Somsikov A.I.

Definition of the DIRECTION

One of the initial or primary concepts considered as the "simplest" (not expressed through other concepts) is considered. The structure of this concept is revealed. Algebraic and geometric consequences are found.

The position of some point on the line can be characterized by the distance from this point to some other point.

This distance is called the *coordinate*, the first point is the point of interest, the second is the reference point.

If we know the position of the point of interest on the line, its coordinate is determined unambiguously.

If, on the contrary, we know the coordinate of the point of interest, its position on the line is ambiguously defined, because there are two different points having the same coordinate value.

If you want to select the only point of interest from the two possible points (unambiguously set the position of the point of interest on the line), you need to specify one more condition, for example, the distance from the point of interest to some other point.

Indeed, entering a second reference point, shifted from the first reference point, immediately breaks the symmetry. In other words, one of the two possible points is closer to the second reference point and the other is further away.

Therefore, the setting of the coordinate of the point of interest relative to the second reference point at the same time as the setting of the coordinate of the point of interest relative to the first reference point quite unambiguously determines the position of the point of interest on the line.

As a result, the position of the point on the line is clearly defined by specifying two numbers, in other words, the line is the space of *two* dimensions.

Actually, it is not necessary to know the exact numerical value of the second (additional) coordinate.

It is enough to specify whether this value is greater or less than the value of the first coordinate, in other words, to set the inequality:

$$O_2A > O_1A \text{ or } O_2A < O_1A,$$

where O_1 is the first reference point, O_2 is the second reference point, A is the point of interest (point A with coordinate $O_1A > 0{,}5\ O_1O_2$).

The position of the reference points should be considered known. So, the equations:

$$O_1A = a,$$
$$O_2A = b,$$

or equality and inequality after eliminating redundant information:

$$O_1A = a,\ (O_1A > 0{,}5\ O_1O_2),\ O_2A > O_1A \text{ or } O_2A < O_1A,$$

unambiguously defines the position of the point of interest on the line.

In addition to eliminating redundant information, replacing the equality $O_2A = b$ with the inequality $O_2A > O_1A$ or $O_2A < O_1A$ gives another great advantage, namely: unlike equality, which can have countless values at different points A, the inequality can have only two values and therefore can be reduced to ">" and "<" symbols or any other pair of symbols, synonyms of this pair, for example, "+" and "-" regardless of the numerical value of the second coordinate.

The last update to the reduction of the record is that these symbols are entered into the first equality containing the coordinate:

$$O_1A = +\, a.$$

So, "+ *a*" should be understood as follows:

$$O_1A = a,$$
$$O_2A > O_1A,$$

In other words, one equality contains two conditions simultaneously.

The position of points A with the coordinate $O_1A < 0{,}5\ O_1O_2$ on the line without specifying the numeric value of the second (additional) coordinate remains undefined.

The approximation of the reference points O_1O_2 reduces the section on which it is necessary to specify the value of the second additional coordinate in extreme case for the endlessly placed reference points O_1 and O_2. The specified section tends to zero, and relative numbers can be considered as unambiguously determining the position of points of interest on the whole line.

So, the signs of relative numbers implicitly specify an additional coordinate. The terms "left", "right" or "direction" cannot be explained without introducing a second reference point. On the contrary, an explanation is achieved easily after such introduction, for example, "right" means the same as "+ *a*", namely:

$$O_1A = a,$$
$$O_2A > O_1A.$$

As a result, the "direction" is, like the "sign", an implicit additional coordinate.

The position of the point on the plane is unambiguously defined by specifying three numbers - distances up to three reference points not lying on a straight line (instead of four Cartesian coordinates, two explicit and two implicit).

The position of a point in Euclidean space is unambiguously defined by the setting of distances up to four reference points not lying in one plane, and each three of them should not lie on one line.

To sum up, the so-called three-dimensional Euclidean space, defined in projections by six Cartesian coordinates (three explicit and three implicit) is the space of *four* dimensions.