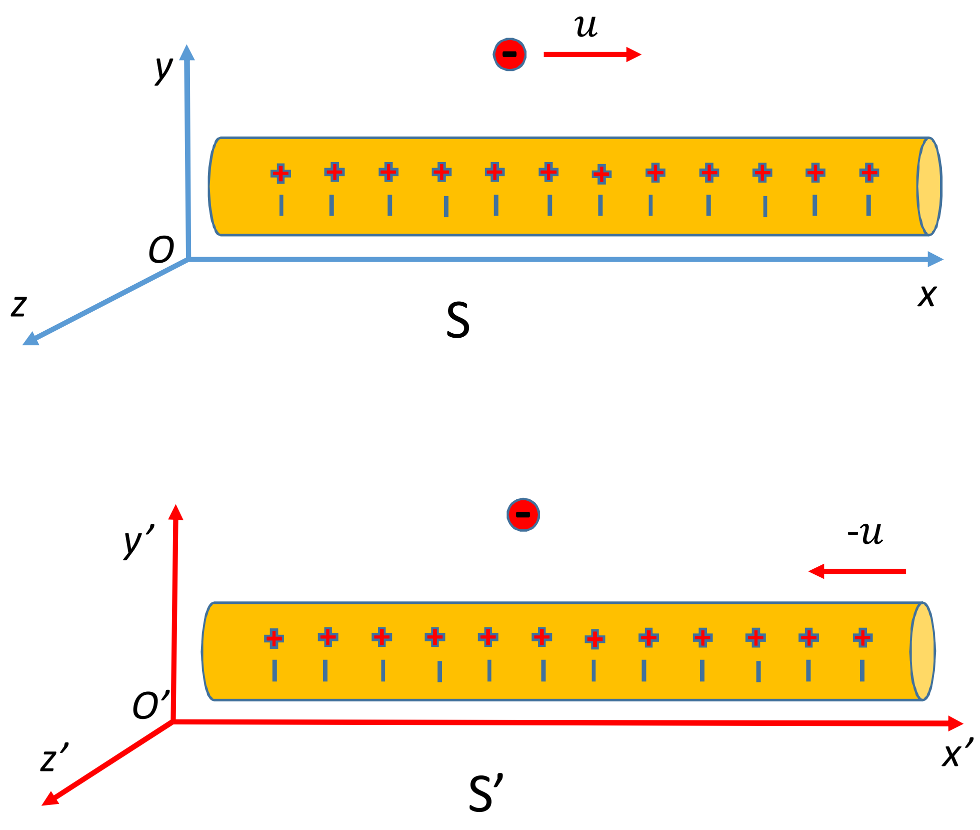

Figure 1

Assumptions in the Lab Frame, S

- Suppose the length of current-carrying wire is L
- Within this length, N *"conduction electrons"* continuously move to the right with the constant speed of v
- Each electron has a negative electric charge of q_-
- The wire is electrically neutral meaning that there are also N numbers of positive charges, q_+. Both charges are equally distributed along the length of the wire

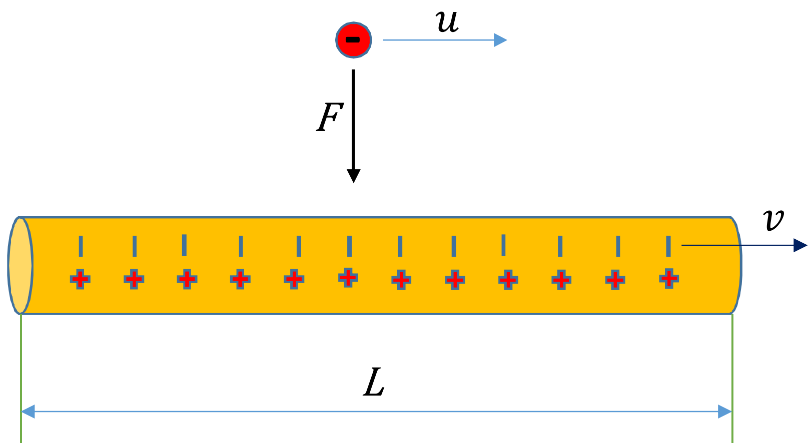

Figure 2

u = Speed of the external charge (for simplicity it can be equal to v)
L = Length of the wire
A = Cross section area of the wire
v = Speed of electrons within the wire
q = Electric charge ($q = 0$)
ρ = Charge density ($\rho = \rho_+ + \rho_- = 0$)
I = Current ($I = \rho_- v\ A$)
J = Current density ($J = \rho_-\ v$ = $N\,q\,v$).
$\boldsymbol{E}$ = Electric field to electric charge within the wire
$\boldsymbol{B}$ = Magnetic field, around the wire due to the flow of current
$\boldsymbol{F} = \boldsymbol{F}_E + \boldsymbol{F}_B = q(\boldsymbol{E} + u \times \boldsymbol{B})$, electromagnetic force on the external charge, q, moving at the constant speed of u parallel to the wire.

Calculation of EM force in S frame

- In the S-frame, the electric field is zero outside the wire as there are equal numbers of positive and negative charges within the wire. Thus, electrostatic force is zero
$$F_E = q\,\boldsymbol{E} = 0$$
- However, due to the flow of current there is a magnetic field around the wire, causing certain magnetic force on the moving charged particle forcing it to curve in towards the wire. The magnitude of this force is

$$F_B = q\,u \times B$$

$$B = \frac{1}{4\pi\epsilon_0 c^2}\frac{2I}{r}$$

$$F_B = \frac{1}{4\pi\epsilon_0 c^2}\frac{2q\rho_- A v u}{r}$$

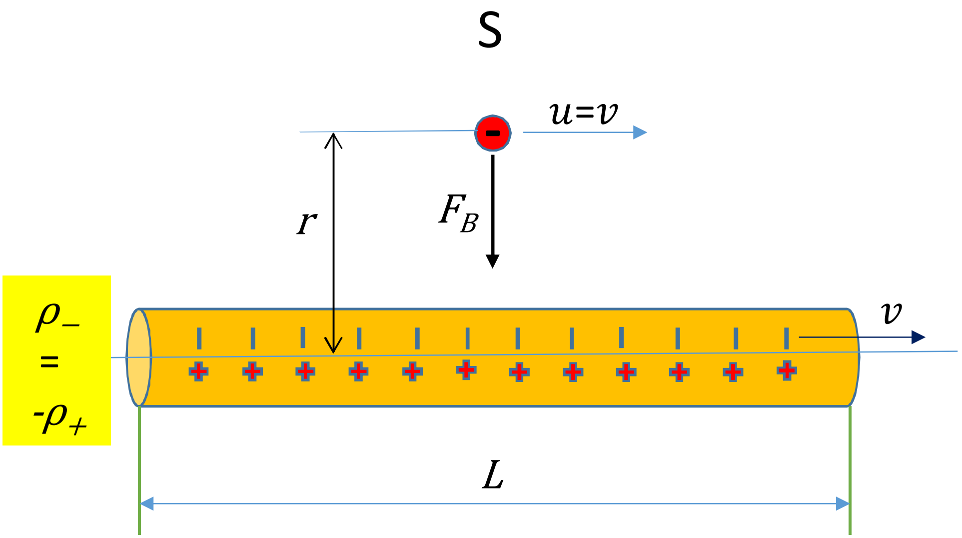

Figure 3

For simplicity we consider a special case when $u = v$, Thus,

$$F = F_B = \frac{q}{2\pi\epsilon_0}\frac{\rho_- A}{r}\frac{v^2}{c^2}$$

For the calculation when $u \neq v$ see [4]

Calculation of EM force in S' frame 1/3

In the S'-frame "***The positive charges moving with the wire will make some magnetic field B' at the external particle. But the particle is now at rest, so there is no magnetic force on it!***"

$$u = 0$$
$$F'_B = q\,u \times B' = 0$$

"***If there is any force on the particle, it must come from an electric field. It must be that the moving wire has produced an electric field. But it can do that only if it appears charged—it must be that a neutral wire with a current appears to be charged when set in motion.***"

"***We know that the apparent mass of a particle changes by*** γ***. Does its charge do something similar? No! Charges are always the same, moving or not. Otherwise we would not always observe that the total charge is conserved.***"

The length of wire in S' is

$$L' = L/\gamma$$

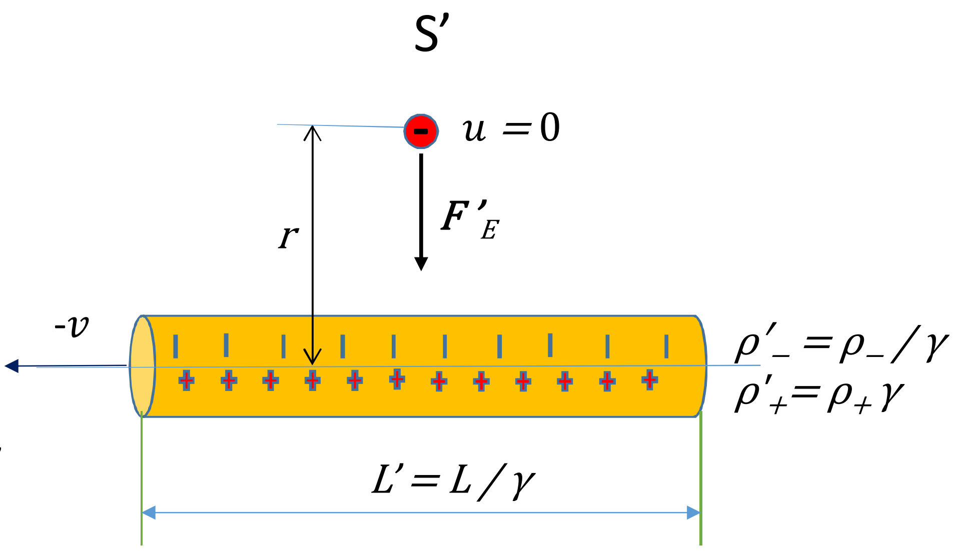

Figure 4

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Calculation of EM force in S' frame 2/3

"***The charge density of a moving distribution of charges varies in the same way as the relativistic mass of a particle.*** *" Thus, "****where the wire moves with the speed , the positive charge density becomes***"

$$\rho'_+ = \rho_+ \gamma$$

On the other hand, "***The negative charges are at rest in S'. So they have their "rest density" in this frame. … For the conduction electrons, we then have that***"

$$\rho'_- = \rho_- / \gamma$$

"***Now we can see why there are electric fields in S'—because in this frame the wire has the net charge density ρ' given by***"

$$\rho' = \rho'_+ + \rho'_-$$

Since the stationary wire is neutral, $\rho_- = -\rho_+$, and we have

$$\rho' = \rho_+ \gamma v^2 / c^2$$

Calculation of EM force in S' frame 3/3

Electric field in S' is thus

$$E' = \frac{\rho' A}{2\pi\epsilon_0 r} \Rightarrow E' = \frac{\rho_+ A}{2\pi\epsilon_0 r} \frac{v^2}{c^2} \gamma$$

The magnitude of the force in S' is

$$F' = F'_E = \frac{q}{2\pi\epsilon_0} \frac{\rho_+ A}{r} \frac{v^2}{c^2} \gamma$$

Comparing the result in S' and S "***we see that the magnitudes of the forces are almost identical from the two points of view. In fact***,"

$$F' = F\gamma$$

Review of Feynman’s analysis 1/3

Let us examine the movement and density of charges in the wire in S and S’ frames
For clear understanding it is better to examine two scenarios in each frame

1. No current flows through the wire
2. Current flows through the wire

Observer in the lab frame, S

The number of negative charges is equal to positive charges irrespective of the amount of current flowing through the wire, as it has been emphasized by Feynman. Cases (a) and (b) in Figure 5,

$$\rho_- = \rho_+ \Rightarrow q = \rho = 0 \text{ for } I \geq 0$$

What does happen if current flows through the wire? Electrons move within the wire but the wire is surely not getting contracted because of these movements. However, based on SR claim, the individual electrons are supposed to get contracted, in the direction of their movements, due to their relative speed along the wire as shown in (b). The same is true for the moving external charge.

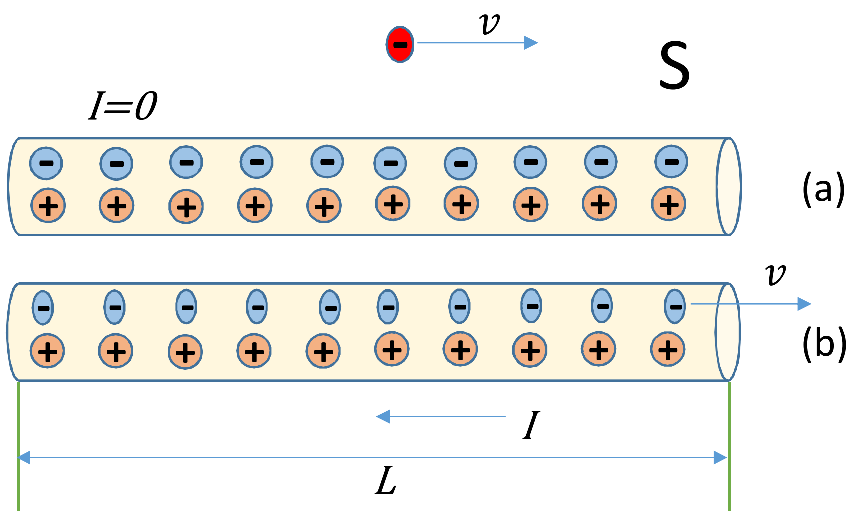

Figure 5

Review of Feynman's analysis 2/3

Observer in the external charge frame, S'

No Current

The whole wire is supposed to get contracted along with all its charges, but still

$$I = 0$$

$$q = \rho = 0$$

Current flows in the wire

The current carrying wire is also contracted but negative charges are not contracting as relative speed between them and the observer is zero.

This case (d) is similar to case (b) in which one of the charges are contracted but the total charge within the length of wire is still zero.

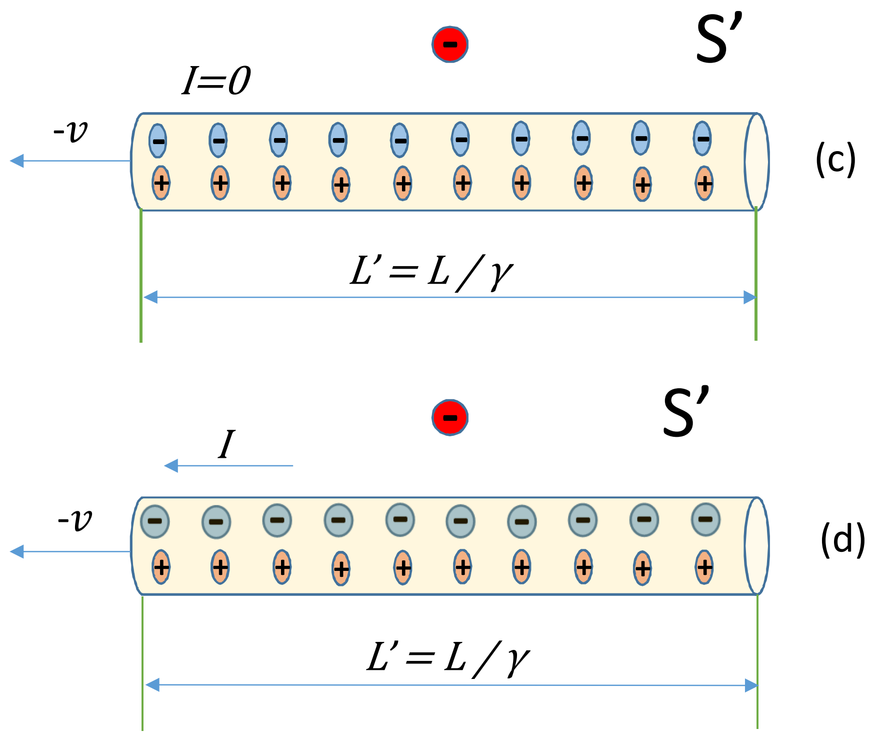

Figure 6

Review of Feynman's analysis 3/3

The charge density in frame S' stays the same as it is in S for any current.

$\rho'_{-} = \rho'_{+} \Rightarrow \rho' = 0$ for $I \geq 0$

Thus there is no electrostatic force in S' to be calculated as well

It seems γ and $1/\gamma$, for positive and negative charges, are produced by Feynman in S' just to comply with Einstein claim

Both terms are needed to produce a result similar to the force in S, as even having one of the two terms result in different outcomes

Even if the Feynman analysis is correct the calculated force are though similar, but not the same

No explanation is given why the resulting forces in the two frames are different. As if having a γ in anything to do with relativity is normal or a must, such as the theory of relativistic mass which is still contentious

The result is in contrast to Einstein's claim that:

"If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by transformation of the field to a system of co-ordinates at rest relatively to the electrical charge."[1]

Tong's Rationalisation

- D. Tong[6] tries to justify the calculated electric force in S' frame by initially introducing a wire in S frame through which the positive charges are moving at the constant speed of v in one direction and the negative charges moving at the same speed in the opposite direction.
- This case is not realistic and is not scientifically and practically justified as only negative charge moves through a wire.

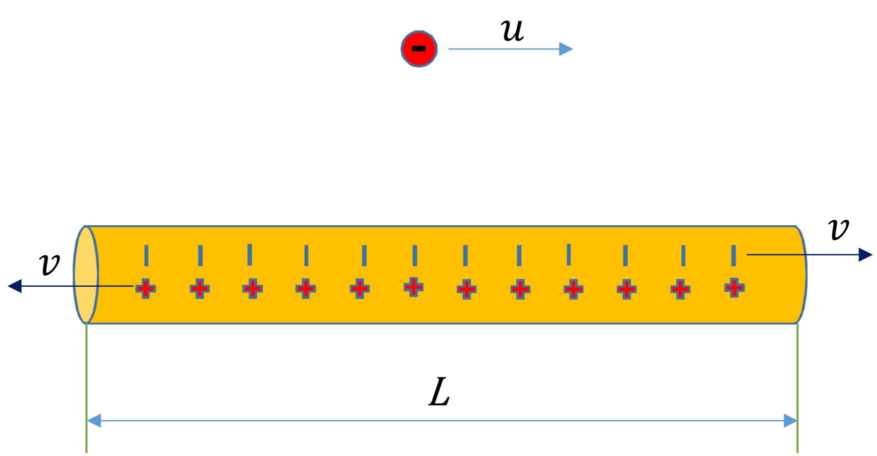

Figure 7

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