·(Kushik) Kaushal Timilsina

Riemann's ~~Hypothesis~~ Zeroes and Manifolds

(- ~~not an attempt to prove, just a~~ new perspective).

"An infinite dimensional manifold is a compact surface by itself" (that encloses nothing)

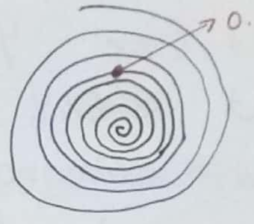
Part 1: The identification of the origin

1.1) Real line $\longleftarrow\!\!\!\underset{0}{+}\!\!\!\longrightarrow$

1.2) Fold the real line into a coil as:

a) Start with a point and fix the origin to it.

b) Rotate clockwise on a spiral about a center (not yet clear), and map $[0,\infty)$ on the clockwise rotation, moving inwards on the spiral.



c) Rotate counterclockwise starting at 0 and map $(-\infty,0)$ ("run to $-\infty$ from 0"); moving outwards on the spiral.

1.3) Define a map from the part of the spiral $[0,\infty)$ to the complex plane, identifying each point with a complex number $s=a+ib$. ("take the spiral to be a disc")

This is pretty sloppy, as the center is not well defined; which should be done but I am unable to do it.

Question: Can you identify the center to the origin (0)? This would mean to convert the real line into a closed curve, given that you can fix ~~the~~ a finite radius (0 and center relation) to define the center of the spiral from the origin (0).

- To accomodate the whole real line, the torsion ("not exactly") of the curve is defined accordingly.

Hypothesis (1) Whether you can form a closed curve by identification will depend on where you start (0, center relation). Only special maps can define the identification.

I'm just being verbose here, but this could probably require non-linear mapping or some kind of non-Cauchy convergence on partitions of the domain $[0,\infty)$.

Part 2: Mysterious : ~~diffeomorphism~~? or homeomorphism? or homomorphism

2.1) consider $V \subset \mathbb{R}^n$ (n dimensional euclidean space).

2.2) Can you construct a vector field, V' at every point in V such that an orthonormal basis (linearly independent could be enough) $\{e_1, e_2, \dots, e_n\}$ at a point p, can span V'? at every point. ("You can construct the basis at other points by linear combination of $\{e_1, e_2, \dots e_n\}$")

2.3) Define $\varphi: V \to M^n$: which is already constructed as $f_\alpha: \mathcal{U}_\alpha \to M^n$, $\alpha: 1, \dots n$ where $\mathcal{U}_\alpha$ are open sets (domains). ("φ is a map from V to an already well defined Manifold M").

2.4) Assign norm 1 vectors $\{f_1, f_2, \dots f_n\}$ to $\mathcal{U}_\alpha$'s such that for the set of points $\{q_1, q_2, q_3, \dots q_n \mid q_i \in \mathcal{U}_i\}$, the map $\phi_i: q_i \to q_i \cdot f_i$ (where dot is the scalar product) is defined such that the set of vectors $\{q_i f_i \mid i=1, \dots n\}$ form an n-dimensional vector space W, such that $q_i f_i$ are linearly independent.

2.5) ~~what~~ ~~sta~~

2.5) A map from $V' \to W$ can be defined by composition. $\{e_i\}$ on V' were defined orthonormal by brute force. $\{q_i h_i\}$ on W were defined ~~orthonormal~~ linearly independent by brute force.

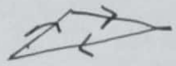
2.6) Define a homomorphism $\omega : V' \to W$ if for any operation ~~defined~~ on ~~$\{e_1, e_2, \ldots, e_n\}$ at p with~~ between a vector in V' at a point p in V and a vector in W, and between a vector in V' at another point q in V and the same vector in W give the same result.

$$\text{“}\operatorname{operation}\left(\sum_i^n \alpha_i e_{i_p}, \sum_i^n q_i h_i\right) = \operatorname{operation}\left(\sum_i^n \alpha_i e_{i_q}, \sum_i^n q_i h_i\right)\text{”}$$

$\forall\ p, q$ in V. (“back to the question in 2.2”)

Part 3: Riemann's ~~hypothesis~~ zeta function.

3.1) n linearly dependent vectors may add upto 0 as do the nth roots of unity.



3.2) The Riemann zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \operatorname{Re}(s) > 1.$$

3.3) Definition:

consider the sum, $1^{-s} + 2^{-s} + 3^{-s} + \ldots + n^{-s}$

define: $Z_1 = (|g_{11}|)^{-s} + (|g_{12}| + |g_{22}|)^{-s} + \ldots$

$\ldots + (|g_{1n}| + |g_{2n}| + \ldots |g_{nn}|)^{-s}$

where g_{ij} is the ith orthonormal basis of

ϑ^j (j dimensional vector space) such that $\vartheta^j \subset \mathbb{F}^n \; \forall j$. ~~and ϑ^j is a subspace of~~ $\mathbb{F}^n$, where $\mathbb{F}^n$ is defined as $\underbrace{\mathbb{F}' \times \mathbb{F}' \times \ldots \mathbb{F}'}_{n}$, $\mathbb{F}'$ is a field.

3.4) Definiton: $Z_2 = \left(\left| (g_{11}) + (g_{12} + g_{22}) + (g_{13} + g_{23} + g_{33}) + \ldots \right.\right.$ $\left.\left. \ldots + (g_{1n} + g_{2n} + \ldots + g_{nn}) \right|^{-s} \right)$ with g_{ij}'s as defined in 3.3.

3.5) If $Z_2 \neq 0$, direct sum of ϑ^j's over j $= \mathbb{F}^n$ and ϑ^j's are subspaces that are disjoint in some sense, $\bigcup_j \vartheta^j$'s is a partion of $\mathbb{F}^n$, which is equal to $\mathbb{F}^n$.

If $Z_2 = 0$, the ϑ^j's are not disjoint.

This establishes some form of compactness.

Part 4: The new perspective.

4.1) Define the construction in part 2 using U_α's (2.3) as different refinements of the closed curve in part 1 (whose origin is identified with the center).

4.2) Define an operation (2.6)

$$\left(\sum_i^n \alpha_i e_{ip}, \sum_j^n q_j h_j\right) = \left(\sum_i^n \alpha_i e_{ip}\right) \wedge \left(\sum_j^n q_j h_j\right)$$

where $\wedge$ is some sort of exterior ~~derivative~~ product (by definition).)

(Because it gives out a vector and would be 0 if $\sum_i^n e_{ip}\alpha_i$ was in some sense parallel to $\sum_i^n q_i h_i$).

~~with the bilinearity of $\wedge$ on the component~~

~~$\Rightarrow$ operation $\left(\sum_i^n \alpha_i e_{ip}, \sum_i^n q_i h_i\right) = \alpha_i q_i \sum_i^n e_i h_i$~~

~~(and now we drop p in e_i using 2.6 definition of homomorphism.)~~

~~4.3) Plug $\alpha_i q_i \sum_i^n e_i h_i$~~

4.3) plug $\left(\sum_i^n \alpha_i e_i\right) \wedge \left(\sum_j^n q_j h_j\right)$

(dropped the p on e_i by 2.6 definition of homomorphism)

into Z_1 and Z_3 such that

g_{ij} ~~α_i~~ is the i'th component of

$$\left(\sum_{i'}^{i} \alpha_{i'} e_{i'}\right) \wedge \left(\sum_{j'}^{j} q_{j'} h_{j'}\right)$$

("without defining the i^{th} unit vector, this is a pretty big formula").

4.4) Hypothesis: With this setup, if $Z_1 = Z_2$ for a finite integer n, (some kind of saturation of the triangle inequality $|\vec{a}+\vec{b}| \leq |\vec{a}|+|\vec{b}|$.), in some sense on a finite dimensional manifold $\mathbb{M}^n$ you can using U_α from the the definition of $\mathbb{M}^n$ you can form a V (as in part 2) that can ~~cover~~ be defined at every point in M. ~~were~~ the relation between (In part 2, ~~things~~ ~~were~~ the relation between U_α's and V was defined by construction. But here we get V from U_α's.)

Hypothesis(2): $Z_1 = Z_2 = 0 \Leftrightarrow \mathbb{M}^n$ is closed ($\mathbb{M}^n$ is a compact surface.)

4.5) Define $Z_3 = (|g_{11}|)^{-s} + (|g_{12}|+|g_{22}|)^{-s} + \dots$

$\dots$ ~~infinitely many~~ such that

$$Z_3 = \lim_{n\to\infty} Z_1$$

$$Z_4 = (|(g_{11}) + (g_{12}+g_{22}) + (g_{13}+g_{23}+g_{33}) \dots |)^{-s}$$

$$Z_4 = \lim_{n\to\infty} Z_2$$

(the definition of limit is sloppy here as indicated at the end of part 1, as it is not clearly defined here based on convergence.)

Hypothesis (3): Riemann's ~~hypothesis~~ $\Leftrightarrow$ $z_3 = z_4 = 0$

which is to say that the existence of Riemann's zeroes ~~hypothesis~~ implies (and vice versa) that an infinite zeroes dimensional manifold is a compact surface (closed). Where? (what is the ambient space?)

Well, by itself is the answer (possibly).

Rephrase: An infinite dimensional manifold is a compact surface by itself. (but there is nothing that it encloses.)

Summary about this perspective

Riemann zeta function.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

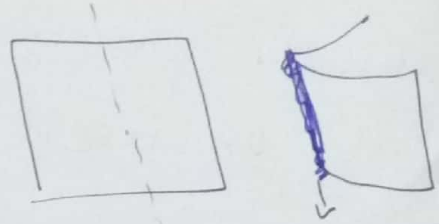
- Many other perspectives on the zeta function try to exple the parameter, s.
- This perspective explore the possibilities on two other parameters of the zeta function – (i) n being natural numbers
  (ii) the sum being infinite.
- Hypothesis (3) is based on Hypothesis (2) and Hypothesis (2) is based on Hypothesis (1).

which is to say that if there exists and can be defined an identification as defined in part 1, Hypothesis (2) and Hypothesis (3) are implied.

- Indeed the zeta function depends on the parameter s as well, which could be describing the uniqueness of Riemann's zeroes.

Part 5: Motivations:

(5.1) If one of the principal curvature of a plane (S^2) blows up → it forms a 1 dimensional line, locally, which is a 1 dimensinal singularity in S^2.



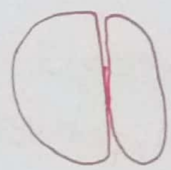
1d singularity.

(5.2) The singularity of a black hole is where the 4d spacetime (lorentzian manifold) forms (collapses into) a singularity (intuitively 0 dimensional, but I don't really know)

(5.3) Speculations on what lies on the "other side" of the singularity include wide range of notions.

(5.4) But if you collapse an infinite dimensional manifold into a singularity, perhaps you might not come out some where but connected to the ~~same~~ singularity is another singularity on the same manifold.

"sloppy figure" to demonstrate, two points both on the "outer surface" of the sphere identified by irregularity.



Even though you cannot (possibly) go there, what lies beyond a singularity in an infinite dimensional manifold is the same manifold.

"Intuitively its just a surface, there is no inside"

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(5.5) Question: Can two manifolds of different dimension be connected by a singularity?

("Is there a 100 dimensional universe beyond that black hole?", well how about the other one?")

(5.6) It can be difficult to start with this question for two finite dimensional ~~surfaces~~ manifolds. So we start with infinite two dimensional manifolds because Hypothesis (3) [the new perspective on the Riemann hypothesis], makes it easier.

Hypothesis:(4) Two infinite dimensional manifolds M^m, N^n can be connected by singularities. | Hypothesis (4) is based on Hypothesis (3).

I have not yet understood how infinite dimensional manifolds are generally treated, but this hypothesis would be constructed based on Cantor's theorem.

(5.7) Hypothesis:(5) An infinite dimensional "sphere" (sloppy) can be smoothly turned inside out or, An infinite dimensional manifold is globally unorientable

"It has the same information on the outer surface as in the inner surface and indeed the two surfaces can be identified the sphere, and hence there is no inside its just the surface"

Hypothesis(5) is based on hypothesis (4).

(5.7). Hypothesis (6): A globally orientable ~~compact~~ manifold can store global information iff it is globally compact. A locally orientable manifold can store local information if it is locally compact.

Question: Can infinite dimensional manifolds store global information?

- it seems like the answer is No, but local information Yes.

- this question has major implications for defining conservation laws on manifolds.

(5.8) I do not yet understand clearly how loops in spacetime are realized. But this setting, can define loops in spacetime as intersection between spacetime manifold and itself. It doesn't always have to be ~~in~~ 0-dimensional singularity, and neither does it even have to be a singularity. If we can work on infinite dimensional manifolds with singularities, we can soon develop ~~into~~ intersections of finite dimensional manifolds (which includes self intersection)

This will help us explore notions of higher dimensions, and ideas of creation, annihilation and conservation.

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(5.9) Aim: the aim of this discussion is build an understanding towards "A Theory of intersection of manifolds"

(5.10) In this discussion, $[0, \infty)$ was collapsed into a disc in complex coordinates which would later be used in the discussion to build $\mathcal{U}_\alpha$'s. It position $s = a+ib$ also plays an important part in the Riemann zeta function,

- Is it possible to collapse the whole complex plane into a similar spiral (or perhaps a stack of discs).

In General, can you collapse any topological space into an analogous structure ("make it closed" is the heart of the problem) and use it to build a more abstract version of a Manifold?

- For example the kähler manifold is defined with a 2-form structure.

Can you ~~define~~ work with constructions from n-form structures?.

(5.11) Corrollary of Hypothesis(6):

Black holes can not store local information. but they store global information. So perhaps one may not traverse beyond the singularity, but two people jumping into "entangled" black holes can meet at the center $\Longleftrightarrow$ (ER = EPR).

-In ~~the~~ the formulation of ER = EPR, "a theory of intersection of manifolds" will help us better understand what entanglement geometrically means.

(5.12) Integrating geometric flows in a theory of intersection of manifolds ~~with~~ could describe the foundations of the dynamics of interactions between manifolds; setting a new path for theories of quantum gravity.

(5.13). The fact that the Riemann zeta function describes the distribution of prime numbers, could hint at the spiral used in part 1 to be described by a map from the real line that preserves the distribution of primes. This could mean that this map could be a function like for example, von Mangoldt function which behaves differently for primes and non-primes. This could describe the relationship between the variable s in the zeta function and the different orders of infinities that we have used in defining infinite dimensional manifolds. The fact that $\xi(s) \neq 0 \; \forall s$ could tell us something like "there is no set whose cardinality is strictly between the cardinality of set of Real numbers and the cardinality of the set of natural numbers"; helping extend this perspective to the Riemann hypothesis.

Updates (1)

Mathematical construction for question following hypothesis (6), in (5.7).

Question: Can infinite dimensional manifolds store global information? No.

The generalized Stokes theorem:

$$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega.$$

if $\int_{\partial\Omega} \omega = 0$: equivalent to talking about the orientability of Ω, based on which we ask the above question.

$\int_{\Omega} d\omega = 0$ which is equivalent to saying that (by exterior derivative = 0) that the variation is constant in some sense. and that is what defines Hypothesis (5) in (5.7).

So, in some way by the Stokes theorem, hypothesis (5) = hypothesis (6), mathematically. previous it was more like guessing, based on intuition

also: $\int_{\partial\Omega} \omega = 0 = \int_{\Omega} d\omega$ also gives a more concrete understanding that an infinite dimensional manifold is a sphere that is (very weirdy) flat.

(1.1) Main point: for an infinite dimensional manifold Ω, $\boxed{\int_{\partial\Omega} \omega = 0 = \int_{\Omega} d\omega}$.

(1.2) Using flatness and local spherical "property" of infinite dimensional manifolds from (1.1) to talk about singularities.

(One of) Hilbert's theorem: S^2 oriented, $K_1 \leq K_2$ principal curvatures on S^2 ~~orientabl.~~
suppose $\exists\, p \in S$ such that:
1> $K(p) > 0$ Gauss Curvature.
2> K_1 has a local minimum at p.
3> K_2 has a local maximum at p.
Then, p is umbilical, $K_1(p) = K_2(p)$
"p is locally spherical".
this theorem is not used directly but should provide some foundation.

Hypothesis (7): ~~Flatness~~ Flat - Umbilical duality:
On a infinite dimensional manifold $(\Omega')^n$, let R be defined (not properly yet) as a curvature tensor field.
define: $\mathcal{D}_{p'} \equiv \dfrac{1}{\text{determinant}(R)}$ at all points $p' \in (\Omega')^n$.

$\mathcal{T}_{p'} = \text{trace}(R)$

- if $\mathcal{D} = 0$ at some point, p' on Ω', p' is a singularity.
- if $\mathcal{T} = 0$ at some point q' on Ω' ~~q' is equivalent~~ the neighborhood of q' is isometric to $\mathbb{R}^n$ the euclidean space of the same dimension.

7. min for f and p_2 max for f.

Hypothesis (7). For every point p in an infinite dimensional manifold Ω,

$\mathcal{D}_p = 0$, $\mathcal{C}_p = 0$.

which is related to hypothesis (5) and update ~~(1A)~~ (1.1) in the sense that information is uninvertibly mapped to $\vec{0}$.

Question (8): How many singular points? - infinitely many, possibily (but we don't know yet the construction) should you give me, so that I can make an infinite dimensional manifold - associated topology? Is it possible?

And the answer we're looking for is Yes? and a better yes in fact in a sense that: if you give me some number (infinity-order) of singularities (remember Cantor) (points) such that they need not all be from the same manifold or even manifolds with same dimensions, we can still construct an infinite dimensional manifold. The key property here is the infinite union of finite set of such points is used to construct the topology for an infinite dimensional manifold.

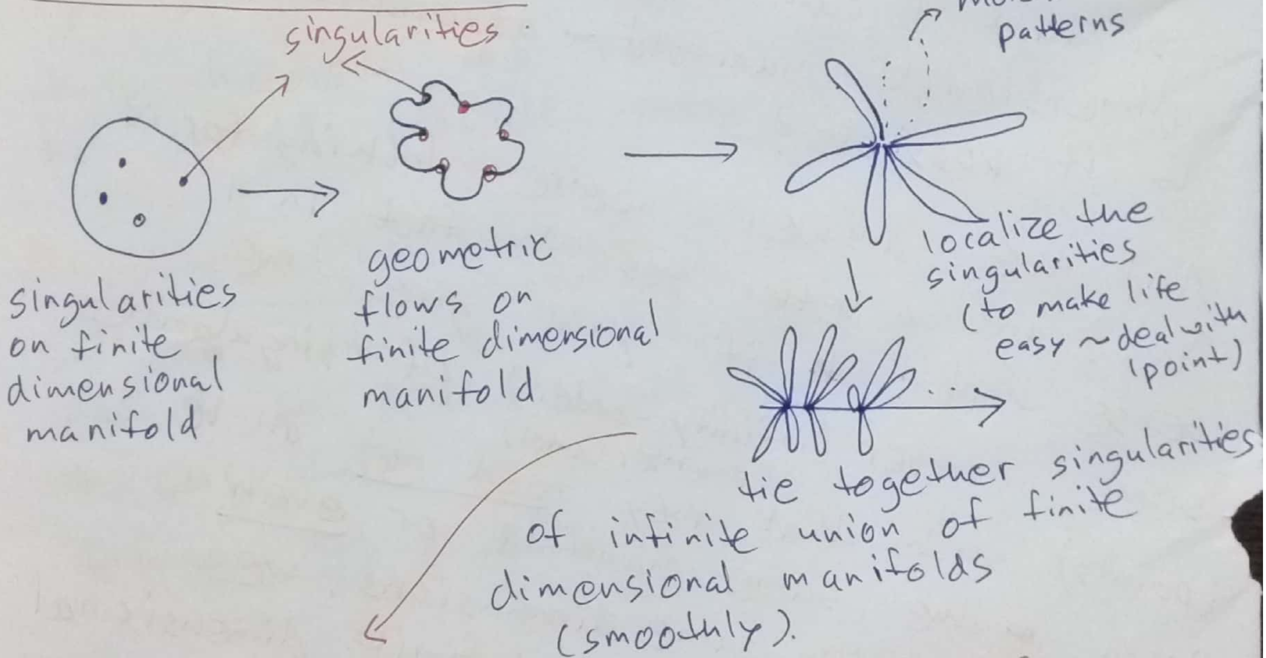
We are trying to construct an infinite dimensional differentiable manifolds by bringing together infinitely many singularities of finite dimensional manifolds.

→ This is a better notion for a static theory of intersecting manifolds.

~ → Add geometric flows to each of the finite dimensional manifolds and we get a dynamic theory of interacting manifolds.

"The challenge really is to define how can such a construction be smooth".

Question 1 in the dynamic theory of interacting manifolds? (from this perspective).



How do non singular points change when you induce geometric flows on this construction of infinite dimensional manifold? How do they intersect or interact?.

~ And Question 2 in the theory: vice versa for some interacting manifolds what are the behavior of their singularities (What happens when two black holes collide? Do singularities merge? what about information? etc of Powerful questions)

- to study this on a finite union, first we try to understand them on infinite union which is more naturally connected to singularities by construction.