Unified Theory of Gravity and Electromagnetic Field in Reissner-Nodstrom Solution, Kerr-Newman Solution

Sangwha-Yi

Department of Math , Taejon University 300-716,South Korea

ABSTRACT

Solutions of unified theory equations of gravity and electromagnetism satisfy Einstein-Maxwell equation. Hence, solutions of the unified theory is Reissner-Nodstrom solution in vacuum. We found in revised Einstein gravity tensor equation, the condition is satisfied by 2-order contravariant metric tensor two times product.

.

.

PACS Number:**04,04.90.+e,41.12**

Key words:General relativity theory,

Unified Theory;

2-order contravariant metric tensor two times product

e-mail address:sangwha1**@nate.com**

Tel:010-2496-3953

1.Introduction

This theory's aim is that we discover the revised Einstein gravity equation had Reissner-Nodstrom solution in vacuum..

First, we can think the following formula (the revised Einstein gravity equation).

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R + \Lambda g_{\mu\upsilon} I (g^{\theta\theta})^2 = -\frac{8\pi G}{c^4} T_{\mu\upsilon}$$

In this time,

$$\Lambda = k\frac{GQ^2}{c^4}, \quad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (1)$$

If Eq(1) take covariant differential operator,

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\mu} + \Lambda g_{\mu\upsilon} I 2 g^{\theta\theta} g^{\theta\theta}{}_{;\mu} = -\frac{8\pi G}{c^4} T_{\mu\upsilon;\mu} = 0 \qquad (2\text{-i})$$

$$(R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R)_{;\upsilon} + \Lambda g_{\mu\upsilon} I 2 g^{\theta\theta} g^{\theta\theta}{}_{;\upsilon} = -\frac{8\pi G}{c^4} T_{\mu\upsilon;\upsilon} = 0 \qquad (2\text{-ii})$$

In this time,

$$g^{\theta\theta}{}_{;\rho} = \frac{\partial g^{\theta\theta}}{\partial x^{\rho}} + 2\Gamma^{\theta}{}_{\sigma\rho} g^{\sigma\theta} = \frac{\partial g^{\theta\theta}}{\partial r} + 2\Gamma^{\theta}{}_{\theta r} g^{\theta\theta}$$

$$= \frac{\partial}{\partial r}(\frac{1}{r^2}) + 2 \cdot \frac{1}{r} \cdot \frac{1}{r^2} = 0 \qquad (3)$$

If $g^{\theta\theta}{}_{;\rho} = V_{\rho}$, the vector transformation is

$$0 = V_{\rho} = \frac{\partial x'^{\alpha}}{\partial x^{\rho}} V'_{\alpha} \quad , \quad V'_{\alpha} = 0 \qquad (4)$$

Therefore, if the coordinate is not the spherical coordinate, the covariant differential of $g^{\theta\theta} = \frac{1}{r^2}$ is still zero in the changed coordinate

2. The revised Einstein gravity equation and Reissner-Nodstrom solution

In this theory, Eq(1) can change the following equation.

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) + \Lambda I g_{\mu\upsilon} (g^{\theta\theta})^2 \qquad (5)$$

In this time, in vacuum, Eq(5) is

$$R_{\mu\upsilon} = \Lambda g_{\mu\upsilon} I (g^{\theta\theta})^2 = \Lambda g_{\mu\upsilon} I \frac{1}{r^4} \qquad (6)$$

Reissner-Nodstrom solution of Einstein-Maxwell equation is

$$g_{00} = -1 + \frac{2GM}{rc^2} - \frac{kGQ^2}{r^2c^4}, g_{11} = 1/(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})$$
$$g_{22} = r^2, g_{33} = r^2 \sin^2\theta \tag{7}$$

The proper time of spherical coordinates is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2]$$

$$lg_{\mu\upsilon} = \begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & -g_{22} & 0 \\ 0 & 0 & 0 & -g_{33} \end{pmatrix} \tag{8}$$

If we use Eq(6),Eq(8), we obtain the Ricci-tensor equations.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = -\Lambda A \frac{1}{r^4} \tag{9}$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = \Lambda B \frac{1}{r^4} \tag{10}$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -\Lambda r^2 \frac{1}{r^4} = -\Lambda \frac{1}{r^2} \tag{11}$$

$$R_{\varphi\varphi} = R_{\theta\theta} \sin^2\theta \tag{12}$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0 \tag{13}$$

$$R_{t\theta} = R_{t\varphi} = R_{r\theta} = R_{r\varphi} = R_{\theta\varphi} = 0 \tag{14}$$

In this time, $' = \frac{\partial}{\partial r}$, $\cdot = \frac{1}{c}\frac{\partial}{\partial t}$

If we calculate,

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}(\frac{A'}{A} + \frac{B'}{B}) = -\frac{(AB)'}{rAB^2} = 0 \tag{15}$$

Hence, we obtain this result.

$$A = \frac{1}{B} \tag{16}$$

If Eq(16) inserts Eq(11),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + (\frac{r}{B})' = -\Lambda \frac{1}{r^2} \tag{17}$$

If we solve Eq(17),

$$\frac{r}{B} = r + C + \frac{\Lambda}{r}$$

$$\rightarrow \; A = \frac{1}{B} = 1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}, \quad \Lambda = k\frac{GQ^2}{c^4}, C = -\frac{2GM}{c^2} \tag{18}$$

Therefore, in vacuum, the spherical solution of the revised Einstein gravity equation is Reissner-Nodstrom solution.

$$d\tau^2 = (1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})dt^2 - \frac{1}{c^2}[\frac{dr^2}{(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2] \tag{19}$$

3. Conclusion

We found the revised Einstein equation of unified theory(the gravity and electromagnetic field).This theory's strong point is 4-dimensional theory. This theory is different from 5-dimensional Kaluza-Klein theory. But as the method of describing universe, Einstein normal gravity equation is equal with the revised Einstein equation of the unified theory because the electric charge has to be zero. .

*APPENDIX A

We think Kerr-Newman equation and solution.

Kerr-Newman solution is

$$ds^2 = g_{00}c^2dt^2 + 2g_{03}cdtd\phi + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2 \tag{A-1}$$

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, Ig_{\mu\upsilon} = \begin{pmatrix} g_{00} & 0 & 0 & g_{03} \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & -g_{22} & 0 \\ -g_{30} & 0 & 0 & -g_{33} \end{pmatrix}, \; \rho^2 = r^2 + a^2\cos^2\theta$$

Hence, Kerr-Newman equation is

$$R_{tt} = \Lambda g_{00}\frac{1}{\rho^4} \tag{A-2}$$

$$R_{rr} = \Lambda g_{11}\frac{1}{\rho^4} \tag{A-3}$$

$$R_{\theta\theta} = -\Lambda g_{22}\frac{1}{\rho^4} = -\Lambda\frac{1}{\rho^2} \tag{A-4}$$

$$R_{\phi\phi} = -\Lambda g_{33} \frac{1}{\rho^4} \tag{A-5}$$

$$R_{t\phi} = R_{\phi t} = \Lambda g_{03} \frac{1}{\rho^4} = \Lambda g_{30} \frac{1}{\rho^4} \tag{A-6}$$

$$R_{\phi t} = R_{t\phi} = -\Lambda g_{03} \frac{1}{\rho^4} = -\Lambda g_{30} \frac{1}{\rho^4} \tag{A-7}$$

Otherwise $R_{\mu\upsilon} = 0$

In this time, for proving (A-6), Maxwell Equation of Einstein-Maxwell equation is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon} = -\frac{2G}{c^5}(F_{\mu\rho}F_{\upsilon}^{\ \rho} - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma}F^{\rho\sigma})$$
$$= -\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma}F^{\rho\sigma}) \tag{A-8}$$

In spherical coordinates, if we calculate (A-8) of plus term of g_{03}, g_{30}

$$-\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma}F^{\rho\sigma})$$
$$= -\frac{2G}{c^5}(g_{03}F_{32}F^{32} - \frac{1}{4} g_{03}(2B^2 - 2E^2))$$
$$= -\frac{2G}{c^5}(g_{03}B^2 - \frac{1}{2} g_{03}(B^2 - E^2)) = -\frac{G}{c^5}E^2 g_{03} = -k\frac{G}{c^4}\frac{Q^2}{\rho^4} g_{03} = \Lambda g_{03}\frac{1}{\rho^4} = \Lambda g_{30}\frac{1}{\rho^4} \tag{A-9}$$

In spherical coordinates, if we calculate (A-8) of minus term of g_{30}, g_{03}

$$-\frac{2G}{c^5}(g_{\mu\upsilon}F_{\upsilon\rho}F^{\upsilon\rho} - \frac{1}{4} g_{\mu\upsilon} F_{\rho\sigma}F^{\rho\sigma})$$
$$= -\frac{2G}{c^5}(g_{30}F_{01}F^{01} - \frac{1}{4} g_{30}(2B^2 - 2E^2))$$
$$= -\frac{2G}{c^5}(-g_{30}E^2 - \frac{1}{2} g_{30}(B^2 - E^2)) = \frac{G}{c^5}E^2 g_{30} = k\frac{G}{c^4}\frac{Q^2}{\rho^4} g_{30} = -\Lambda g_{30}\frac{1}{\rho^4} = -\Lambda g_{03}\frac{1}{\rho^4}$$

(A-10)

Hence, Riemannian Curvature tensor $R_{t\phi} = R_{\phi t}$ have two value term (plus and minus term),

$$R_{t\phi} = \pm\Lambda g_{03}\frac{1}{\rho^4} = \pm\Lambda g_{30}\frac{1}{\rho^4}, \ R_{\phi t} = \pm\Lambda g_{03}\frac{1}{\rho^4} = \pm\Lambda g_{30}\frac{1}{\rho^4} \tag{A-11}$$

The reason of two value term is maybe the orientation of rotating (right hand or left hand)..

According to Eq(A-9),Eq(A-10), we prove Eq(A-6),Eq(A-7).Therefore, in Kerr-Newman equation and solution, the electromagnetic energy-momentum tensor's formula is equal with the revised Einstein equation's added term of our unified theory. Hence, we know if we solve the revised Einstein's equation, we can obtain Kerr-Newman solution.

References

[1]S.Yi, "Spherical Solution of Classical Quantum Gravity", International Journal of Advanced Research in Physical Science,**6**,8,2019,pp3-6

[2]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[3]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[4]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[5]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[6]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)