Simpson's rule and Simpson's 3/8 rule

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ABSTRACT. Numerical evaluation of the integral: $\int_0^1 f(x)\,dx = \frac{\pi}{2}$.

keywords: definite integral,number Pi,numerical integration.

I. Introduction .

Recall that

$$\pi = 4 \sum_{k=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.141592653589793238 \ldots \tag{1}$$

$$\frac{\pi}{2} = \int_0^1 \frac{1}{(1-x)^2 + x^2} (\cosh(\sin(1-x)\cosh(x))\cos(\cos(1-x)\sinh(x)) - (1-2x)\sinh(\sin(1-x)\cosh(x))\sin(\cos(1-x)\sinh(x)))\,dx \tag{2}$$

In this note we give the numerical evaluation of (2) via Simpson's rule and Simpson's 3/8 rule.

II. Simpson's rule

If $f : [0, 1] \longrightarrow \mathbb{R}$, the Simpson's rule is

$$\int_0^1 f(x)\,dx \approx I_n = \frac{1}{6n}\left(f(0) + 4\sum_{k=1}^{n} f\left(\frac{2k-1}{2n}\right) + 2\sum_{k=1}^{n-1} f\left(\frac{k}{n}\right) + f(1)\right) \;,\; n = 1, 2, 3, \ldots \tag{3}$$

III. Simpson's 3/8 rule

If $f : [0, 1] \longrightarrow \mathbb{R}$, the Simpson's 3/8 rule is

$$\int_0^1 f(x)\,dx \approx J_n = \frac{1}{8n}\left(f(0) + 2\sum_{k=1}^{n-1} f\left(\frac{k}{n}\right) + 3\sum_{k=1}^{n}\left(f\left(\frac{3k-2}{3n}\right) + f\left(\frac{3k-1}{3n}\right)\right) + f(1)\right)\ , \ n = 1, 2, 3, \ldots \qquad (4)$$

IV. Evaluation of (2)

Let

$$f(x) = \frac{1}{(1-x)^2 + x^2}(\cosh(\sin(1-x)\cosh(x))\cos(\cos(1-x)\sinh(x)) - (1-2x)\sinh(\sin(1-x)\cosh(x))\sin(\cos(1-x)\sinh(x))) \qquad (5)$$

then

n	I_n	$\left\lvert \frac{\pi}{2} - I_n \right\rvert$	J_n	$\left\lvert \frac{\pi}{2} - J_n \right\rvert$
1	1.6689121914993193999	$9.81 \cdot 10^{-2}$	1.6008888847214758506	$3.00 \cdot 10^{-2}$
2	1.5667029078361742276	$4.09 \cdot 10^{-3}$	1.5692450545499859736	$1.55 \cdot 10^{-3}$
5	1.5707955296500062498	$7.97 \cdot 10^{-7}$	1.5707960052500544409	$3.21 \cdot 10^{-7}$
10	1.5707963091788461020	$1.76 \cdot 10^{-8}$	1.5707963198351681488	$6.95 \cdot 10^{-9}$
50	1.5707963267937681597	$1.12 \cdot 10^{-12}$	1.5707963267944508080	$4.45 \cdot 10^{-13}$
100	1.5707963267948789870	$1.76 \cdot 10^{-14}$	1.5707963267948896534	$6.96 \cdot 10^{-15}$

Table 1. Simpson’s rule and Simpson’s 3/8 rule

- Conclusions: $\left\lvert \frac{\pi}{2} - J_n \right\rvert < \left\lvert \frac{\pi}{2} - I_n \right\rvert$, $n = 1, 2, 5, 10, 50, 100$.

V. References

1. Olver, Frank W.J.;Lozier, Daniel W.; Boisvert, Ronald F.;and Clark, Charles W.: NIST Handbook of Mathematical Functions.Cambridge University Press, 2010.