

Formula of ζ odd numbers

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Abstract

Three new ζ odd number formulas were built.
And up to $\zeta(333)$, using that formula.
Using this, it may be possible to prove irrationality of ζ odd number.

key words

ζ odd-number formula, $\zeta(1)$, $\zeta(333)$

1 Introduction

Write the formula I finally got in advance.

$$\zeta(2m+1) = \frac{(2^{2m+1} - 4)}{(2^{2m+1} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{2m-1}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}} \quad (1)$$

m is a positive integer.

and

$$\zeta(2m+1) = \zeta(2m-1) \frac{(2^{2m+1} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}}} \quad (2)$$

m is a positive integer.

and

Eq.(1) and Eq.(2) are equation these are modification of Eq.(3).

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1} - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \quad (3)$$

m is a positive integer.

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In detail

$$\zeta(1) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1} = \frac{1}{2^1} \sum_{n=1}^{\infty} \frac{1}{n^1} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = \frac{1}{2^1} \zeta(1) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (4)$$

$$\zeta(1) = \frac{2^1}{2^1 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (5)$$

Use these for $\zeta(3), \zeta(5), \zeta(7)$ etc.

2 Discussion

Example 1

from Eq.(3)

if m=1

$$\zeta(1) = \frac{2^1}{2^1 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (6)$$

if m=2

$$\zeta(3) = \frac{2^3}{2^3 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (7)$$

Multiply $\zeta(1)$ and $\zeta(3)$

$$\zeta(3) 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = \zeta(1) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (8)$$

$$2\zeta(3) = \frac{8}{7} \zeta(1) \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \quad (9)$$

$$\zeta(3) = \frac{4}{7} \left[\frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{4}{7} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^1}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (10)$$

and

from Eq.(3)

if m=2

$$\zeta(3) = \frac{2^3}{2^3 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (11)$$

if $m=3$

$$\zeta(5) = \frac{2^5}{2^5 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (12)$$

Multiply $\zeta(3)$ and $\zeta(5)$.

$$\zeta(5) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \zeta(3) \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (13)$$

$$\zeta(5) = \zeta(3) \frac{7}{8} \frac{32}{31} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (14)$$

$$\zeta(5) = \zeta(3) \frac{28}{31} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{28}{31} \zeta(3) \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (15)$$

$$= \frac{28}{31} \left[\frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{28}{31} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (16)$$

The new formula Eq.(1) has been followed.

Do the same for $\zeta(7), \zeta(9), \zeta(11)$ etc.

$$\zeta(3) = \zeta(1) \frac{(2^3 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{(2^3 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \frac{(2^3 - 4)}{(2^3 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^3}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (17)$$

$$\zeta(5) = \zeta(3) \frac{(2^5 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{(2^5 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \frac{(2^5 - 4)}{(2^5 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (18)$$

$$\zeta(7) = \zeta(5) \frac{(2^7 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{(2^7 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} = \frac{(2^7 - 4)}{(2^7 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^7}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (19)$$

$$\zeta(9) = \zeta(7) \frac{(2^9 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{(2^9 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}} = \frac{(2^9 - 4)}{(2^9 - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^9}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (20)$$

$$\zeta(11) = \zeta(9) \frac{(2^{11} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{(2^{11} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}} = \frac{(2^{11} - 4)}{(2^{11} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{11}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (21)$$

$$\zeta(13) = \zeta(11) \frac{(2^{13} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{(2^{13} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}} = \frac{(2^{13} - 4)}{(2^{13} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{13}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (22)$$

$$\zeta(15) = \zeta(13) \frac{(2^{15} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{(2^{15} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}} = \frac{(2^{15} - 4)}{(2^{15} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{13}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (23)$$

$$\zeta(17) = \zeta(15) \frac{(2^{17} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{(2^{17} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}} = \frac{(2^{17} - 4)}{(2^{17} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{15}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (24)$$

$$\zeta(19) = \zeta(17) \frac{(2^{19} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{(2^{19} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}} = \frac{(2^{19} - 4)}{(2^{19} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{17}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (25)$$

$$\zeta(21) = \zeta(19) \frac{(2^{21} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{(2^{21} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}} = \frac{(2^{21} - 4)}{(2^{21} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{19}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}} \quad (26)$$

$$\zeta(23) = \zeta(21) \frac{(2^{23} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{(2^{23} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}} = \frac{(2^{23} - 4)}{(2^{23} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{21}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}} \quad (27)$$

$$\zeta(25) = \zeta(23) \frac{(2^{25} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{(2^{25} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}} = \frac{(2^{25} - 4)}{(2^{25} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{23}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}} \quad (28)$$

$$\zeta(27) = \zeta(25) \frac{(2^{27} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{(2^{27} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}} = \frac{(2^{27} - 4)}{(2^{27} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{25}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}} \quad (29)$$

$$\zeta(29) = \zeta(27) \frac{(2^{29} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{(2^{29} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}} = \frac{(2^{29} - 4)}{(2^{29} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{27}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}} \quad (30)$$

$$\zeta(31) = \zeta(29) \frac{(2^{31} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{(2^{31} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}} = \frac{(2^{31} - 4)}{(2^{31} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{29}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}} \quad (31)$$

$$\zeta(33) = \zeta(31) \frac{(2^{33} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{(2^{33} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}} = \frac{(2^{33} - 4)}{(2^{33} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{31}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}} \quad (32)$$

$$\zeta(35) = \zeta(33) \frac{(2^{35} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{(2^{35} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}} = \frac{(2^{35} - 4)}{(2^{35} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{33}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}} \quad (33)$$

$$\zeta(37) = \zeta(35) \frac{(2^{37} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{(2^{37} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}} = \frac{(2^{37} - 4)}{(2^{37} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{35}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}} \quad (34)$$

$$\zeta(39) = \zeta(37) \frac{(2^{39} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{(2^{39} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}} = \frac{(2^{39} - 4)}{(2^{39} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{37}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}} \quad (35)$$

$$\zeta(41) = \zeta(39) \frac{(2^{41} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{(2^{41} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}} = \frac{(2^{41} - 4)}{(2^{41} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{39}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}} \quad (36)$$

$$\zeta(43) = \zeta(41) \frac{(2^{43} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{(2^{43} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}} = \frac{(2^{43} - 4)}{(2^{43} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{41}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}} \quad (37)$$

$$\zeta(45) = \zeta(43) \frac{(2^{45} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{(2^{45} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}} = \frac{(2^{45} - 4)}{(2^{45} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{43}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}} \quad (38)$$

$$\zeta(47) = \zeta(45) \frac{(2^{47} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{(2^{47} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}} = \frac{(2^{47} - 4)}{(2^{47} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{45}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}} \quad (39)$$

$$\zeta(49) = \zeta(47) \frac{(2^{49} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{(2^{49} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}} = \frac{(2^{49} - 4)}{(2^{49} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{47}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}} \quad (40)$$

$$\zeta(51) = \zeta(49) \frac{(2^{51} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{(2^{51} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}} = \frac{(2^{51} - 4)}{(2^{51} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{49}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}} \quad (41)$$

$$\zeta(53) = \zeta(51) \frac{(2^{53} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{(2^{53} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}} = \frac{(2^{53} - 4)}{(2^{53} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{51}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}} \quad (42)$$

$$\zeta(55) = \zeta(53) \frac{(2^{55} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{(2^{55} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}} = \frac{(2^{55} - 4)}{(2^{55} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{53}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}} \quad (43)$$

$$\zeta(57) = \zeta(55) \frac{(2^{57} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{(2^{57} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}} = \frac{(2^{57} - 4)}{(2^{57} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{55}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}} \quad (44)$$

$$\zeta(59) = \zeta(57) \frac{(2^{59} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{(2^{59} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}} = \frac{(2^{59} - 4)}{(2^{59} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{57}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}} \quad (45)$$

$$\zeta(61) = \zeta(59) \frac{(2^{61} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{(2^{61} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}} = \frac{(2^{61} - 4)}{(2^{61} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{59}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}} \quad (46)$$

$$\zeta(63) = \zeta(61) \frac{(2^{63} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{(2^{63} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}} = \frac{(2^{63} - 4)}{(2^{63} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{61}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}} \quad (47)$$

$$\zeta(65) = \zeta(63) \frac{(2^{65} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{(2^{65} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}} = \frac{(2^{65} - 4)}{(2^{65} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{63}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}} \quad (48)$$

$$\zeta(67) = \zeta(65) \frac{(2^{67} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{(2^{67} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}} = \frac{(2^{67} - 4)}{(2^{67} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{65}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}} \quad (49)$$

$$\zeta(69) = \zeta(67) \frac{(2^{69} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{(2^{69} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}} = \frac{(2^{69} - 4)}{(2^{69} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{67}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}} \quad (50)$$

$$\zeta(71) = \zeta(69) \frac{(2^{71} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{(2^{71} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}} = \frac{(2^{71} - 4)}{(2^{71} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{69}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}} \quad (51)$$

$$\zeta(73) = \zeta(71) \frac{(2^{73} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{(2^{73} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}} = \frac{(2^{73} - 4)}{(2^{73} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{71}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}} \quad (52)$$

$$\zeta(75) = \zeta(73) \frac{(2^{75} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{(2^{75} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}} = \frac{(2^{75} - 4)}{(2^{75} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{73}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}} \quad (53)$$

$$\zeta(77) = \zeta(75) \frac{(2^{77} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{(2^{77} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}} = \frac{(2^{77} - 4)}{(2^{77} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{75}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}} \quad (54)$$

$$\zeta(79) = \zeta(77) \frac{(2^{79} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{(2^{79} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}} = \frac{(2^{79} - 4)}{(2^{79} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{77}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}} \quad (55)$$

$$\zeta(81) = \zeta(79) \frac{(2^{81} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{(2^{81} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}} = \frac{(2^{81} - 4)}{(2^{81} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{79}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}} \quad (56)$$

$$\zeta(83) = \zeta(81) \frac{(2^{83} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{(2^{83} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}} = \frac{(2^{83} - 4)}{(2^{83} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{81}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}} \quad (57)$$

$$\zeta(85) = \zeta(83) \frac{(2^{85} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{(2^{85} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}} = \frac{(2^{85} - 4)}{(2^{85} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{83}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}} \quad (58)$$

$$\zeta(87) = \zeta(85) \frac{(2^{87} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{(2^{87} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}} = \frac{(2^{87} - 4)}{(2^{87} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{85}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}} \quad (59)$$

$$\zeta(89) = \zeta(87) \frac{(2^{89} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{(2^{89} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}} = \frac{(2^{89} - 4)}{(2^{89} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{87}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}} \quad (60)$$

$$\zeta(91) = \zeta(89) \frac{(2^{91} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{(2^{91} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}} = \frac{(2^{91} - 4)}{(2^{91} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{89}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}} \quad (61)$$

$$\zeta(93) = \zeta(91) \frac{(2^{93} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{(2^{93} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}} = \frac{(2^{93} - 4)}{(2^{93} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{91}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}} \quad (62)$$

$$\zeta(95) = \zeta(93) \frac{(2^{95} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{(2^{95} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}} = \frac{(2^{95} - 4)}{(2^{95} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{93}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}} \quad (63)$$

$$\zeta(97) = \zeta(95) \frac{(2^{97} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{(2^{97} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}} = \frac{(2^{97} - 4)}{(2^{97} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{95}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}} \quad (64)$$

$$\zeta(99) = \zeta(97) \frac{(2^{99} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{(2^{99} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}} = \frac{(2^{99} - 4)}{(2^{99} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{97}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}} \quad (65)$$

$$\zeta(101) = \zeta(99) \frac{(2^{101} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{(2^{101} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}} = \frac{(2^{101} - 4)}{(2^{101} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{99}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}} \quad (66)$$

$$\zeta(103) = \zeta(101) \frac{(2^{103} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}}{(2^{103} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}} = \frac{(2^{103} - 4)}{(2^{103} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{101}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}} \quad (67)$$

$$\zeta(105) = \zeta(103) \frac{(2^{105} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}}{(2^{105} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}} = \frac{(2^{105} - 4)}{(2^{105} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{103}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}} \quad (68)$$

$$\zeta(107) = \zeta(105) \frac{(2^{107} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}}{(2^{107} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}} = \frac{(2^{107} - 4)}{(2^{107} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{105}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}} \quad (69)$$

$$\zeta(109) = \zeta(107) \frac{(2^{109} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}}{(2^{109} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}} = \frac{(2^{109} - 4)}{(2^{109} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{107}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}} \quad (70)$$

$$\zeta(135) = \zeta(133) \frac{(2^{135} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{(2^{135} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}} = \frac{(2^{135} - 4)}{(2^{135} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{133}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{133}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}} \quad (83)$$

$$\zeta(137) = \zeta(135) \frac{(2^{137} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{(2^{137} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}} = \frac{(2^{137} - 4)}{(2^{137} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{135}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}} \quad (84)$$

$$\zeta(139) = \zeta(137) \frac{(2^{139} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{(2^{139} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}} = \frac{(2^{139} - 4)}{(2^{139} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{137}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}} \quad (85)$$

$$\zeta(141) = \zeta(139) \frac{(2^{141} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{(2^{141} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}} = \frac{(2^{141} - 4)}{(2^{141} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{139}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}} \quad (86)$$

$$\zeta(143) = \zeta(141) \frac{(2^{143} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{(2^{143} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}} = \frac{(2^{143} - 4)}{(2^{143} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{141}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}} \quad (87)$$

$$\zeta(145) = \zeta(143) \frac{(2^{145} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{(2^{145} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}} = \frac{(2^{145} - 4)}{(2^{145} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{143}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}} \quad (88)$$

$$\zeta(147) = \zeta(145) \frac{(2^{147} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{(2^{147} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}} = \frac{(2^{147} - 4)}{(2^{147} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{145}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}} \quad (89)$$

$$\zeta(149) = \zeta(147) \frac{(2^{149} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{(2^{149} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}} = \frac{(2^{149} - 4)}{(2^{149} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{147}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}} \quad (90)$$

$$\zeta(151) = \zeta(149) \frac{(2^{151} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}}{(2^{151} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}} = \frac{(2^{151} - 4)}{(2^{151} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{149}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}} \quad (91)$$

$$\zeta(153) = \zeta(151) \frac{(2^{153} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}}{(2^{153} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}} = \frac{(2^{153} - 4)}{(2^{153} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{151}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}} \quad (92)$$

$$\zeta(155) = \zeta(153) \frac{(2^{155} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}}{(2^{155} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}} = \frac{(2^{155} - 4)}{(2^{155} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{153}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}} \quad (93)$$

$$\zeta(157) = \zeta(155) \frac{(2^{157} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}}{(2^{157} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}} = \frac{(2^{157} - 4)}{(2^{157} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{155}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}} \quad (94)$$

$$\zeta(159) = \zeta(157) \frac{(2^{159} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}}{(2^{159} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}} = \frac{(2^{159} - 4)}{(2^{159} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{157}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}} \quad (95)$$

$$\zeta(161) = \zeta(159) \frac{(2^{161} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}}{(2^{161} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}} = \frac{(2^{161} - 4)}{(2^{161} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{159}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{159}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}} \quad (96)$$

$$\zeta(163) = \zeta(161) \frac{(2^{163} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}}{(2^{163} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}} = \frac{(2^{163} - 4)}{(2^{163} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{161}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{161}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}} \quad (97)$$

$$\zeta(165) = \zeta(163) \frac{(2^{165} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}}{(2^{165} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}} = \frac{(2^{165} - 4)}{(2^{165} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{163}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{163}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}} \quad (98)$$

$$\zeta(167) = \zeta(165) \frac{(2^{167} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}}{(2^{167} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}} = \frac{(2^{167} - 4)}{(2^{167} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{165}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{165}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}} \quad (99)$$

$$\zeta(169) = \zeta(167) \frac{(2^{169} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}}{(2^{169} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}} = \frac{(2^{169} - 4)}{(2^{169} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{167}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{167}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}} \quad (100)$$

$$\zeta(171) = \zeta(169) \frac{(2^{171} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}}{(2^{171} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}} = \frac{(2^{171} - 4)}{(2^{171} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{169}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{169}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}} \quad (101)$$

$$\zeta(173) = \zeta(171) \frac{(2^{173} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}}{(2^{173} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}} = \frac{(2^{173} - 4)}{(2^{173} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{171}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{171}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}} \quad (102)$$

$$\zeta(175) = \zeta(173) \frac{(2^{175} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}}{(2^{175} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}} = \frac{(2^{175} - 4)}{(2^{175} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{173}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{173}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}} \quad (103)$$

$$\zeta(177) = \zeta(175) \frac{(2^{177} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}}{(2^{177} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}} = \frac{(2^{177} - 4)}{(2^{177} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{175}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{175}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}} \quad (104)$$

$$\zeta(179) = \zeta(177) \frac{(2^{179} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}}{(2^{179} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}} = \frac{(2^{179} - 4)}{(2^{179} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{177}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{177}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}} \quad (105)$$

$$\zeta(181) = \zeta(179) \frac{(2^{181} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}}{(2^{181} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}} = \frac{(2^{181} - 4)}{(2^{181} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{179}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{179}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}} \quad (106)$$

$$\zeta(183) = \zeta(181) \frac{(2^{183} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{(2^{183} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}} = \frac{(2^{183} - 4)}{(2^{183} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{181}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{181}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}} \quad (107)$$

$$\zeta(185) = \zeta(183) \frac{(2^{185} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}}{(2^{185} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}} = \frac{(2^{185} - 4)}{(2^{185} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{183}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}} \quad (108)$$

$$\zeta(187) = \zeta(185) \frac{(2^{187} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}}{(2^{187} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}} = \frac{(2^{187} - 4)}{(2^{187} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{185}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}} \quad (109)$$

$$\zeta(189) = \zeta(187) \frac{(2^{189} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}}{(2^{189} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}} = \frac{(2^{189} - 4)}{(2^{189} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{187}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}} \quad (110)$$

$$\zeta(191) = \zeta(189) \frac{(2^{191} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}}{(2^{191} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}} = \frac{(2^{191} - 4)}{(2^{191} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{189}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}} \quad (111)$$

$$\zeta(193) = \zeta(191) \frac{(2^{193} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}}{(2^{193} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}} = \frac{(2^{193} - 4)}{(2^{193} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{191}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}} \quad (112)$$

$$\zeta(195) = \zeta(193) \frac{(2^{195} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}}{(2^{195} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}} = \frac{(2^{195} - 4)}{(2^{195} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{193}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}} \quad (113)$$

$$\zeta(197) = \zeta(195) \frac{(2^{197} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}}{(2^{197} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}} = \frac{(2^{197} - 4)}{(2^{197} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{195}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}} \quad (114)$$

$$\zeta(199) = \zeta(197) \frac{(2^{199} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{(2^{199} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}} = \frac{(2^{199} - 4)}{(2^{199} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{197}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}} \quad (115)$$

$$\zeta(201) = \zeta(199) \frac{(2^{201} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{(2^{201} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}} = \frac{(2^{201} - 4)}{(2^{201} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{199}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}} \quad (116)$$

$$\zeta(203) = \zeta(201) \frac{(2^{203} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{(2^{203} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}} = \frac{(2^{203} - 4)}{(2^{203} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{201}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}} \quad (117)$$

$$\zeta(205) = \zeta(203) \frac{(2^{205} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{(2^{205} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}} = \frac{(2^{205} - 4)}{(2^{205} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{203}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}} \quad (118)$$

$$\zeta(207) = \zeta(205) \frac{(2^{207} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{(2^{207} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}} = \frac{(2^{207} - 4)}{(2^{207} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{205}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}} \quad (119)$$

$$\zeta(209) = \zeta(207) \frac{(2^{209} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{(2^{209} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}} = \frac{(2^{209} - 4)}{(2^{209} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{207}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}} \quad (120)$$

$$\zeta(211) = \zeta(209) \frac{(2^{211} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{(2^{211} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}} = \frac{(2^{211} - 4)}{(2^{211} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{209}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}} \quad (121)$$

$$\zeta(213) = \zeta(211) \frac{(2^{213} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{(2^{213} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}} = \frac{(2^{213} - 4)}{(2^{213} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{211}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}} \quad (122)$$

$$\zeta(215) = \zeta(213) \frac{(2^{215} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{(2^{215} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}} = \frac{(2^{215} - 4)}{(2^{215} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{213}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}} \quad (123)$$

$$\zeta(217) = \zeta(215) \frac{(2^{217} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{(2^{217} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}} = \frac{(2^{217} - 4)}{(2^{217} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{215}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}} \quad (124)$$

$$\zeta(219) = \zeta(217) \frac{(2^{219} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{(2^{219} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}} = \frac{(2^{219} - 4)}{(2^{219} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{217}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}} \quad (125)$$

$$\zeta(221) = \zeta(219) \frac{(2^{221} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{(2^{221} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}} = \frac{(2^{221} - 4)}{(2^{221} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{219}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}} \quad (126)$$

$$\zeta(223) = \zeta(221) \frac{(2^{223} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{(2^{223} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}} = \frac{(2^{223} - 4)}{(2^{223} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{221}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}} \quad (127)$$

$$\zeta(225) = \zeta(223) \frac{(2^{225} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{(2^{225} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}} = \frac{(2^{225} - 4)}{(2^{225} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{223}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}} \quad (128)$$

$$\zeta(227) = \zeta(225) \frac{(2^{227} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}}{(2^{227} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}} = \frac{(2^{227} - 4)}{(2^{227} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{225}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}} \quad (129)$$

$$\zeta(229) = \zeta(227) \frac{(2^{229} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}}{(2^{229} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}} = \frac{(2^{229} - 4)}{(2^{229} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{227}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}} \quad (130)$$

$$\zeta(279) = \zeta(277) \frac{(2^{279} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{(2^{279} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}} = \frac{(2^{279} - 4)}{(2^{279} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{277}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{277}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}} \quad (155)$$

$$\zeta(281) = \zeta(279) \frac{(2^{281} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{(2^{281} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}} = \frac{(2^{281} - 4)}{(2^{281} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}} \quad (156)$$

$$\zeta(283) = \zeta(281) \frac{(2^{283} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{(2^{283} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}} = \frac{(2^{283} - 4)}{(2^{283} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}} \quad (157)$$

$$\zeta(285) = \zeta(283) \frac{(2^{285} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{(2^{285} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}} = \frac{(2^{285} - 4)}{(2^{285} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}} \quad (158)$$

$$\zeta(287) = \zeta(285) \frac{(2^{287} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{(2^{287} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}} = \frac{(2^{287} - 4)}{(2^{287} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}} \quad (159)$$

$$\zeta(289) = \zeta(287) \frac{(2^{289} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{(2^{289} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}} = \frac{(2^{289} - 4)}{(2^{289} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}} \quad (160)$$

$$\zeta(291) = \zeta(289) \frac{(2^{291} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{(2^{291} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}} = \frac{(2^{291} - 4)}{(2^{291} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}} \quad (161)$$

$$\zeta(293) = \zeta(291) \frac{(2^{293} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{(2^{293} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}} = \frac{(2^{293} - 4)}{(2^{293} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}} \quad (162)$$

$$\zeta(295) = \zeta(293) \frac{(2^{295} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{(2^{295} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}} = \frac{(2^{295} - 4)}{(2^{295} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}} \quad (163)$$

$$\zeta(297) = \zeta(295) \frac{(2^{297} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{(2^{297} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}} = \frac{(2^{297} - 4)}{(2^{297} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}} \quad (164)$$

$$\zeta(299) = \zeta(297) \frac{(2^{299} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{(2^{299} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}} = \frac{(2^{299} - 4)}{(2^{299} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}} \quad (165)$$

$$\zeta(301) = \zeta(299) \frac{(2^{301} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{(2^{301} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}} = \frac{(2^{301} - 4)}{(2^{301} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{299}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}} \quad (166)$$

$$\zeta(303) = \zeta(301) \frac{(2^{303} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{(2^{303} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}} = \frac{(2^{303} - 4)}{(2^{303} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{301}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}} \quad (167)$$

$$\zeta(305) = \zeta(303) \frac{(2^{305} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{(2^{305} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}} = \frac{(2^{305} - 4)}{(2^{305} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{303}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}} \quad (168)$$

$$\zeta(307) = \zeta(305) \frac{(2^{307} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{(2^{307} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}} = \frac{(2^{307} - 4)}{(2^{307} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{305}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}} \quad (169)$$

$$\zeta(309) = \zeta(307) \frac{(2^{309} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{(2^{309} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}} = \frac{(2^{309} - 4)}{(2^{309} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{307}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}} \quad (170)$$

$$\zeta(311) = \zeta(309) \frac{(2^{311} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{(2^{311} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}} = \frac{(2^{311} - 4)}{(2^{311} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{309}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}} \quad (171)$$

$$\zeta(313) = \zeta(311) \frac{(2^{313} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{(2^{313} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}} = \frac{(2^{313} - 4)}{(2^{313} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{311}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}} \quad (172)$$

$$\zeta(315) = \zeta(313) \frac{(2^{315} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{(2^{315} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}} = \frac{(2^{315} - 4)}{(2^{315} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{313}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}} \quad (173)$$

$$\zeta(317) = \zeta(315) \frac{(2^{317} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{(2^{317} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}} = \frac{(2^{317} - 4)}{(2^{317} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{315}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}} \quad (174)$$

$$\zeta(319) = \zeta(317) \frac{(2^{319} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{(2^{319} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}} = \frac{(2^{319} - 4)}{(2^{319} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{317}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}} \quad (175)$$

$$\zeta(321) = \zeta(319) \frac{(2^{321} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{(2^{321} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}} = \frac{(2^{321} - 4)}{(2^{321} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{319}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}} \quad (176)$$

$$\zeta(323) = \zeta(321) \frac{(2^{323} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{(2^{323} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}} = \frac{(2^{323} - 4)}{(2^{323} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{321}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}} \quad (177)$$

$$\zeta(325) = \zeta(323) \frac{(2^{325} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{(2^{325} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}} = \frac{(2^{325} - 4)}{(2^{325} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{323}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}} \quad (178)$$

$$\zeta(327) = \zeta(325) \frac{(2^{327} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{(2^{327} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}} = \frac{(2^{327} - 4)}{(2^{327} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{325}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}} \quad (179)$$

$$\zeta(329) = \zeta(327) \frac{(2^{329} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{(2^{329} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}} = \frac{(2^{329} - 4)}{(2^{329} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{327}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}} \quad (180)$$

$$\zeta(331) = \zeta(329) \frac{(2^{331} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{(2^{331} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}} = \frac{(2^{331} - 4)}{(2^{331} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{329}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}} \quad (181)$$

$$\zeta(333) = \zeta(331) \frac{(2^{333} - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{(2^{333} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}} = \frac{(2^{333} - 4)}{(2^{333} - 1)} \left[1 + \frac{\sum_{n=1}^{\infty} \frac{1}{(2n)^{331}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}} \right] \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}} \quad (182)$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations.

$\zeta(5), \zeta(7), \dots, \zeta(331), \zeta(333)$ are irrational numbers.

Example 2

Detailed description

$$\zeta(1) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \sum_{n=1}^{\infty} \frac{1}{(2n)^1} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{(n)^1} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} + \frac{1}{2} \zeta(1) \quad (183)$$

$$\frac{1}{2} \zeta(1) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (184)$$

$$\zeta(1) = 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} \quad (185)$$

$$\zeta(3) = \frac{2^3}{2^3 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (186)$$

Multiply $\zeta(1)$ and $\zeta(3)$

$$\zeta(3) 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1} = \zeta(1) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (187)$$

$$\zeta(3) = \zeta(1) 2 \frac{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \quad (188)$$

and

$$\zeta(3) = \frac{2^3}{2^3 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (189)$$

$$\zeta(5) = \frac{2^5}{2^5 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (190)$$

Multiply $\zeta(3)$ and $\zeta(5)$

$$\zeta(5) \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \zeta(3) \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (191)$$

$$\zeta(5) = \zeta(3) \frac{7}{8} \frac{32}{31} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{28}{31} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (192)$$

Do the same for $\zeta(7), \zeta(9), \zeta(11)$ etc.

$$\zeta(3) = \zeta(1) \frac{4 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} = \zeta(1) \frac{(2^3 - 4) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}}{(2^3 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^1}} \quad (193)$$

$$\zeta(5) = \zeta(3) \frac{2^5}{(2^5 - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^2 \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5}}{(2^5 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (194)$$

$$\zeta(7) = \zeta(3) \frac{2^7}{(2^7 - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^4 \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7}}{(2^7 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (195)$$

$$\zeta(9) = \zeta(3) \frac{2^9}{(2^9 - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^6 \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9}}{(2^9 - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (196)$$

$$\zeta(11) = \zeta(3) \frac{2^{11}}{(2^{11} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^8 \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}}}{(2^{11} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (197)$$

$$\zeta(13) = \zeta(3) \frac{2^{13}}{(2^{13} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{10} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}}}{(2^{13} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (198)$$

$$\zeta(15) = \zeta(3) \frac{2^{15}}{(2^{15} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{12} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}}}{(2^{15} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (199)$$

$$\zeta(17) = \zeta(3) \frac{2^{17}}{(2^{17} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{14} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}}}{(2^{17} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (200)$$

$$\zeta(19) = \zeta(3) \frac{2^{19}}{(2^{19} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{16} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}}}{(2^{19} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (201)$$

$$\zeta(21) = \zeta(3) \frac{2^{21}}{(2^{21} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{18} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{21}}}{(2^{21} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (202)$$

$$\zeta(23) = \zeta(3) \frac{2^{23}}{(2^{23} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{20} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{23}}}{(2^{23} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (203)$$

$$\zeta(25) = \zeta(3) \frac{2^{25}}{(2^{25} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{22} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{25}}}{(2^{25} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (204)$$

$$\zeta(27) = \zeta(3) \frac{2^{27}}{(2^{27} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{24} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{27}}}{(2^{27} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (205)$$

$$\zeta(29) = \zeta(3) \frac{2^{29}}{(2^{29} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{26} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{29}}}{(2^{29} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (206)$$

$$\zeta(31) = \zeta(3) \frac{2^{31}}{(2^{31} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{28} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{31}}}{(2^{31} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (207)$$

$$\zeta(33) = \zeta(3) \frac{2^{33}}{(2^{33} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{30} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{33}}}{(2^{33} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (208)$$

$$\zeta(35) = \zeta(3) \frac{2^{35}}{(2^{35} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{32} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{35}}}{(2^{35} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (209)$$

$$\zeta(37) = \zeta(3) \frac{2^{37}}{(2^{37} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{34} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{37}}}{(2^{37} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (210)$$

$$\zeta(39) = \zeta(3) \frac{2^{39}}{(2^{39} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{36} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{39}}}{(2^{39} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (211)$$

$$\zeta(41) = \zeta(3) \frac{2^{41}}{(2^{41} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{38} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{41}}}{(2^{41} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (212)$$

$$\zeta(43) = \zeta(3) \frac{2^{43}}{(2^{43} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{40} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{43}}}{(2^{43} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (213)$$

$$\zeta(45) = \zeta(3) \frac{2^{45}}{(2^{45} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{42} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{45}}}{(2^{45} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (214)$$

$$\zeta(47) = \zeta(3) \frac{2^{47}}{(2^{47} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{44} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{47}}}{(2^{47} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (215)$$

$$\zeta(49) = \zeta(3) \frac{2^{49}}{(2^{49} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{46} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{49}}}{(2^{49} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (216)$$

$$\zeta(51) = \zeta(3) \frac{2^{51}}{(2^{51} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{48} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{51}}}{(2^{51} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (217)$$

$$\zeta(53) = \zeta(3) \frac{2^{53}}{(2^{53} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{50} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{53}}}{(2^{53} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (218)$$

$$\zeta(55) = \zeta(3) \frac{2^{55}}{(2^{55} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{52} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{55}}}{(2^{55} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (219)$$

$$\zeta(57) = \zeta(3) \frac{2^{57}}{(2^{57} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{54} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{57}}}{(2^{57} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (220)$$

$$\zeta(59) = \zeta(3) \frac{2^{59}}{(2^{59} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{56} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{59}}}{(2^{59} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (221)$$

$$\zeta(61) = \zeta(3) \frac{2^{61}}{(2^{61} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{58} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{61}}}{(2^{61} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (222)$$

$$\zeta(63) = \zeta(3) \frac{2^{63}}{(2^{63} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{60} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{63}}}{(2^{63} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (223)$$

$$\zeta(65) = \zeta(3) \frac{2^{65}}{(2^{65} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{62} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{65}}}{(2^{65} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (224)$$

$$\zeta(67) = \zeta(3) \frac{2^{67}}{(2^{67} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{64} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{67}}}{(2^{67} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (225)$$

$$\zeta(69) = \zeta(3) \frac{2^{69}}{(2^{69} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{66} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{69}}}{(2^{69} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (226)$$

$$\zeta(71) = \zeta(3) \frac{2^{71}}{(2^{71} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{68} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{71}}}{(2^{71} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (227)$$

$$\zeta(73) = \zeta(3) \frac{2^{73}}{(2^{73} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{70} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{73}}}{(2^{73} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (228)$$

$$\zeta(75) = \zeta(3) \frac{2^{75}}{(2^{75} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{72} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{75}}}{(2^{75} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (229)$$

$$\zeta(77) = \zeta(3) \frac{2^{77}}{(2^{77} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{74} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{77}}}{(2^{77} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (230)$$

$$\zeta(79) = \zeta(3) \frac{2^{79}}{(2^{79} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{76} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{79}}}{(2^{79} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (231)$$

$$\zeta(81) = \zeta(3) \frac{2^{81}}{(2^{81} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{78} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{81}}}{(2^{81} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (232)$$

$$\zeta(83) = \zeta(3) \frac{2^{83}}{(2^{83} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{80} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{83}}}{(2^{83} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (233)$$

$$\zeta(85) = \zeta(3) \frac{2^{85}}{(2^{85} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{82} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{85}}}{(2^{85} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (234)$$

$$\zeta(87) = \zeta(3) \frac{2^{87}}{(2^{87} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{84} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{87}}}{(2^{87} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (235)$$

$$\zeta(89) = \zeta(3) \frac{2^{89}}{(2^{89} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{86} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{89}}}{(2^{89} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (236)$$

$$\zeta(91) = \zeta(3) \frac{2^{91}}{(2^{91} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{88} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{91}}}{(2^{91} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (237)$$

$$\zeta(93) = \zeta(3) \frac{2^{93}}{(2^{93} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{90} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{93}}}{(2^{93} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (238)$$

$$\zeta(95) = \zeta(3) \frac{2^{95}}{(2^{95} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{92} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{95}}}{(2^{95} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (239)$$

$$\zeta(97) = \zeta(3) \frac{2^{97}}{(2^{97} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{94} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{97}}}{(2^{97} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (240)$$

$$\zeta(99) = \zeta(3) \frac{2^{99}}{(2^{99} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{96} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{99}}}{(2^{99} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (241)$$

$$\zeta(101) = \zeta(3) \frac{2^{101}}{(2^{101} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{98} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{101}}}{(2^{101} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (242)$$

$$\zeta(103) = \zeta(3) \frac{2^{103}}{(2^{103} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{100} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{103}}}{(2^{103} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (243)$$

$$\zeta(105) = \zeta(3) \frac{2^{105}}{(2^{105} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{102} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{105}}}{(2^{105} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (244)$$

$$\zeta(107) = \zeta(3) \frac{2^{107}}{(2^{107} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{104} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{107}}}{(2^{107} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (245)$$

$$\zeta(109) = \zeta(3) \frac{2^{109}}{(2^{109} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{106} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{109}}}{(2^{109} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (246)$$

$$\zeta(135) = \zeta(3) \frac{2^{135}}{(2^{135} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{132} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{135}}}{(2^{135} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (259)$$

$$\zeta(137) = \zeta(3) \frac{2^{137}}{(2^{137} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{134} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{137}}}{(2^{137} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (260)$$

$$\zeta(139) = \zeta(3) \frac{2^{139}}{(2^{139} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{136} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{139}}}{(2^{139} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (261)$$

$$\zeta(141) = \zeta(3) \frac{2^{141}}{(2^{141} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{138} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{141}}}{(2^{141} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (262)$$

$$\zeta(143) = \zeta(3) \frac{2^{143}}{(2^{143} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{140} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{143}}}{(2^{143} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (263)$$

$$\zeta(145) = \zeta(3) \frac{2^{145}}{(2^{145} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{142} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{145}}}{(2^{145} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (264)$$

$$\zeta(147) = \zeta(3) \frac{2^{147}}{(2^{147} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{144} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{147}}}{(2^{147} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (265)$$

$$\zeta(149) = \zeta(3) \frac{2^{149}}{(2^{149} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{146} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{149}}}{(2^{149} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (266)$$

$$\zeta(151) = \zeta(3) \frac{2^{151}}{(2^{151} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{148} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{151}}}{(2^{151} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (267)$$

$$\zeta(153) = \zeta(3) \frac{2^{153}}{(2^{153} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{150} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{153}}}{(2^{153} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (268)$$

$$\zeta(155) = \zeta(3) \frac{2^{155}}{(2^{155} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{152} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{155}}}{(2^{155} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (269)$$

$$\zeta(157) = \zeta(3) \frac{2^{157}}{(2^{157} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{154} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{157}}}{(2^{157} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (270)$$

$$\zeta(183) = \zeta(3) \frac{2^{183}}{(2^{183} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{180} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{183}}}{(2^{183} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (283)$$

$$\zeta(185) = \zeta(3) \frac{2^{185}}{(2^{185} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{182} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{185}}}{(2^{185} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (284)$$

$$\zeta(187) = \zeta(3) \frac{2^{187}}{(2^{187} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{184} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{187}}}{(2^{187} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (285)$$

$$\zeta(189) = \zeta(3) \frac{2^{189}}{(2^{189} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{186} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{189}}}{(2^{189} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (286)$$

$$\zeta(191) = \zeta(3) \frac{2^{191}}{(2^{191} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{188} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{191}}}{(2^{191} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (287)$$

$$\zeta(193) = \zeta(3) \frac{2^{193}}{(2^{193} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{190} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{193}}}{(2^{193} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (288)$$

$$\zeta(195) = \zeta(3) \frac{2^{195}}{(2^{195} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{192} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{195}}}{(2^{195} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (289)$$

$$\zeta(197) = \zeta(3) \frac{2^{197}}{(2^{197} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{194} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{197}}}{(2^{197} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (290)$$

$$\zeta(199) = \zeta(3) \frac{2^{199}}{(2^{199} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{196} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{199}}}{(2^{199} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (291)$$

$$\zeta(201) = \zeta(3) \frac{2^{201}}{(2^{201} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{198} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{201}}}{(2^{201} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (292)$$

$$\zeta(203) = \zeta(3) \frac{2^{203}}{(2^{203} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{200} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{203}}}{(2^{203} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (293)$$

$$\zeta(205) = \zeta(3) \frac{2^{205}}{(2^{205} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{202} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{205}}}{(2^{205} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (294)$$

$$\zeta(207) = \zeta(3) \frac{2^{207}}{(2^{207} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{204} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{207}}}{(2^{207} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (295)$$

$$\zeta(209) = \zeta(3) \frac{2^{209}}{(2^{209} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{206} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{209}}}{(2^{209} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (296)$$

$$\zeta(211) = \zeta(3) \frac{2^{211}}{(2^{211} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{208} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{211}}}{(2^{211} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (297)$$

$$\zeta(213) = \zeta(3) \frac{2^{213}}{(2^{213} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{210} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{213}}}{(2^{213} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (298)$$

$$\zeta(215) = \zeta(3) \frac{2^{215}}{(2^{215} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{212} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{215}}}{(2^{215} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (299)$$

$$\zeta(217) = \zeta(3) \frac{2^{217}}{(2^{217} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{214} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{217}}}{(2^{217} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (300)$$

$$\zeta(219) = \zeta(3) \frac{2^{219}}{(2^{219} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{216} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{219}}}{(2^{219} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (301)$$

$$\zeta(221) = \zeta(3) \frac{2^{221}}{(2^{221} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{218} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{221}}}{(2^{221} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (302)$$

$$\zeta(223) = \zeta(3) \frac{2^{223}}{(2^{223} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{220} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{223}}}{(2^{223} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (303)$$

$$\zeta(225) = \zeta(3) \frac{2^{225}}{(2^{225} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{222} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{225}}}{(2^{225} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (304)$$

$$\zeta(227) = \zeta(3) \frac{2^{227}}{(2^{227} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{224} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{227}}}{(2^{227} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (305)$$

$$\zeta(229) = \zeta(3) \frac{2^{229}}{(2^{229} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{226} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{229}}}{(2^{229} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (306)$$

$$\zeta(231) = \zeta(3) \frac{2^{231}}{(2^{231} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{231}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{228} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{231}}}{(2^{231} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (307)$$

$$\zeta(233) = \zeta(3) \frac{2^{233}}{(2^{233} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{233}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{230} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{233}}}{(2^{233} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (308)$$

$$\zeta(235) = \zeta(3) \frac{2^{235}}{(2^{235} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{235}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{232} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{235}}}{(2^{235} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (309)$$

$$\zeta(237) = \zeta(3) \frac{2^{237}}{(2^{237} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{237}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{234} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{237}}}{(2^{237} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (310)$$

$$\zeta(239) = \zeta(3) \frac{2^{239}}{(2^{239} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{239}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{236} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{239}}}{(2^{239} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (311)$$

$$\zeta(241) = \zeta(3) \frac{2^{241}}{(2^{241} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{238} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{241}}}{(2^{241} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (312)$$

$$\zeta(243) = \zeta(3) \frac{2^{243}}{(2^{243} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{240} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{243}}}{(2^{243} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (313)$$

$$\zeta(245) = \zeta(3) \frac{2^{245}}{(2^{245} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{242} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{245}}}{(2^{245} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (314)$$

$$\zeta(247) = \zeta(3) \frac{2^{247}}{(2^{247} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{244} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{247}}}{(2^{247} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (315)$$

$$\zeta(249) = \zeta(3) \frac{2^{249}}{(2^{249} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{246} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{249}}}{(2^{249} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (316)$$

$$\zeta(251) = \zeta(3) \frac{2^{251}}{(2^{251} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{248} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{251}}}{(2^{251} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (317)$$

$$\zeta(253) = \zeta(3) \frac{2^{253}}{(2^{253} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{250} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{253}}}{(2^{253} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (318)$$

$$\zeta(279) = \zeta(3) \frac{2^{279}}{(2^{279} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{276} \times 7}{(2^{279} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{279}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (331)$$

$$\zeta(281) = \zeta(3) \frac{2^{281}}{(2^{281} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{278} \times 7}{(2^{281} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{281}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (332)$$

$$\zeta(283) = \zeta(3) \frac{2^{283}}{(2^{283} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{280} \times 7}{(2^{283} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{283}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (333)$$

$$\zeta(285) = \zeta(3) \frac{2^{285}}{(2^{285} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{282} \times 7}{(2^{285} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{285}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (334)$$

$$\zeta(287) = \zeta(3) \frac{2^{287}}{(2^{287} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{284} \times 7}{(2^{287} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{287}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (335)$$

$$\zeta(289) = \zeta(3) \frac{2^{289}}{(2^{289} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{286} \times 7}{(2^{289} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{289}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (336)$$

$$\zeta(291) = \zeta(3) \frac{2^{291}}{(2^{291} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{288} \times 7}{(2^{291} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{291}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (337)$$

$$\zeta(293) = \zeta(3) \frac{2^{293}}{(2^{293} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{290} \times 7}{(2^{293} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{293}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (338)$$

$$\zeta(295) = \zeta(3) \frac{2^{295}}{(2^{295} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{292} \times 7}{(2^{295} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{295}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (339)$$

$$\zeta(297) = \zeta(3) \frac{2^{297}}{(2^{297} - 1)} \frac{7}{8} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{294} \times 7}{(2^{297} - 1)} \frac{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^{297}}}{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (340)$$

$$\zeta(299) = \zeta(3) \frac{2^{299}}{(2^{299} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{296} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{299}}}{(2^{299} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (341)$$

$$\zeta(301) = \zeta(3) \frac{2^{301}}{(2^{301} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{298} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{301}}}{(2^{301} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (342)$$

$$\zeta(303) = \zeta(3) \frac{2^{303}}{(2^{303} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{300} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{303}}}{(2^{303} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (343)$$

$$\zeta(305) = \zeta(3) \frac{2^{305}}{(2^{305} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{302} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{305}}}{(2^{305} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (344)$$

$$\zeta(307) = \zeta(3) \frac{2^{307}}{(2^{307} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{304} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{307}}}{(2^{307} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (345)$$

$$\zeta(309) = \zeta(3) \frac{2^{309}}{(2^{309} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{306} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{309}}}{(2^{309} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (346)$$

$$\zeta(311) = \zeta(3) \frac{2^{311}}{(2^{311} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{308} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{311}}}{(2^{311} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (347)$$

$$\zeta(313) = \zeta(3) \frac{2^{313}}{(2^{313} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{310} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{313}}}{(2^{313} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (348)$$

$$\zeta(315) = \zeta(3) \frac{2^{315}}{(2^{315} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{312} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{315}}}{(2^{315} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (349)$$

$$\zeta(317) = \zeta(3) \frac{2^{317}}{(2^{317} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{314} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{317}}}{(2^{317} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (350)$$

$$\zeta(319) = \zeta(3) \frac{2^{319}}{(2^{319} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{316} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{319}}}{(2^{319} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (351)$$

$$\zeta(321) = \zeta(3) \frac{2^{321}}{(2^{321} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{318} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{321}}}{(2^{321} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (352)$$

$$\zeta(323) = \zeta(3) \frac{2^{323}}{(2^{323} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{320} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{323}}}{(2^{323} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (353)$$

$$\zeta(325) = \zeta(3) \frac{2^{325}}{(2^{325} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{322} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{325}}}{(2^{325} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (354)$$

$$\zeta(327) = \zeta(3) \frac{2^{327}}{(2^{327} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{324} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{327}}}{(2^{327} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (355)$$

$$\zeta(329) = \zeta(3) \frac{2^{329}}{(2^{329} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{326} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{329}}}{(2^{329} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (356)$$

$$\zeta(331) = \zeta(3) \frac{2^{331}}{(2^{331} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{328} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{331}}}{(2^{331} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (357)$$

$$\zeta(333) = \zeta(3) \frac{2^{333}}{(2^{333} - 1)} \frac{7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{8 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} = \zeta(3) \frac{2^{330} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{333}}}{(2^{333} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (358)$$

$\zeta(335), \zeta(337)$ etc. can also be expressed by these equations

3 Conclusion

$\zeta(3), \zeta(5), \dots, \zeta(331), \zeta(333)$ are irrational numbers.

And I declare a new formula.

$$\zeta(2m + 1) = \zeta(3) \frac{2^{2m-2} \times 7 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m+1}}}{(2^{2m+1} - 1) \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}} \quad (359)$$

m is a positive integer of 2 or more.

4 Postscript

The figures in this paper have been fully verified by WolframAlpha.

References

- [1] B.Riemann.: Uber die Anzahl der Primzahlen unter einer gegebenen Grosse, Mon. Not. Berlin Akad pp.671-680, 1859
- [2] John Derbyshire.: Prime Obsession: Bernhard Riemann and The Greatest Unsolved Problem in Mathematics, Joseph Henry Press, 2003
- [3] S.Kurokawa.: Riemann hypothesis, Japan Hyoron Press, 2009
- [4] Marcus du Sautoy.: The Music of The Primes, Zahar Press, 2007