Einstein-Whitehead model of GW 150914
by Lluís Bel (retired from CNRS, France)
A fully relativistic descriptiony of the encounter of 35 solar masses point-like objects, somehow mimicking the binary merger GW1508144 described by LIGO Scientific and VIRGO collaboration (Ann. Phys. (Berlin) 529, No 1-2)

!! Formulas for odevx, odevy, odevxp and odevyp have been corrected)
The Lagrangian and Hamiltonian used have been included.

```
> restart:
> with(tensor) :
```

Whitehead Lagrangian 05/07/2019

```
> g[4,4]:=(-1+2*mp/r)*c^2;
```

$$g_{4,4} := \left(-1 + \frac{2\,mp}{r}\right) c^2 \tag{1}$$

```
> g[4,1]:=(2*mp*x/r^2)*c;
```

$$g_{4,1} := \frac{2\,mp\,x\,c}{r^2} \tag{2}$$

```
> g[4,2]:=(2*mp*y/r^2)*c;
```

$$g_{4,2} := \frac{2\,mp\,y\,c}{r^2} \tag{3}$$

```
> g[4,3]:=(2*mp*z/r^2)*c;
```

$$g_{4,3} := \frac{2\,mp\,z\,c}{r^2} \tag{4}$$

```
> g[1,1]:=1+2*mp*x^2/r^3;
```

$$g_{1,1} := 1 + \frac{2\,mp\,x^2}{r^3} \tag{5}$$

```
> g[2,2]:=1+2*mp*y^2/r^3;
```

$$g_{2,2} := 1 + \frac{2\,mp\,y^2}{r^3} \tag{6}$$

```
> g[3,3]:=1+2*mp*z^2/r^3;
```

$$g_{3,3} := 1 + \frac{2\,mp\,z^2}{r^3} \tag{7}$$

```
> g[1,2]:=2*mp*x*y/r^3;
```

$$g_{1,2} := \frac{2\,mp\,x\,y}{r^3} \tag{8}$$

```
> g[2,3]:=2*mp*y*z/r^3;
```

$$g_{2,3} := \frac{2\,mp\,y\,z}{r^3} \tag{9}$$

```
> g[3,1]:=2*mp*x*z/r^3;
```

$$g_{3,1} := \frac{2\,mp\,x\,z}{r^3} \tag{10}$$

Kontuz---

```
> L2:=+M^2*c^2*(g[4,4]+g[1,1]*vx^2+g[2,2]*vy^2+g[3,3]*vz^2
   +2*(g[4,1]*vx+g[4,2]*vy+g[4,3]*vz
```

```
+2*(g[1,2]*vx*vy+g[2,3]*vy*vz+g[3,1]*vz*vx))); epsilon^2=1
```

$$L2 := M^2 c^2 \left(vx^2 \left(1 + \frac{2\,mp\,x^2}{r^3}\right) + \frac{8\,vx\,vy\,mp\,x\,y}{r^3} + \frac{8\,vx\,vz\,mp\,x\,z}{r^3} + vy^2 \left(1 + \frac{2\,mp\,y^2}{r^3}\right) + \frac{8\,vy\,vz\,mp\,y\,z}{r^3} + vz^2 \left(1 + \frac{2\,mp\,z^2}{r^3}\right) + \frac{4\,vx\,mp\,x\,c}{r^2} + \frac{4\,vy\,mp\,y\,c}{r^2} + \frac{4\,vz\,mp\,z\,c}{r^2} + \left(-1 + \frac{2\,mp}{r}\right) c^2 \right) \quad (11)$$

```
> L2x:=diff(L2,x)+diff(L2,r)*x/r;
```

$$L2x := M^2 c^2 \left(\frac{4\,vx^2\,mp\,x}{r^3} + \frac{8\,vx\,vy\,mp\,y}{r^3} + \frac{8\,vx\,vz\,mp\,z}{r^3} + \frac{4\,vx\,mp\,c}{r^2} \right) + \frac{1}{r} \left(M^2 c^2 \left(-\frac{6\,vx^2\,mp\,x^2}{r^4} - \frac{24\,vx\,vy\,mp\,x\,y}{r^4} - \frac{24\,vx\,vz\,mp\,x\,z}{r^4} - \frac{6\,vy^2\,mp\,y^2}{r^4} - \frac{24\,vy\,vz\,mp\,y\,z}{r^4} - \frac{6\,vz^2\,mp\,z^2}{r^4} - \frac{8\,vx\,mp\,x\,c}{r^3} - \frac{8\,vy\,mp\,y\,c}{r^3} - \frac{8\,vz\,mp\,z\,c}{r^3} - \frac{2\,mp\,c^2}{r^2} \right) x \right) \quad (12)$$

```
> L2y:=diff(L2,y)+diff(L2,r)*y/r;
```

$$L2y := M^2 c^2 \left(\frac{8\,vx\,vy\,mp\,x}{r^3} + \frac{4\,vy^2\,mp\,y}{r^3} + \frac{8\,vy\,vz\,mp\,z}{r^3} + \frac{4\,vy\,mp\,c}{r^2} \right) + \frac{1}{r} \left(M^2 c^2 \left(-\frac{6\,vx^2\,mp\,x^2}{r^4} - \frac{24\,vx\,vy\,mp\,x\,y}{r^4} - \frac{24\,vx\,vz\,mp\,x\,z}{r^4} - \frac{6\,vy^2\,mp\,y^2}{r^4} - \frac{24\,vy\,vz\,mp\,y\,z}{r^4} - \frac{6\,vz^2\,mp\,z^2}{r^4} - \frac{8\,vx\,mp\,x\,c}{r^3} - \frac{8\,vy\,mp\,y\,c}{r^3} - \frac{8\,vz\,mp\,z\,c}{r^3} - \frac{2\,mp\,c^2}{r^2} \right) y \right) \quad (13)$$

```
> L2z:=diff(L2,z)+diff(L2,r)*z/r;
```

$$L2z := M^2 c^2 \left(\frac{8\,vx\,vz\,mp\,x}{r^3} + \frac{8\,vy\,vz\,mp\,y}{r^3} + \frac{4\,vz^2\,mp\,z}{r^3} + \frac{4\,vz\,mp\,c}{r^2} \right) + \frac{1}{r} \left(M^2 c^2 \left(-\frac{6\,vx^2\,mp\,x^2}{r^4} - \frac{24\,vx\,vy\,mp\,x\,y}{r^4} - \frac{24\,vx\,vz\,mp\,x\,z}{r^4} - \frac{6\,vy^2\,mp\,y^2}{r^4} - \frac{24\,vy\,vz\,mp\,y\,z}{r^4} - \frac{6\,vz^2\,mp\,z^2}{r^4} - \frac{8\,vx\,mp\,x\,c}{r^3} - \frac{8\,vy\,mp\,y\,c}{r^3} - \frac{8\,vz\,mp\,z\,c}{r^3} - \frac{2\,mp\,c^2}{r^2} \right) z \right) \quad (14)$$

```
> px:=1/(2*L)*diff(L2,vx);  # px;=diff(sqrt(L2)),x):
```

$$px := \frac{1}{2} \, \frac{M^2 c^2 \left(2\,vx \left(1 + \frac{2\,mp\,x^2}{r^3}\right) + \frac{8\,vy\,mp\,x\,y}{r^3} + \frac{8\,vz\,mp\,x\,z}{r^3} + \frac{4\,mp\,x\,c}{r^2} \right)}{L} \quad (15)$$

```
> py:=1/(2*L)*diff(L2,vy);
```

$$py := \frac{1}{2} \, \frac{M^2 c^2 \left(\frac{8\,vx\,mp\,x\,y}{r^3} + 2\,vy \left(1 + \frac{2\,mp\,y^2}{r^3}\right) + \frac{8\,vz\,mp\,y\,z}{r^3} + \frac{4\,mp\,y\,c}{r^2} \right)}{L} \quad (16)$$

```
> pz:=1/(2*L)*diff(L2,vz);
```

(17)

$$pz := \frac{1}{2} \frac{M^2 c^2 \left(\frac{8\, vx\, mp\, x\, z}{r^3} + \frac{8\, vy\, mp\, y\, z}{r^3} + 2\, vz \left(1 + \frac{2\, mp\, z^2}{r^3}\right) + \frac{4\, mp\, z\, c}{r^2} \right)}{L} \quad \textbf{(17)}$$

```
> H:=(px*vx+py*vy+pz*vz)-L;
```

$$H := \frac{1}{2} \frac{M^2 c^2 \left(2\, vx \left(1 + \frac{2\, mp\, x^2}{r^3}\right) + \frac{8\, vy\, mp\, x\, y}{r^3} + \frac{8\, vz\, mp\, x\, z}{r^3} + \frac{4\, mp\, x\, c}{r^2} \right) vx}{L} \quad \textbf{(18)}$$
$$+ \frac{1}{2} \frac{M^2 c^2 \left(\frac{8\, vx\, mp\, x\, y}{r^3} + 2\, vy \left(1 + \frac{2\, mp\, y^2}{r^3}\right) + \frac{8\, vz\, mp\, y\, z}{r^3} + \frac{4\, mp\, y\, c}{r^2} \right) vy}{L}$$
$$+ \frac{1}{2} \frac{M^2 c^2 \left(\frac{8\, vx\, mp\, x\, z}{r^3} + \frac{8\, vy\, mp\, y\, z}{r^3} + 2\, vz \left(1 + \frac{2\, mp\, z^2}{r^3}\right) + \frac{4\, mp\, z\, c}{r^2} \right) vz}{L} - L$$

```
> L:=sqrt(L2);
```

$$L := \textbf{(19)}$$
$$\left(M^2 c^2 \left(vx^2 \left(1 + \frac{2\, mp\, x^2}{r^3}\right) + \frac{8\, vx\, vy\, mp\, x\, y}{r^3} + \frac{8\, vx\, vz\, mp\, x\, z}{r^3} + vy^2 \left(1 \right.\right.\right.$$
$$\left. + \frac{2\, mp\, y^2}{r^3}\right) + \frac{8\, vy\, vz\, mp\, y\, z}{r^3} + vz^2 \left(1 + \frac{2\, mp\, z^2}{r^3}\right) + \frac{4\, vx\, mp\, x\, c}{r^2} + \frac{4\, vy\, mp\, y\, c}{r^2}$$
$$\left.\left. + \frac{4\, vz\, mp\, z\, c}{r^2} + \left(-1 + \frac{2\, mp}{r}\right) c^2 \right)\right)^{1/2}$$

```
> H:=simplify(H);
```

$$H := -\left(M^2 c^3 \left(2\, c\, mp\, r - c\, r^2 + 2\, mp\, vx\, x + 2\, mp\, vy\, y + 2\, mp\, vz\, z\right)\right) \Bigg/ \quad \textbf{(20)}$$
$$\left(r^2 \right.$$
$$\left(\frac{1}{r^3} \left(M^2 c^2 \left(2\, c^2\, mp\, r^2 - c^2\, r^3 + 4\, c\, mp\, r\, vx\, x + 4\, c\, mp\, r\, vy\, y + 4\, c\, mp\, r\, vz\, z\right.\right.\right.$$
$$+ 2\, mp\, vx^2\, x^2 + 8\, mp\, vx\, vy\, x\, y + 8\, mp\, vx\, vz\, x\, z + 2\, mp\, vy^2\, y^2 + 8\, mp\, vy\, vz\, y\, z$$
$$\left.\left.\left.\left. + 2\, mp\, vz^2\, z^2 + r^3\, vx^2 + r^3\, vy^2 + r^3\, vz^2\right)\right)\right)^{1/2} \right)$$

```
> #limit(H,r=infinity);
```

```
> H2:=subs(z=0,vz=0,H);
```

$$H2 := -\left(M^2 c^3 \left(2\, c\, mp\, r - c\, r^2 + 2\, mp\, vx\, x + 2\, mp\, vy\, y\right)\right) \Bigg/ \quad \textbf{(21)}$$
$$\left(r^2 \right.$$

$$\left(\frac{1}{r^3}\left(M^2 c^2 \left(2 c^2 mp\, r^2 - c^2 r^3 + 4 c\, mp\, r\, vx\, x + 4 c\, mp\, r\, vy\, y + 2\, mp\, vx^2 x^2 + 8\, mp\, vx\, vy\, x\, y + 2\, mp\, vy^2 y^2 + r^3 vx^2 + r^3 vy^2\right)\right)\right)^{1/2}\Bigg)$$

```
> H2:=convert(series(H2,mp,2),polynom);
```

$$H2 := \frac{M^2 c^4}{\sqrt{-M^2 c^2 (c^2 - vx^2 - vy^2)}} + \left(\frac{M^2 c^4 (c^2 r^2 + 2 c\, r\, vx\, x + 2 c\, r\, vy\, y + vx^2 x^2 + 4\, vx\, vy\, x\, y + vy^2 y^2)}{\sqrt{-M^2 c^2 (c^2 - vx^2 - vy^2)}\; r^3 (c^2 - vx^2 - vy^2)} - \frac{M^2 c^3 (2 c\, r + 2\, vx\, x + 2\, vy\, y)}{r^2 \sqrt{-M^2 c^2 (c^2 - vx^2 - vy^2)}}\right) mp \tag{22}$$

```
> H2:=convert(asympt(H2,c,4),polynom);
```

$$H2 := \left(\frac{M^2}{\sqrt{-M^2}} - \frac{M^2\, mp}{\sqrt{-M^2}\; r}\right) c^2 + \left(\frac{M^2 (2 r\, vx\, x + 2 r\, vy\, y)}{\sqrt{-M^2}\; r^3} - \frac{M^2 (2\, vx\, x + 2\, vy\, y)}{r^2 \sqrt{-M^2}}\right) mp\, c \tag{23}$$

```
> simplify(H2);
```

$$-\frac{M^2 (mp - r)\, c^2}{\sqrt{-M^2}\; r} \tag{24}$$

```
> Hnumer:=collect(numer(H),mp);
```

$$Hnumer := -M^2 c^3 (2 c\, r + 2\, vx\, x + 2\, vy\, y + 2\, vz\, z)\, mp + M^2 c^4 r^2 \tag{25}$$

```
> Hdenom:=collect(denom(H),mp);
```

$$Hdenom := r^2 \left(\frac{1}{r^3}\left(M^2 c^2 \left(2 c^2 mp\, r^2 - c^2 r^3 + 4 c\, mp\, r\, vx\, x + 4 c\, mp\, r\, vy\, y + 4 c\, mp\, r\, vz\, z + 2\, mp\, vx^2 x^2 + 8\, mp\, vx\, vy\, x\, y + 8\, mp\, vx\, vz\, x\, z + 2\, mp\, vy^2 y^2 + 8\, mp\, vy\, vz\, y\, z + 2\, mp\, vz^2 z^2 + r^3 vx^2 + r^3 vy^2 + r^3 vz^2\right)\right)\right)^{1/2} \tag{26}$$

```
> r:=sqrt(x^2+y^2+z^2);
```

$$r := \sqrt{x^2 + y^2 + z^2} \tag{27}$$

```
> coord := [x,y,z,t]:
> g_compts := array(symmetric,sparse,1..4,1..4):
> g_compts[4,4]:=g[4,4]:
> g_compts[4,1]:=g[4,1]:
> g_compts[4,2]:=g[4,2]:
> g_compts[4,3]:=g[4,3]:
> g_compts[1,1]:=g[1,1]:
```

```
> g_compts[2,2]:=g[2,2]:
> g_compts[3,3]:=g[3,3]:
> g_compts[1,2]:=g[1,2]:
> g_compts[2,3]:=g[2,3]:
> g_compts[3,1]:=g[3,1]:
> g := create([-1,-1],eval(g_compts)):
> ginv:=invert(g,'detg'):
> D1g:=d1metric(g,coord): D2g:=d2metric(D1g,coord):
> Cf1 :=Christoffel1(D1g):
```

```
> #RMN:=Riemann(ginv, D2g, Cf1):
> #RICCI:=simplify(Ricci(ginv,RMN));
```

```
> `tensor/Christoffel2/simp`:=proc(x) simplify(x,trig) end proc:
> Cf2:=Christoffel2 (ginv,Cf1):
> Cf2c:=get_compts(Cf2):
> Cf2p:=act(subs,mp=m,Cf2):
> Cf2pc:=get_compts(Cf2p):
```

Two bodies plane motion---

```
> z:=0; vz:=0;
```

$$z := 0$$

$$vz := 0 \tag{28}$$

```
> Cf2c[4,4,4]:=simplify(Cf2c[4,4,4]):
> Cf2c[4,4,1]:=simplify(Cf2c[4,4,1]):
> Cf2c[4,4,2]:=simplify(Cf2c[4,4,2]):
> Cf2c[4,1,1]:=simplify(Cf2c[4,1,1]):
> Cf2c[4,2,2]:=simplify(Cf2c[4,2,2]):
> Cf2c[4,1,2]:=simplify(Cf2c[4,1,2]):
```

```
> Cf2c[1,4,4]:=simplify(Cf2c[1,4,4]):
> Cf2c[1,4,1]:=simplify(Cf2c[1,4,1]):
> Cf2c[1,4,2]:=simplify(Cf2c[1,4,2]):
> Cf2c[1,1,1]:=simplify(Cf2c[1,1,1]):
> Cf2c[1,2,2]:=simplify(Cf2c[1,2,2]):
> Cf2c[1,1,2]:=simplify(Cf2c[1,1,2]):
```

```
> Cf2c[2,4,4]:=simplify(Cf2c[2,4,4]);
```

$$Cf2c_{2,\,4,\,4} := \frac{\left(-2\,mp+\sqrt{x^2+y^2}\right)mp\,y\,c^2}{x^4+2\,x^2\,y^2+y^4} \tag{29}$$

```
> Cf2c[2,4,1]:=simplify(Cf2c[2,4,1]);
```

$$Cf2c_{2,\,4,\,1} := -\frac{2\,y\,c\,x\,mp^2}{\left(x^2+y^2\right)^{5/2}} \tag{30}$$

```
> Cf2c[2,4,2]:=simplify(Cf2c[2,4,2]);
```

$$Cf2c_{2,\,4,\,2} := -\frac{2\,c\,y^2\,mp^2}{\left(x^2+y^2\right)^{5/2}} \tag{31}$$

```
> Cf2c[2,1,1]:=simplify(Cf2c[2,1,1]);
```

$$Cf2c_{2,\,1,\,1} := -\frac{\left(2\,mp\,x^2 + x^2\sqrt{x^2+y^2} - 2\sqrt{x^2+y^2}\,y^2\right)y\,mp}{x^6 + 3\,x^4\,y^2 + 3\,x^2\,y^4 + y^6} \quad (32)$$

```
> Cf2c[2,2,2]:=simplify(Cf2c[2,2,2]);
```

$$Cf2c_{2,\,2,\,2} := \frac{\left(-2\,mp\,y^2 + 2\,x^2\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\,y^2\right)y\,mp}{x^6 + 3\,x^4\,y^2 + 3\,x^2\,y^4 + y^6} \quad (33)$$

```
> Cf2c[2,1,2]:=simplify(Cf2c[2,1,2]);
```

$$Cf2c_{2,\,1,\,2} := -\frac{\left(2\,mp + 3\sqrt{x^2+y^2}\right)y^2\,x\,mp}{x^6 + 3\,x^4\,y^2 + 3\,x^2\,y^4 + y^6} \quad (34)$$

```
> Cf2pc[4,4,4]:=simplify(Cf2pc[4,4,4]);
```

$$Cf2pc_{4,\,4,\,4} := \frac{2\,m^2\,c}{\left(x^2+y^2\right)^{3/2}} \quad (35)$$

```
> Cf2pc[4,4,1]:=simplify(Cf2pc[4,4,1]);
```

$$Cf2pc_{4,\,4,\,1} := \frac{\left(2\,m + \sqrt{x^2+y^2}\right)m\,x}{x^4 + 2\,x^2\,y^2 + y^4} \quad (36)$$

```
> Cf2pc[4,4,2]:=simplify(Cf2pc[4,4,2]);
```

$$Cf2pc_{4,\,4,\,2} := \frac{\left(2\,m + \sqrt{x^2+y^2}\right)m\,y}{x^4 + 2\,x^2\,y^2 + y^4} \quad (37)$$

```
> Cf2pc[4,1,1]:=simplify(Cf2pc[4,1,1]);
```

$$Cf2pc_{4,\,1,\,1} := \frac{2\left(m\,x^2 + x^2\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\,y^2\right)m}{\left(x^2+y^2\right)^{5/2}\,c} \quad (38)$$

```
> Cf2pc[4,2,2]:=simplify(Cf2pc[4,2,2]);
```

$$Cf2pc_{4,\,2,\,2} := -\frac{2\left(-m\,y^2 + x^2\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\,y^2\right)m}{\left(x^2+y^2\right)^{5/2}\,c} \quad (39)$$

```
> Cf2pc[4,1,2]:=simplify(Cf2pc[4,1,2]);
```

$$Cf2pc_{4,\,1,\,2} := \frac{2\left(m + 2\sqrt{x^2+y^2}\right)m\,x\,y}{\left(x^2+y^2\right)^{5/2}\,c} \quad (40)$$

```
> Cf2pc[1,4,4]:=simplify(Cf2pc[1,4,4]);
```

$$Cf2pc_{1,\,4,\,4} := \frac{\left(-2\,m + \sqrt{x^2+y^2}\right)m\,x\,c^2}{x^4 + 2\,x^2\,y^2 + y^4} \quad (41)$$

```
> Cf2pc[1,4,1]:=simplify(Cf2pc[1,4,1]);
```

$$Cf2pc_{1,\,4,\,1} := -\frac{2\,c\,x^2\,m^2}{\left(x^2+y^2\right)^{5/2}} \quad (42)$$

```
> Cf2pc[1,4,2]:=simplify(Cf2pc[1,4,2]);
```

$$Cf2pc_{1,\,4,\,2} := -\frac{2\,y\,c\,x\,m^2}{\left(x^2+y^2\right)^{5/2}} \quad (43)$$

```
> Cf2pc[1,1,1]:=simplify(Cf2pc[1,1,1]);
```

(44)

$$Cf2pc_{1,1,1} := -\frac{\left(2\,m\,x^2 + x^2\sqrt{x^2+y^2} - 2\sqrt{x^2+y^2}\;y^2\right)x\,m}{x^6+3\,x^4\,y^2+3\,x^2\,y^4+y^6} \quad \textbf{(44)}$$

```
> Cf2pc[1,2,2]:=simplify(Cf2pc[1,2,2]);
```

$$Cf2pc_{1,2,2} := \frac{\left(-2\,m\,y^2 + 2\,x^2\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\;y^2\right)x\,m}{x^6+3\,x^4\,y^2+3\,x^2\,y^4+y^6} \quad \textbf{(45)}$$

```
> Cf2pc[1,1,2]:=simplify(Cf2pc[1,1,2]);
```

$$Cf2pc_{1,1,2} := -\frac{\left(2\,m+3\sqrt{x^2+y^2}\right)y\,x^2\,m}{x^6+3\,x^4\,y^2+3\,x^2\,y^4+y^6} \quad \textbf{(46)}$$

```
> Cf2pc[2,4,4]:=simplify(Cf2pc[2,4,4]);
```

$$Cf2pc_{2,4,4} := \frac{\left(-2\,m+\sqrt{x^2+y^2}\right)m\,y\,c^2}{x^4+2\,x^2\,y^2+y^4} \quad \textbf{(47)}$$

```
> Cf2pc[2,4,1]:=simplify(Cf2pc[2,4,1]);
```

$$Cf2pc_{2,4,1} := -\frac{2\,y\,c\,x\,m^2}{\left(x^2+y^2\right)^{5/2}} \quad \textbf{(48)}$$

```
> Cf2pc[2,4,2]:=simplify(Cf2pc[2,4,2]);
```

$$Cf2pc_{2,4,2} := -\frac{2\,c\,y^2\,m^2}{\left(x^2+y^2\right)^{5/2}} \quad \textbf{(49)}$$

```
> Cf2pc[2,4,2]:=simplify(Cf2pc[2,4,2]);
```

$$Cf2pc_{2,4,2} := -\frac{2\,c\,y^2\,m^2}{\left(x^2+y^2\right)^{5/2}} \quad \textbf{(50)}$$

```
> Cf2pc[2,1,1]:=simplify(Cf2pc[2,1,1]);
```

$$Cf2pc_{2,1,1} := -\frac{\left(2\,m\,x^2 + x^2\sqrt{x^2+y^2} - 2\sqrt{x^2+y^2}\;y^2\right)y\,m}{x^6+3\,x^4\,y^2+3\,x^2\,y^4+y^6} \quad \textbf{(51)}$$

```
> Cf2pc[2,2,2]:=simplify(Cf2pc[2,2,2]);
```

$$Cf2pc_{2,2,2} := \frac{\left(-2\,m\,y^2 + 2\,x^2\sqrt{x^2+y^2} - \sqrt{x^2+y^2}\;y^2\right)y\,m}{x^6+3\,x^4\,y^2+3\,x^2\,y^4+y^6} \quad \textbf{(52)}$$

```
> Cf2pc[2,1,2]:=simplify(Cf2pc[2,1,2]);
```

$$Cf2pc_{2,1,2} := -\frac{\left(2\,m+3\sqrt{x^2+y^2}\right)y^2\,x\,m}{x^6+3\,x^4\,y^2+3\,x^2\,y^4+y^6} \quad \textbf{(53)}$$

```
> b:=simplify(Cf2c[4,4,4]+2*Cf2c[4,4,1]*vx+Cf2c[4,1,1]*vx^2
          +2*Cf2c[4,4,2]*vy+Cf2c[4,2,2]*vy^2+2*Cf2c[4,1,2]*vx*
  vy);
```

$$b := \frac{1}{\left(x^4+2\,x^2\,y^2+y^4\right)c\sqrt{x^2+y^2}}\Big(2\,\big(c^2\,mp\,x^2 + c^2\,mp\,y^2 + 2\,c\,mp\,vx\,x\sqrt{x^2+y^2} + 2\,c\,mp\,vy\sqrt{x^2+y^2}\;y + c\,vx\,x^3 + c\,vx\,x\,y^2 + c\,vy\,x^2\,y + c\,vy\,y^3 + mp\,vx^2\,x^2 + 2\,vx\,vy\,mp\,x\,y + mp\,vy^2\,y^2 + vx^2\,x^2\sqrt{x^2+y^2} - vx^2\sqrt{x^2+y^2}\;y^2 + 4\,vx\,vy\,x\sqrt{x^2+y^2}\;y - vy^2\,x^2\sqrt{x^2+y^2} + vy^2\sqrt{x^2+y^2}\;y^2\big)\,mp\Big) \quad \textbf{(54)}$$

```
> bp:=simplify(Cf2pc[4,4,4]+2*Cf2pc[4,4,1]*vxp+Cf2pc[4,1,1]*vxp^2
```

```
+2*Cf2c[4,4,2]*vyp+Cf2pc[4,2,2]*vyp^2+2*Cf2pc[4,1,2]*
vxp*vyp);
```

$$bp := \frac{1}{\left(x^4+2\,x^2\,y^2+y^4\right)\,c\,\sqrt{x^2+y^2}}\left(2\,\left(c^2\,m^2\,x^2+c^2\,m^2\,y^2+2\,c\,m^2\,vxp\,x\,\sqrt{x^2+y^2}+c\,m\,vxp\,x^3+c\,m\,vxp\,x\,y^2+2\,c\,mp^2\,vyp\,\sqrt{x^2+y^2}\,y+c\,mp\,vyp\,x^2\,y+c\,mp\,vyp\,y^3+m^2\,vxp^2\,x^2+2\,m^2\,vxp\,vyp\,x\,y+m^2\,vyp^2\,y^2+m\,vxp^2\,x^2\,\sqrt{x^2+y^2}-m\,vxp^2\,\sqrt{x^2+y^2}\,y^2+4\,m\,vxp\,vyp\,x\,\sqrt{x^2+y^2}\,y-m\,vyp^2\,x^2\,\sqrt{x^2+y^2}+m\,vyp^2\,\sqrt{x^2+y^2}\,y^2\right)\right) \quad (55)$$

```
> odevx:=simplify(ax+Cf2c[1,4,4]+Cf2c[1,1,1]*vx^2+Cf2c[1,2,2]*
vy^2
      +2*(Cf2c[1,1,4]*vx +Cf2c[1,1,2]*vx*vy+Cf2c[1,2,4]*vy)=b*
vx):
```

```
> odevy:=simplify(ay+Cf2c[2,4,4]+Cf2c[2,1,1]*vx^2+Cf2c[2,2,2]*
vy^2
      +2*(Cf2c[2,1,4]*vx +Cf2c[2,1,2]*vx*vy+Cf2c[2,2,4]*vy)=b*
vy):
```

```
> odevxp:=simplify(axp+Cf2c[1,4,4]+Cf2c[1,1,1]*vxp^2+Cf2c[1,2,2]*
vyp^2
      +2*(Cf2c[1,1,4]*vxp +Cf2c[1,1,2]*vxp*vyp+Cf2c[1,2,4]*vyp)
=bp*vxp):
```

```
> odevyp:=simplify(ayp+Cf2c[2,4,4]+Cf2c[2,1,1]*vxp^2+Cf2c[2,2,2]*
vyp^2
      +2*(Cf2c[2,1,4]*vxp +Cf2c[2,1,2]*vxp*vyp+Cf2c[2,2,4]*vyp)
=bp*vyp):
```

DATA --

```
> c:=299792458.0; # m/s
```

$$c := 2.997924580\ 10^{8} \quad (56)$$

```
> G:=6.6738480*10^(-11);      #   *m^3/(kg*s^2);
```

$$G := 6.673848000\ 10^{-11} \quad (57)$$

```
> DTS:=149597870700;  #   m
```

$$DTS := 149597870700 \quad (58)$$

```
> MS:=2.0*10^30;      # *kg
```

$$MS := 2.000000000\ 10^{30} \quad (59)$$

```
> R:=DTS; Rp:=DTS;
```

$$R := 149597870700$$

$$Rp := 149597870700 \quad (60)$$

```
> M:=MS;  Mp:=MS;  # kg
```

$$M := 2.000000000\ 10^{30}$$

$$Mp := 2.000000000\ 10^{30} \quad (61)$$

```
> GM:=G*M;     #   *kg*s^2;
```

$$GM := 1.334769600\ 10^{20} \quad (62)$$

```
> GMp:=G*Mp;   #   *kg*s^2;
```

$$GMp := 1.334769600\ 10^{20} \quad (63)$$

KONTUZ Gravity waves--

```
> M:=35*M;  GM:=35*GM;
```

$$M := 7.000000000\ 10^{31}$$

$$GM := 4.671693600\ 10^{21} \quad (64)$$

KONTUZ--

```
> #M:=1; GM:=G;
```

```
> Mp:=35*Mp; GMp :=35*GMp;
```

$$Mp := 7.000000000\ 10^{31}$$

$$GMp := 4.671693600\ 10^{21} \quad (65)$$

```
> R:=175*10^3; # R:=256*4*R;              # m
```

$$R := 175000 \quad (66)$$

```
> Rp:=175*10^3; # :=256*4*Rp;             # m
```

$$Rp := 175000 \quad (67)$$

```
> m:=GM/c^2; mp:=GMp/c^2;
```

$$m := 51979.60146$$

$$mp := 51979.60146 \quad (68)$$

KONTUZ--

```
> #m:=0;
```

```
> Omega:=sqrt(GMp/(4*R^3));
```
Initial value. Guess from Newton's theory

$$\Omega := 466.8207016 \quad (69)$$

```
> Omega  := Omega;
```

$$\Omega := 466.8207016 \quad (70)$$

```
> #Omega:=0.001;
> #Omega:=0.0005;
> #Omega:=0.0004;
```

```
> #Omega:=0.0003;
```
No answer

```
> #Omega:=0.0002;
```
No answer

```
> #Omega:=0.0001;
```
No answer

```
> Per:=evalf(2*Pi/Omega);   #  s
```

$$Per := 0.01345952587 \quad (71)$$

```
> Per/(60*24*365);   # years
```

$$2.560792593\ 10^{-8} \quad (72)$$

```
> x0:=R; xp0:=-x0; y0:=0; yp0:=0;
```

$$x0 := 175000$$

$$xp0 := -175000$$

$$y0 := 0$$

$$yp0 := 0 \quad (73)$$

```
> vx0:=0; vxp0:=-vx0; vy0:=x0*Omega; vyp0:=-vy0;
```

$$vx0 := 0$$
$$vxp0 := 0$$
$$vy0 := 8.169362278\ 10^7$$
$$vyp0 := -8.169362278\ 10^7 \quad (74)$$

Kontuz: Initial rotation data

```
> #vy0:=0; vyp0:=0;
```

```
> odex := diff(x(t), t) = vx(t); odexp := diff(xp(t), t) = vxp(t)
  ;
```

$$odex := \frac{d}{dt}\, x(t) = vx(t)$$
$$odexp := \frac{d}{dt}\, xp(t) = vxp(t) \quad (75)$$

```
> odey := diff(y(t), t) = vy(t); odeyp := diff(yp(t), t) = vyp(t)
  ;
```

$$odey := \frac{d}{dt}\, y(t) = vy(t)$$
$$odeyp := \frac{d}{dt}\, yp(t) = vyp(t) \quad (76)$$

Whitehead---

```
> odevx:=subs(x=(x(t)-xp(t)),y=(y(t)-yp(t)),vx=vx(t),vy=vy(t),ax=
  diff(vx(t),t),odevx):
> odevy:=subs(x=(x(t)-xp(t)),y=(y(t)-yp(t)),vx=vx(t), vy=vy(t),
  ay=diff(vy(t),t),odevy):
> odevxp:=subs(x=(xp(t)-x(t)),y=(yp(t)-y(t)),vxp=vxp(t),vyp=vyp
  (t),axp=diff(vxp(t),t),odevxp):
> odevyp:=subs(x=(xp(t)-x(t)),y=(yp(t)-y(t)),vxp=vxp(t), vyp=vyp
  (t),ayp=diff(vyp(t),t),odevyp):
```

Newton ---

```
> sys:=odex,odexp,odey,odeyp,odevx,odevxp,odevy,odevyp:
> fncs:={x(t),xp(t),vx(t),vxp(t),y(t),yp(t),vy(t),vyp(t)};
```

$$fncs := \{vx(t), vxp(t), vy(t), vyp(t), x(t), xp(t), y(t), yp(t)\} \quad (77)$$

```
> ics:=x(0)=x0, xp(0)=xp0, y(0)=0, yp(0)=0, vx(0)=vx0, vxp(0)=
  vxp0, vy(0)=vy0,
  vyp(0)=-vy0;
```

$$ics := x(0) = 175000, xp(0) = -175000, y(0) = 0, yp(0) = 0, vx(0) = 0, vxp(0) = 0, vy(0) = 8.169362278\ 10^7, vyp(0) = -8.169362278\ 10^7 \quad (78)$$

```
> dsn:=dsolve({sys,ics},fncs,numeric,method=gear,abserr=1.*10^
  (-10),relerr=1.*10^(-10),output=listprocedure);
```

dsn := [*t* = **proc**(*t*) ... **end proc**, *vx*(*t*) = **proc**(*t*) ... **end proc**, *vxp*(*t*) = **proc**(*t*) ... **end proc**, *vy*(*t*) = **proc**(*t*) ... **end proc**, *vyp*(*t*) = **proc**(*t*) ... **end proc**, *x*(*t*) = **proc**(*t*) ... **end proc**, *xp*(*t*) = **proc**(*t*) ... **end proc**, *y*(*t*) = **proc**(*t*) ... **end proc**, *yp*(*t*) = **proc**(*t*) ... **end proc**] **(79)**

```
> x,xp,vx,vxp,y,yp,vy,vyp:=op(subs(dsn,[x(t),xp(t),vx(t),vxp(t),y
  (t),yp(t),vy(t),vyp(t)]));
```

$x, xp, vx, vxp, y, yp, vy, vyp :=$ **proc**(t) ... **end proc**, **proc**(t) ... **end proc**, **proc**(t) **(80)**

...

end proc, **proc**(t) ... **end proc**, **proc**(t) ... **end proc**, **proc**(t) ... **end proc**, **proc**(t)

...

end proc, **proc**(t) ... **end proc**

```
> Omega:=Omega; Per:=evalf(2*Pi/Omega);
```

$$\Omega := 466.8207016$$

$$Per := 0.01345952587 \quad \textbf{(81)}$$

```
> x(Per); xp(Per); y(Per); yp(Per);
```

$$-4.50140020452582\ 10^5$$

$$4.50140020476698\ 10^5$$

$$-11420.2679878751$$

$$11420.2679727990 \quad \textbf{(82)}$$

```
> Ti:=-Per; Tf:=9*Per;
```

$$Ti := -0.01345952587$$

$$Tf := 0.1211357328 \quad \textbf{(83)}$$

```
> r:=t->sqrt((x(t)-xp(t))^2+(y(t)-yp(t))^2);
```

$$r := t \rightarrow \sqrt{(x(t) - xp(t))^2 + (y(t) - yp(t))^2} \quad \textbf{(84)}$$

```
> r(0);
```

$$3.500000000\ 10^5 \quad \textbf{(85)}$$

```
> Omega:=Omega;
```

$$\Omega := 466.8207016 \quad \textbf{(86)}$$

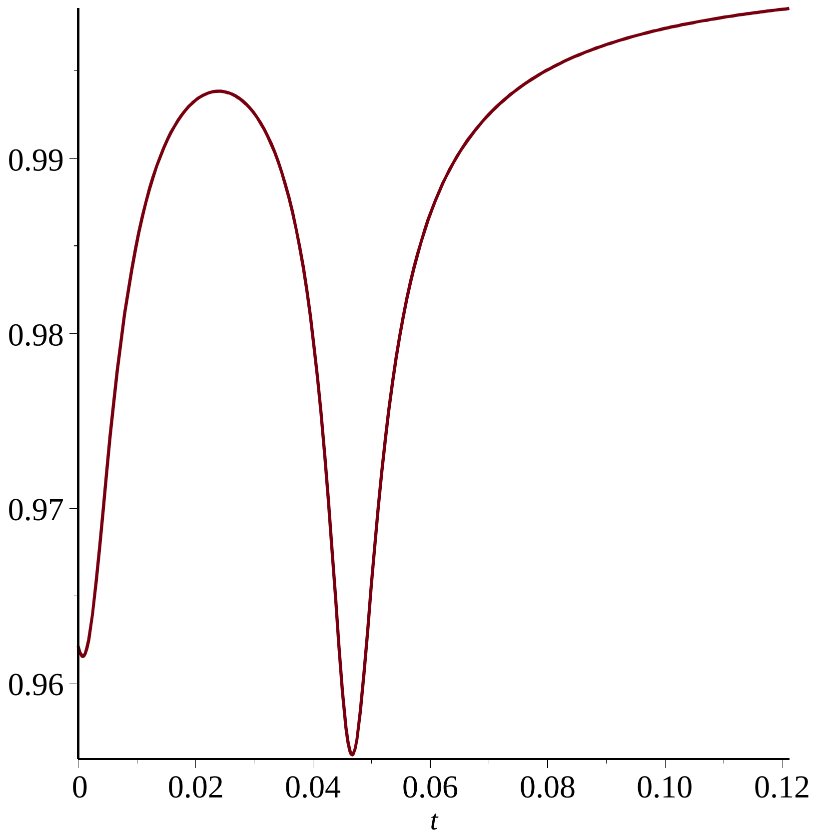
```
> sqrt(1-(vx(0)^2+vy(0)^2)/c^2);
```

$$0.9621556144 \quad \textbf{(87)}$$

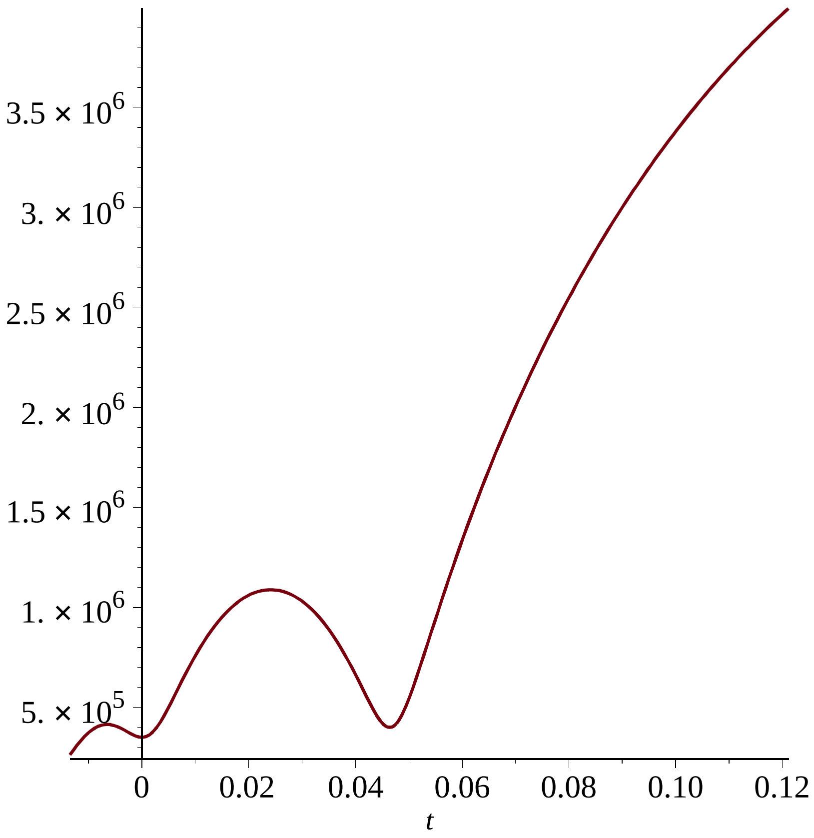
```
> TestV:=t->sqrt(1-(vx(t)^2+vy(t)^2)/c^2);
```

$$TestV := t \rightarrow \sqrt{1 - \frac{vx(t)^2 + vy(t)^2}{c^2}} \quad \textbf{(88)}$$

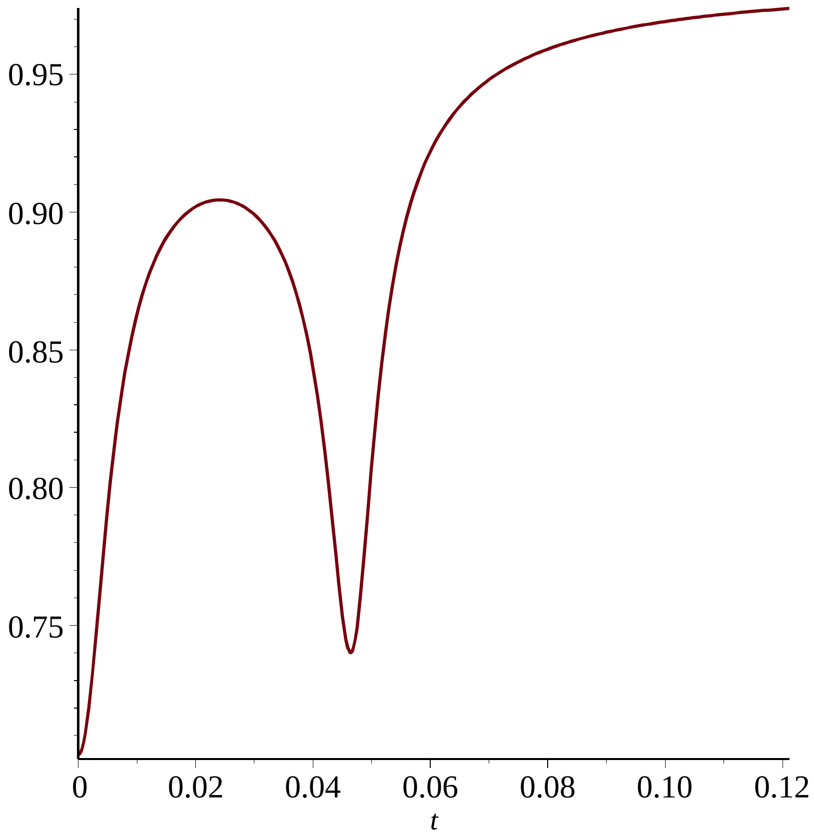
```
> plot(TestV(t),t=0..Tf);
```



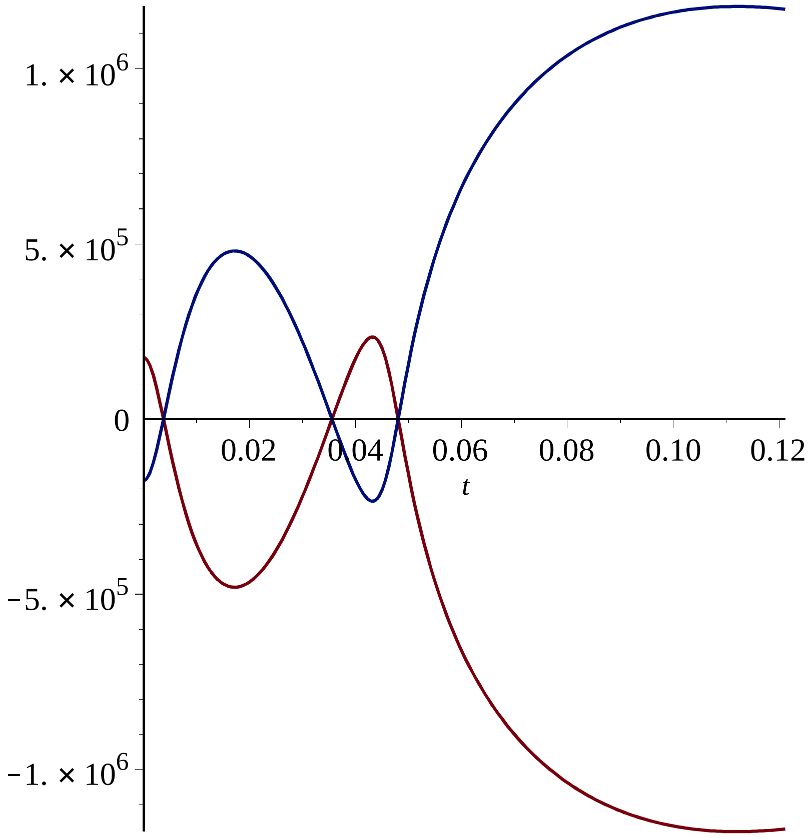

```
> plot(r(t),t=Ti..Tf);
```



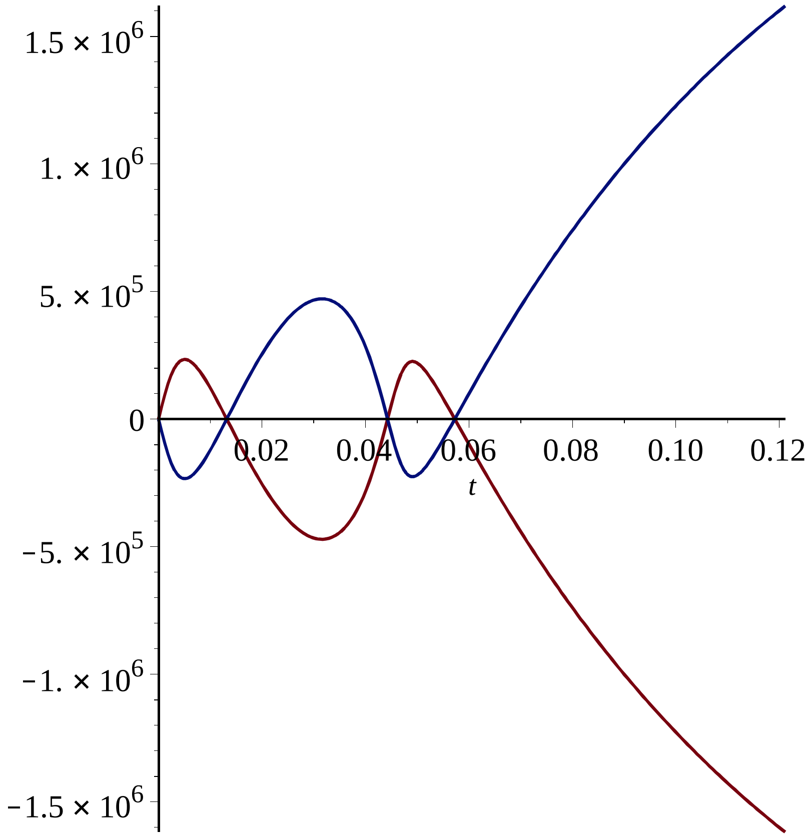

```
> plot(1-2*m/r(t),t=0..Tf);
```



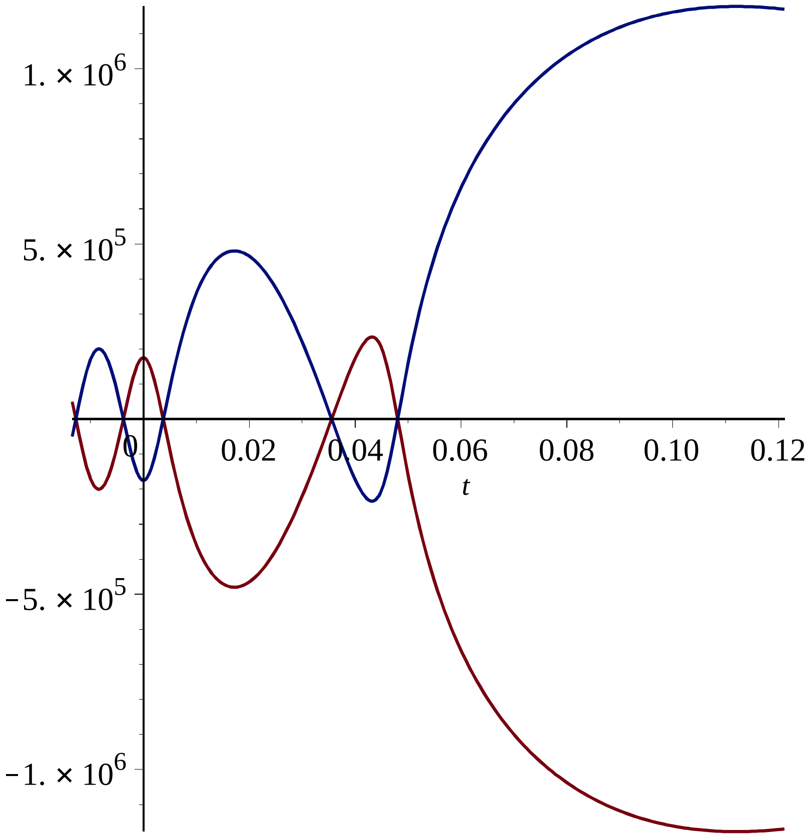

```
> plot([x(t),xp(t)],t=0..Tf);
```



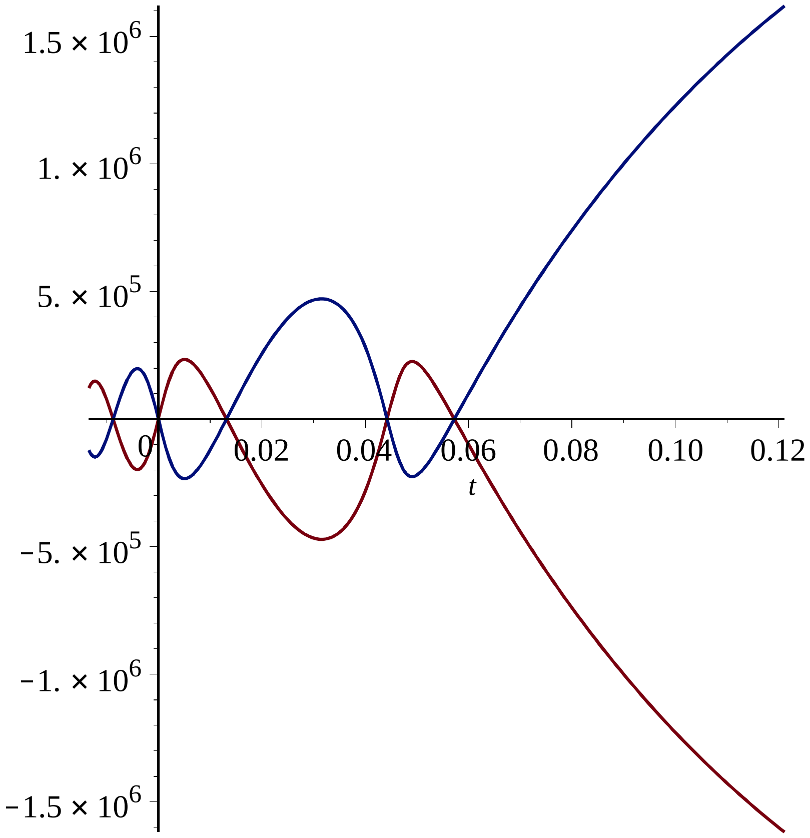

```
> plot([y(t),yp(t)],t=0..Tf);
```



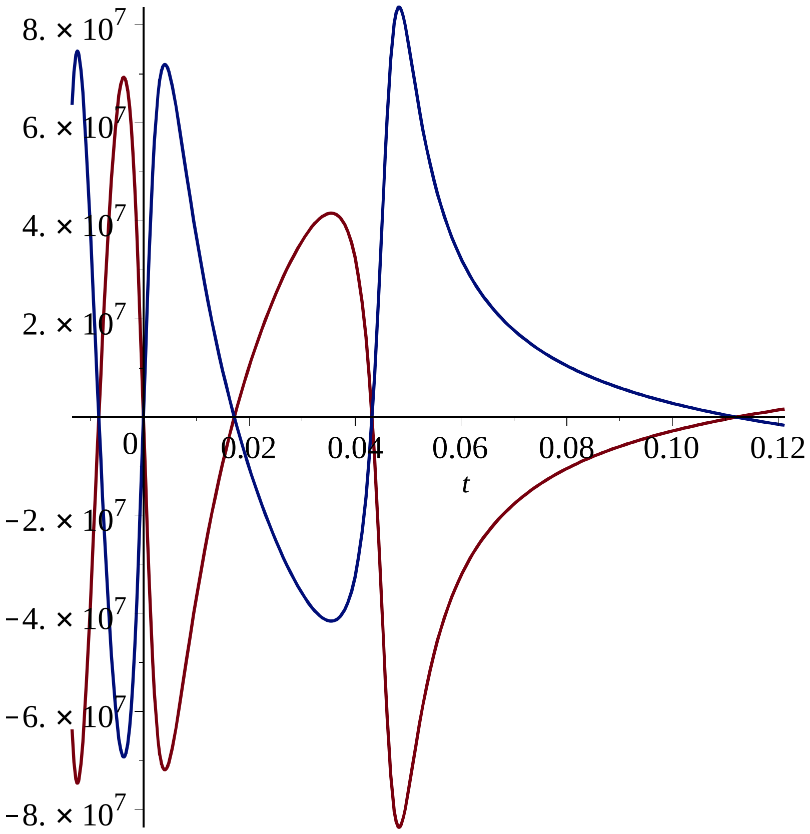

```
> plot([x(t),xp(t)],t=Ti..Tf);
```



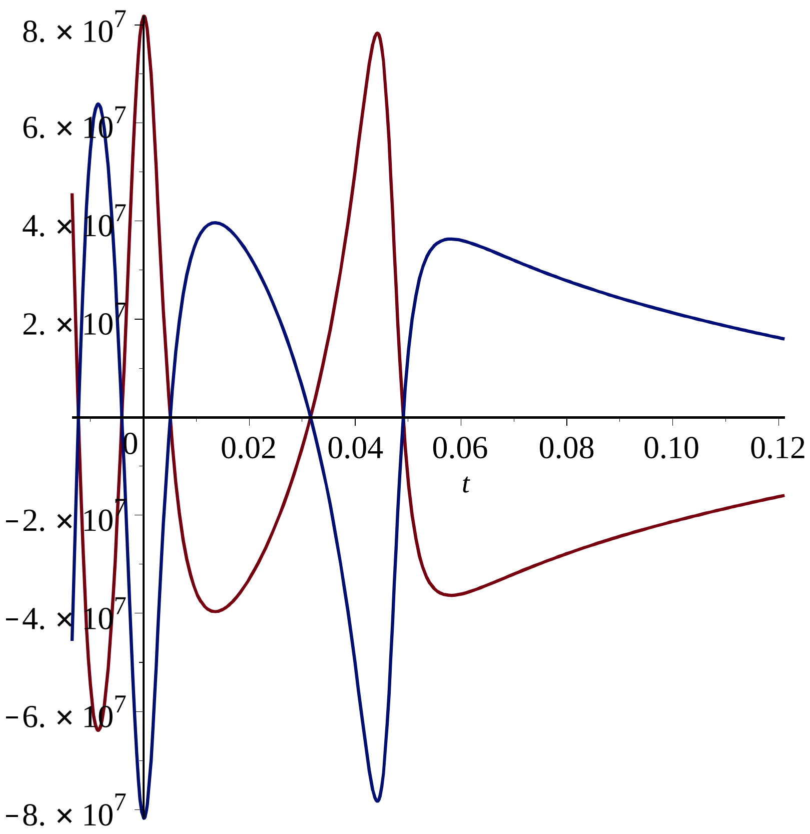

```
> plot([y(t),yp(t)],t=Ti..Tf);
```




```
> plot([vx(t),vxp(t)],t=Ti..Tf);
```




```
> plot([vy(t),vyp(t)],t=Ti..Tf);
```



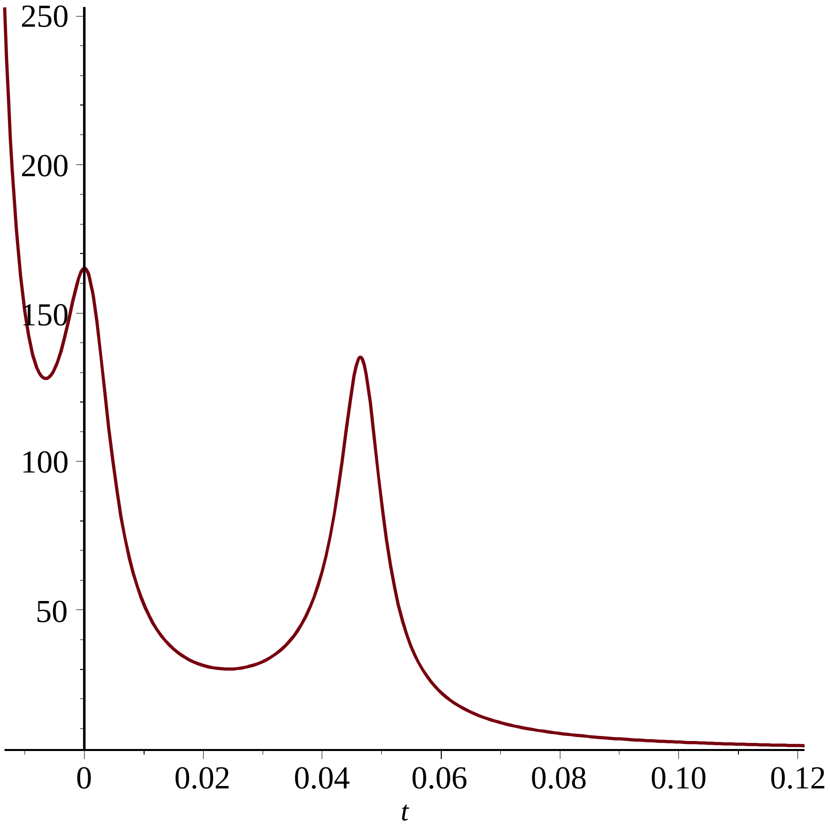

```
> Tf:=Tf;
```

$$Tf := 0.1211357328 \quad (89)$$

```
> Omega:=t->sqrt(GM/(4*r(t)^3));
```

$$\Omega := t \rightarrow \sqrt{\frac{1}{4} \frac{GM}{r(t)^3}} \quad (90)$$

```
> plot([Omega(t)],t=Ti..Tf);
```




```
> theta:=t->arctan(y(t)/x(t));
```

$$\theta := t \rightarrow \arctan\left(\frac{y(t)}{x(t)}\right) \quad \textbf{(91)}$$

```
> #plot([theta(t)],t=Ti..Tf);
> evalf(Pi/2);
```

$$1.570796327 \quad \textbf{(92)}$$

Kontuz: +M^2, ----------- ---

```
> H:=t->(M^2*c^3*(2*c*r(t)+2*vx(t)*x(t)+2*vy(t)*y(t)*mp-M^2*c^4*r
  (t)^2)/(r(t)^2*sqrt(-M^2*c^2*(2*c^2*mp*r(t)^2-c^2*r(t)^3+4*c*
  mp*r(t)*vx(t)*x(t)+4*c*mp*r(t)*vy(t)*y(t)+2*mp*vx(t)^2*x(t)
  ^2+8*mp*vx(t)*vy(t)*x(t)*y(t)+2*mp*vy(t)^2*y(t)^2+r(t)^3*vx(t)
  ^2+r(t)^3*vy(t)^2)/r(t)^3)));
```

$$H := t \rightarrow \left(M^2 c^3 \left(2\, c\, r(t) + 2\, vx(t)\, x(t) + 2\, vy(t)\, y(t)\, mp - M^2 c^4 r(t)^2\right)\right) \Big/ \quad \textbf{(93)}$$

$$\left(r(t)^2 \left(-\frac{1}{r(t)^3} \left(M^2 c^2 \left(2\, c^2\, mp\, r(t)^2 - c^2\, r(t)^3 + 4\, c\, mp\, r(t)\, vx(t)\, x(t)\right.\right.\right.\right.$$

$$+ 4\, c\, mp\, r(t)\, vy(t)\, y(t) + 2\, mp\, vx(t)^2\, x(t)^2 + 8\, mp\, vx(t)\, vy(t)\, x(t)\, y(t)$$

$$+ 2\, mp\, vy(t)^2\, y(t)^2 + r(t)^3\, vx(t)^2 + r(t)^3\, vy(t)^2)))^{1/2} \Big)$$

```
> H(0);
```

$$-1.047538485\ 10^{138} \quad \textbf{(94)}$$

Kontuz: +Mp^2, --

```
> Hp:=t->(Mp^2*c^3*(2*c*r(t)+2*vxp(t)*xp(t)+2*vyp(t)*yp(t)*m-
  Mp^2*c^4*r(t)^2)/(r(t)^2*sqrt(-Mp^2*c^2*(2*c^2*m*r(t)^2-c^2*r
  (t)^3+4*c*m*r(t)*vxp(t)*xp(t)+4*c*m*r(t)*vyp(t)*yp(t)+2*m*vxp
  (t)^2*xp(t)^2+8*m*vxp(t)*vyp(t)*xp(t)*yp(t)+2*m*vyp(t)^2*yp(t)
  ^2+r(t)^3*vxp(t)^2+r(t)^3*vyp(t)^2)/r(t)^3)));
```

$$Hp := t \rightarrow \left(Mp^2\, c^3\, (2\, c\, r(t) + 2\, vxp(t)\, xp(t) + 2\, vyp(t)\, yp(t)\, m - Mp^2\, c^4\, r(t)^2) \right) \Bigg/ \Bigg(r(t)^2 \Big(- \frac{1}{r(t)^3} (Mp^2\, c^2\, (2\, c^2\, m\, r(t)^2 - c^2\, r(t)^3 + 4\, c\, m\, r(t)\, vxp(t)\, xp(t) + 4\, c\, m\, r(t)\, vyp(t)\, yp(t) + 2\, m\, vxp(t)^2\, xp(t)^2 + 8\, m\, vxp(t)\, vyp(t)\, xp(t)\, yp(t) + 2\, m\, vyp(t)^2\, yp(t)^2 + r(t)^3\, vxp(t)^2 + r(t)^3\, vyp(t)^2)) \Big)^{1/2} \Bigg) \quad \textbf{(95)}$$

```
> subs(m=0,Hp(0)); subs(mp=0,H(0));
```

$$-1.047538485\ 10^{138}$$

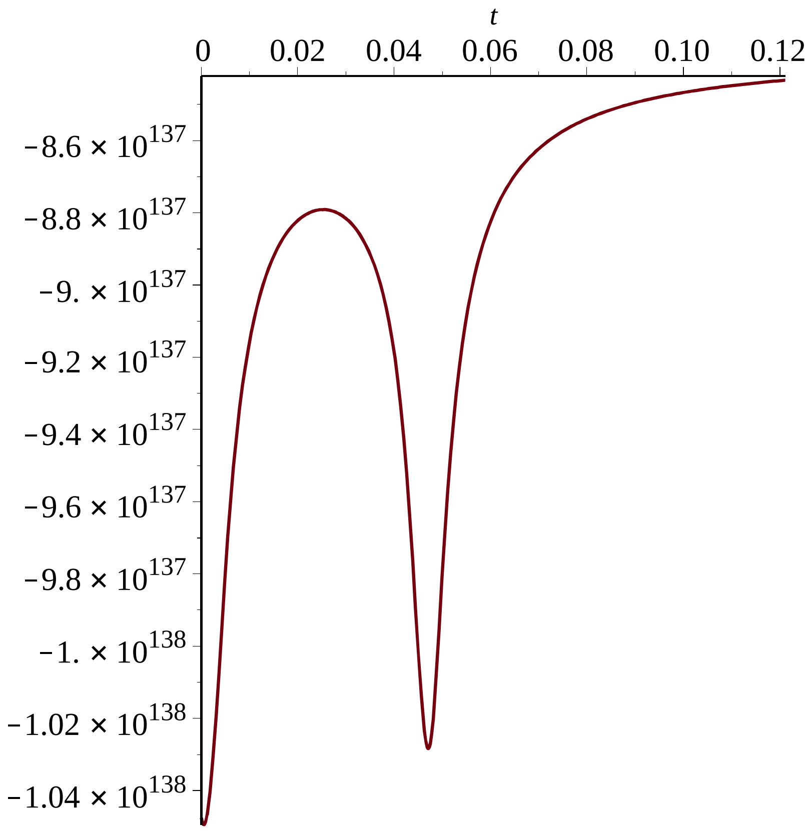
$$-1.047538485\ 10^{138} \quad \textbf{(96)}$$

```
> H(0); Hp(0);
```

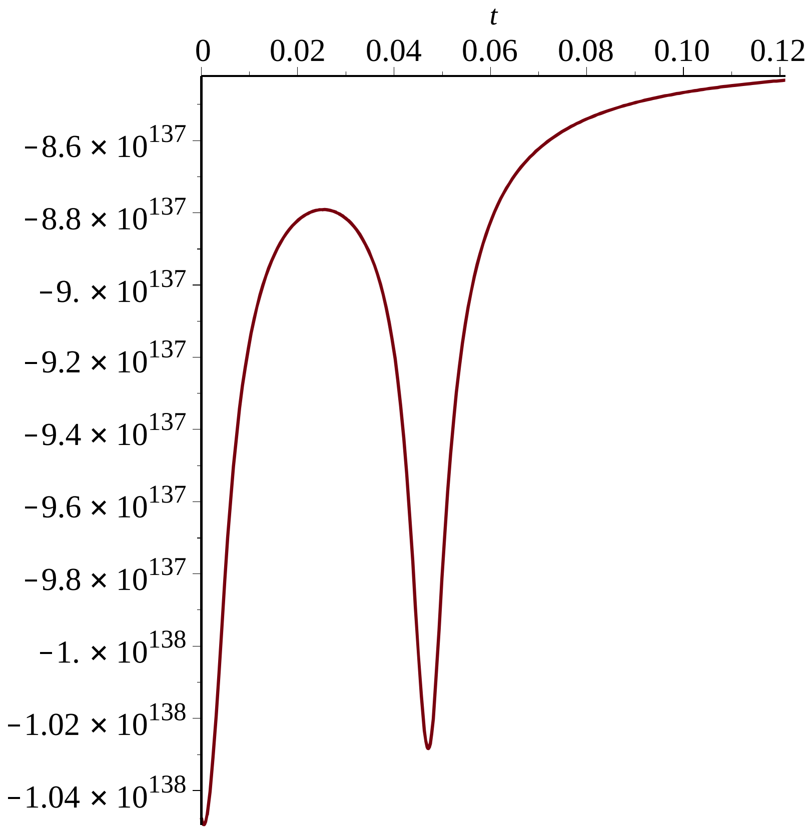
$$-1.047538485\ 10^{138}$$

$$-1.047538485\ 10^{138} \quad \textbf{(97)}$$

```
> plot([H(t)], t=0..Tf);
```




```
> plot([Hp(t)], t=0..Tf);
```




```
> (H(Tf)-H(0))/(H(Tf)+H(0));
```

$$-0.107998748081240 \quad \textbf{(98)}$$

```
> H(0);
```

$$-1.047538485\ 10^{138} \quad \textbf{(99)}$$

```
> Tf:=Tf; H(Tf);
```

$$Tf := 0.1211357328$$

$$-8.43327342807222\ 10^{137} \quad \textbf{(100)}$$