# Resolving Schrödinger's Cat, Wigner's Friend and Frauchiger-Renner's Paradoxes at a Single-Quantum Level

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Schrödinger's cat and Wigner's friend paradoxes are analyzed using the 'wave-particle non-dualistic interpretation of quantum mechanics at a single-quantum level' and are shown to be non-paradoxes within the quantum formalism. Then, the extended version of Wigner's friend thought experiment, proposed in a recent article titled, "*Quantum theory cannot consistently describe the use of itself*", Nature Communications **9**, 3711 (2018), by Frauchiger and Renner (FR) is considered. In quantum mechanics, it's well-known that, statistically observing a large number of identical quantum systems at some particular quantum state, which results in Born's probability, and merely inferring its presence in the same quantum state with the same probability yield distinct physical phenomena. If this fact is not taken care while interpreting any experimental outcomes, then FR type paradoxes pop up. '*What an astonishingly self-consistent the Quantum Theory is*!' - is explicitly worked out in the case of FR gedankenexperiment. The present work shows the importance of single-quantum phenomenon for the non-paradoxical interpretation of statistically observed experimental outcomes.

## 1. INTRODUCTION

Schrödinger's cat [1] and Wigner's friend [2] paradoxes are well-known and deeply studied gedankenexperiments in the literature of quantum mechanics. An extended version of Wigner's friend paradox was recently considered in great detail by Frauchiger and Renner (FR) and was analyzed using various interpretations of quantum mechanics [3]. In the present article, these three paradoxes are resolved using the 'wave-particle non-dualistic interpretation of quantum mechanics at a single-quantum level' (NI) [4–6], showing that the quantum formalism never admits any paradoxes and is applicable to physical systems of any scale.

## SCHRÖDINGER'S CAT AND WIGNER'S FRIEND PARADOXES

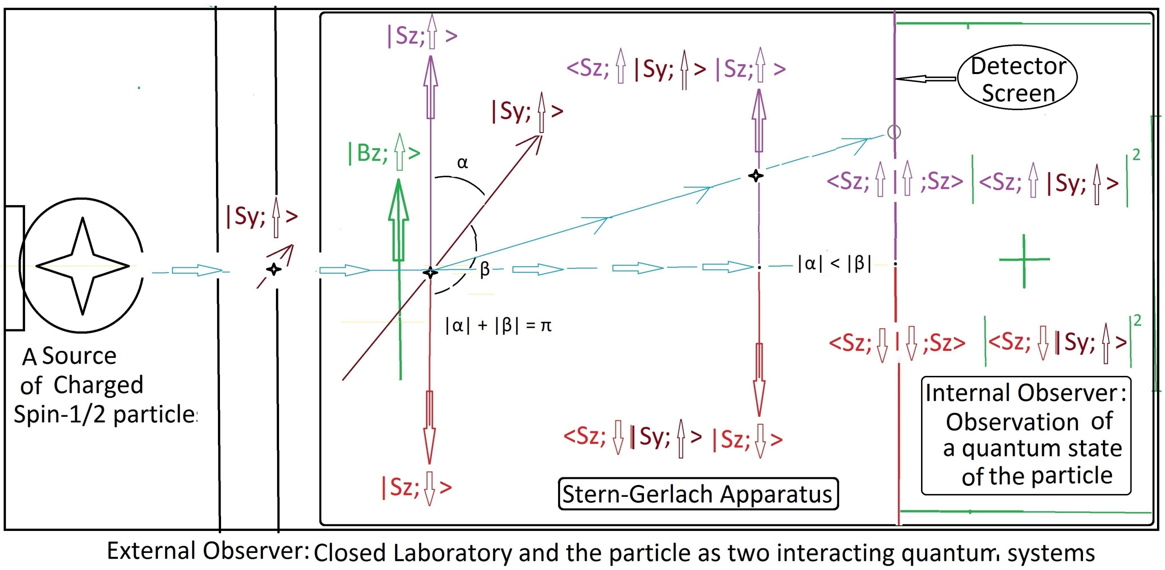


FIG. 1: **Gedankenexperiment Analogues to Schrödinger's Cat and Wigner's Friend Paradoxes**: A source emits a charged spin-$\frac{1}{2}$ particle whose initial state is prepared 'up along Y-axis', $|S_y;\uparrow>$. Then the particle is subjected to the Stern-Gerlach measurement along Z-axis. For the case of $|\alpha| < |\beta|$, the particle enters into $|S_z;\uparrow>$ and the ontological state, $|S_z;\downarrow>$, remains without a particle. During the observation, the particle contributes a point to $| < S_z;\uparrow |S_y;\uparrow> |^2$ while $|S_z;\downarrow>$ yields nothing. See the main text for the details of all mathematical symbols.

One of the spin eigenstate, $|S_y;\uparrow>$, of an observable $\hat{S}_y$ associated with a charged spin-$\frac{1}{2}$ particle - subjected to an observation by the Stren-Gerlach apparatus [7, 8] having the magnetic field direction along $Z$-axis ($SG_z$) [see FIG. 1] - can be taken instead of the radioactive atom in Schrödinger's thought experiment [1]. All the sequential events leading to the death or/and live states of the cat are dropped except the first two, i.e., preparation of the particle in the initial state $|S_y;\uparrow>$ and its detection by $SG_z$ either in $|S_z;\downarrow>$ or in $|S_z;\uparrow>$. The NI does not distinguish between the microscopic and macroscopic systems, since, they are all represented by the complex vector spaces (CVSs) of suitable observables, because, the observables of the particle and the detector must commute with each other in order to the detection to happen. Now, $|S_z;\downarrow>$ and $|S_z;\uparrow>$ represent the decayed and undecayed states of the radioactive atom, respectively, result in the $SG_z$ detecting the particle either in $|S_z;\downarrow>$ or in $|S_z;\uparrow>$, which represents the death or live states of the Cat.

According to the NI, the Schrödinger wave function (or in general, the simultaneous eigenstate of a maximally commuting set of observables) is interpreted as an 'instantaneous resonant spatial mode' in which a quantum moves akin to the case of a test particle in the curved space-time of the general theory of relativity. The unavoidable initial phase of the state vector is related to a particular eigenstate of the observable.

The unit operator, $\hat{I}_z$, in the CVS of the $SG_z$ can be written using the eigenstates of the observable $\hat{S}_z$, $|S_z;\uparrow>$ and $|S_z;\downarrow>$:

$$\hat{I}_z = |S_z;\uparrow>< S_z;\uparrow| + |S_z;\downarrow>< S_z;\downarrow|. \tag{1}$$

Therefore, at the moment $|S_y;\uparrow>$ encounters $SG_z$, it becomes a superposition of $|S_z;\downarrow>$ and $|S_z;\uparrow>$:

$$\begin{aligned} |S_y;\uparrow> &= |S_z;\uparrow>< S_z;\uparrow|S_y;\uparrow> + |S_z;\downarrow>< S_z;\downarrow|S_y;\uparrow> \\ &= |S_z;\uparrow>.|< S_z;\uparrow|S_y;\uparrow>|.e^{i\alpha} + |S_z;\downarrow>.|< S_z;\downarrow|S_y;\uparrow>|.e^{\beta} \\ &= \frac{1}{\sqrt{2}}e^{i\alpha}|S_z;\uparrow> + \frac{1}{\sqrt{2}}e^{\beta}|S_z;\downarrow>, \end{aligned} \tag{2}$$

where, $|< S_z;\uparrow|S_y;\uparrow>| = |< S_z;\downarrow|S_y;\uparrow>| = \frac{1}{\sqrt{2}}$ and $\alpha$ and $\beta$ are the phase angles between $|S_y;\uparrow>$ and $|S_z;\uparrow>$ and $|S_y;\uparrow>$ and $|S_z;\downarrow>$, respectively [6]. According to the 'principle of minimum phase' introduced in the NI, if $|\alpha| < |\beta|$ ($|\alpha| > |\beta|$), then the particle will be present in $|S_z;\uparrow>$ ($|S_z;\uparrow>$). The state $|S_y;\uparrow>$ induces a dual state at the screen and interacts according to the inner-product:

$$< S_y;\uparrow|S_y;\uparrow>= |< S_z;\uparrow|S_y;\uparrow>|^2 + |< S_z;\downarrow|S_y;\uparrow>|^2 \xrightarrow[|\alpha|<|\beta|]{\text{observation}} |< S_z;\uparrow|S_y;\uparrow>|^2 \tag{3}$$

yielding the eigenvalue $+\frac{1}{2}$; the particle itself contributes a point to $|< S_z;\uparrow|S_y;\uparrow>|^2$ [see FIG. 1]. Therefore, in a single quantum event, depending on the initial phase of $|S_y;\uparrow>$, $SG_z$ will detect the particle either in $|S_z;\uparrow>$ or $|S_z;\downarrow>$ [6].

Though during the time interval between the particle entering the magnetic field and hitting the detector screen, the state of the particle and hence the $SG_z$ apparatus is indeed described by the superposition state as given in Eq. (2), but, the particle and the $SG_z$ themselves will be in any one of the states but not in the both at the same time. Therefore, the conclusion is that the Schrödinger cat paradox doesn't exist in the quantum mechanics - according to the NI and the same can be said about Wigner's friend (WF) as well, which is analyzed in the following:

With respect to Wigner, the particle and the whole laboratory are like two interacting quantum systems. Let $|L; s>$ be the spin eigenstate of the observable, $\hat{L}$, associated with the lab such that the joint state can be written using Eq. (2) as,

$$|S_y;\uparrow> \otimes|L; s>= \frac{1}{\sqrt{2}}e^{i\alpha}|S_z;\uparrow> \otimes|L; s> + \frac{1}{\sqrt{2}}e^{\beta}|S_z;\downarrow> \otimes|L; s>, \tag{4}$$

where, $L$ stands for labeling the lab and $s$, its spin eigenvalue. Considering the case $|\alpha| < |\beta|$ resulting in the observation of particle in $|S_z;\uparrow>$ inside $L$ which is an interaction from outside $L$ for Wigner (FIG. 1):

$$|S_y;\uparrow> \otimes|L; s> \xrightarrow{\text{interaction}} \frac{1}{\sqrt{2}}e^{i\alpha}|S_z;\uparrow> |L; s'>, \tag{5}$$

which is subjected to the following 'total spin conservation' law:

$$(\hat{S}_y + \hat{L})(|S_y;\uparrow> \otimes|L; s>) = (\hat{S}_z + \hat{L}')|S_z;\uparrow> |L; s'>, \tag{6}$$

where, $|L; s'>$ is the resulting state of L after interaction, which is an eigenstate of $\hat{L}'$; $[\hat{L}\ ,\ \hat{L}'] \neq 0$. Since, WF observed only the eigenvalue of $\hat{S}_z$, one has $\hat{L}'|L; s'>= 0$:

$$(\hat{S}_z + \hat{L}')(|S_z;\uparrow> |L; s'>) = \hat{S}_z(|S_z;\uparrow> |L; s'>) = \frac{1}{2}\hbar(|L; s'> |S_z;\uparrow>). \tag{7}$$

Therefore, the combined system of the particle and the lab is also an eigenstate of Wigner's $\hat{S}_z$. Hence, there is no Wigner's friend paradox either.

Notice that, the NI statistically reproduces the standard Born's probabilistic description of quantum mechanics and hence naturally explains the observed 'cat states' [9]. As examples, see the reference [5], where, the entanglement swapping, both in space and time, are analyzed at individual quantum level which is in perfect agreement with the experimental findings.

## ANALYSIS OF FRAUCHIGER-RENNER'S PARADOX AT A SINGLE-QUANTUM LEVEL

Briefly, the FR gedankenexperiment is as follows: Consider two copies of Wigner's friend paradoxes, where, $\bar{W}$ and $W$ are Wigners and $\bar{F}$ and $F$ are their friends, respectively. $\bar{F}$

tosses a coin system, say $C$. Whenever the head was the outcome, then he prepares a spin system, say $S$, in a state $|\downarrow>$ and if tail was the outcome, then $S$ is prepared in the state $|\rightarrow>$. The initial state of $C$ is such that it yields $\frac{1}{3}$ probability for heads and $\frac{2}{3}$ for tails. $S$ is then sent to $F$, who measures it in his own basis. Similarly, $\bar{W}$ and $W$ make observations on $\bar{F}$ and $F$ in their own basis, respectively. The toss of coin is repeated over a large number of times to arrive at the probabilities. See reference [3] for more details.

Let $X$ be a generic agent whose observable is $\hat{X}_X$ with eigenstates $|u>$ and $|v>$ corresponding to the eigenvalues $u$ and $v$, respectively:

$$\hat{X}_X = u|u><u| + v|v><v|, \tag{8}$$

and having the unit operator,

$$\hat{I}_X = |u><u| + |v><v| = \hat{P}_u + \hat{P}_v, \tag{9}$$

where, $\hat{I}_X = (\hat{P}_u, \hat{P}_v)$; $\hat{P}_u = |u><u|$ and $\hat{P}_v = |v><v|$ are the projectors, respectively. Now, $X \in \{\bar{W}, \bar{F}, F, W\}$ and the measurement basis for Wigners and their friends are $\bar{W} = (-, +)$, $\bar{F} = (h, t)$, $F = (\downarrow, \uparrow)$ and $W = (\leftarrow, \rightarrow)$; $X = (u, v)$; where,

$$|\pm>= \frac{1}{\sqrt{2}}(|h> \pm |t>)\,;\, |\rightarrow>= \frac{1}{\sqrt{2}}(|\downarrow> + |\uparrow>)\,;\, |\leftarrow>= \frac{1}{\sqrt{2}}(|\downarrow> - |\uparrow>). \tag{10}$$

Notice that, as described earlier in the case of $SG_z$ apparatus, the NI allows one to write down both $C$ and $S$ as a single entangled state (this will be justified little later) as,

$$|\psi>>= \frac{1}{\sqrt{3}}|h>|\downarrow> + \sqrt{\frac{2}{3}}|t>|\rightarrow> \tag{11}$$

such that, when $X = \bar{F}$,

$$\hat{I}_X|\psi>>= (\hat{P}_u\ ,\ \hat{P}_v)|\psi>>= \left(\frac{1}{\sqrt{3}}|h>|\downarrow>\ ,\ \sqrt{\frac{2}{3}}|t>|\rightarrow>\right) \tag{12}$$

and

$$\hat{\bar{F}}_{\bar{F}}|\psi>>= \hat{\bar{F}}_{\bar{F}}(\hat{P}_u\ ,\ \hat{P}_v)|\psi>>= (h\ ,\ t)|\psi>>, \tag{13}$$

correctly produce the outcomes of $\bar{F}$, because, he will observe either $h$ or $t$ during a single quantum event. Note that, since, $C$ and $S$ are two independently prepared systems, there are no conservation laws governing the combined system and hence, Einstein's spooky action-at-a-distance will be absent here [5].

Let $\bar{t}_2$, $t_1$, $t_2$ and $t_3$ be the instances of time at which $\bar{W}$, $\bar{F}$, $F$ and $W$ make their observations, respectively; here, $\bar{t}_2 > t_1 < t_2 < t_3$. The four statements given in the reference [3] are rewritten below for the clarity, following the notations in the present article:

**Statement 1 by $\bar{F}$:** "If I get $|t>$, then I am certain that $W$ will observe $|\rightarrow>$"

**Statement 2 by $F$:** "If I get $|\uparrow>$ , then I am certain that $\bar{F}$ had observed $|t>$"

**Statement 3 by $\bar{W}$:** "If I get $|->$, then I am certain that $F$ had observed $|\uparrow>$"

**Statement 4 by $W$:** "If I get $<\leftarrow|$, I know that there exists one round of the experiment in which $\bar{W}$ also gets $|->$".

Before proceeding to analyze the above statements using the NI, it's useful to note the following facts:

1. As already shown earlier, there is no Schrödinger's cat paradox at the level of a single-quantum's single event.

   If $\bar{W}$ uses the same basis as $\bar{F}$, then both Schrödinger's cat and Wigner's friend paradoxes are one and the same.

2. When $\bar{W}$ uses a different measurement basis and treats $\bar{F}$'s state as a superposition, he knows that $\bar{F}$ himself will be present in a particular eigenstate which will result during the observation, though he can't predict it with certainty. This situation of $\bar{W}$ is the same as that of $\bar{F}$, who can't predicted the exact outcome of the toss of a coin.

3. Let $\bar{W} = \bar{F}$ and $F = W$. Then $\bar{F}$ can predict the outcome of $F$ as $|\downarrow>$ if his observed eigenvalue is $h$ and $F$ can infer the observed eigenstate of $\bar{F}$ as $|t>$ if his measurement results in the eigenvalue $\frac{1}{2}$. Since, $\bar{F}$'s prediction and $F$'s inference do not commute, this FR system doesn't carry any information, i.e., it's useless for the transfer of information.

   This fundamental drawback will be further amplified if $\bar{W}$ and $W$ use the measurement basis different from $\bar{F}$ and $F$, respectively, i.e.,

   (a) The certainty of $\bar{F}$ in predicting the definite outcome of $F$ can't be extended to that of $W$.

   (b) The inference of $F$ can't be extrapolated to $\bar{W}$ and vice versa.

See TABLE I describing the propagation of a single-quantum event observed by the sequential observers, $\bar{W}$, $\bar{F}$, $F$ and $W$ in the time sequence, $\bar{t}_2 > t_1 < t_2 < t_3$, respectively; here, $\bar{t}_2$ doesn't have to be necessarily lie in the interval $(t_1, t_3]$.

As an example, consider $\bar{F}$'s measurement at $t_1$. If $h$ is the outcome, then $\bar{F}$ himself enters into a state of both $|h>$ and $|\downarrow>$ which is available for $\bar{W}$'s and $F$'s measurements, respectively. Since, $[\hat{\bar{W}}_{\bar{W}}\ ,\ \hat{F}_F] = 0$, both $\bar{W}$ and $F$ can make simultaneous measurements on $\bar{F}$ after his measurement. It's important to note that, similar to the observation of $h$, which throws $\bar{F}$ into $|h>$, the preparation of $|\downarrow>$ also puts him into the same spin state. Therefore, whether $F$ observes $S$ or $\bar{F}$ are all one and the same.

$\bar{W}$'s measurement at $\bar{t}_2$ can yield an outcome of either $|+>$ or $|->$, resulting in a state given at S.No. $a_1$ or $a_3$, respectively, with relative frequency of detection (probability) $\frac{1}{6}$ for each state.

Later measurement of $W$ on the resulting state of $F$ will have no effect on the outcome of $\bar{W}$, since, as already mentioned, $|\psi>>$ does not exhibit Einstein's spooky action-at-a-distance. $W$ can observe $F$ either in $|\rightarrow>$ or $|\leftarrow>$ with probability $\frac{1}{6}$ for each one. Similarly, all other outcomes are listed in S.No. $b_1$ to S.No. $b_7$.

**Statement 1 by $\bar{F}$: TABLE II**

<u>Statement 1 by $\bar{F}$</u>: "If I get $|t>$, then I am certain that $W$ will observe $|\rightarrow>$".

From the Table I & II, it can be seen that if S.No. $=\sum_{i=1}^{7} b_i$, then $|\rightarrow>$ becomes a definite outcome for W because, the contribution at $|\leftarrow>$ is zero. But, this necessarily implies no measurement by $F$ at $t_2$. In this case, $\bar{W}'s$ measurement can yield either $|+>$ or $|->$ with probability $\frac{1}{3}$ for each one, irrespective of whether $\bar{t}_2 > t_3$ or $\bar{t}_2 < t_3$.

**Statement 2 by $F$: TABLE III**

<u>Statement 2 by $F$</u>: "If I get $|\uparrow>$ , then I am certain that $\bar{F}$ had observed $|t>$"- implying S.No. $=\sum_{i=1}^{7} b_i$ of TABLE III.

The statement is is true at a single-quantum's single event, but, contradicts the 1st statement by $\bar{F}$ about $W$, because, $W$'s outcome now will be either $|\rightarrow>$ or $|\leftarrow>$ with a probability $\frac{1}{6}$ for each one.

TABLE I: Propagation of a single-quantum event observed by the sequential observers, $\bar{W}$, $\bar{F}$, $F$ and $W$ in the time sequence, $\bar{t}_2 > t_1 < t_2 < t_3$, respectively. $\bar{t}_2$ need not be necessarily lie in the interval $(t_1, t_3]$.

| S. No. | $\bar{W}'s$ Measurement at $\bar{t}_2$ | $\bar{W}$'s $\hat{I}_{\bar{W}}$ | $\bar{F}$'s $\hat{I}_{\bar{F}}$ | $\bar{F}$'s Measurement at $t_1$ | $F$'s $\hat{I}_F$ | $F$'s Measurement at $t_2$ | $W$'s $\hat{I}_W$ | $W$'s Measurement at $t_3$ |
|---|---|---|---|---|---|---|---|---|
| $a_1$ | $\frac{1}{\sqrt{6}}\|+>\|\downarrow>$ | $\hat{P}_+$ | | | | | $\hat{P}_\rightarrow$ | $\frac{1}{\sqrt{6}}\|h>\|\rightarrow>$ |
| $a_2$ | | | $\hat{P}_h$ | $\frac{1}{\sqrt{3}}\|h>\|\downarrow>$ | $\hat{P}_\downarrow$ | $\frac{1}{\sqrt{3}}\|h>\|\downarrow>$ | | |
| $a_3$ | $\frac{1}{\sqrt{6}}\|->\|\downarrow>$ | $\hat{P}_-$ | | | | | $\hat{P}_\leftarrow$ | $\frac{1}{\sqrt{6}}\|h>\|\leftarrow>$ |
| $b_1$ | | | | | | | $\hat{P}_\rightarrow$ | $\frac{1}{\sqrt{6}}\|t>\|\rightarrow>$ |
| $b_2$ | $\frac{1}{\sqrt{3}}\|+>\|\rightarrow>$ | $\hat{P}_+$ | | | $\hat{P}_\downarrow$ | $\frac{1}{\sqrt{3}}\|t>\|\downarrow>$ | | |
| $b_3$ | | | | | | | $\hat{P}_\leftarrow$ | $\frac{1}{\sqrt{6}}\|t>\|\leftarrow>$ |
| $b_4$ | | | $\hat{P}_t$ | $\sqrt{\frac{2}{3}}\|t>\|\rightarrow>$ | | | | |
| $b_5$ | | | | | | | $\hat{P}_\rightarrow$ | $\frac{1}{\sqrt{6}}\|t>\|\rightarrow>$ |
| $b_6$ | $-\frac{1}{\sqrt{3}}\|->\|\rightarrow>$ | $\hat{P}_-$ | | | $\hat{P}_\uparrow$ | $\frac{1}{\sqrt{3}}\|t>\|\uparrow>$ | | |
| $b_7$ | | | | | | | $\hat{P}_\leftarrow$ | $-\frac{1}{\sqrt{6}}\|t>\|\leftarrow>$ |

TABLE II: Statement 1 by $\bar{F}$ : "If I get $|t>$, then I am certain that $W$ will observe $|\rightarrow>$"; (provided, $F$ didn't make any observation).

| S. No. | $\bar{W}'s$ Measurement at $\bar{t}_2$ | $\bar{W}$'s $\hat{I}_{\bar{W}}$ | $\bar{F}$'s $\hat{I}_{\bar{F}}$ | $\bar{F}$'s Measurement at $t_1$ | $F$'s $\hat{I}_F$ | No Measurement by $F$ at $t_2$ | $W$'s $\hat{I}_W$ | $W$'s Measurement at $t_3$ |
|---|---|---|---|---|---|---|---|---|
| | $\frac{1}{\sqrt{3}}\|+>\|\rightarrow>$ | $\hat{P}_+$ | | | $\hat{P}_\downarrow$ | $\frac{1}{\sqrt{3}}\|t>\|\downarrow>$ | $\hat{P}_\rightarrow$ | $\sqrt{\frac{2}{3}}\|t>\|\rightarrow>$ |
| $\sum_{i=1}^{7} b_i$ | | | $\hat{P}_t$ | $\sqrt{\frac{2}{3}}\|t>\|\rightarrow>$ | + | + | | |
| | $-\frac{1}{\sqrt{3}}\|->\|\rightarrow>$ | $\hat{P}_-$ | | | $\hat{P}_\uparrow$ | $\frac{1}{\sqrt{3}}\|t>\|\uparrow>$ | $\hat{P}_\leftarrow$ | 0 |

In this case, $\bar{W}'s$ measurement can yield either $|+>$ or $|->$ with probability $\frac{1}{3}$ for each one, irrespective of whether $\bar{t}_2 > t_3$ or $\bar{t}_2 < t_3$ akin to the situation in 1st statement.

TABLE III: Statement 2 by $F$ : "If I get $|\uparrow>$ , then I am certain that $\bar{F}$ had observed $|t>$"; true at a single event level, but, makes the Statement 1 of $\bar{F}$ about $W$ as FALSE.

| S. No. | $\bar{W}'s$ Measurement at $\bar{t}_2$ | $\bar{W}$'s $\hat{I}_{\bar{W}}$ | $\bar{F}$'s $\hat{I}_{\bar{F}}$ | $\bar{F}$'s Measurement at $t_1$ | $F$'s $\hat{I}_F$ | $F$'s Measurement at $t_2$ | $W$'s $\hat{I}_W$ | $W$'s Measurement at $t_3$ |
|---|---|---|---|---|---|---|---|---|
| $b_1$ | | | | | | | $\hat{P}_{\rightarrow}$ | 0 |
| $b_2$ | $\frac{1}{\sqrt{3}}\|+>\|\rightarrow>$ | $\hat{P}_+$ | | | $\hat{P}_{\downarrow}$ | 0 | | |
| $b_3$ | | | | | | | $\hat{P}_{\leftarrow}$ | 0 |
| $b_4$ | | | $\hat{P}_t$ | $\sqrt{\frac{2}{3}}\|t>\|\rightarrow>$ | | | | |
| $b_5$ | | | | | | | $\hat{P}_{\rightarrow}$ | $\frac{1}{\sqrt{6}}\|t>\|\rightarrow>$ |
| $b_6$ | $-\frac{1}{\sqrt{3}}\|->\|\rightarrow>$ | $\hat{P}_-$ | | | $\hat{P}_{\uparrow}$ | $\frac{1}{\sqrt{3}}\|t>\|\uparrow>$ | | |
| $b_7$ | | | | | | | $\hat{P}_{\leftarrow}$ | $-\frac{1}{\sqrt{6}}\|t>\|\leftarrow>$ |

**Statement 3 by $\bar{W}$: TABLE IV**

Statement 3 by $\bar{W}$: "If I get $|->$, then I am certain that $F$ had observed $|\uparrow>$".

As it can be seen from TABLE I, this happens if and only if S.No. $= \sum_{i=1}^{3} a_i + \sum_{i=1}^{7} b_i$ (see TABLE IV - (A)). Since, the system is observed in $|->$, it will not be present in $|+>$, resulting in the state $-\frac{1}{\sqrt{6}}|->|\uparrow>$ for $F$, provided $\bar{F}$ didn't make any observation on $|\psi>>$, which is a non-existent situation (see TABLE IV - (B)). Now, $W$ can find $F$ in either $|\rightarrow>$ or $|\leftarrow>$ with probability $\frac{1}{12}$ for each one. But, now, $F$ can't infer the outcome of $\bar{F}$ as $|t>$ at time $t_1$ - contradicting the statement 2. One way of making the statement to be true is by interpreting the state of $\bar{F}$ to be in the superposition $|\psi>>$, while $\bar{F}$ himself is in the particular state, $-\frac{1}{\sqrt{6}}|->|\uparrow>$, and the remaining part of $|\psi>>$, i.e., $|+>(\sqrt{\frac{2}{3}}|\downarrow> + \frac{1}{\sqrt{6}}|\uparrow>)$ is an ontological empty mode - though this is in agreement with the NI, but such a situation is excluded in FR's experiment.

**Statement 4 by $W$: TABLE V**

Statement 4 by $W$: "If I get $<\leftarrow|$, I know that there exists one round of the experiment in which $\bar{W}$ also gets $|->$".

This statement is true as shown in TABLE V, but, contradicts the Statement 1, because,

TABLE IV: Statement 3 by $\bar{W}$: "If I get $\vert - >$, then I am certain that $F$ had observed $\vert\uparrow>$" - using this, $\bar{W}$ can't conclude that $\bar{F}$ was in $\vert t>$ (Statement 2) which again results in $W$ being in $\vert\rightarrow>$ (Statement 1), because, in this case, $W$ will be in either $\vert\rightarrow>$ or $\vert\leftarrow>$ with probability $\frac{1}{12}$ for each one.

| S. No. | $\bar{W}$'s Measurement at $\bar{t}_2$ | $\bar{W}$'s $\hat{I}_{\bar{W}}$ | $\bar{F}$'s $\hat{I}_{\bar{F}}$ | No Measurement by $\bar{F}$ | $F$'s $\hat{I}_F$ | $F$'s Measurement at $t_2$ | $W$ 's $\hat{I}_W$ | $W$'s Measurement at $t_3$ |
|---|---|---|---|---|---|---|---|---|
| (A) | (A) | (A) | (A) | (A) | (A) | (A) | (A) | (A) |
| $\sum_{i=1}^{3} a_i$ | $\sqrt{\frac{2}{3}}\vert + >\vert\downarrow>$ + $\frac{1}{\sqrt{6}}\vert + >\vert\uparrow>$ | $\hat{P}_+$ | $\hat{P}_h$ | $\frac{1}{\sqrt{3}}\vert h>\vert\downarrow>$ | $\hat{P}_\downarrow$ | $\sqrt{\frac{2}{3}}\vert + >\vert\downarrow>$ | $\hat{P}_\rightarrow$ | $\frac{\sqrt{3}}{2}\vert + >\vert\rightarrow>$ − $\frac{1}{\sqrt{12}}\vert - >\vert\rightarrow>$ |
| + | | | + | + | | | | |
| $\sum_{i=1}^{7} b_i$ | $-\frac{1}{\sqrt{6}}\vert - >\vert\uparrow>$ | $\hat{P}_-$ | $\hat{P}_t$ | $\sqrt{\frac{2}{3}}\vert t>\vert\rightarrow>$ | $\hat{P}_\uparrow$ | $\frac{1}{\sqrt{6}}\vert + >\vert\uparrow>$ − $\frac{1}{\sqrt{6}}\vert - >\vert\uparrow>$ | $\hat{P}_\leftarrow$ | $\frac{1}{\sqrt{3}}\vert + >\vert\leftarrow>$ |
| (B) | (B) | (B) | (B) | (B) | (B) | (B) | (B) | (B) |
| $\sum_{i=1}^{3} a_i$ | 0 + 0 | $\hat{P}_+$ | $\hat{P}_h$ | $\frac{1}{\sqrt{3}}\vert h>\vert\downarrow>$ | $\hat{P}_\downarrow$ | 0 | $\hat{P}_\rightarrow$ | 0 − $\frac{1}{\sqrt{12}}\vert - >\vert\rightarrow>$ |
| + | | | + | + | | | | |
| $\sum_{i=1}^{7} b_i$ | $-\frac{1}{\sqrt{6}}\vert - >\vert\uparrow>$ | $\hat{P}_-$ | $\hat{P}_t$ | $\sqrt{\frac{2}{3}}\vert t>\vert\rightarrow>$ | $\hat{P}_\uparrow$ | 0 − $\frac{1}{\sqrt{6}}\vert - >\vert\uparrow>$ | $\hat{P}_\leftarrow$ | $\frac{1}{\sqrt{12}}\vert - >\vert\leftarrow>$ |

$\bar{F}$ will become uncertain about the outcome of $W$ since there is a probability of $\frac{1}{6}$ to get $\vert\leftarrow>$ as well.

TABLE V: Statement 4 by $W$ : If I get $<\leftarrow|$, I know that there exists one round of the experiment in which $\bar{W}$ also gets $|->$.

| S. No. | $\bar{W}$'s Measurement at $\bar{t}_2$ | $\bar{W}$'s $\hat{I}_{\bar{W}}$ | $\bar{F}$'s $\hat{I}_{\bar{F}}$ | 1st Measurement at $t_1$ | $F$'s $\hat{I}_F$ | 2nd Measurement at $t_2$ | $W$'s $\hat{I}_W$ | 3rd Measurement at $t_3$ |
|---|---|---|---|---|---|---|---|---|
| $a_1$ | $\frac{1}{\sqrt{12}}\|+>\|\rightarrow> + \frac{1}{\sqrt{12}}\|+>\|\leftarrow>$ | $\hat{P}_+$ | | | | | $\hat{P}_\rightarrow$ | $\frac{1}{\sqrt{12}}\|+>\|\rightarrow> + \frac{1}{\sqrt{12}}\|->\|\rightarrow>$ |
| $a_2$ | | | $\hat{P}_h$ | $\frac{1}{\sqrt{3}}\|h>\|\downarrow>$ | $\hat{P}_\downarrow$ | $\frac{1}{\sqrt{3}}\|h>\|\downarrow>$ | | |
| $a_3$ | $\frac{1}{\sqrt{12}}\|->\|\rightarrow> + \frac{1}{\sqrt{12}}\|->\|\leftarrow>$ | $\hat{P}_-$ | | | | | $\hat{P}_\leftarrow$ | $\frac{1}{\sqrt{12}}\|+>\|\leftarrow> + \frac{1}{\sqrt{12}}\|->\|\leftarrow>$ |

## CONCLUSIONS

Schrödinger's cat and Wigner's friend paradoxes are analyzed using the 'wave-particle non-dualistic intrepretaion of quantum mechanics at a single quantum level' and are shown to be non-paradoxes. These results are then used to study Frauchiger-Renner's paradox and found that the four statements given in their article [3] are either mutually or self contradictory ones, which themselves are the actual paradoxes, but, not the Quantum Mechanics.

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