Galactic *Rotation Curves* and *Spiral Form*

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Abstract

The two problems in the title, concerning massive core galaxies, unexplained by the original gravity law of Newton, are normal features according to the same law *Relative-Velocity Dependence* completed, thus—among other hypotheses—the *dark matter* is no longer necessary.

Keywords: spiral galaxy; rotation curve; dark matter; interactions relative-velocity dependence; gravitational refractive index; black captor/hole; atom radius.

1 Introduction

Galactic *rotation curves*, in gross disagreement with the original Newton law of gravitation, first observed by Oort (1931) [1], then, at a greater scale, by Zwicky (1937) [2] and, more extensive and precise, by Rubin and co-workers (1970) [3], (1980) [4], are currently explained, as mostly accepted, by the hypothetical *dark matter* coined by Zwicky.

The MOND theory by Milgrom [5], an attempt to replace the *dark matter* hypothesis in explaining the galactic *rotation curves*, is rather complicated and artificial, created for this single purpose; however, this is the best known "revolt" against the illusive notion.

The hypothesis in question has been partially accepted and listed as an unsolved problem in physics, because nothing has been learned about *dark matter* since it was proposed for gravitational property only.

The galactic spiral form is not well accounted for by the *density waves* theory [6] specially proposed for this purpose, but incomplete and not convincing, only maintained for lack of something better.

The RVD[1] solution we now put forward, (i) is simpler than each of *MOND*, and *Density waves* theories, (ii) solves both problems, and (iii) is not made specially for these purposes, but solves more problems, including in the solar system, like: gravitational index of refraction; perihelion advance; solar cycle; earth secular retardation; tectonic plates drift; continental drift, and more [**?**].

2 The RVD[1] completion of Newton's law of gravitation

Let M and m be two point masses, and $\vec{r}$ the position vector of m with respect to M, i.e., $\vec{r}$ has its initial point at M and the terminal point at m or, in other words, m lies in the gravitational field of M; denote as usually $\vec{v} = \dot{\vec{r}}$ the velocity of m with respect to M. Newton's law of gravitation writes $\vec{F}_N = -GMm\vec{r}/r^3 = m\vec{g}_N$. *Newton's gravitational law* (empirically) *RVD completed* is

$$\vec{F} = \vec{F}_N \left[1 + 3\frac{v^2}{c^2} + 4\left(\frac{v}{c}\right)^{\gamma} \frac{v_{\shortparallel}}{c}\right], \qquad (1)$$

where $v_{\shortparallel}$ is the component of $\vec{v}$ along the field, $v_{\shortparallel} = \dot{r}$, and $\gamma = 1.8$ (or $\gamma = 9/5$); using $\vec{g} = \vec{F}/m$ (*force per unit mass*, or *gravitational field strength*, or *gravitational acceleration*), Eq. (1) writes

$$\vec{g} = \vec{g}_N \left[1 + 3\frac{v^2}{c^2} + 4\left(\frac{v}{c}\right)^{\gamma} \frac{v_{\shortparallel}}{c}\right], \qquad (1')$$

[1] RVD stands for Relative-Velocity Dependence/Dependent (according to context).

whence the gravitational potential, defined as a function U having the property $\vec{g} = -\nabla U$, is

$$U = c^2 - U_N \left[1 + 3\frac{v^2}{c^2} + 4\left(\frac{v}{c}\right)^\gamma \frac{v_{\shortparallel}}{c} \right] , \tag{2}$$

where U_N is the Newton gravitational potential, $U_N = GM/r$; the arbitrary constant is taken as c^2 since the dimension of U is that of a square velocity, and at a point far away from any mass U should be just the maximum square velocity, i.e., c^2 is the gravitational potential with respect to all masses in the universe, at a point far away from any mass, hence the energy

$$\mathcal{E} = mc^2 \tag{3}$$

is the potential energy of a mas m far away from any mass, with respect to all masses in the universe, unlike in Special Theory of Relativity where this is related to the kinetic energy.

The fact must be stressed that *this is not a theory of gravitation, but simply a completion of Newton's law*.

2.1 Gravitational Index of Refraction

Intending to use the gravitational potential for ***light*** instead of mass (herein light is used as a generic term for electromagnetic waves), note that an optics principle excludes the last term between square brackets of (2), as odd with respect to $\vec{v}$, implying non reversibility, inadmissible for light, so that

$$U_{optic} = c^2 - U_N \left(1 + 3\frac{v^2}{c^2} \right). \tag{4}$$

As dimensionally the potential is a square velocity, we are suggested to put $U_{optic} = v^2$, that is,

$$v^2 = c^2 - U_N \left(1 + 3\frac{v^2}{c^2} \right), \tag{5}$$

whence, dividing both sides by c^2, replacing $c/v = n$, and solving the equation for $1/n^2$, one finally finds the index of refraction,

$$n = \sqrt{\frac{1 + 3U_N/c^2}{1 - U_N/c^2}} \, . \tag{6}$$

Use twice the binomial series $(1+\xi)^\nu = 1 + \nu\xi + [\nu(\nu-1)/2!]\xi^2 + [\nu(\nu-1)(\nu-2)/3!]\xi^3 + \ldots$, $\xi < 1$, neglecting the terms of powers $\nu \geq 2$: once for $\xi = -U_N/c^2$ and $\nu = -1$, obtaining $n \approx (1+4U_N/c^2)^{1/2}$, then for $\xi = 4U_N/c^2$ and $\nu = 1/2$, obtaining

$$n \approx 1 + 2U_N/c^2, \tag{6'}$$

just the gravitational index of refraction known from General Theory of Relativity, and therefore we call (6) the *relativistic approximation* of the gravitational index of refraction.

3 Two *galactic interdictions*: for mass, and for light

For a circular motion of an m mass celestial body around an M mass central body write the equality between centrifugal and centripetal (gravity) forces, $mv^2/r = mg$, where g is given by (1′),

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \left(1 + 3\frac{v^2}{c^2} \right) , \tag{7}$$

since $v_{\shortparallel} = 0$, and solve this equation for v^2,

$$v^2 = \frac{GM/r}{1 - 3GM/(rc^2)} \, , \tag{8}$$

whence some radical conclusions, of which the first two correspond to the problems in the article title:

1. **the rotation speed is greater than predicted by Newton's original law, never equal**, as the denominator in (8) is smaller than 1, so that the apparently strange behavior of the galactic *rotation curves* with respect to the original law of Newton is actually normal, and the "dark matter" hypothesis is no longer necessary; the (almost) Newtonian behavior in the solar system is due to the fact that $3GM/(rc^2) << 1$ in all cases (see subsection 3.1);

2. **for gravitational rotation around M the inequality $3GM/(rc^2) < 1$ must be fulfilled**, that is

$$\frac{M}{r} < \frac{c^2}{3G} \approx \frac{(3\times10^8)^2}{3\times6.674\times10^{-11}} \approx 4.5\times10^{26}, \tag{9}$$

which can be called the *galactic interdiction for mass*, because it imposes the spiral form (to massive core galaxies). A region/locus in which this interdiction is not fulfilled, that is, $M/r \geq c^2/(3G)$, is an orbiting interdicted locus that can be called a *mass captor*—and is located between spiral arms—as any mass cannot rotate, but is captured by M.

3. **in the locus/region in which $U_N/c^2 \geq 1$ not only that masses cannot orbit, but even light does not propagate, but is captured**, according to (6) (see also the figure), hence the name of *captor of light* or *black captor*—by analogy to the known *black hole* from General Theory of Relativity, having these two properties, but altogether differently defined. Note that (i) the *relativistic approximation* of the gravitational index of refraction (6′)—unlike the exact (6)—does no imply any *light/black captor*; (ii) the absence in these reasonings of something like a hole.

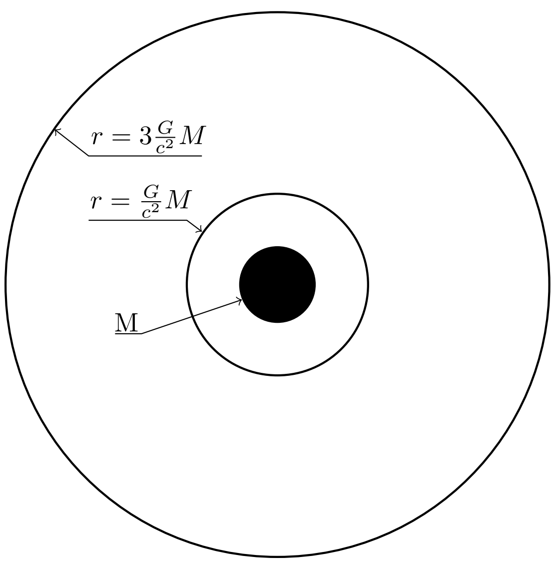

Figure 1: **The simplest galactic central structure inferred from Newton's gravity law RVD completed.** $r = 3\frac{G}{c^2}M$: frontier of *mass captor*; $r = \frac{G}{c^2}M$: frontier of *light captor*; M: frontier of mass M, depending on galaxy.

Let M_{core} and r_{core} be a mass and radius of its spheric distribution, such that M_{core}/r_{core} does not satisfy the *galactic interdiction* (9). As mentioned, the region between $r = r_{core}$ and r for which M_{core}/r satisfies (9) is a *mass captor*, i.e., an orbiting interdicted locus. Beyond this edge, masses (stars) can orbit with a compromise between the great speed (great kinetic energy) near the edge, and greater potential energy at greater distance. According as these masses increase by ΔM, the ratio $(M_{core}+\Delta M)/r$ increases making the place a mass captor, and so on, resulting a sequence of rings of stars, separated by rings as frontiers of *mass captors*. This is the simplest structure, but in general passages from one ring to its neighbor is done continuously, not by discrete steps (or leaps), thus rings are deformed into spirals.

A scenario for the process of formation of the central bar of barred spiral galaxies seems difficult to infer; this task waits for a more inspired confrère.

3.1 Galactic *mass interdiction* and solar system

Consider the SI system of measure units, and no other mention. In the solar system, every member is far from having *mass captor* (and *spiral form*) behavior. Indeed, according to (9), for such a behavior, as a *central* body:

> **Earth**, with its mass ($\approx 6 \times 10^{24}$), should have a radius of 13 millimeters; and with its radius ($\approx 6.38{\times}10^{6}$), should have a mass of $\approx 2.9{\times}10^{33}$.
>
> **Jupiter**, with its mass ($\approx 1.9{\times}10^{27}$) should have a radius of about 4.2 meters; and with its actual radius ($\approx 7{\times}10^{7}$) should have a mass of $\approx 3{\times}10^{34}$.
>
> **Sun** with the about 2×10^{30} mass, should have a radius of about 4.4 km; and with its radius ($\approx 7{\times}10^{8}$) should have a mass of about $3{\times}10^{35}$.

4 On atom radius

As Coulomb law (1785) is analogous to Newton law (1686)—electric charges instead of masses—

$$\vec{F}_C = \frac{1}{4\pi\varepsilon}\frac{Q\,q\,\vec{r}}{r^3}\,, \tag{10}$$

one expects the same RVD completion (supposing Nature not to play tricks),

$$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{Q\,q\,\vec{r}}{r^3}\left[1 + 3\frac{v^2}{c^2} + 4\left(\frac{v}{c}\right)^{\gamma}\frac{\dot{r}}{c}\right]. \tag{11}$$

and should see how the analogous of the interdiction/restriction (9) works on atom, that is: $m = m_e$, the mass of the electron; $q = q_e \approx 1.6\times10^{-19}$, the electric charge of the electron; and $Q = Nq_e$, where N is the number of protons in the nucleus. Write the analogue of (7)—equality between the centrifugal force and that centripetal (Coulombian RVD completed),

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\varepsilon}\frac{Nq_e^2}{r^2}\left(1 + 3\frac{v^2}{c^2}\right), \tag{12}$$

whence

$$v^2 = \frac{1}{4\pi\varepsilon}\frac{Nq_e^2/m_e}{r}\Big/\left(1 - \frac{3}{4\pi\varepsilon c^2}\frac{Nq_e^2/m_e}{r}\right), \tag{13}$$

whence the RVD interdiction for atom,

$$\frac{3}{4\pi\varepsilon c^2}\frac{Nq_e^2/m_e}{r} < 1\,, \tag{14}$$

that is,

$$r > \frac{3}{4\pi\varepsilon c^2}\frac{Nq_e^2}{m_e}\,. \tag{14$'$}$$

As $1/(4\pi\varepsilon) \approx 9\times10^9$, $c \approx 3\times10^8$, $q_e \approx 1.6\times10^{-19}$, and $m_e \approx 9.1{\times}10^{-31}$, the inequality/interdiction (14′) writes

$$r > N{\times}8.44{\times}10^{-16}. \tag{14$''$}$$

In case of Hydrogen, $N = 1$, hence $r > 8.44\times10^{-16}$, in agreement with the known value $r_{hydrogen} = 5.3\times10^{-11}$. This information appears as not quite interesting, but it lets us know in non-quantum way why such relatively great distances between nucleus and electrons. In fact, an atom nucleus is an electric captor of electrons (the known K-capture).

Rutherford and Bohr (1913) thought the atom configuration by analogy with the solar system, but now it turns out that a better analogy is with massive core galaxies: in the solar system, there is no enough-massive body to activate the RVD interdiction (9), as discussed in section 3.1, while the atom nucleus activates (15) in all cases.

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