In this pdf there is my point of view about successions and series. I talk about when a succession (and so a serie) is said convergent making attention to the succession's definition.

Natural set is $\mathbb{N} = \{1, 2, 3, 4, 5, 6, ...\}$
We define a special sum between sets "+" where {1,4,3,2} "+" {1,3,2}={1,4,3,2,4+1,4+3,4+2}

A succession is a map $f : U \to \mathbb{R}$ with U=$\mathbb{N}$ or U=$\mathbb{N}$"+" ..."+" $\mathbb{N}$ where n$\in U \mapsto a_n$
A succession is said convergent if $f(U) \subsetneqq \mathbb{R}$.
The convergence of a succession depends by the domain of f. For example we take the succession {1,1,2,2,3,3,4,4,5,5,6,6,7,7,...}. If $f : \mathbb{N} \to \mathbb{R}$ (in this case we can assume $f : \mathbb{N} \to \mathbb{N}$) we obtain $a_n = \lceil \frac{n}{2} \rceil$ than for n> 1 $a_n < n$ so f($\mathbb{N}$) $\subsetneqq \mathbb{N}$ and the succession is convergent. Otherwise, if $f : \mathbb{N}``+" \mathbb{N} \to \mathbb{N}$ we obtain for $n \in \mathbb{N}$ and $s \in \mathbb{N}``+" \mathbb{N}$ $a_s = a_{2n} = \lceil \frac{2n}{2} \rceil = n$ than $f(\mathbb{N}``+" \mathbb{N}) = \mathbb{N}$ so the succession diverges.

The same is also true for the series. A serie is the sum of the terms of a succession.
A serie is convergente if the partial sum succession is convergent so the serie 1+0+1+0+1+0+... has {1,1,2,2,3,3,4,4,5,5,...} like a partial sum succession so if the index of the serie is in $\mathbb{N}$ the serie converges, but if index is in $\mathbb{N}``+" \mathbb{N}$ the serie diverges.

If there are mistakes in my reasoning, you will tell me on mary.v95@live.it please.