

# **On the Ramanujan Modular Equations, Class Invariants and Mock Theta Functions: new mathematical connections with some particle-like solutions, Black Holes entropies, $\zeta(2)$ and Golden Ratio**

**Michele Nardelli<sup>1</sup>, Antonio Nardelli**

## **Abstract**

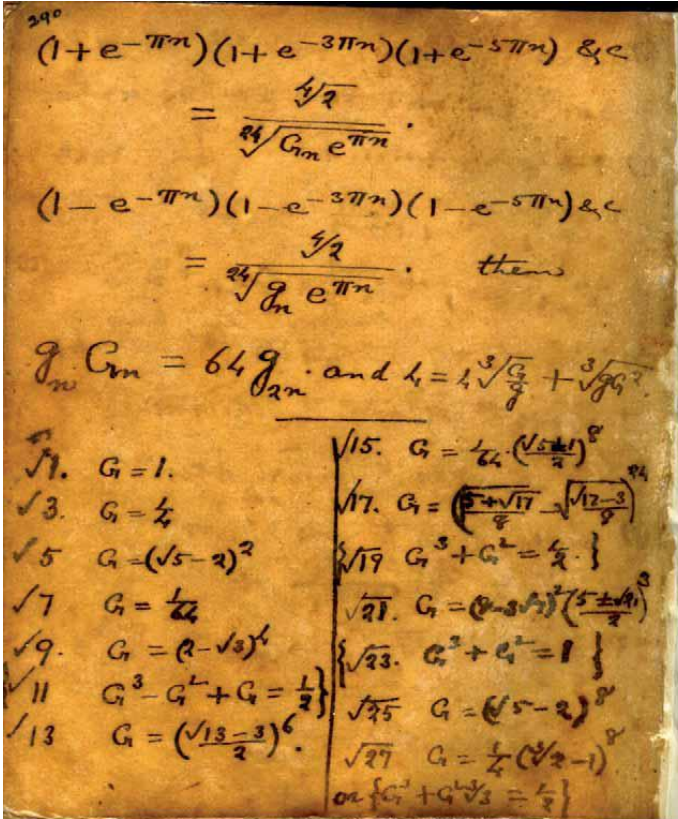
*In the present research thesis, we have obtained various interesting new possible mathematical connections between the Ramanujan Modular Equations, Class Invariants, the Mock Theta Functions, some particle-like solutions, Black Holes entropies,  $\zeta(2)$  and Golden Ratio*

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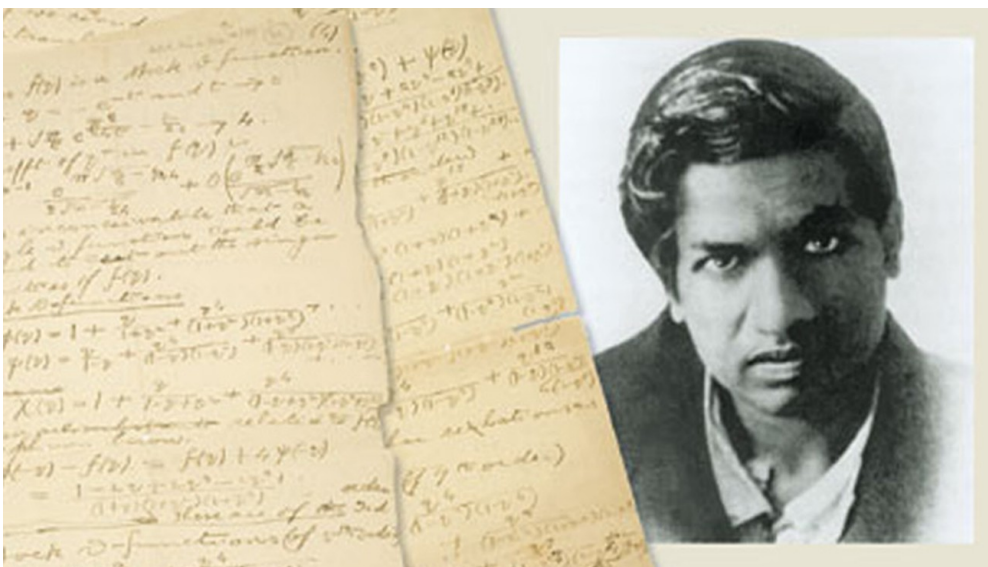
From:

<https://math.stackexchange.com/questions/516203/problems-in-the-ramanujan-class-invariant-g-n>



From:

<https://www.pourlascience.fr/sd/mathematiques/les-notes-de-ramanujan-un-tresor-inepuise-7955.php>



We begin this paper by analyzing Ramanujan's modular equations and class invariants according to our interpretation inspired by Ramanujan

From:

**MODULAR EQUATIONS IN THE SPIRIT OF RAMANUJAN**

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June 25, 2012

If

$$P = \frac{f(-q)}{q^{1/24}f(-q^2)} \quad \text{and} \quad Q_n = \frac{f(-q^n)}{q^{n/24}f(-q^{2n})}, \quad (12)$$

then

$$(PQ_3)^3 + \frac{8}{(PQ_3)^3} = \left(\frac{Q_3}{P}\right)^6 - \left(\frac{P}{Q_3}\right)^6, \quad (13)$$

Using (12) with  $n = 3$  in the above identity, we find that

$$(PQ_3)^6 + \frac{64}{(PQ_3)^6} + 32 = \left(P^{12} + \frac{64}{P^{12}}\right)^{1/2} \left(Q_3^{12} + \frac{64}{Q_3^{12}}\right)^{1/2}.$$

For  $q = 0.5$ , we obtain, from (12)

$$(-0.5)/(0.5^{1/24} * 0.5^2)$$

**Input:**

$$\frac{0.5}{\sqrt[24]{0.5} \times 0.5^2}$$

**Result:**

-2.05860...

-2.05860...

$$(-0.5)^3 / (0.5^{3/24} * 0.5^6)$$

**Input:**

$$\frac{(-0.5)^3}{0.5^{3/24} \times 0.5^6}$$

**Result:**

-8.72406...

-8.72406...

$$(-2.05860 * -8.72406)^3 + 8 / (((-2.05860 * -8.72406)^3))$$

**Input interpretation:**

$$(-2.05860 \times (-8.72406))^3 + \frac{8}{(-2.05860 \times (-8.72406))^3}$$

**Result:**

5792.578663442205228118634186199274133619469045373719515158...

5792.57866... result very near to the rest mass of bottom Xi baryon 5791.1

$$(-2.05860 * -8.72406)^6 + 64 / (((-2.05860 * -8.72406)^6)) + 32$$

**Input interpretation:**

$$(-2.05860 \times (-8.72406))^6 + \frac{64}{(-2.05860 \times (-8.72406))^6} + 32$$

**Result:**

3.35539835721658847073296415031360221237116577207601366... × 10<sup>7</sup>

3.355398... \* 10<sup>7</sup>

$$(((((-2.05860 * -8.72406)^3 + 8 / (((-2.05860 * -8.72406)^3))))))^{1/18}$$

**Input interpretation:**

$$\sqrt[18]{(-2.05860 \times (-8.72406))^3 + \frac{8}{(-2.05860 \times (-8.72406))^3}}$$

**Result:**



1.618260528050015977687190912963540280597100599303237610424...

1.61826052...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

Or:

$((((( -2.05860 * -8.72406 )^6 + 64 / ((( -2.05860 * -8.72406 )^6 ) + 32 ) ) ) ) )^{1/36}$

**Input interpretation:**

$$\sqrt[36]{(-2.05860 \times (-8.72406))^6 + \frac{64}{(-2.05860 \times (-8.72406))^6} + 32}$$

**Result:**

1.618260549484939674642933735554797388483238650931317143813...

1.61826054...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

## THEOREM

If  $P = \frac{f(-q)f(-q^{13})}{q^{14/3}f(-q^9)f(-q^{117})}$  and  $Q = \frac{f(-q)f(-q^{117})}{q^{-4}f(-q^9)f(-q^{13})}$  then

$$Q^7 + \frac{1}{Q^7} - 65 \left( Q^6 + \frac{1}{Q^6} \right) + 910 \left( Q^5 + \frac{1}{Q^5} \right) - 1417 \left( Q^4 + \frac{1}{Q^4} \right) - 6994 \left( Q^3 + \frac{1}{Q^3} \right) + 10049 \left( Q^2 + \frac{1}{Q^2} \right) + 6981 \left( Q + \frac{1}{Q} \right) = 17472$$

$$P^6 + \frac{9^6}{P^6} - 13 \left( \sqrt{P^9} + \frac{3^9}{\sqrt{P^9}} \right) \left[ \left( \sqrt{Q} + \frac{1}{\sqrt{Q}} \right) - \left( \sqrt{Q^3} + \frac{1}{\sqrt{Q^3}} \right) \right] - 13 \left( \sqrt{P^3} + \frac{3^3}{\sqrt{P^3}} \right) \left[ 139 \left( \sqrt{Q} + \frac{1}{\sqrt{Q}} \right) - 179 \left( \sqrt{Q^3} + \frac{1}{\sqrt{Q^3}} \right) - 2 \left( \sqrt{Q^5} + \frac{1}{\sqrt{Q^5}} \right) + 52 \left( \sqrt{Q^7} + \frac{1}{\sqrt{Q^7}} \right) + 10 \left( \sqrt{Q^9} + \frac{1}{\sqrt{Q^9}} \right) \right] + 13 \left( P^3 + \frac{9^3}{P^3} \right) \left[ 5 \left( Q^3 + \frac{1}{Q^3} \right) - 12 \left( Q^2 + \frac{1}{Q^2} \right) - 6 \left( Q + \frac{1}{Q} \right) + 26 \right]. \quad (100)$$

$$Q^7 + \frac{1}{Q^7} - 65 \left( Q^6 + \frac{1}{Q^6} \right) + 910 \left( Q^5 + \frac{1}{Q^5} \right) - 1417 \left( Q^4 + \frac{1}{Q^4} \right) - 6994 \left( Q^3 + \frac{1}{Q^3} \right) + 10049 \left( Q^2 + \frac{1}{Q^2} \right) + 6981 \left( Q + \frac{1}{Q} \right) = 17472$$

$$1417 - 910 - 65 + 10049 + 6981 = 17472$$

$$17472 - 6981 - 10049 = 1417 - 910 - 65 = 442$$

$$(442)^{1/12} = 1.6613145\dots$$

For  $P = Q = 1$ , we obtain:

$$(1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2)$$

**Input:**

$$1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2$$

**Result:**

18930

18930

Note that:

$$((((1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2)/2)))-144-21$$

**Input:**

$$\frac{1}{2} (1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2) - 144 - 21$$

**Result:**

9300

9300 result equal to the rest mass of Bottom eta meson

Now:

$$17472/(1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2)$$

**Input:**

$$\frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2}$$

**Exact result:**

2912

3155

**Decimal approximation:**

0.922979397781299524564183835182250396196513470681458003169...

0.922979...

And:

$$17472/0.922979397781299524564183835182250396196513470681458003169$$

**Input interpretation:**

$$\frac{17472}{0.922979397781299524564183835182250396196513470681458003169}$$

**Result:**



$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \frac{1}{\left(\frac{17472}{18930}\right)^{360^\circ}}$$

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \frac{1}{\left(\frac{17472}{18930}\right)^{-2i \log(-1)}}$$

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \frac{1}{\left(\frac{17472}{18930}\right)^{2 \cos^{-1}(-1)}}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

### Series representations:

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \left(\frac{2912}{3155}\right)^{-8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \left(\frac{2912}{3155}\right)^{8 \sum_{k=0}^{\infty} ((-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})) / (1+2k)}$$

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \left(\frac{2912}{3155}\right)^{-2 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))}$$

### Integral representations:

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \left(\frac{2912}{3155}\right)^{-8 \int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \left(\frac{2912}{3155}\right)^{-4} \int_0^{\infty} \frac{1}{(1+t^2)} dt$$

$$\frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^{2\pi}} = \left(\frac{2912}{3155}\right)^{-4} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$55+10^3[1/((((17472/(1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2)))))]^6$$

**Input:**

$$55 + 10^3 \times \frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^6}$$

**Exact result:**

$$\frac{127475848374720601759765}{76218120810002382848}$$

**Decimal approximation:**

1672.513662367696158027720527054348874626859874674635042818...

1672.51366.... result practically equal to the rest mass of Omega baryon 1672.45

$$-2.103786766-34+10^2[1/((((17472/(1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2)))))]^6$$

where 2.103786766... is a Ramanujan mock theta function

**Input interpretation:**

$$-2.103786766 - 34 + 10^2 \times \frac{1}{\left(\frac{17472}{1+1-65 \times 2+910 \times 2-1417 \times 2-6994 \times 2+10049 \times 2+6981 \times 2}\right)^6}$$

**Result:**

125.6475794707696158027720527054348874626859874674635042818...

125.647579... result very near to the Higgs boson mass 125.18

$$13 - 1.22734321771259 - 34 + 10^2 \left[ \frac{1}{\left( \frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2} \right)^6} \right]^6$$

here  $f(q) = 1.22734321771259\dots$  is a Ramanujan mock theta function

**Input interpretation:**

$$13 - 1.22734321771259 - 34 + 10^2 \times \frac{1}{\left( \frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2} \right)^6}$$

**Result:**

139.5240230190570258027720527054348874626859874674635042818...

139.52402.... result practically equal to the rest mass of Pion meson 139.57

$$(-144 - 13 - 21^2) + 10^3 \left[ \frac{1}{\left( \frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2} \right)^6} \right]^6$$

**Input:**

$$(-144 - 13 - 21^2) + 10^3 \times \frac{1}{\left( \frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2} \right)^6}$$

**Exact result:**

$$\frac{77705415485789045760021}{76218120810002382848}$$

**Decimal approximation:**

1019.513662367696158027720527054348874626859874674635042818...

1019.513662... result practically equal to the rest mass of Phi meson 1019.445

$$\left[ \left[ \left[ \left[ (-144 - 13 - 21^2) + \left[ \frac{10^3}{\left( \frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2} \right)^6} \right]^6 \right]^6 \right]^6 \right]^6 \right]^6$$

**Input:**

$$\sqrt[14]{(-144 - 13 - 21^2) + \frac{10^3}{\left( \frac{17472}{1 + 1 - 65 \times 2 + 910 \times 2 - 1417 \times 2 - 6994 \times 2 + 10049 \times 2 + 6981 \times 2} \right)^6}}$$

**Result:**



$$\frac{\sqrt[14]{77705415485789045760021}}{2 \times 2^{13/14} \times 91^{3/7}}$$

**Decimal approximation:**

1.640156229772129765747784256473260022997596705858787698650...

$$1.64015622... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

**Alternate form:**

$$\frac{1}{364} \sqrt[14]{155410830971578091520042} 91^{4/7}$$

$$(-21-1)/(10^3)+\left[\left[\left[\left[(-144-13-21^2)+\left[10^3/\left(\left(\left(17472/(1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2)\right)\right)^6\right]\right]\right]\right]\right]^{1/14}$$

**Input:**

$$\frac{-21-1}{10^3} + \sqrt[14]{(-144-13-21^2) + \frac{10^3}{\left(\frac{17472}{1+1-65*2+910*2-1417*2-6994*2+10049*2+6981*2}\right)^6}}$$

**Result:**

$$\frac{\sqrt[14]{77705415485789045760021}}{2 \times 2^{13/14} \times 91^{3/7}} - \frac{11}{500}$$

**Decimal approximation:**

1.618156229772129765747784256473260022997596705858787698650...

1.61815622...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Alternate forms:**

$$\frac{125 \times 91^{4/7} \sqrt[14]{155410830971578091520042} - 1001}{45500}$$

$$\frac{11 \times 2^{13/14} \times 91^{3/7} - 250 \sqrt[14]{77705415485789045760021}}{500 \times 2^{13/14} \times 91^{3/7}}$$



For P and Q = 1, we obtain:

$$1-1+14(2+0+10*2)+1+125+7((((1+5)(1+1)+6*(2(1+1)+9)))) = 1064$$

$$1064 = 0$$

Thence:

$$1064 / 0$$

**Input:**

$$\frac{1064}{0}$$

**Result:**

$\infty$  (supersymmetric condition  $\rightarrow \infty$ )

Or:

$$0 / 1064$$

**Input:**

$$\frac{0}{1064}$$

**Exact result:**

$$0$$

0 (supersymmetric condition  $\rightarrow 0$ )

If we take only the expression, we obtain 1064. Thence, we can to obtain:

$$((((1-1+14(2+0+10*2)+1+125+7((((1+5)(1+1)+6*(2(1+1)+9))))))))^{1/14}$$

**Input:**

$$\sqrt[14]{1-1+14(2+0+10 \times 2)+1+125+7((1+5)(1+1)+6(2(1+1)+9))}$$

**Result:**

$$2^{3/14} \sqrt[14]{133}$$

**Decimal approximation:**

$$1.645167480201061406938865705004925835649619237793470364697...$$

$$1.64516748\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

If  $P := \frac{\psi(-q)}{q^{1/2}\psi(-q^5)}$  and  $Q := \frac{\psi(-q^5)}{q^{5/2}\psi(-q^{25})}$  then

$$\frac{Q^3}{P^3} - \frac{5Q^2}{P^2} - \frac{15Q}{P} - 5 \left( PQ + \frac{5}{PQ} \right) - 5 \left( Q^2 + \frac{5}{P^2} \right) - P^2Q^2 + \frac{5^2}{P^2Q^2} - 15 = 0. \quad (68)$$

For  $P = 1$  and  $Q = 2$ , we obtain:

$$(((8-5*4-15*2-5(2+5/2))-5(4+5)-2^2+25/4)))$$

**Input:**

$$8 - 5 \times 4 - 15 \times 2 - 5 \left( 2 + \frac{5}{2} \right) - 5(4+5) - 2^2 + \frac{25}{4}$$

**Exact result:**

$$-\frac{429}{4}$$

**Decimal form:**

$$-107.25$$

$$-107.25$$

$$(((8-5*4-15*2-5(2+5/2))-5(4+5)-2^2+25/4)))/0$$

**Input:**

$$\frac{1}{0} \left( 8 - 5 \times 4 - 15 \times 2 - 5 \left( 2 + \frac{5}{2} \right) - 5(4+5) - 2^2 + \frac{25}{4} \right)$$

**Result:**

$\infty$  (supersymmetric condition  $\rightarrow \infty$ )

$\infty$  is complex infinity

If we take only the result -107.25, we obtain:

$$55 + \frac{1}{16} \left( (8 - 5 \times 4 - 15 \times 2 - 5(2 + \frac{5}{2}) - 5(4 + 5) - 2^2 + \frac{25}{4}) \right)^2$$

**Input:**

$$55 + \frac{1}{16} \left( 8 - 5 \times 4 - 15 \times 2 - 5 \left( 2 + \frac{5}{2} \right) - 5(4 + 5) - 2^2 + \frac{25}{4} \right)^2$$

**Exact result:**

$$\frac{198\,121}{256}$$

**Decimal form:**

773.91015625

773.910... result very near to the rest mass of Charged rho meson 775.4

$$\frac{1}{84} \left( (8 - 5 \times 4 - 15 \times 2 - 5(2 + \frac{5}{2}) - 5(4 + 5) - 2^2 + \frac{25}{4}) \right)^2$$

**Input:**

$$\frac{1}{84} \left( 8 - 5 \times 4 - 15 \times 2 - 5 \left( 2 + \frac{5}{2} \right) - 5(4 + 5) - 2^2 + \frac{25}{4} \right)^2$$

**Exact result:**

$$\frac{61\,347}{448}$$

**Decimal approximation:**

136.9352678571428571428571428571428571428571428571428...

136.93526...

This result is very near to the inverse of fine-structure constant 137,035

And:

$$- \left( (8 - 5 \times 4 - 15 \times 2 - 5(2 + \frac{5}{2}) - 5(4 + 5) - 2^2 + \frac{25}{4}) \right) + 21 + 8 + 3$$

**Input:**

$$- \left( 8 - 5 \times 4 - 15 \times 2 - 5 \left( 2 + \frac{5}{2} \right) - 5(4 + 5) - 2^2 + \frac{25}{4} \right) + 21 + 8 + 3$$

**Exact result:**

$$\frac{557}{4}$$

**Decimal form:**

139.25

139.25 result very near to the rest mass of Pion meson 139.57

From:

**Schwarzschild meets Ramanujan:**

**From quantum black holes to mock modular forms**

Boris Pioline, LPTHE, Paris - Math-Physics Colloquium

University of Amsterdam, 7/06/2019



Letter to G. H. Hardy (Jan 1920, Madras): *I am extremely sorry for not writing you a single letter up to now... I discovered very interesting functions recently which I call "Mock" theta-functions. Unlike the "False" theta-functions... they enter into mathematics as beautifully as the ordinary theta functions. I am sending you with this letter some examples.*



$$f(\tau) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(1+q)^2(1+q^2)^2 \dots (1+q^n)^2} = 1 + q - 2q^2 + 3q^3 + \dots$$

- Recall that  $f(\tau) = \sum_{n \geq 0} a_n q^{n-\Delta}$  (with  $q = e^{2\pi i \tau}$ ,  $\text{Im} \tau > 0$ ) is a modular form of weight  $k$  under  $\Gamma \subset SL(2, \mathbb{Z})$  if

$$f\left(\frac{a\tau + b}{c\tau + d}\right) = \chi(\gamma) (c\tau + d)^k f(\tau) \quad \forall \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$$



- Examples include *Dedekind's* function  $\eta(\tau) = q^{1/24} \prod_{n \geq 1} (1 - q^n)$  and *Jacobi's* theta function  $\theta(\tau) = \sum_{n=0}^{\infty} q^{n^2}$  which are modular forms of weight  $1/2$  under  $SL(2, \mathbb{Z})$  and  $\Gamma_0(4)$ , respectively.

From:

$$f(\tau) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(1+q)^2(1+q^2)^2 \dots (1+q^n)^2} = 1 + q - 2q^2 + 3q^3 + \dots$$

For  $n = -1$  and  $q = e^{-2\pi}$ , we obtain:



$((e^{-2\pi}))$

**Input:**

$$e^{-2\pi}$$

**Decimal approximation:**

0.001867442731707988814430212934827030393422805002475317199...

0.001867442731...

**Property:**

$e^{-2\pi}$  is a transcendental number

**Series representations:**

$$e^{-2\pi} = e^{-8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{-2\pi} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-2\pi}$$

$$e^{-2\pi} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-2\pi}$$

n! is the factorial function

**Integral representations:**

$$e^{-2\pi} = e^{-8 \int_0^1 \sqrt{1-t^2} dt}$$

$$e^{-2\pi} = e^{-4 \int_0^1 1/\sqrt{1-t^2} dt}$$

$$e^{-2\pi} = e^{-4 \int_0^{\infty} 1/(1+t^2) dt}$$

$$(0.001867442731)/(((((((1+0.001867442731)^2 * (1+0.001867442731^2)^2 * (1+1/0.001867442731^2))))))))))$$

**Input interpretation:**

$$\frac{0.001867442731}{(1 + 0.001867442731)^2 (1 + 0.001867442731^2)^2 \left(1 + \frac{1}{0.001867442731}\right)^2}$$

**Result:**

6.4639470824775064808643561686889066041221937180408037... × 10<sup>-9</sup>

6.463947... × 10<sup>-9</sup>

Or:

$$\frac{((e^{-2\pi}))}{(((1 + (e^{-2\pi})))^2 * (((1 + [(e^{-2\pi}])^2])))^2 * ((1 + ((1/e^{-2\pi}))))^2}$$

**Input:**

$$\frac{e^{-2\pi}}{((1 + e^{-2\pi})^2 (1 + (e^{-2\pi})^2))^2 \left(1 + \frac{1}{e^{-2\pi}}\right)^2}$$

**Exact result:**

$$\frac{e^{-2\pi}}{(1 + e^{-4\pi})^2 (1 + e^{-2\pi})^4 (1 + e^{2\pi})^2}$$

**Decimal approximation:**

6.4398724457871043047944550696621109774959482098774950... × 10<sup>-9</sup>

6.43987244... × 10<sup>-9</sup>

**Property:**

$$\frac{e^{-2\pi}}{(1 + e^{-4\pi})^2 (1 + e^{-2\pi})^4 (1 + e^{2\pi})^2} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{256} e^{4\pi} \operatorname{sech}^6(\pi) \operatorname{sech}^2(2\pi)$$

$$\frac{e^{14\pi}}{(1 + e^{2\pi})^6 (1 + e^{4\pi})^2}$$

$$-\frac{1}{4(1 + e^{2\pi})^6} + \frac{5}{4(1 + e^{2\pi})^5} - \frac{9}{4(1 + e^{2\pi})^4} + \frac{3}{2(1 + e^{2\pi})^3} + \frac{1}{16(1 + e^{2\pi})^2} - \frac{3}{16(1 + e^{2\pi})} + \frac{1}{8(1 + e^{4\pi})^2} + \frac{3e^{2\pi} - 4}{16(1 + e^{4\pi})}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

**Alternative representations:**

$$\frac{e^{-2\pi}}{((1 + e^{-2\pi})^2 (1 + (e^{-2\pi})^2))^2 \left(1 + \frac{1}{e^{-2\pi}}\right)^2} = \frac{e^{-360^\circ}}{\left(1 + \frac{1}{e^{-360^\circ}}\right)^2 ((1 + (e^{-360^\circ})^2)(1 + e^{-360^\circ})^2)^2}$$

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{\exp^{-2\pi}(z)}{\left(\left(1+\exp^{-2\pi}(z)\right)^2\left(1+\exp^{-2\pi}(z)^2\right)\right)^2\left(1+\frac{1}{\exp^{-2\pi}(z)}\right)^2} \text{ for } z = 1$$

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{e^{2i\log(-1)}}{\left(1+\frac{1}{e^{2i\log(-1)}}\right)^2\left(\left(1+\left(e^{2i\log(-1)}\right)^2\right)\left(1+e^{2i\log(-1)}\right)^2\right)^2}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

### Series representations:

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{e^{56\sum_{k=0}^{\infty}(-1)^k/(1+2k)}}{\left(1+e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^6\left(1+e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)^2}$$

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{14\pi}}{\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\right)^6\left(1+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}\right)^2}$$

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{14\pi}}{\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}\right)^6\left(1+\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}\right)^2}$$

$n!$  is the factorial function

### Integral representations:

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{e^{28\int_0^{\infty}1/(1+t^2)dt}}{\left(1+e^{4\int_0^{\infty}1/(1+t^2)dt}\right)^6\left(1+e^{8\int_0^{\infty}1/(1+t^2)dt}\right)^2}$$

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{e^{28} \int_0^{\infty} \sin(t)/t dt}{\left(1+e^4 \int_0^{\infty} \sin(t)/t dt\right)^6 \left(1+e^8 \int_0^{\infty} \sin(t)/t dt\right)^2}$$

$$\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2} = \frac{e^{56} \int_0^1 \sqrt{1-t^2} dt}{\left(1+e^8 \int_0^1 \sqrt{1-t^2} dt\right)^6 \left(1+e^{16} \int_0^1 \sqrt{1-t^2} dt\right)^2}$$

- By viewing black holes as **black strings wrapped on a circle**, micro-states can be viewed as excitations in a 2D **superconformal field theory**. The partition function  $Z_{T^2(\tau)}$  is then manifestly modular. BPS ground states are chiral excitations of the SCFT, counted (with sign) by a weakly holomorphic modular form.
- E.g. in type II string theory compactified on  $T^6$ ,  $\Omega(Q) = c(l_4(Q))$  where  $l_4$  is a quartic polynomial in  $Q$  and  $c(n)$  are Fourier coefficients of the modular form

$$\frac{\theta(\tau)}{\eta^6(4\tau)} = \sum_{n \geq -1} c(n) q^n = \frac{1}{q} + 2 + 8q^3 + 12q^4 + 39q^7 + 56q^8 + \dots$$

*Moore Maldacena Strominger 1999, BP 2005, Shih Strominger Yin 2005*

- The Hardy-Ramanujan formula (aka. Cardy's formula in physics) gives  $c(n) \sim e^{\pi\sqrt{n}}$ , in perfect agreement with gravity's prediction  $\Omega(Q) \sim e^{\pi\sqrt{l_4(Q)}}$ .

From:

$$\frac{\theta(\tau)}{\eta^6(4\tau)} = \sum_{n \geq -1} c(n) q^n = \frac{1}{q} + 2 + 8q^3 + 12q^4 + 39q^7 + 56q^8 + \dots$$

For  $q = e^{-2\pi}$

$$1/(e^{-2\pi}) + 2 + 8*(e^{-2\pi})^3 + 12*(e^{-2\pi})^4 + 39*(e^{-2\pi})^7 + 56*(e^{-2\pi})^8$$

**Input:**

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8$$

**Exact result:**

$$2 + 56 e^{-16\pi} + 39 e^{-14\pi} + 12 e^{-8\pi} + 8 e^{-6\pi} + e^{2\pi}$$

**Decimal approximation:**

537.4916555770099722752985714201740043996997680857133693461...

**Property:**

$2 + 56 e^{-16\pi} + 39 e^{-14\pi} + 12 e^{-8\pi} + 8 e^{-6\pi} + e^{2\pi}$  is a transcendental number

**Alternate forms:**

$$2 + e^{-16\pi} (56 + 39 e^{2\pi} + 12 e^{8\pi} + 8 e^{10\pi} + e^{18\pi})$$

$$e^{-16\pi} (56 + 39 e^{2\pi} + 12 e^{8\pi} + 8 e^{10\pi} + 2 e^{16\pi} + e^{18\pi})$$

**Alternative representations:**

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 = \frac{1}{\exp^{-2\pi}(z)} + 2 + 8 \exp^{-2\pi}(z)^3 + 12 \exp^{-2\pi}(z)^4 + 39 \exp^{-2\pi}(z)^7 + 56 \exp^{-2\pi}(z)^8 \text{ for } z = 1$$

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 = 2 + \frac{1}{e^{-360^\circ}} + 8(e^{-360^\circ})^3 + 12(e^{-360^\circ})^4 + 39(e^{-360^\circ})^7 + 56(e^{-360^\circ})^8$$

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 = \frac{1}{\exp^{-2 \cos^{-1}(-1)}(z)} + 2 + 8 \exp^{-2 \cos^{-1}(-1)}(z)^3 + 12 \exp^{-2 \cos^{-1}(-1)}(z)^4 + 39 \exp^{-2 \cos^{-1}(-1)}(z)^7 + 56 \exp^{-2 \cos^{-1}(-1)}(z)^8 \text{ for } z = 1$$

$\cos^{-1}(x)$  is the inverse cosine function

**Series representations:**

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 = 2 + 56 e^{-64 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + 39 e^{-56 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + 12 e^{-32 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + 8 e^{-24 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-16\pi}$$

$$\left(56 + 39\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} + 12\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8\pi} + 8\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10\pi} + 2\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16\pi} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{18\pi}\right)$$

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 =$$

$$\left(56 + 39\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi} + 12\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi} + 8\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi} +$$

$$2\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{16\pi} + \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{18\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-16\pi}$$

$n!$  is the factorial function

### Integral representations:

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 = 2 + 56e^{-64} \int_0^1 \sqrt{1-t^2} dt +$$

$$39e^{-56} \int_0^1 \sqrt{1-t^2} dt + 12e^{-32} \int_0^1 \sqrt{1-t^2} dt + 8e^{-24} \int_0^1 \sqrt{1-t^2} dt + e^8 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 =$$

$$2 + 56e^{-128/3} \int_0^{\infty} \frac{\sin^3(t)}{t^3} dt + 39e^{-112/3} \int_0^{\infty} \frac{\sin^3(t)}{t^3} dt +$$

$$12e^{-64/3} \int_0^{\infty} \frac{\sin^3(t)}{t^3} dt + 8e^{-16} \int_0^{\infty} \frac{\sin^3(t)}{t^3} dt + e^{16/3} \int_0^{\infty} \frac{\sin^3(t)}{t^3} dt$$

$$\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 =$$

$$e^{-32} \int_0^{\infty} \frac{1}{(1+t^2)} dt \left(56 + 39e^4 \int_0^{\infty} \frac{1}{(1+t^2)} dt + 12e^{16} \int_0^{\infty} \frac{1}{(1+t^2)} dt +$$

$$8e^{20} \int_0^{\infty} \frac{1}{(1+t^2)} dt + 2e^{32} \int_0^{\infty} \frac{1}{(1+t^2)} dt + e^{36} \int_0^{\infty} \frac{1}{(1+t^2)} dt\right)$$

Note that, inverting the formula, we obtain:

$$1/\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{e^{-2\pi}}\right) + 2 + 8*(e^{-2\pi})^3 + 12*(e^{-2\pi})^4 + 39*(e^{-2\pi})^7 + 56*(e^{-2\pi})^8\right)\right)\right)\right)\right)\right)\right)\right)\right)$$

**Input:**

$$\frac{1}{\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8}$$

**Exact result:**

$$\frac{1}{2 + 56 e^{-16\pi} + 39 e^{-14\pi} + 12 e^{-8\pi} + 8 e^{-6\pi} + e^{2\pi}}$$

**Decimal approximation:**

0.001860493999532841879956621007555187996639000019261084218...  
0.0018604939...

**Property:**

$$\frac{1}{2 + 56 e^{-16\pi} + 39 e^{-14\pi} + 12 e^{-8\pi} + 8 e^{-6\pi} + e^{2\pi}} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{2 + e^{-16\pi} (56 + 39 e^{2\pi} + 12 e^{8\pi} + 8 e^{10\pi} + e^{18\pi})}$$

$$\frac{e^{16\pi}}{56 + 39 e^{2\pi} + 12 e^{8\pi} + 8 e^{10\pi} + 2 e^{16\pi} + e^{18\pi}}$$

**Alternative representations:**

$$\frac{1}{\frac{1}{e^{-2\pi}} + 2 + 8 (e^{-2\pi})^3 + 12 (e^{-2\pi})^4 + 39 (e^{-2\pi})^7 + 56 (e^{-2\pi})^8} =$$

$$\frac{1}{2 + \frac{1}{e^{-360^\circ}} + 8 (e^{-360^\circ})^3 + 12 (e^{-360^\circ})^4 + 39 (e^{-360^\circ})^7 + 56 (e^{-360^\circ})^8}$$

$$\frac{1}{\frac{1}{e^{-2\pi}} + 2 + 8 (e^{-2\pi})^3 + 12 (e^{-2\pi})^4 + 39 (e^{-2\pi})^7 + 56 (e^{-2\pi})^8} =$$

$$\frac{1}{\frac{1}{\exp^{-2\pi(z)}} + 2 + 8 \exp^{-2\pi(z)^3} + 12 \exp^{-2\pi(z)^4} + 39 \exp^{-2\pi(z)^7} + 56 \exp^{-2\pi(z)^8}} \text{ for } z = 1$$

$$\frac{1}{\frac{1}{e^{-2\pi}} + 2 + 8 (e^{-2\pi})^3 + 12 (e^{-2\pi})^4 + 39 (e^{-2\pi})^7 + 56 (e^{-2\pi})^8} =$$

$$1 / \left( \frac{1}{\exp^{-2 \cos^{-1}(-1)(z)}} + 2 + 8 \exp^{-2 \cos^{-1}(-1)(z)^3} + 12 \exp^{-2 \cos^{-1}(-1)(z)^4} + \right.$$

$$\left. 39 \exp^{-2 \cos^{-1}(-1)(z)^7} + 56 \exp^{-2 \cos^{-1}(-1)(z)^8} \right) \text{ for } z = 1$$

$\cos^{-1}(x)$  is the inverse cosine function

**Series representations:**



$$\frac{1}{e^{-2\pi} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8} =$$

$$1 / \left( 2 + 56 e^{-64} \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)} + 39 e^{-56} \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)} + \right.$$

$$\left. 12 e^{-32} \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)} + 8 e^{-24} \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)} + e^8 \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)} \right)$$

$$\frac{1}{e^{-2\pi} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{16\pi} /$$

$$\left( 56 + 39 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} + 12 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{8\pi} + 8 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10\pi} + 2 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{16\pi} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{18\pi} \right)$$

$$\frac{1}{e^{-2\pi} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8} =$$

$$\left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{16\pi} / \left( 56 + 39 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} + 12 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{8\pi} + \right.$$

$$\left. 8 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{10\pi} + 2 \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{16\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{18\pi} \right)$$

$n!$  is the factorial function

### Integral representations:

$$\frac{1}{e^{-2\pi} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8} =$$

$$1 / \left( 2 + 56 e^{-64} \int_0^1 \sqrt{1-t^2} dt + 39 e^{-56} \int_0^1 \sqrt{1-t^2} dt + \right.$$

$$\left. 12 e^{-32} \int_0^1 \sqrt{1-t^2} dt + 8 e^{-24} \int_0^1 \sqrt{1-t^2} dt + e^8 \int_0^1 \sqrt{1-t^2} dt \right)$$

$$\frac{1}{e^{-2\pi} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8} =$$

$$e^{32} \int_0^{\infty} \sin(t)/t dt / \left( 56 + 39 e^4 \int_0^{\infty} \sin(t)/t dt + 12 e^{16} \int_0^{\infty} \sin(t)/t dt + \right.$$

$$\left. 8 e^{20} \int_0^{\infty} \sin(t)/t dt + 2 e^{32} \int_0^{\infty} \sin(t)/t dt + e^{36} \int_0^{\infty} \sin(t)/t dt \right)$$

$$\frac{1}{\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8} = \frac{e^{32} \int_0^{\infty} \frac{1}{(1+t^2)} dt}{\left( 56 + 39 e^4 \int_0^{\infty} \frac{1}{(1+t^2)} dt + 12 e^{16} \int_0^{\infty} \frac{1}{(1+t^2)} dt + 8 e^{20} \int_0^{\infty} \frac{1}{(1+t^2)} dt + 2 e^{32} \int_0^{\infty} \frac{1}{(1+t^2)} dt + e^{36} \int_0^{\infty} \frac{1}{(1+t^2)} dt \right)}$$

a result equal to the value of  $q = 0.0018604939\dots$

From the expression from which we obtain the value  $6.43987244\dots \cdot 10^{-9}$ , we have:

$$\text{colog} \left( \frac{(((((e^{-2\pi}))) / (((((1 + ((e^{-2\pi}))))^2 * (((((1 + [((e^{-2\pi}))))^2))))))))))^2 * (((1 + (((1/e^{-2\pi}))))^2))) \right)$$

**Input:**

$$-\log \left( \frac{e^{-2\pi}}{\left( (1 + e^{-2\pi})^2 (1 + (e^{-2\pi})^2) \right)^2 \left( 1 + \frac{1}{e^{-2\pi}} \right)^2} \right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$-\log \left( \frac{e^{-2\pi}}{(1 + e^{-4\pi})^2 (1 + e^{-2\pi})^4 (1 + e^{2\pi})^2} \right)$$

**Decimal approximation:**

18.86075710358109877419657920223437390281214331253857949495...

18.8607571... result very near to the black hole entropy 18.7328

**Alternate forms:**

$$2(-7\pi + 3 \log(1 + e^{2\pi}) + \log(1 + e^{4\pi}))$$

$$-14\pi + 6 \log(1 + e^{2\pi}) + 2 \log(1 + e^{4\pi})$$

$$-2(7\pi - 3 \log(1 + e^{2\pi}) - \log(1 + e^{4\pi}))$$

**Alternative representations:**

$$\begin{aligned}
& -\log\left(\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2}\right) = \\
& -\log_e\left(\frac{e^{-2\pi}}{\left(1+\frac{1}{e^{-2\pi}}\right)^2\left(\left(1+\left(e^{-2\pi}\right)^2\right)\left(1+e^{-2\pi}\right)^2\right)^2}\right) \\
& -\log\left(\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2}\right) = \\
& -\log(a)\log_a\left(\frac{e^{-2\pi}}{\left(1+\frac{1}{e^{-2\pi}}\right)^2\left(\left(1+\left(e^{-2\pi}\right)^2\right)\left(1+e^{-2\pi}\right)^2\right)^2}\right) \\
& -\log\left(\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2}\right) = \\
& \text{Li}_1\left(1-\frac{e^{-2\pi}}{\left(1+\frac{1}{e^{-2\pi}}\right)^2\left(\left(1+\left(e^{-2\pi}\right)^2\right)\left(1+e^{-2\pi}\right)^2\right)^2}\right)
\end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm  
 $\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$-\log\left(\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2}\right) = \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{e^{14\pi}}{(1+e^{2\pi})^6(1+e^{4\pi})^2}\right)^k}{k}$$

$$-\log\left(\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2}\right) = \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{e^{-2\pi}}{(1+e^{-4\pi})^2(1+e^{-2\pi})^4(1+e^{2\pi})^2}\right)^k}{k}$$

$$-\log\left(\frac{e^{-2\pi}}{\left(\left(1+e^{-2\pi}\right)^2\left(1+\left(e^{-2\pi}\right)^2\right)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2}\right) = -2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{e^{-2\pi}}{(1+e^{-4\pi})^2(1+e^{-2\pi})^4(1+e^{2\pi})^2} - z_0\right)^k}{k} z_0^{-k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representation:

$$-\log\left(\frac{e^{-2\pi}}{\left(\left((1+e^{-2\pi})^2(1+(e^{-2\pi})^2)\right)^2\left(1+\frac{1}{e^{-2\pi}}\right)^2\right)}\right) = -\int_1^{e^{14\pi}} \frac{1}{(1+e^{2\pi})^6(1+e^{4\pi})^2} \frac{1}{t} dt$$

We take the result as a value of entropy (indeed we have the value of BH entropy 18.73, see Table) and obtain from the Hawking radiation calculator the following values:

$$\text{Mass} = 4.046535e-8$$

$$\text{Radius} = 6.008504e-35$$

$$\text{Temperature} = 3.032726e+30$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}\left[\left[\left[\left[\frac{1}{\left(\left(\left(\left(4 \times 1.962364415e+19\right) / \left(5 \times 0.0864055^2\right)\right)\right) \times \frac{1}{4.046535e-8}\right) \times \text{sqrt}\left[\left[\frac{3.032726e+30 \times 4 \times \pi \times \left(6.008504e-35\right)^3 - \left(6.008504e-35\right)^2\right]}{\left(6.67 \times 10^{-11}\right)}\right]\right]\right]\right]\right]$$

### Input interpretation:

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.046535 \times 10^{-8}}\right)\right) \sqrt{\left(\frac{3.032726 \times 10^{30} \times 4 \pi \left(6.008504 \times 10^{-35}\right)^3 - \left(6.008504 \times 10^{-35}\right)^2}{6.67 \times 10^{-11}}\right)}}$$

### Result:

$$1.618249195139212749989026860015151305427028314306734727343...$$

$$1.61824919...$$

From the result 537.49165..., we obtain good approximations to the values of circle length with unitary radius and of  $\zeta(2)$ :

$$\ln\left(\left(\left(\left(\left(\frac{1}{e^{-2\pi}}\right) + 2 + 8 \times \left(e^{-2\pi}\right)^3 + 12 \times \left(e^{-2\pi}\right)^4 + 39 \times \left(e^{-2\pi}\right)^7 + 56 \times \left(e^{-2\pi}\right)^8\right)\right)\right)\right)$$

**Input:**

$$\log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log(2 + 56e^{-16\pi} + 39e^{-14\pi} + 12e^{-8\pi} + 8e^{-6\pi} + e^{2\pi})$$

**Decimal approximation:**

$$6.286913235373423019790878185469644688178126417823403437264\dots$$

$$6.2869132353 \approx 2\pi$$

**Alternate forms:**

$$\log(2 + e^{-16\pi}(56 + 39e^{2\pi} + 12e^{8\pi} + 8e^{10\pi} + e^{18\pi}))$$

$$\log(56 + 39e^{2\pi} + 12e^{8\pi} + 8e^{10\pi} + 2e^{16\pi} + e^{18\pi}) - 16\pi$$

**Alternative representations:**

$$\begin{aligned} \log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) = \\ \log_e\left(2 + \frac{1}{e^{-2\pi}} + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) \end{aligned}$$

$$\begin{aligned} \log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) = \\ \log(a) \log_a\left(2 + \frac{1}{e^{-2\pi}} + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) \end{aligned}$$

$$\begin{aligned} \log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) = \\ -\text{Li}_1\left(-1 - \frac{1}{e^{-2\pi}} - 8(e^{-2\pi})^3 - 12(e^{-2\pi})^4 - 39(e^{-2\pi})^7 - 56(e^{-2\pi})^8\right) \end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) =$$

$$\log(1 + 56e^{-16\pi} + 39e^{-14\pi} + 12e^{-8\pi} + 8e^{-6\pi} + e^{2\pi}) -$$

$$\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1+56e^{-16\pi}+39e^{-14\pi}+12e^{-8\pi}+8e^{-6\pi}+e^{2\pi}}\right)^k}{k}$$

$$\log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) =$$

$$2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k (2 + 56e^{-16\pi} + 39e^{-14\pi} + 12e^{-8\pi} + 8e^{-6\pi} + e^{2\pi} - z_0)^k z_0^{-k}}{k}$$

$$\log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) =$$

$$2i\pi \left[ \frac{\arg(2 + 56e^{-16\pi} + 39e^{-14\pi} + 12e^{-8\pi} + 8e^{-6\pi} + e^{2\pi} - x)}{2\pi} \right] + \log(x) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k (2 + 56e^{-16\pi} + 39e^{-14\pi} + 12e^{-8\pi} + 8e^{-6\pi} + e^{2\pi} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) =$$

$$\int_1^{2+56e^{-16\pi}+39e^{-14\pi}+12e^{-8\pi}+8e^{-6\pi}+e^{2\pi}} \frac{1}{t} dt$$

$$\log\left(\frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8\right) =$$

$$-\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(1 + 56e^{-16\pi} + 39e^{-14\pi} + 12e^{-8\pi} + 8e^{-6\pi} + e^{2\pi})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

$\Gamma(x)$  is the gamma function

And:

$$1/24(((((((\ln(((((((1/(e^{(-2\pi)}) + 2 + 8*(e^{(-2\pi)})^3 + 12*(e^{(-2\pi)})^4 + 39*(e^{(-2\pi)})^7 + 56*(e^{(-2\pi)})^8)))))))))))))^2$$

**Input:**

$$\frac{1}{24} \log^2 \left( \frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 \right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{1}{24} \log^2 \left( 2 + 56 e^{-16\pi} + 39 e^{-14\pi} + 12 e^{-8\pi} + 8 e^{-6\pi} + e^{2\pi} \right)$$

**Decimal approximation:**

$$1.646886584546396728162221781029730265112214318179980282531\dots$$

$$1.64688658\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

**Alternate forms:**

$$\frac{1}{24} \log^2 \left( 2 + e^{-16\pi} (56 + 39 e^{2\pi} + 12 e^{8\pi} + 8 e^{10\pi} + e^{18\pi}) \right)$$

$$\frac{1}{24} \left( 16\pi - \log(56 + 39 e^{2\pi} + 12 e^{8\pi} + 8 e^{10\pi} + 2 e^{16\pi} + e^{18\pi}) \right)^2$$

**Alternative representations:**

$$\begin{aligned} \frac{1}{24} \log^2 \left( \frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 \right) = \\ \frac{1}{24} \log_e^2 \left( 2 + \frac{1}{e^{-2\pi}} + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 \right) \end{aligned}$$

$$\begin{aligned} \frac{1}{24} \log^2 \left( \frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 \right) = \\ \frac{1}{24} \left( \log(a) \log_a \left( 2 + \frac{1}{e^{-2\pi}} + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 \right) \right)^2 \end{aligned}$$

$$\begin{aligned} \frac{1}{24} \log^2 \left( \frac{1}{e^{-2\pi}} + 2 + 8(e^{-2\pi})^3 + 12(e^{-2\pi})^4 + 39(e^{-2\pi})^7 + 56(e^{-2\pi})^8 \right) = \\ \frac{1}{24} \left( -\text{Li}_1 \left( -1 - \frac{1}{e^{-2\pi}} - 8(e^{-2\pi})^3 - 12(e^{-2\pi})^4 - 39(e^{-2\pi})^7 - 56(e^{-2\pi})^8 \right) \right)^2 \end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function



Thence the colog of the result of mock theta function analyzed and equated with the value of an entropy, corresponds to a Black Hole of mass =  $4.046535 \times 10^{-8}$  kg equivalent to a mass of  $2.26994 \times 10^{19}$  GeV, **practically near to the mean value  $1.962 \times 10^{19}$  of DM particle that has a Planck scale mass:  $m \approx 10^{19}$  GeV (Planck mass =  $1,2209 \times 10^{19}$  GeV/c<sup>2</sup> = 21,76  $\mu$ g Wikipedia) and is very nearly to the result of the following Ramanujan mock theta function:  $\chi(\mathbf{q}) = 1.962364415$**

Indeed:

**Input interpretation:**

convert  $4.046535 \times 10^{-8}$  kg (kilograms)  
to gigaelectronvolts per speed of light squared

**Result:**

$2.26994 \times 10^{19}$  GeV/c<sup>2</sup>  
 $2.26994 \times 10^{19}$  GeV

From the following 14-th root of Ramanujan class invariant 1164.2696

$$\sqrt[14]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,65578 \dots$$

((((((((sqrt((1/8(113+5sqrt(505)))))+sqrt((1/8(105+5sqrt(505))))))^3))))))^1/14

**Input:**

$$\sqrt[14]{\left(\sqrt{\frac{1}{8}(113 + 5\sqrt{505})} + \sqrt{\frac{1}{8}(105 + 5\sqrt{505})}\right)^3}$$

**Exact result:**

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}(105 + 5\sqrt{505})} + \frac{1}{2}\sqrt{\frac{1}{2}(113 + 5\sqrt{505})}\right)^{3/14}$$

**Decimal approximation:**

1.655784548804744724619349561761107639558068114480697960239...  
1.6557845488.....

**Alternate forms:**

$$\sqrt[28]{338881 + 15080\sqrt{505} + 4\sqrt{5(2871007052 + 127758137\sqrt{505})}}$$

$$\frac{(5\sqrt{5} + \sqrt{101} + \sqrt{105 - 40i} + \sqrt{105 + 40i})^{3/14}}{2^{3/7}}$$

$$\frac{\left(\sqrt{5(21 + \sqrt{505})} + \sqrt{113 + 5\sqrt{505}}\right)^{3/14}}{2^{9/28}}$$

**Minimal polynomial:**

$$x^{112} - 1355524x^{84} + 400646x^{56} - 1355524x^{28} + 1$$

From the mass  $2.26994 \times 10^{19}$ , we obtain:

$$(2.26994 \times 10^{19}) / \left( \left( \left( \left( \left( \left( \sqrt{\frac{1}{8}(113 + 5\sqrt{505})} \right) + \sqrt{\frac{1}{8}(105 + 5\sqrt{505})} \right) \right) \right) \right) \right)^{1/14} \times 1/0.69897$$

Where 0.69897 is a Hausdorff dimension  $\log_{10}(5) = 0.698970004336...$

**Input interpretation:**

$$\frac{2.26994 \times 10^{19}}{\sqrt[14]{\left(\sqrt{\frac{1}{8}(113 + 5\sqrt{505})} + \sqrt{\frac{1}{8}(105 + 5\sqrt{505})}\right)^3}} \times \frac{1}{0.69897}$$

**Result:**

$$1.96134... \times 10^{19}$$

$$1.96134... * 10^{19} \text{ GeV}$$

practically very near to the mean value  $1.962 \times 10^{19}$  of DM particle that has a Planck scale mass:  $m \approx 10^{19}$  GeV (Planck mass =  $1,2209 \times 10^{19}$  GeV/c<sup>2</sup> = 21,76 μg Wikipedia)

from:

**MODULAR EQUATIONS IN THE SPIRIT OF RAMANUJAN**  
M. S. Mahadeva Naika

We consider P and Q that are equal to P = -2.05860 Q = -8.72406

$$\begin{aligned}
 & \text{If } P = \frac{\varphi(q)}{\varphi(q^2)} \frac{\varphi(q^7)}{\varphi(q^{14})} \text{ and } Q = \frac{\varphi(q)}{\varphi(q^2)} \frac{\varphi(q^{14})}{\varphi(q^7)}, \text{ then} \\
 & Q^4 + \frac{1}{Q^4} + 56 \left( Q^3 + \frac{1}{Q^3} \right) + 252 \left( Q^2 + \frac{1}{Q^2} \right) + 1064 \left( Q + \frac{1}{Q} \right) \\
 & - 64 \left( P^3 + \frac{8}{P^3} \right) + 1078 = 112 \left\{ \left( P + \frac{2}{P} \right) \left[ 2 \left( Q^2 + \frac{1}{Q^2} \right) \right. \right. \\
 & \left. \left. + 3 \left( Q + \frac{1}{Q} \right) + 8 \right] - \left( P^2 + \frac{4}{P^2} \right) \left[ 2 \left( Q + \frac{1}{Q} \right) + 1 \right] \right\}. \tag{43}
 \end{aligned}$$

$$112 * (((((((((-2.05860 + 2 / (-2.05860)) * (((2 * (-8.72406^2) + 1 / (-8.72406)^2) + 3 * (-8.72406 + 1 / (-8.72406)) + 8)))))) - (((((-2.05860^2 + 4 / (-2.05860^2)) * ((2 * (-8.72406 + 1 / (-8.72406)) + 1))))))))))$$

**Input interpretation:**

$$\begin{aligned}
 & 112 \left( \left( -2.05860 + -\frac{2}{2.05860} \right. \right. \\
 & \quad \left. \left( \left( 2 * (-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) \right) - \\
 & \quad \left( -2.05860^2 + -\frac{4}{2.05860^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right)
 \end{aligned}$$

**Result:**

8667.195929413593108789613720368221235040594427173860111278...

8667.1959...

And:

$$-5 + 1/5 * 112 * (((((((((-2.05860 + 2 / (-2.05860)) * (((2 * (-8.72406^2) + 1 / (-8.72406)^2) + 3 * (-8.72406 + 1 / (-8.72406)) + 8)))))) - (((((-2.05860^2 + 4 / (-2.05860^2)) * ((2 * (-8.72406 + 1 / (-8.72406)) + 1))))))))))$$

**Input interpretation:**

$$-5 + \frac{1}{5} \times 112 \left( \left( -2.05860 + -\frac{2}{2.05860} \right) \left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) - \left( -2.05860^2 + -\frac{4}{2.05860^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right)$$

**Result:**

1728.439185882718621757922744073644247008118885434772022255...

1728.43918...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$\ln[112 * (((((((((-2.05860 + 2/(-2.05860)) * (((2(-8.72406^2) + 1/(-8.72406)^2) + 3(-8.72406 + 1/(-8.72406)) + 8)))) - ((((-2.05860^2 + 4/(-2.05860^2)) * ((2(-8.72406 + 1/(-8.72406)) + 1)))))))]$$

**Input interpretation:**

$$\log \left( 112 \left( \left( -2.05860 + -\frac{2}{2.05860} \right) \left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) - \left( -2.05860^2 + -\frac{4}{2.05860^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right)$$

$\log(x)$  is the natural logarithm

**Result:**

9.06730...

9.06730... result near to the black hole entropy 9.3664

**Alternative representations:**

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3(-8.72406 + -\frac{1}{8.72406}) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \\ \log_e \left( 112 \left( -2.0586 + -\frac{2 \left( 8 + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) - 2 \times 8.72406^2 + \frac{1}{(-8.72406)^2} \right)}{2.0586} \right. \right. \\ \left. \left. \left( 1 + 2 \left( -8.72406 + -\frac{1}{8.72406} \right) \right) \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \right) \right)$$

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3(-8.72406 + -\frac{1}{8.72406}) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \log(a) \\ \log_a \left( 112 \left( -2.0586 + -\frac{2 \left( 8 + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) - 2 \times 8.72406^2 + \frac{1}{(-8.72406)^2} \right)}{2.0586} \right. \right. \\ \left. \left. \left( 1 + 2 \left( -8.72406 + -\frac{1}{8.72406} \right) \right) \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \right) \right)$$

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3(-8.72406 + -\frac{1}{8.72406}) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = -\text{Li}_1 \left[ \right. \\ \left. 1 - 112 \left( -2.0586 + -\frac{2 \left( 8 + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) - 2 \times 8.72406^2 + \frac{1}{(-8.72406)^2} \right)}{2.0586} \right. \right. \\ \left. \left. \left( 1 + 2 \left( -8.72406 + -\frac{1}{8.72406} \right) \right) \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \right) \right]$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \\ \log(8666.2) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-9.06719k}}{k}$$

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \\ 2i\pi \left\lfloor \frac{\arg(8667.2 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8667.2 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \\ \left\lfloor \frac{\arg(8667.2 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(8667.2 - z_0)}{2\pi} \right\rfloor \log(z_0) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k (8667.2 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \int_1^{8667.2} \frac{1}{t} dt$$

$$\log \left( 112 \left( \left( -2.0586 + -\frac{\left( \left( 2(-8.72406^2) + \frac{1}{(-8.72406)^2} \right) + 3 \left( -8.72406 + -\frac{1}{8.72406} \right) + 8 \right) 2}{2.0586} \right) \right. \right. \\ \left. \left. \left( -2.0586^2 + -\frac{4}{2.0586^2} \right) \left( 2 \left( -8.72406 + -\frac{1}{8.72406} \right) + 1 \right) \right) \right) = \\ \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-9.06719s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

With an entropy equal to the result 9.067301 that we have obtained from the ln of the value of expression, by the Hawking radiation calculator, we have the following black hole parameters:

Mass = 2.805710e-8

Radius = 4.166063e-35

Temperature = 4.373949e+30

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[ \left[ \left[ \frac{1}{\left( \left( \left( \left( \left( 4 * 1.962364415e+19 \right) / \left( 5 * 0.0864055^2 \right) \right) \right) * 1 / \left( 2.805710e-8 \right) * \sqrt{\left[ \left[ - \left( \left( \left( 4.373949e+30 * 4 * \pi * \left( 4.166063e-35 \right)^3 - \left( 4.166063e-35 \right)^2 \right) \right) \right] / \left( \left( 6.67 * 10^{-11} \right) \right) \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{2.805710 \times 10^{-8}}\right) \sqrt{-\frac{4.373949 \times 10^{30} \times 4 \pi (4.166063 \times 10^{-35})^3 - (4.166063 \times 10^{-35})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.618249195953930216361799175981036602633174350182042907988...  
 1.61824919...

Now, we have:

$$h_{4,13} = -\sqrt{\frac{1279 + 355\sqrt{13} + 12\sqrt{a_1}}{2}} + \sqrt{\frac{1281 + 355\sqrt{13} + 12\sqrt{a_1}}{2}},$$

$$h_{4,1/13} = \sqrt{\frac{1279 + 355\sqrt{13} + 12\sqrt{a_1}}{2}} + \sqrt{\frac{1281 + 355\sqrt{13} + 12\sqrt{a_1}}{2}},$$

where  $a_1 = 22733 + 6305\sqrt{13}$ ,

We obtain:

$$\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{2} \cdot (1279 + 355\sqrt{13} + 12 \cdot \sqrt{22733 + 6305\sqrt{13}})\right)\right)\right)\right)\right)\right)\right)\right)\right)^{0.5}\right) + \left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{2} \cdot (1281 + 355\sqrt{13} + 12 \cdot \sqrt{22733 + 6305\sqrt{13}})\right)\right)\right)\right)\right)\right)\right)\right)\right)^{0.5}\right)$$

**Input:**

$$\sqrt{\frac{1}{2} \left( 1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right) + \sqrt{\frac{1}{2} \left( 1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)}}$$

**Decimal approximation:**

101.1800534855957098546733135864456977523922232456688420658...  
 101.18...

**Alternate forms:**



root of  $x^8 - 100x^7 - 120x^6 + 60x^5 + 94x^4 - 60x^3 - 120x^2 + 100x + 1$   
near  $x = 101.18$

$$\frac{1}{\sqrt{2}} \left( \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)$$

$$\frac{1}{\sqrt{2}} \left( \sqrt{1279 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} + \sqrt{1281 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} \right)$$

**Minimal polynomial:**

$$x^8 - 100x^7 - 120x^6 + 60x^5 + 94x^4 - 60x^3 - 120x^2 + 100x + 1$$

$$-\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{2}*(1279+355\sqrt{13})+12*\sqrt{(22733+6305\sqrt{13})}\right)\right)\right)\right)\right)\right)\right)\right)^{0.5}\right) - \left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{2}*(1281+355\sqrt{13})+12*\sqrt{(22733+6305\sqrt{13})}\right)\right)\right)\right)\right)\right)\right)\right)^{0.5}\right)$$

**Input:**

$$-\left( \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} - \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right)$$

**Exact result:**

$$\sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} - \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)}$$

**Decimal approximation:**

0.009883370936766335497016430294461115920248564878851856440...

0.00988337...

**Alternate forms:**

root of  $x^8 + 100x^7 - 120x^6 - 60x^5 + 94x^4 + 60x^3 - 120x^2 - 100x + 1$   
 near  $x = 0.00988337$

$$-\frac{1}{\sqrt{2}} \left( \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} - \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)$$

$$-\frac{1}{\sqrt{2}} \left( \sqrt{1279 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} - \sqrt{1281 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} \right)$$

**Minimal polynomial:**

$$x^8 + 100x^7 - 120x^6 - 60x^5 + 94x^4 + 60x^3 - 120x^2 - 100x + 1$$

Now, inverting the above expression, we obtain:

$$1 / \left( \left( \left( \left( \left( \left( \left( \left( \left( \frac{1}{2} * (1279 + 355\sqrt{13}) + 12 * \sqrt{(22733 + 6305\sqrt{13})} \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) - \left( \left( \left( \left( \left( \left( \left( \left( \left( \frac{1}{2} * (1281 + 355\sqrt{13}) + 12 * \sqrt{(22733 + 6305\sqrt{13})} \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)$$

**Input:**

$$-\left[ 1 / \left( \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} - \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) \right]$$

**Exact result:**

$$1/\left(\sqrt{\frac{1}{2}\left(1281+355\sqrt{13}+12\sqrt{22733+6305\sqrt{13}}\right)}-\sqrt{\frac{1}{2}\left(1279+355\sqrt{13}+12\sqrt{22733+6305\sqrt{13}}\right)}\right)$$

**Decimal approximation:**

101.1800534855957098546733135864456977523922232456688420658...

101.18...

**Alternate forms:**

root of  $x^8 - 100x^7 - 120x^6 + 60x^5 + 94x^4 - 60x^3 - 120x^2 + 100x + 1$   
near  $x = 101.18$

$$-\left(\sqrt{2}\right)/\left(\sqrt{1279+355\sqrt{13}+12\sqrt{22733+6305\sqrt{13}}}-\sqrt{1281+355\sqrt{13}+12\sqrt{22733+6305\sqrt{13}}}\right)$$

$$-\left(\sqrt{2}\right)/\left(\sqrt{1279+355\sqrt{13}+6\sqrt{45466-12i}+6\sqrt{45466+12i}}-\sqrt{1281+355\sqrt{13}+6\sqrt{45466-12i}+6\sqrt{45466+12i}}\right)$$

**Minimal polynomial:**

$$x^8 - 100x^7 - 120x^6 + 60x^5 + 94x^4 - 60x^3 - 120x^2 + 100x + 1$$

From the sum of the two expressions, we obtain, multiplying by 8 and dividing by  $10^3$ , the following result:

$$8/10^3 * (((101.180053485595709854673313586 + (((((((((1/2 * (1279 + 355 * \sqrt{13}) + 12 * \sqrt{(22733 + 6305 * \sqrt{13})})))^0.5)))))) + (((((((((1/2 * (1281 + 355 * \sqrt{13}) + 12 * \sqrt{(22733 + 6305 * \sqrt{13})})))^0.5))))))$$

**Input interpretation:**

$$\frac{8}{10^3} \left( 101.180053485595709854673313586 + \sqrt{\frac{1}{2} \left( 1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left( 1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} \right)$$

**Result:**

1.61888085576953135767477301738...

1.6188808...

This result is a very good approximation to the value of the golden ratio

1,618033988749...

From the sum of the two expressions, we obtain also:

$$\left( \left( \frac{\sqrt{5} + 1}{2} \right) + \ln^2 \left( \left( 101.18005 + \left( \left( \left( \left( \left( \left( \frac{1}{2} \left( 1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right) \right) \right) \right) \right) \right) \right) \right) \right)^{0.5} \right) \right) + \left( \left( \left( \left( \left( \left( \left( \frac{1}{2} \left( 1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right) \right) \right) \right) \right) \right) \right) \right)^{0.5} \right) \right)$$

**Input interpretation:**

$$\frac{1}{2} (\sqrt{5} + 1) + \log^2 \left( 101.18005 + \sqrt{\frac{1}{2} \left( 1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left( 1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} \right)$$

log(x) is the natural logarithm

**Result:**

29.8146523...

29.8146523... result very near to the black hole entropy 29.7668

**Alternative representations:**

$$\begin{aligned} & \frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \\ & \quad \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \\ & \log_e^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \\ & \quad \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) + \frac{1}{2}(1 + \sqrt{5}) \end{aligned}$$

$$\begin{aligned} & \frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \\ & \quad \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \\ & \left( \log(a) \log_a \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \right. \\ & \quad \left. \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) \right)^2 + \frac{1}{2}(1 + \sqrt{5}) \end{aligned}$$

$$\begin{aligned} & \frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \\ & \quad \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \\ & \left( -\text{Li}_1 \left( -100.18 - \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} - \right. \right. \\ & \quad \left. \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) \right)^2 + \frac{1}{2}(1 + \sqrt{5}) \end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left( 1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} \right) =$$

$$\frac{1}{2} \left( 1 + 2 \log^2 \left( 101.18 + \frac{1}{\sqrt{2}} \left( \sqrt{\left( 1279 + \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left( 355 \times 12^{-k} \sqrt{12} + 12 \left( 22732 + 6305 \sqrt{13} \right)^{-k} \sqrt{22732 + 6305 \sqrt{13}} \right)} \right)} \right) + \frac{1}{\sqrt{2}} \left( \sqrt{\left( 1281 + \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left( 355 \times 12^{-k} \sqrt{12} + 12 \left( 22732 + 6305 \sqrt{13} \right)^{-k} \sqrt{22732 + 6305 \sqrt{13}} \right)} \right)} \right) + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \right)$$

$$\begin{aligned}
& \frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \\
& \quad \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \\
& \frac{1}{2} \left( 1 + 2 \log^2 \left( 100.18 + \frac{\sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}}}{\sqrt{2}} + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}}}{\sqrt{2}} \right) + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} - \right. \\
& \quad \left. 4 \log \left( 100.18 + \frac{\sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}}}{\sqrt{2}} + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}}}{\sqrt{2}} \right) \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \right. \\
& \quad \left. \left( 100.18 + 0.707107 \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \right. \right. \\
& \quad \left. \left. 0.707107 \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)^{-k} + \right. \\
& \quad \left. 2 \left( \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left( 100.18 + 0.707107 \right. \right. \right. \\
& \quad \left. \left. \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \right. \right. \\
& \quad \left. \left. 0.707107 \right. \right. \\
& \quad \left. \left. \left. \left. \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)^{-k} \right)^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \right. \\
& \quad \left. \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \\
& \frac{1}{2} \left( 1 + 2 \log^2 \left( 100.18 + 0.707107 \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \right. \right. \\
& \quad \left. \left. 0.707107 \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right) + \right. \\
& \quad \left. \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \\
& \quad 4 \log \left( 100.18 + 0.707107 \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \right. \\
& \quad \left. \left. 0.707107 \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right) \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \right. \\
& \quad \left( 100.18 + 0.707107 \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \right. \\
& \quad \left. \left. 0.707107 \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)^{-k} + \right. \\
& \quad \left. 2 \left( \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left( 100.18 + 0.707107 \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \right. \right. \right. \\
& \quad \left. \left. \left. 0.707107 \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)^{-k} \right)^2 \right)
\end{aligned}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function



$(a)_n$  is the Pochhammer symbol (rising factorial)

### Integral representations:

$$\frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \frac{1}{2} + \left( \int_1^{101.18 + 0.707107\sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + 0.707107\sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}}} \frac{1}{t} dt \right)^2 + \frac{\sqrt{5}}{2}$$

$$\frac{1}{2}(\sqrt{5} + 1) + \log^2 \left( 101.18 + \sqrt{\frac{1}{2} \left( 1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left( 1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} \right) = \frac{1}{2} + \frac{1}{4i^2\pi^2} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) \left( 100.18 + 0.707107 \sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + 0.707107 \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)^{-s} ds \right)^2 + \frac{\sqrt{5}}{2} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$i$  is the imaginary unit

For the above result 29.81465 considered an entropy, we obtain:

Mass = 5.087666e-8

Radius = 7.554429e-35

Temperature = 2.412114e+30

From the Ramanujan-Nardelli mock formula, we obtain:

$\sqrt{\left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{5.087666 \times 10^{-8}} \right) \times \sqrt{\left[ -\frac{\left( \frac{2.412114 \times 10^{30} \times 4 \pi (7.554429 \times 10^{-35})^3 - (7.554429 \times 10^{-35})^2 \right)}{6.67 \times 10^{-11}} \right]} \right] \right] \right] \right] \right]$

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{5.087666 \times 10^{-8}}} \right) \sqrt{-\frac{2.412114 \times 10^{30} \times 4 \pi (7.554429 \times 10^{-35})^3 - (7.554429 \times 10^{-35})^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618249199205496114879158681348057708738955142401748452971...  
1.61824919...

Now, we have that:

If  $P = \frac{\varphi(q)}{\varphi(q^3)}$  and  $Q = \frac{\varphi(q^9)}{\varphi(q^{27})}$ , then

$$\begin{aligned} & \frac{Q^5}{P^5} + 9 \left( \frac{Q^3}{P^3} - \frac{Q^4}{P^4} \right) + 9 \left( \frac{5Q}{P} + \frac{9P}{Q} \right) + 9 \left( Q^4 + \frac{9}{P^4} \right) + 3(Q^2 + 3P^2) \\ & \left( PQ + \frac{9}{P^3Q^3} \right) = 135 + 3 \left( \frac{Q^2}{P^2} + \frac{9P^2}{Q^2} \right) + \left( P^4Q^4 + \frac{3^4}{P^4Q^4} \right) \\ & + 9 \left( P^2Q^2 + \frac{3^2}{P^2Q^2} \right). \end{aligned} \tag{52}$$

We consider P and Q that are equal to  $P = -2.05860$   $Q = -8.72406$

$$\begin{aligned} & 135 + 3(8.72406^2/2.05860^2 + 9 * 2.05860^2/8.72406^2) + (((2.05860^4 * 8.72406^4 + 3^4) / (2.05860^4 * 8.72406^4))) + 9(((2.05860^2 * 8.72406^2 + 3^2) / (2.05860^2 * 8.72406^2))) \\ & ) \\ & 135 + 3(8.72406^2/2.05860^2 + 9 * 2.05860^2/8.72406^2) + (((2.05860^4 * 8.72406^4 + 3^4) / (2.05860^4 * 8.72406^4))) + 9(((2.05860^2 * 8.72406^2 + 3^2) / (2.05860^2 * 8.72406^2))) \\ & ) \end{aligned}$$

**Input interpretation:**

$$\begin{aligned} & 135 + 3 \left( \frac{8.72406^2}{2.05860^2} + 9 \times \frac{2.05860^2}{8.72406^2} \right) + \left( \left( \frac{2.05860^4 \times 8.72406^4 + \frac{3^4}{2.05860^4 \times 8.72406^4}}{2.05860^4 \times 8.72406^4} \right) + \right. \\ & \left. 9 \left( \frac{2.05860^2 \times 8.72406^2 + \frac{3^2}{2.05860^2 \times 8.72406^2}}{2.05860^2 \times 8.72406^2} \right) \right) \end{aligned}$$

**Result:**

107124.4002608978574012936563175966559864904668572081854653...

107124.40026...

$$\begin{aligned} & ((((((135 + 3(8.72406^2/2.05860^2 + 9 * 2.05860^2/8.72406^2) + (((2.05860^4 * 8.72406^4 + 3^4) / (2.05860^4 * 8.72406^4))) + 9(((2.05860^2 * 8.72406^2 + 3^2) / (2.05860^2 * 8.72406^2)))))))))^{1/4} \end{aligned}$$

**Input interpretation:**

$$\begin{aligned} & \left( 135 + 3 \left( \frac{8.72406^2}{2.05860^2} + 9 \times \frac{2.05860^2}{8.72406^2} \right) + \right. \\ & \left( \left( \frac{2.05860^4 \times 8.72406^4 + \frac{3^4}{2.05860^4 \times 8.72406^4}}{2.05860^4 \times 8.72406^4} \right) + \right. \\ & \left. \left. 9 \left( \frac{2.05860^2 \times 8.72406^2 + \frac{3^2}{2.05860^2 \times 8.72406^2}}{2.05860^2 \times 8.72406^2} \right) \right) \right)^{(1/4)} \end{aligned}$$

**Result:**

18.0914...

18.0914.... result very near to the black hole entropy 18.0524

Now, we have:

$$\begin{aligned}
 & \text{If } X = \frac{\varphi(q)\varphi(q^5)}{\varphi(q^4)\varphi(q^{20})} \text{ and } Y = \frac{\varphi(q)\varphi(q^{20})}{\varphi(q^4)\varphi(q^5)}, \text{ then} \\
 & Y^3 + \frac{1}{Y^3} - 70 \left[ Y^2 + \frac{1}{Y^2} \right] - 785 \left[ Y + \frac{1}{Y} \right] + 160 \left[ \sqrt{Y^3} + \frac{1}{\sqrt{Y^3}} \right] \\
 & \times \left[ \sqrt{X} + \frac{2}{\sqrt{X}} \right] + 80 \left[ \sqrt{Y} + \frac{1}{\sqrt{Y}} \right] \left[ \sqrt{X^3} + \frac{8}{\sqrt{X^3}} + 10 \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right) \right] \quad (61) \\
 & = 80 \left[ X + \frac{4}{X} \right] \left[ 5 + 2 \left( Y + \frac{1}{Y} \right) \right] + 16 \left[ X^2 + \frac{16}{X^2} \right] + 1620.
 \end{aligned}$$

For X and Y equal to 8, we obtain:

$$\begin{aligned}
 & Y^3 + \frac{1}{Y^3} - 70 \left[ Y^2 + \frac{1}{Y^2} \right] - 785 \left[ Y + \frac{1}{Y} \right] + 160 \left[ \sqrt{Y^3} + \frac{1}{\sqrt{Y^3}} \right] \\
 & \times \left[ \sqrt{X} + \frac{2}{\sqrt{X}} \right] + 80 \left[ \sqrt{Y} + \frac{1}{\sqrt{Y}} \right] \left[ \sqrt{X^3} + \frac{8}{\sqrt{X^3}} + 10 \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & 512 + 1/512 - 70 (64 + 1/64) - 785 (8 + 1/8) + 160 (\sqrt{512} + 1/(\sqrt{512})) * \\
 & (\sqrt{8} + 2/(\sqrt{8})) + 80 (\sqrt{8} + 1/(\sqrt{8})) * (((\sqrt{512} + 8/(\sqrt{512}) + 10 \\
 & ((\sqrt{8} + 2/(\sqrt{8}))))))
 \end{aligned}$$

$$80 (\sqrt{8} + 1/\sqrt{8}) * (((\sqrt{512} + 8/\sqrt{512}) + 10 ((\sqrt{8} + 2/\sqrt{8}))))$$

**Input:**

$$80 \left( \sqrt{8} + \frac{1}{\sqrt{8}} \right) \left( \sqrt{512} + \frac{8}{\sqrt{512}} + 10 \left( \sqrt{8} + \frac{2}{\sqrt{8}} \right) \right)$$

**Result:**

14850

$$512 + \frac{1}{512} - 70 \left(64 + \frac{1}{64}\right) - 785 \left(8 + \frac{1}{8}\right) + 160 \left(\sqrt{512} + \frac{1}{\sqrt{512}}\right) \left(\sqrt{8} + \frac{2}{\sqrt{8}}\right)$$

**Input:**

$$512 + \frac{1}{512} - 70 \left(64 + \frac{1}{64}\right) - 785 \left(8 + \frac{1}{8}\right) + 160 \left(\sqrt{512} + \frac{1}{\sqrt{512}}\right) \left(\sqrt{8} + \frac{2}{\sqrt{8}}\right)$$

**Result:**

$$\frac{1268625}{512}$$

**Decimal form:**

2477.783203125

$$2477.783203125 + 14850$$

**Input interpretation:**

$$2477.783203125 + 14850$$

**Result:**

$$17327.783203125$$

$$17327.783203125$$

Note that:

$17327.783203125 / 10 = 1732.778\dots$  result very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$80 \left[ X + \frac{4}{X} \right] \left[ 5 + 2 \left( Y + \frac{1}{Y} \right) \right] + 16 \left[ X^2 + \frac{16}{X^2} \right] + 1620.$$

$$[(((16(64+1/4))))] + 1620 + 80(((8+1/2)*(5+16+1/4)))$$

**Input:**

$$16 \left(64 + \frac{1}{4}\right) + 1620 + 80 \left( \left(8 + \frac{1}{2}\right) \left(5 + 16 + \frac{1}{4}\right) \right)$$

**Exact result:**

17098

17098

We have that:

$$\frac{((((((((16(64+1/4)))) + 1620 + 80(((8+1/2)*(5+16+1/4)))))))))}{(1.0061571663)} + 233 + 89 + 8$$

Where 1.0061571663 is a Ramanujan mock theta function

**Input interpretation:**

$$\frac{16\left(64 + \frac{1}{4}\right) + 1620 + 80\left(\left(8 + \frac{1}{2}\right)\left(5 + 16 + \frac{1}{4}\right)\right)}{1.0061571663} + 233 + 89 + 8$$

**Result:**

17323.36900106318906809937015304246111594782565531680793436...

17323.369...

Note that:

$$17098 / (\pi^2/10)$$

**Input:**

$$\frac{17098}{\frac{\pi^2}{10}}$$

**Result:**

$$\frac{170980}{\pi^2}$$

**Decimal approximation:**

17323.89597916691216147451061959923169986726329346071492162...

17323.89597...

**Alternative representations:**

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{17098}{\frac{1}{10} (180^\circ)^2}$$

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{17098}{\frac{6\zeta(2)}{10}}$$

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{17098}{\frac{1}{10} (-i \log(-1))^2}$$

$\zeta(s)$  is the Riemann zeta function

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

### Series representations:

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{42745}{4 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2}$$

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{42745}{4 \left( \sum_{k=0}^{\infty} \frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2}$$

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{170980}{\left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2}$$

### Integral representations:

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{42745}{\left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2}$$

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{42745}{\left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2}$$

$$\frac{17098}{\frac{\pi^2}{10}} = \frac{42745}{4 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2}$$

Now, we have that:

$\ln 17327.783203125$

**Input interpretation:**

$\log(17327.783203125)$

$\log(x)$  is the natural logarithm

**Result:**

9.76006645780084...

9.76006... result near to the black hole entropy 9.9340

**Alternative representations:**

$\log(17327.7832031250000) = \log_e(17327.7832031250000)$

$\log(17327.7832031250000) = \log(a) \log_a(17327.7832031250000)$

$\log(17327.7832031250000) = -\text{Li}_1(-17326.7832031250000)$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\log(17327.7832031250000) = \log(17326.7832031250000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-9.76000874534882782 k}}{k}$$



$$\log(17327.7832031250000) = 2i\pi \left\lfloor \frac{\arg(17327.7832031250000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (17327.7832031250000 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\log(17327.7832031250000) = \left\lfloor \frac{\arg(17327.7832031250000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(17327.7832031250000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (17327.7832031250000 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$\log(17327.7832031250000) = \int_1^{17327.7832031250000} \frac{1}{t} dt$$

$$\log(17327.7832031250000) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-9.76000874534882782s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

For X and Y equal to 5, we obtain:

$$Y^3 + \frac{1}{Y^3} - 70 \left[ Y^2 + \frac{1}{Y^2} \right] - 785 \left[ Y + \frac{1}{Y} \right] + 160 \left[ \sqrt{Y^3} + \frac{1}{\sqrt{Y^3}} \right] \\ \times \left[ \sqrt{X} + \frac{2}{\sqrt{X}} \right] + 80 \left[ \sqrt{Y} + \frac{1}{\sqrt{Y}} \right] \left[ \sqrt{X^3} + \frac{8}{\sqrt{X^3}} + 10 \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right) \right]$$

$$125 + \frac{1}{125} - 70 \left(25 + \frac{1}{25}\right) - 785 \left(5 + \frac{1}{5}\right) + 160 \left(\left(\sqrt{125} + \frac{1}{\sqrt{125}}\right) \cdot \left(\sqrt{5} + \frac{2}{\sqrt{5}}\right)\right) + 80 \left(\sqrt{5} + \frac{1}{\sqrt{5}}\right) \cdot \left(\left(\left(\sqrt{125} + \frac{8}{\sqrt{125}}\right) + 10 \left(\sqrt{5} + \frac{2}{\sqrt{5}}\right)\right)\right)$$

**Input:**

$$125 + \frac{1}{125} - 70 \left(25 + \frac{1}{25}\right) - 785 \left(5 + \frac{1}{5}\right) + 160 \left(\sqrt{125} + \frac{1}{\sqrt{125}}\right) \left(\sqrt{5} + \frac{2}{\sqrt{5}}\right) + 80 \left(\sqrt{5} + \frac{1}{\sqrt{5}}\right) \left(\left(\sqrt{125} + \frac{8}{\sqrt{125}}\right) + 10 \left(\sqrt{5} + \frac{2}{\sqrt{5}}\right)\right)$$

**Result:**

$$160 \left(5\sqrt{5} + \frac{\frac{2}{\sqrt{5}} + \sqrt{5}}{5\sqrt{5}} + 80 \left(\sqrt{5} + \frac{\frac{8}{5\sqrt{5}} + 5\sqrt{5} + 10\left(\frac{2}{\sqrt{5}} + \sqrt{5}\right)}{\sqrt{5}}\right)\right) - \frac{713724}{125}$$

**Decimal approximation:**

272041.5324939971398711647618947453568019924096907167498500...  
272041...

**Alternate forms:**

$$\frac{4}{125} \left(7550969 + 425000\sqrt{5}\right)$$

$$\frac{30203876}{125} + 13600\sqrt{5}$$

$$13600\sqrt{5} + \frac{30203876}{125}$$

**Minimal polynomial:**

$$15625x^2 - 7550969000x + 897824125423376$$

$$80 \left[X + \frac{4}{X}\right] \left[5 + 2 \left(Y + \frac{1}{Y}\right)\right] + 16 \left[X^2 + \frac{16}{X^2}\right] + 1620.$$

$$80 \left(\left(5 + \frac{4}{5}\right) \cdot \left(5 + 10 + \frac{2}{5}\right)\right) + \left[\left(\left(16 \left(25 + \frac{16}{25}\right)\right)\right)\right] + 1620$$

**Input:**

$$80 \left(\left(5 + \frac{4}{5}\right) \left(5 + 10 + \frac{2}{5}\right)\right) + 16 \left(25 + \frac{16}{25}\right) + 1620$$

**Exact result:**

$$\frac{229\,396}{25}$$

**Decimal form:**

9175.84

9175.84

Now, we have that:

$$1.7168646^{2\pi} * ((((((80(((5+4/5)*(5+10+2/5)))+ [(((16(25+16/25)))]+1620))))))$$

Where 1.7168646 is a Ramanujan mock theta function

**Input interpretation:**

$$1.7168646^{2\pi} \left( 80 \left( \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right)$$

**Result:**

273864.6...

273864.6

**Alternative representations:**

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) =$$

$$1.71686^{360^\circ} \left( 1620 + 80 \left( 15 + \frac{2}{5} \right) \left( 5 + \frac{4}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) \right)$$

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) =$$

$$1.71686^{-2i \log(-1)} \left( 1620 + 80 \left( 15 + \frac{2}{5} \right) \left( 5 + \frac{4}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) \right)$$

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) =$$

$$1.71686^{2 \cos^{-1}(-1)} \left( 1620 + 80 \left( 15 + \frac{2}{5} \right) \left( 5 + \frac{4}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) \right)$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

### Series representations:

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) = \frac{229\,396}{25} \times 1.71686^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) = 1056.09 e^{2.162 \times \sum_{k=1}^{\infty} 2^k / \binom{2k}{k}}$$

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) = \frac{229\,396}{25} \times 1.71686^{2 \sum_{k=0}^{\infty} (2^{-k} (-6+50k)) / \binom{3k}{k}}$$

$\binom{n}{m}$  is the binomial coefficient

### Integral representations:

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) = \frac{229\,396}{25} e^{2.162 \int_0^{\infty} 1/(1+t^2) dt}$$

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) = \frac{229\,396}{25} e^{4.324 \int_0^1 \sqrt{1-t^2} dt}$$

$$1.71686^{2\pi} \left( 80 \left( 5 + \frac{4}{5} \right) \left( 5 + 10 + \frac{2}{5} \right) + 16 \left( 25 + \frac{16}{25} \right) + 1620 \right) = \frac{229\,396}{25} e^{2.162 \int_0^{\infty} \sin(t)/t dt}$$

$$9175.84 * ((9\pi + \sqrt{2}))$$

**Input interpretation:**

$$9175.84 (9\pi + \sqrt{2})$$

**Result:**

$$2.72417... \times 10^5$$

$$272417...$$

**Series representations:**

$$9175.84 (9\pi + \sqrt{2}) = 82582.6\pi + 9175.84 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$9175.84 (9\pi + \sqrt{2}) =$$

$$82582.6\pi + 9175.84 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$9175.84 (9\pi + \sqrt{2}) = 82582.6\pi +$$

$$9175.84 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!}$$

Now, we have that:

$$\ln 272041.5324939971398711647618947453568019924096907167498500$$

**Input interpretation:**

$$\log(272041.5324939971398711647618947453568019924096907167498500)$$

$\log(x)$  is the natural logarithm

**Result:**

$$12.51371002661438246425795557965170346312224343945379598480...$$

12.51371... result very near to the black hole entropy 12.5664

**Alternative representations:**

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = \log_e(272\,041.53249399713987116476189474535680199240969071674985000000)$$

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = \log(a) \log_a(272\,041.53249399713987116476189474535680199240969071674985000000)$$

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = -\text{Li}_1(-272\,040.5324939971398711647618947453568019924096907167498500000)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = \log(272\,040.53249399713987116476189474535680199240969071674985000000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-12.5137063506983236498875908724238963677044530078367515941068384k}}{k}$$

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = 2i\pi \left[ \frac{1}{2\pi} \arg(272\,041.53249399713987116476189474535680199240969071674985000000 - x) \right] + \log(x) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (272\,041.53249399713987116476189474535680199240969071674985000000 - x)^k x^{-k} \text{ for } x < 0$$

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = \left\lfloor \frac{1}{2\pi} \arg(272\,041.5324939971398711647618947453568019924096907167498500000 - z_0) \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{1}{2\pi} \arg(272\,041.5324939971398711647618947453568019924096907167498500000 - z_0) \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (272\,041.5324939971398711647618947453568019924096907167498500000 - z_0)^k z_0^{-k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = \int_1^{272\,041.53249399713987116476189474535680199240969071674985000000} \frac{1}{t} dt$$

$$\log(272\,041.53249399713987116476189474535680199240969071674985000000) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} e^{-12.5137063506983236498875908724238963677044530078367515941068384s} \Gamma(-s)^2 \Gamma(1+s) ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

For X and Y equal to 0.5, we obtain:

$$0.125 + 1/0.125 - 70 (0.25 + 1/0.25) - 785 (0.5 + 1/0.5) + 160 ((\sqrt{0.125} + 1/(\sqrt{0.125})) * (\sqrt{0.5} + 2/(\sqrt{0.5})))$$

**Input:**

$$0.125 + \frac{1}{0.125} - 70 \left( 0.25 + \frac{1}{0.25} \right) - 785 \left( 0.5 + \frac{1}{0.5} \right) + 160 \left( \sqrt{0.125} + \frac{1}{\sqrt{0.125}} \left( \sqrt{0.5} + \frac{2}{\sqrt{0.5}} \right) \right)$$

**Result:**

-595.306...

$$-595.306 + 80 (\text{sqrt}(0.5)+1/(\text{sqrt}(0.5))*(((\text{sqrt}(0.125)+8/(\text{sqrt}(0.125))))+10((\text{sqrt}(0.5)+2/(\text{sqrt}(0.5)))))$$

**Input interpretation:**

$$-595.306 + 80 \left( \sqrt{0.5} + \frac{1}{\sqrt{0.5}} \left( \left( \sqrt{0.125} + \frac{8}{\sqrt{0.125}} \right) + 10 \left( \sqrt{0.5} + \frac{2}{\sqrt{0.5}} \right) \right) \right)$$

**Result:**

6061.26...

6061.26...

$$((((([80*(((0.5+4/0.5)*(0.5+2(0.5+1/0.5)))]+ [(((16*(0.25+16/0.25)))])))))) + 1620$$

**Input:**

$$80 \left( \left( 0.5 + \frac{4}{0.5} \right) \left( 0.5 + 2 \left( 0.5 + \frac{1}{0.5} \right) \right) + 16 \left( 0.25 + \frac{16}{0.25} \right) \right) + 1620$$

**Result:**

87600

87600

The ratio of the two results is:

$$87600 / ((((((((-595.306 + 80 (\text{sqrt}(0.5)+1/(\text{sqrt}(0.5))*(((\text{sqrt}(0.125)+8/(\text{sqrt}(0.125))))+10((\text{sqrt}(0.5)+2/(\text{sqrt}(0.5)))))$$

**Input interpretation:**



$$\frac{87600}{-595.306 + 80 \left( \sqrt{0.5} + \frac{1}{\sqrt{0.5}} \left( \left( \sqrt{0.125} + \frac{8}{\sqrt{0.125}} \right) + 10 \left( \sqrt{0.5} + \frac{2}{\sqrt{0.5}} \right) \right) \right)}$$

**Result:**

14.4524...

14.4524...

Note that:

14.4524 + 2\*1.61803398 = 17.68846796 result very near to the black hole entropy  
17.7715

Now, we have that:

$$\exp(2.6709253774829) \left( \left( \left( \left( \left( \left( -595.306 + 80 \left( \sqrt{0.5} + \frac{1}{\sqrt{0.5}} \left( \left( \sqrt{0.125} + \frac{8}{\sqrt{0.125}} \right) + 10 \left( \sqrt{0.5} + \frac{2}{\sqrt{0.5}} \right) \right) \right) \right) \right) \right) \right) \right) \right)$$

where 2.67092537... is a Ramanujan mock theta function

**Input interpretation:**

$$\exp(2.6709253774829) \left( -595.306 + 80 \left( \sqrt{0.5} + \frac{1}{\sqrt{0.5}} \left( \left( \sqrt{0.125} + \frac{8}{\sqrt{0.125}} \right) + 10 \left( \sqrt{0.5} + \frac{2}{\sqrt{0.5}} \right) \right) \right) \right)$$

**Result:**

87605.5...

87605.5...

For X and Y equal to 1, we obtain:

$$1+1 - 70 (1+1) - 785 (1+1) + 160 (1+1) ((1+2)) + 80 (((1+1))*(((1+8) + 10 ((1+2))))$$

**Input:**

$$1 + 1 - 70 (1 + 1) - 785 (1 + 1) + 160 (1 + 1) (1 + 2) + 80 ((1 + 1) ((1 + 8) + 10 (1 + 2)))$$

**Result:**

5492

5492

$$80(1+4)*(5+2(1+1)) + 16(1+16) + 1620$$

**Input:**

$$80(1+4)(5+2(1+1)) + 16(1+16) + 1620$$

**Result:**

$$5492$$

$$5492$$

$$13 + \frac{1}{\pi} [1 + 1 - 70(1+1) - 785(1+1) + 160(1+1) + 80(((1+2))) + 10((1+2))] + 80((((1+1))*(((1+8) + 10((1+2))))$$

**Input:**

$$13 + \frac{1}{\pi} (1 + 1 - 70(1+1) - 785(1+1) + 160(1+1)(1+2) + 80((1+1)((1+8) + 10(1+2))))$$

**Result:**

$$13 + \frac{5492}{\pi}$$

**Decimal approximation:**

$$1761.157894921378368085419256883697752586504748813173633044...$$

1761.15789... result in the range of the mass of candidate “glueball”  $f_0(1710)$  (“glueball” =  $1760 \pm 15$  MeV).

**Property:**

$$13 + \frac{5492}{\pi} \text{ is a transcendental number}$$

**Alternate form:**

$$\frac{13\pi + 5492}{\pi}$$

**Alternative representations:**

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{5492}{180^\circ}$$

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + -\frac{5492}{i \log(-1)}$$

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{5492}{\cos^{-1}(-1)}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

### Series representations:

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{1373}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{1373}{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{5492}{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}$$

**Integral representations:**

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{1373}{\int_0^1 \sqrt{1-t^2} dt}$$

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{2746}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$13 + \frac{1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2)))}{\pi} =$$

$$13 + \frac{2746}{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

$$e \ln[1+1 - 70(1+1) - 785(1+1) + 160(1+1) + 80(((1+2))) + 80((((1+1))*(((1+8) + 10((1+2))))]$$

**Input:**

$$e \log(1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80((1 + 1)((1 + 8) + 10(1 + 2))))$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$e \log(5492)$$

### Decimal approximation:

23.40725466872411493540999322570609428158243499995425792366...

23.40725... result very near to the black hole entropy 23.3621

### Alternate forms:

$$e (2 \log(2) + \log(1373))$$

$$2 e \log(2) + e \log(1373)$$

### Alternative representations:

$$e \log(1 + 1 - 70 (1 + 1) - 785 (1 + 1) + 160 (1 + 1) (1 + 2) + 80 (1 + 1) ((1 + 8) + 10 (1 + 2))) = e \log_e(5492)$$

$$e \log(1 + 1 - 70 (1 + 1) - 785 (1 + 1) + 160 (1 + 1) (1 + 2) + 80 (1 + 1) ((1 + 8) + 10 (1 + 2))) = e \log(a) \log_a(5492)$$

$$e \log(1 + 1 - 70 (1 + 1) - 785 (1 + 1) + 160 (1 + 1) (1 + 2) + 80 (1 + 1) ((1 + 8) + 10 (1 + 2))) = -e \operatorname{Li}_1(-5491)$$

$\log_b(x)$  is the base- $b$  logarithm

$\operatorname{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$e \log(1 + 1 - 70 (1 + 1) - 785 (1 + 1) + 160 (1 + 1) (1 + 2) + 80 (1 + 1) ((1 + 8) + 10 (1 + 2))) = e \log(5491) - e \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{5491}\right)^k}{k}$$

$$e \log(1 + 1 - 70 (1 + 1) - 785 (1 + 1) + 160 (1 + 1) (1 + 2) + 80 (1 + 1) ((1 + 8) + 10 (1 + 2))) = 2 i e \pi \left[ \frac{\arg(5492 - x)}{2 \pi} \right] + e \log(x) - e \sum_{k=1}^{\infty} \frac{(-1)^k (5492 - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$e \log(1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80(1 + 1)((1 + 8) + 10(1 + 2))) =$$

$$e \left\lfloor \frac{\arg(5492 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + e \log(z_0) +$$

$$e \left\lfloor \frac{\arg(5492 - z_0)}{2\pi} \right\rfloor \log(z_0) - e \sum_{k=1}^{\infty} \frac{(-1)^k (5492 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$e \log(1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80(1 + 1)((1 + 8) + 10(1 + 2))) =$$

$$e \int_1^{5492} \frac{1}{t} dt$$

$$e \log(1 + 1 - 70(1 + 1) - 785(1 + 1) + 160(1 + 1)(1 + 2) + 80(1 + 1)((1 + 8) + 10(1 + 2))) =$$

$$-\frac{ie}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{5491^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

Now, we have:

(Formula 62)

$$\text{If } X = \frac{\varphi(q)\varphi(q^7)}{\varphi(q^4)\varphi(q^{28})} \text{ and } Y = \frac{\varphi(q)\varphi(q^{28})}{\varphi(q^4)\varphi(q^7)}, \text{ then}$$

$$Y^4 + \frac{1}{Y^4} - 280 \left( Y^3 + \frac{1}{Y^3} \right) - 28 \left( Y^2 + \frac{1}{Y^2} \right) \left[ 349 + 78 \left( X + \frac{4}{X} \right) \right]$$

$$- 56 \left( Y + \frac{1}{Y} \right) \left[ 1079 + 310 \left( X + \frac{4}{X} \right) + 24 \left( X^2 + \frac{16}{X^2} \right) \right]$$

$$+ 112 \left( \sqrt{Y} + \frac{1}{\sqrt{Y}} \right) \left[ 515 \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right) + 90 \left( \sqrt{X^3} + \frac{8}{\sqrt{X^3}} \right) \right]$$

$$+ 4 \left( \sqrt{X^5} + \frac{32}{\sqrt{X^5}} \right) + 56 \left( \sqrt{Y^3} + \frac{1}{\sqrt{Y^3}} \right) \left[ 40 \left( \sqrt{X^3} + \frac{8}{\sqrt{X^3}} \right) \right]$$

$$+ 313 \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right) + 1176 \left( \sqrt{Y^5} + \frac{1}{\sqrt{Y^5}} \right) \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right)$$

$$= 16 \left[ 4 \left( X^3 + \frac{64}{X^3} \right) + 203 \left( X^2 + \frac{16}{X^2} \right) + 2023 \left( X + \frac{4}{X} \right) \right] + 106330.$$

From:

$$16 \left[ 4 \left( X^3 + \frac{64}{X^3} \right) + 203 \left( X^2 + \frac{16}{X^2} \right) + 2023 \left( X + \frac{4}{X} \right) \right] + 106330.$$

For  $X = 2$ , we obtain:

$$16[(4(8+8)+203(4+4)+2023(2+2))+106330]$$

$$(((((((16(((((((4(8+8)+203(4+4)+2023(2+2)))))))))))))))+106330$$

**Input:**

$$16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330$$

**Result:**

262810

262810

$$\ln(((((((((((16(((((((4(8+8)+203(4+4)+2023(2+2)))))))))))))))+106330))))))$$

**Input:**

$$\log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log(262810)$$

**Decimal approximation:**

12.47918661661902352771816136572337198184574578742303288516...

12.479186... result very near to the black hole entropy 12.5663

**Property:**

$\log(262810)$  is a transcendental number

**Alternate form:**

$$\log(2) + \log(5) + \log(41) + \log(641)$$

**Alternative representations:**

$$\log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) = \log_e(262 810)$$

$$\log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) = \log(a) \log_a(262 810)$$

$$\log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) = -\text{Li}_1(-262 809)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Integral representations:

$$\log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) = \int_1^{262 810} \frac{1}{t} dt$$

$$\log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) = -\frac{i}{2 \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{262 809^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

We have also:

$$e^{e \cdot \pi^{-(1-e)} \sin(e \cdot \pi)} \left( \frac{1}{4} \ln(\ln(\ln(\ln(\ln(\ln(\ln(\ln(\ln(\ln(16(\ln(\ln(\ln(\ln(\ln(4(8+8)+203(4+4)+2023(2+2)))))\dots)))\dots)))\dots)))\dots))\dots))\dots))\dots))\right)^2$$

### Input:

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2$$

$\log(x)$  is the natural logarithm

### Exact result:

$$\frac{1}{16} e^e \pi^{-1-e} \log^2(262 810) \sin(e \pi)$$

### Decimal approximation:

1.617904327401028985503733217552857226635659278696535559569...

1.61790432...



This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\frac{1}{32} i e^{e-i e \pi} \pi^{-1-e} \log^2(262810) - \frac{1}{32} i e^{e+i e \pi} \pi^{-1-e} \log^2(262810)$$

$$\frac{1}{16} e^e \pi^{-1-e} (\log(2) + \log(5) + \log(41) + \log(641))^2 \sin(e \pi)$$

**Alternative representations:**

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106330) \right)^2 = \cos\left(\frac{\pi}{2} - e \pi\right) e^e \pi^{-1-e} \left( \frac{\log(262810)}{4} \right)^2$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106330) \right)^2 = \cos\left(\frac{\pi}{2} - e \pi\right) e^e \pi^{-1-e} \left( \frac{\log_e(262810)}{4} \right)^2$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106330) \right)^2 = -\cos\left(\frac{\pi}{2} + e \pi\right) e^e \pi^{-1-e} \left( \frac{\log_e(262810)}{4} \right)^2$$

$\log_b(x)$  is the base- $b$  logarithm

**Series representations:**

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106330) \right)^2 = \frac{1}{16} e^e \pi^{-1-e} \log^2(262810) \sum_{k=0}^{\infty} \frac{(-1)^k (e \pi)^{1+2k}}{(1+2k)!}$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106330) \right)^2 = \frac{1}{16} e^e \pi^{-1-e} \log^2(262810) \sum_{k=0}^{\infty} \frac{(-1)^k \left( \left( -\frac{1}{2} + e \right) \pi \right)^{2k}}{(2k)!}$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$\frac{1}{16} e^e \pi^{-1-e} \sin(e \pi) \left( \log(262 809) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{262 809}\right)^k}{k} \right)^2$$

$n!$  is the factorial function

### Integral representations:

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$\frac{1}{16} e^{1+e} \pi^{-e} \log^2(262 810) \int_0^1 \cos(e \pi t) dt$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$\frac{1}{16} e^{1+e} \pi^{-e} \left( \int_1^{262 810} \frac{1}{t} dt \right)^2 \int_0^1 \cos(e \pi t) dt$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$-\frac{1}{64} i e^e \pi^{-1/2-e} \log^2(262 810) \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-(e^2 \pi^2)/(4s)+s}}{s^{3/2}} ds \text{ for } \gamma > 0$$

### Multiple-argument formulas:

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$-\frac{1}{32} i e^e \left( (-1)^e + (-1)^{1+e} e^{-2ie\pi} \right) \pi^{-1-e} \log^2(262 810)$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$\frac{1}{8} e^e \pi^{-1-e} \cos\left(\frac{e \pi}{2}\right) \log^2(262 810) \sin\left(\frac{e \pi}{2}\right)$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$\frac{1}{16} e^e \pi^{-1-e} \log^2(262 810) (-\sin((-2 + e) \pi) - 2 \sin((-1 + e) \pi))$$

$$e^e \pi^{-1-e} \sin(e \pi) \left( \frac{1}{4} \log(16 (4 (8 + 8) + 203 (4 + 4) + 2023 (2 + 2)) + 106 330) \right)^2 =$$

$$\frac{1}{16} e^e \pi^{-1-e} \left( 1 + 2 \cos\left(\frac{2 e \pi}{3}\right) \right) \log^2(262 810) \sin\left(\frac{e \pi}{3}\right)$$

From 12.479186 as entropy, we obtain:

Mass = 3.291523e-8 (equivalent to 1.846409×10<sup>19</sup> GeV, practically near to the mean value 1.962 \* 10<sup>19</sup> of DM particle that has a Planck scale mass: m ≈ 10<sup>19</sup> GeV (Planck mass = 1,2209 × 10<sup>19</sup> GeV/c<sup>2</sup> = 21,76 μg Wikipedia) and is very nearly to the result of the following Ramanujan mock theta function: **χ(q) = 1.962364415**)

Radius = 4.887423e-35

Temperature = 3.728375e+30

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[ $\left[ \left[ \left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \left( \left( \left( \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{3.291523 \times 10^{-8}} \right) \right) \right) \right) \right] \right] \right] \right] \right] \sqrt{\left[ \left[ \left[ \left[ \frac{3.728375 \times 10^{30} \times 4 \pi \left( 4.887423 \times 10^{-35} \right)^3 - \left( 4.887423 \times 10^{-35} \right)^2}{6.67 \times 10^{-11}} \right] \right] \right] \right] \right] \right]$ ]

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.291523 \times 10^{-8}} \right) \sqrt{\frac{3.728375 \times 10^{30} \times 4 \pi \left( 4.887423 \times 10^{-35} \right)^3 - \left( 4.887423 \times 10^{-35} \right)^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249231206197105000126055818074317231564893932325563655...

1.6182492...

From:

$$Q^2 + \frac{1}{Q^2} + \left(P^2 + \frac{9^2}{P^2}\right) + 2\left(P + \frac{9}{P}\right) \left[4 + 3\left(Q + \frac{1}{Q}\right)\right] = 8\left(Q + \frac{1}{Q}\right) + 4 \left[ \left(\sqrt{P^3} + \frac{3^3}{\sqrt{P^3}}\right) \left(\sqrt{Q} + \frac{1}{\sqrt{Q}}\right) - \left(\sqrt{P} + \frac{3}{\sqrt{P}}\right) \left(\sqrt{Q^3} + \frac{1}{\sqrt{Q^3}}\right) + 3 \right]. \quad (92)$$

$$Q = 2, P = 3$$

$$4 + 1/4 + (9+9) + 2(3+3) (4+3(2+1/2))$$

$$8(2+1/2) + 4[(((\sqrt{27} + 27/\sqrt{27})) * ((\sqrt{2} + 1/\sqrt{2}))) - ((\sqrt{3} + 3/\sqrt{3})) * ((2+1/2)+3)]]$$

**Input:**

$$8\left(2 + \frac{1}{2}\right) + 4 \left[ \left( \sqrt{27} + \frac{27}{\sqrt{27}} \right) \left( \sqrt{2} + \frac{1}{\sqrt{2}} \right) - \left( \sqrt{3} + \frac{3}{\sqrt{3}} \right) \left( \left(2 + \frac{1}{2}\right) + 3 \right) \right]$$

**Result:**

$$20 + 24\sqrt{3} \left( \frac{1}{\sqrt{2}} + \sqrt{2} - 11\sqrt{3} \right)$$

**Decimal approximation:**

-683.818369259805588464897773310587909889225890696359875376...

-683.81836...

**Alternate forms:**

$$4(9\sqrt{6} - 193)$$

$$36\sqrt{6} - 772$$

$$36\sqrt{6} - 772$$

**Minimal polynomial:**

$$x^2 + 1544x + 588208$$

$$4 + 1/4 + (9+9) + 2(3+3) (4+3(2+1/2))$$

**Input:**

$$4 + \frac{1}{4} + (9 + 9) + 2(3 + 3) \left( 4 + 3 \left( 2 + \frac{1}{2} \right) \right)$$

**Exact result:**

$$\frac{641}{4}$$

**Decimal form:**

160.25

160.25

-683.818369259805588464897773310587909889225890696359875376/160.25

**Input interpretation:**

$$\frac{683.818369259805588464897773310587909889225890696359875376}{160.25}$$

**Result:**

-4.26719731207366981881371465404423032692184643180255772465...

-4.2671973...

$(-4.26719731207366981881371465404423032692184643180255772465)^2$

**Input interpretation:**

$(-4.26719731207366981881371465404423032692184643180255772465)^2$

**Result:**

18.20897290016875264964024764314914470589644301820447857474...

18.208972... result very near to the black hole entropy 18.2773

$24/\exp(-4.2671973120736698)$

**Input interpretation:**

$$\frac{24}{\exp(-4.2671973120736698)}$$

**Result:**

1711.715122457020...

1711.715... result very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$((((24/\exp(-4.2671973120736698))))))^{1/15}$$

**Input interpretation:**

$$\sqrt[15]{\frac{24}{\exp(-4.2671973120736698)}}$$

**Result:**

1.64271453236822648...

$$1.642714... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$2*((((6*(((24/\exp(-4.2671973120736698))))^{1/15}))))^{1/2}$$

**Input interpretation:**

$$2\sqrt{6\sqrt[15]{\frac{24}{\exp(-4.2671973120736698)}}}$$

**Result:**

6.27894487767152129...

$$6.2789448... \approx 2\pi$$

$$-24/(10^3)+((((24/\exp(-4.2671973120736698))))^{1/15}$$

**Input interpretation:**

$$-\frac{24}{10^3} + \sqrt[15]{\frac{24}{\exp(-4.2671973120736698)}}$$

**Result:**

1.61871453236822648...

1.618714...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

Now, we have that:

$$\begin{aligned}
 & \text{If } P = \frac{\varphi(q)\varphi(q^5)}{\varphi(q^9)\varphi(q^{45})} \text{ and } Q = \frac{\varphi(q)\varphi(q^{45})}{\varphi(q^9)\varphi(q^5)}, \text{ then} \\
 & Q^3 + \frac{1}{Q^3} - 15 \left( Q^2 + \frac{1}{Q^2} \right) - 45 \left( Q + \frac{1}{Q} \right) - \left( P^2 + \frac{81}{P^2} \right) - 10 \left( P + \frac{9}{P} \right) \\
 & \times \left[ 2 + Q + \frac{1}{Q} \right] + 5 \left( \sqrt{P^3} + \frac{3^3}{\sqrt{P^3}} \right) \left( \sqrt{Q} + \frac{1}{\sqrt{Q}} \right) + 15 \left( \sqrt{P} + \frac{3}{\sqrt{P}} \right) \\
 & \times \left[ \left( \sqrt{Q^3} + \frac{1}{\sqrt{Q^3}} \right) + 2 \left( \sqrt{Q} + \frac{1}{\sqrt{Q}} \right) \right] = 40.
 \end{aligned} \tag{95}$$

For P and Q = 1, we obtain:

$$(2-30-90-82-10(10))*(4)+5*28*2+15*4*6$$

**Input:**

$$2 - 30 - 90 - 82 - 10 \times 10 \times 4 + 5 \times 28 \times 2 + 15 \times 4 \times 6$$

**Result:**

40

40

Now, we have that:

$$\begin{aligned}
 & \text{If } P = \frac{f(-q)f(-q^7)}{q^{8/3}f(-q^9)f(-q^{63})} \text{ and } Q = \frac{f(-q)f(-q^{63})}{q^{-2}f(-q^9)f(-q^7)} \text{ then} \\
 & Q^4 + \frac{1}{Q^4} - 14 \left( Q^3 + \frac{1}{Q^3} \right) + 28 \left( Q^2 + \frac{1}{Q^2} \right) + 7 \left( Q + \frac{1}{Q} \right) = P^3 + \frac{9^3}{P^3} \\
 & + 7 \left( \sqrt{P^3} + \frac{27}{\sqrt{P^3}} \right) \left[ \left( \sqrt{Q^3} + \frac{1}{\sqrt{Q^3}} \right) - \left( \sqrt{Q} + \frac{1}{\sqrt{Q}} \right) \right] + 98.
 \end{aligned} \tag{98}$$

For Q and P = 1, we obtain:

$$2-14*2+28*2+7*2$$

**Input:**

$$2 - 14 \times 2 + 28 \times 2 + 7 \times 2$$

**Result:**

44

44

$$1 + 9^3 + 7 * 28 * (2 - 2) + 98$$

**Input:**

$$1 + 9^3 + 7 \times 28 (2 - 2) + 98$$

**Result:**

828

828

$$(((1 + 9^3 + 7 * 28 * (2 - 2) + 98))) / 44$$

**Input:**

$$\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)$$

**Exact result:**

$$\frac{207}{11}$$

**Decimal approximation:**

18.81...

18.81... result very near to the black hole entropy 18.7328

$$55 + 13 + 10^3 \left( \ln \left( \frac{1}{44} (1 + 9^3 + 7 * 28 * (2 - 2) + 98) \right) \right)^{1/2}$$

**Input:**

$$55 + 13 + 10^3 \sqrt{\log \left( \frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98) \right)}$$

$\log(x)$  is the natural logarithm



**Exact result:**

$$68 + 1000 \sqrt{\log\left(\frac{207}{11}\right)}$$

**Decimal approximation:**

1781.132662833500040068869287279782048923999774828652698398...

1781.13266... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

**Property:**

$68 + 1000 \sqrt{\log\left(\frac{207}{11}\right)}$  is a transcendental number

**Alternate forms:**

$$4 \left( 17 + 250 \sqrt{\log\left(\frac{207}{11}\right)} \right)$$

$$4 \left( 17 + 250 \sqrt{\log\left(\frac{23}{11}\right) + 2 \log(3)} \right)$$

$$68 + 1000 \sqrt{2 \log(3) - \log(11) + \log(23)}$$

**Alternative representations:**

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} = 68 + 10^3 \sqrt{\log_e\left(\frac{1}{44} (99 + 9^3)\right)}$$

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} =$$

$$68 + 10^3 \sqrt{\log(a) \log_a\left(\frac{1}{44} (99 + 9^3)\right)}$$

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} =$$

$$68 + 10^3 \sqrt{-\text{Li}_1\left(1 - \frac{1}{44} (99 + 9^3)\right)}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} =$$

$$68 + 1000 \sqrt{\log\left(\frac{196}{11}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{11}{196}\right)^k}{k}}$$

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} =$$

$$68 + 1000 \sqrt{2i\pi \left\lfloor \frac{\arg\left(\frac{207}{11} - x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{207}{11} - x\right)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} =$$

$$68 + 1000 \sqrt{\log(z_0) + \left\lfloor \frac{\arg\left(\frac{207}{11} - z_0\right)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{207}{11} - z_0\right)^k z_0^{-k}}{k}}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} = 68 + 1000 \sqrt{\int_1^{\frac{207}{11}} \frac{1}{t} dt}$$

$$55 + 13 + 10^3 \sqrt{\log\left(\frac{1}{44} (1 + 9^3 + 7 \times 28 (2 - 2) + 98)\right)} =$$

$$68 + 500 \sqrt{\frac{2}{\pi}} \sqrt{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{11}{196}\right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0}$$

$\Gamma(x)$  is the gamma function

We have also that:

$$(((1+9^3+7*28*(2-2)+98)))-(((2-14*2+28*2+7*2)))$$

**Input:**

$$(1 + 9^3 + 7 \times 28 (2 - 2) + 98) - (2 - 14 \times 2 + 28 \times 2 + 7 \times 2)$$

**Result:**

784

784 result very near to the rest mass of Omega meson 782.65

And:

$$10^3 + (((1+9^3+7*28*(2-2)+98)))-(((2-14*2+28*2+7*2)))-55$$

**Input:**

$$10^3 + (1 + 9^3 + 7 \times 28 (2 - 2) + 98) - (2 - 14 \times 2 + 28 \times 2 + 7 \times 2) - 55$$

**Result:**

1729

1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Now, we have that:

$$\text{If } P = \frac{f(-q)f(-q^{11})}{q^4 f(-q^9)f(-q^{99})} \text{ and } Q = \frac{f(-q)f(-q^{99})}{q^{-10/3} f(-q^9)f(-q^{11})} \text{ then}$$

$$Q^6 + \frac{1}{Q^6} = 165 \left( P + \frac{9}{P} \right) + 66 \left( P^2 + \frac{9^2}{P^2} \right) + 11 \left( P^3 + \frac{9^3}{P^3} \right) + 1848$$

$$+ \left( P^5 + \frac{9^5}{P^5} \right) + 22 \left( Q^3 + \frac{1}{Q^3} \right) \left[ 2 \left( P^2 + \frac{9^2}{P^2} \right) + 3 \left( P + \frac{9}{P} \right) + 26 \right]$$

$$+ 11 \left( \sqrt{Q^3} + \frac{1}{\sqrt{Q^3}} \right) \left[ 15 \left( \sqrt{P} + \frac{3}{\sqrt{P}} \right) + 14 \left( \sqrt{P^3} + \frac{3^3}{\sqrt{P^3}} \right) \right.$$

$$\left. + \left( \sqrt{P^5} + \frac{3^5}{\sqrt{P^5}} \right) + \left( \sqrt{P^7} + \frac{3^7}{\sqrt{P^7}} \right) \right] + 55 \left( \sqrt{P} + \frac{3}{P} \right) \left( \sqrt{Q^9} + \frac{1}{\sqrt{Q^9}} \right).$$

(99)

For  $P = Q = 1$ , we obtain:

$$165 \cdot 10 + 66 \cdot 82 + 11 \cdot (1 + 9^3) + 1848 + 1 + 9^5 + 22 \cdot 2 \cdot ((2 \cdot 82) + 3 \cdot (10) + 26) + 11 \cdot 2 \cdot ((15 \cdot 4 + 14 \cdot 28 + 1 + 243 + 1 + 2187)) + 55 \cdot 4 \cdot 2$$

**Input:**

$$165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2$$

**Result:**

149558

149558

For  $Q = 1$ , we have  $2x = 149558$ , thence:

$$\frac{(((((165 \cdot 10 + 66 \cdot 82 + 11 \cdot (1 + 9^3) + 1848 + 1 + 9^5 + 22 \cdot 2 \cdot ((2 \cdot 82) + 3 \cdot (10) + 26))) + 11 \cdot 2 \cdot ((15 \cdot 4 + 14 \cdot 28 + 1 + 243 + 1 + 2187))) + 55 \cdot 4 \cdot 2))))))}{2}$$

**Input:**

$$\frac{1}{2} ((165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2))$$

**Result:**

74779

74779

$$0.61803398 + \ln(\frac{1}{2}(((165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2))))))$$

**Input interpretation:**

$$0.61803398 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right)$$

log(x) is the natural logarithm

**Result:**

11.8403264...

11.8403264... result practically equal to the black hole entropy 11.8458

**Alternative representations:**

$$0.618034 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right) = 0.618034 + \log_e\left(\frac{1}{2} (82\,479 + 11(1 + 9^3) + 9^5)\right)$$

$$0.618034 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right) = 0.618034 + \log(a) \log_a\left(\frac{1}{2} (82\,479 + 11(1 + 9^3) + 9^5)\right)$$

$$0.618034 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right) = 0.618034 - \text{Li}_1\left(1 + \frac{1}{2} (-82\,479 - 11(1 + 9^3) - 9^5)\right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$0.618034 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right) =$$

$$0.618034 + \log(74778) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{74778}\right)^k}{k}$$

$$0.618034 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right) =$$

$$0.618034 + 2i\pi \left\lfloor \frac{\arg(74779 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (74779 - x)^k x^{-k}}{k}$$

for  
 $x < 0$

$$0.618034 + \log\left(\frac{1}{2} \left( (165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2 \right)\right) =$$

$$0.618034 + \left\lfloor \frac{\arg(74779 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(74779 - z_0)}{2\pi} \right\rfloor \log(z_0) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k (74779 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$0.618034 + \log\left(\frac{1}{2} ((165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2)\right) = 0.618034 + \int_1^{74779} \frac{1}{t} dt$$

$$0.618034 + \log\left(\frac{1}{2} ((165 \times 10 + 66 \times 82 + 11(1 + 9^3) + 1848 + 1 + 9^5 + 22 \times 2(2 \times 82 + 3 \times 10 + 26)) + 11 \times 2(15 \times 4 + 14 \times 28 + 1 + 243 + 1 + 2187) + 55 \times 4 \times 2)\right) = 0.618034 + \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{74778^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

From the entropy 11.84033, we obtain:

Mass = 3.206163e-8 (equivalent to 1.798526×10<sup>19</sup> GeV, practically near to the mean value 1.962 \* 10<sup>19</sup> of DM particle that has a Planck scale mass: m ≈ 10<sup>19</sup> GeV (Planck mass = 1,2209 × 10<sup>19</sup> GeV/c<sup>2</sup> = 21,76 μg Wikipedia) and is very nearly to the result of the following Ramanujan mock theta function: **χ(q) = 1.962364415**)

$$\text{Radius} = 4.760676e-35$$

$$\text{Temperature} = 3.827638e+30$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[\left[\left[\frac{1}{\left(\left(\left(\left(\left(\left(4 \times 1.962364415e+19\right)/\left(5 \times 0.0864055^2\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right] \times \frac{1}{3.206163e-8} \times \sqrt{\left[\left[\left[\left[\frac{3.827638e+30 \times 4 \times \pi \times \left(4.760676e-35\right)^3 - \left(4.760676e-35\right)^2}{6.67 \times 10^{-11}}\right]\right]\right]\right] \right]}$$

**Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.206163 \times 10^{-8}}\right)\right) \sqrt{\frac{3.827638 \times 10^{30} \times 4 \pi \times \left(4.760676 \times 10^{-35}\right)^3 - \left(4.760676 \times 10^{-35}\right)^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618249243446674171830932681608481937883915561867254129325...

1.6182492....

Now, we have that:

$$l'_{9,28} = 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2 + \sqrt{7})(2 + \sqrt{3})\sqrt{9 + 2\sqrt{21}}.$$

$$7*\text{sqrt}(7)+11*\text{sqrt}(3)+4*\text{sqrt}(21)+18+(2+\text{sqrt}(7))(2+\text{sqrt}(3))(\text{sqrt}(9+2\text{sqrt}(21)))$$

**Input:**

$$7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2 + \sqrt{7})(2 + \sqrt{3})\sqrt{9 + 2\sqrt{21}}$$

**Decimal approximation:**

147.7994757572030489260258422133272414024971164505156043244...

147.79947...

**Alternate forms:**

$$4\sqrt{9 + 2\sqrt{21}} + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + 2\sqrt{3(9 + 2\sqrt{21})} + 2\sqrt{7(9 + 2\sqrt{21})} + \sqrt{21(9 + 2\sqrt{21})} + 18$$

root of  $x^8 - 144x^7 - 556x^6 - 816x^5 - 858x^4 - 816x^3 - 556x^2 - 144x + 1$   
near  $x = 147.799$

$$\frac{1}{\sqrt{2}} \left( \sqrt{9 - i\sqrt{3}} (2 + \sqrt{3})(2 + \sqrt{7}) + \sqrt{2} \left( 18 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + \sqrt{\frac{1}{2}i(\sqrt{3} + -9i)} (2 + \sqrt{3})(2 + \sqrt{7}) \right) \right)$$

**Minimal polynomial:**

$$x^8 - 144x^7 - 556x^6 - 816x^5 - 858x^4 - 816x^3 - 556x^2 - 144x + 1$$

$$(\text{sqrt}(9+2\text{sqrt}(21))) * ((2+\text{sqrt}(3)) * (2+\text{sqrt}(7)) * (7*\text{sqrt}(7)+11*\text{sqrt}(3)+4*\text{sqrt}(21)+18)) - 13$$



**Input:**

$$\left(7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2 + \sqrt{7})(2 + \sqrt{3})\sqrt{9 + 2\sqrt{21}}\right) - 13$$

**Result:**

$$5 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2 + \sqrt{3})(2 + \sqrt{7})\sqrt{9 + 2\sqrt{21}}$$

**Decimal approximation:**

134.7994757572030489260258422133272414024971164505156043244...

134.79947... result very near to the rest mass of Pion meson 134.9766

**Alternate forms:**

$$4\sqrt{9 + 2\sqrt{21}} + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + 2\sqrt{3(9 + 2\sqrt{21})} + 2\sqrt{7(9 + 2\sqrt{21})} + \sqrt{21(9 + 2\sqrt{21})} + 5$$

root of  $x^8 - 40x^7 - 8928x^6 - 432208x^5 - 10536968x^4 - 149010144x^3 - 1244668032x^2 - 5726566080x - 11233130544$  near  $x = 134.799$

$$\frac{1}{\sqrt{2}} \left( \sqrt{9 - i\sqrt{3}} (2 + \sqrt{3})(2 + \sqrt{7}) + \sqrt{2} \left( 5 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + \sqrt{\frac{1}{2}i(\sqrt{3} + -9i)(2 + \sqrt{3})(2 + \sqrt{7})} \right) \right)$$

**Minimal polynomial:**

$$x^8 - 40x^7 - 8928x^6 - 432208x^5 - 10536968x^4 - 149010144x^3 - 1244668032x^2 - 5726566080x - 11233130544$$

$$\text{Pi} * \ln((((7 * \text{sqrt}(7) + 11 * \text{sqrt}(3) + 4 * \text{sqrt}(21) + 18 + (2 + \text{sqrt}(7))(2 + \text{sqrt}(3))(((\text{sqrt}(9 + 2 * \text{sqrt}(21))))))))))$$

**Input:**

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2 + \sqrt{7})(2 + \sqrt{3})\sqrt{9 + 2\sqrt{21}} \right)$$

log(x) is the natural logarithm

**Decimal approximation:**

15.69494595794287476941223299476101498896739951728274910293...

15.694945... result practically equal to the black hole entropy 15.6730

**Alternate forms:**

$$\pi \log \left( \boxed{\begin{array}{l} \text{root of } x^8 - 144x^7 - 556x^6 - 816x^5 - 858x^4 - 816x^3 - 556x^2 - 144x + 1 \\ \text{near } x = 147.799 \end{array}} \right)$$

$$\pi \left( -\frac{\log(2)}{2} + \log \left( 18\sqrt{2} + 11\sqrt{6} + 7\sqrt{14} + 4\sqrt{42} + 4\sqrt{9-i\sqrt{3}} + \right. \right. \\ \left. \left. 2\sqrt{3(9-i\sqrt{3})} + 2\sqrt{7(9-i\sqrt{3})} + \sqrt{21(9-i\sqrt{3})} + \right. \right. \\ \left. \left. 4\sqrt{9+i\sqrt{3}} + 2\sqrt{3(9+i\sqrt{3})} + 2\sqrt{7(9+i\sqrt{3})} + \sqrt{21(9+i\sqrt{3})} \right) \right)$$

**Alternative representations:**

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) = \\ \pi \log_e \left( 18 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} \right)$$

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) = \\ \pi \log(a) \log_a \left( 18 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} \right)$$

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) = \\ -\pi \text{Li}_1 \left( -17 - 11\sqrt{3} - 7\sqrt{7} - 4\sqrt{21} - (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) =$$

$$\pi \log \left( 17 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} \right) -$$

$$\pi \sum_{k=1}^{\infty} \frac{\left( \frac{1}{17+11\sqrt{3}+7\sqrt{7}+4\sqrt{21}+(2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}}} \right)^k}{k}$$

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) =$$

$$2i\pi^2 \left[ \frac{\arg \left( 18 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} - x \right)}{2\pi} \right] +$$

$$\pi \log(x) - \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left( 18 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + \right.$$

$$\left. (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} - x \right)^k x^{-k} \text{ for } x < 0$$

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) =$$

$$2i\pi^2 \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \pi \log(z_0) - \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k$$

$$\left( 18 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} - z_0 \right)^k z_0^{-k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

**Integral representations:**

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) =$$

$$\pi \int_1^{18+11\sqrt{3}+7\sqrt{7}+4\sqrt{21}+(2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}}} \frac{1}{t} dt$$

$$\pi \log \left( 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2+\sqrt{7})(2+\sqrt{3})\sqrt{9+2\sqrt{21}} \right) =$$

$$-\frac{i}{2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)}$$

$$\left( 17 + 11\sqrt{3} + 7\sqrt{7} + 4\sqrt{21} + (2+\sqrt{3})(2+\sqrt{7})\sqrt{9+2\sqrt{21}} \right)^{-s}$$

$$\Gamma(-s)^2 \Gamma(1+s) ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

From the entropy 15.69495, we obtain:

Mass = 3.691337e-8 (equivalent to 2.070689 × 10<sup>19</sup> GeV, practically near to the mean value 1.962 × 10<sup>19</sup> of DM particle that has a Planck scale mass: m ≈ 10<sup>19</sup> GeV (Planck mass = 1,2209 × 10<sup>19</sup> GeV/c<sup>2</sup> = 21,76 μg Wikipedia) and is very nearly to the result of the following Ramanujan mock theta function:  **$\chi(\mathbf{q}) = 1.962364415$** )

Radius = 5.481087e-35

Temperature = 3.324550e+30

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[(((1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))))\*1/(3.691337e-8)\* sqrt[[-(((3.324550e+30\* 4\*Pi\*(5.481087e-35)^3-(5.481087e-35)^2))])) / ((6.67\*10^-11)))))))]

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.691337 \times 10^{-8}} \right) \sqrt{\frac{3.324550 \times 10^{30} \times 4 \pi (5.481087 \times 10^{-35})^3 - (5.481087 \times 10^{-35})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249295402398408610748086813731512731385001452972280144...

1.6182492...

We have that:

$$\begin{aligned}
 & \text{If } P = \frac{\varphi(q)}{\varphi(q^2)} \frac{\varphi(q^8)}{\varphi(q^{16})} \text{ and } Q = \frac{\varphi(q)}{\varphi(q^2)} \frac{\varphi(q^{16})}{\varphi(q^8)}, \text{ then} \\
 & 16Q^4 + \frac{1}{Q^4} + 8 \left( 8Q^3 - \frac{11}{Q^3} \right) + 64 \left( 4Q^2 + \frac{13}{Q^2} \right) + 16 \left( 74Q + \frac{113}{Q} \right) \\
 & + \left( P^4 + \frac{16}{P^4} \right) - 4 \left( P^3 + \frac{8}{P^3} \right) \left[ \left( 2Q + \frac{1}{Q} \right) + 6 \right] + 2 \left( P^2 + \frac{4}{P^2} \right) \\
 & \times \left[ 4 \left( 14Q + \frac{19}{Q} \right) + 3 \left( 4Q^2 + \frac{1}{Q^2} \right) + 160 \right] + 1936 - 4 \left( P + \frac{2}{P} \right) \\
 & \times \left[ 12 \left( 10Q + \frac{21}{Q} \right) + 2 \left( 20Q^2 + \frac{21}{Q^2} \right) + \left( 8Q^3 + \frac{1}{Q^3} \right) + 168 \right] = 0. \tag{44}
 \end{aligned}$$

$$\begin{aligned}
 & 16+1+8(8-11)+64(4+13)+16(74+113)+(1+16)- \\
 & 4(1+8)[(3+6)]+2(1+4)*[(((4(14+19)+3(4+1)+160)))]+1936- \\
 & 4(1+2)*[(((12(10+21)+2(20+21)+(8+1)+168)))]
 \end{aligned}$$

**Input:**

$$\begin{aligned}
 & 16 + 1 + 8(8 - 11) + 64(4 + 13) + 16(74 + 113) + \\
 & (1 + 16) - 4(1 + 8)(3 + 6) + 2(1 + 4)(4(14 + 19) + 3(4 + 1) + 160) + \\
 & 1936 - 4(1 + 2)(12(10 + 21) + 2(20 + 21) + (8 + 1) + 168)
 \end{aligned}$$

**Result:**

$$\begin{aligned}
 & 1200 \\
 & 1200
 \end{aligned}$$

We have:

$$\begin{aligned}
 & ((((((16+1+8(-3)+64(17)+16(187)+(1+16)-4(9)(9)+2(5)*(((4(33)+3(5)+160)))+1936- \\
 & 4(3)*(((12(31)+2(41)+(9)+168)))))))))))-144-34-3
 \end{aligned}$$

**Input:**

$$\begin{aligned}
 & (16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + \\
 & 2 \times 5(4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3(12 \times 31 + 2 \times 41 + 9 + 168)) - 144 - 34 - 3
 \end{aligned}$$

**Result:**

1019

1019 result practically equal to the rest mass of Phi meson 1019.445

$$((((((16+1+8(-3)+64(17)+16(187)+(1+16)-4(9)(9)+2(5)*(((4(33)+3(5)+160))))+1936-4(3)*(((12(31)+2(41)+(9)+168))))))))+233+144+89+34+21+8$$

**Input:**

$$(16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168)) + 233 + 144 + 89 + 34 + 21 + 8$$

**Result:**

1729

1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$[((((((16+1+8(-3)+64(17)+16(187)+(1+16)-4(9)(9)+2(5)*(((4(33)+3(5)+160))))+1936-4(3)*(((12(31)+2(41)+(9)+168))))))))+233+144+89+34+21+8]^{1/15}$$

**Input:**

$$((16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168)) + 233 + 144 + 89 + 34 + 21 + 8)^{(1/15)}$$

**Result:**

$$\sqrt[15]{1729}$$

**Decimal approximation:**

1.643815228748728130580088031324769514329283143699940172645...

$$1.6438152287\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$\sqrt{6 \left( (16 + 1 + 8(-3) + 64(17) + 16(187) + (1 + 16) - 4(9)(9) + 2(5) * ((4(33) + 3(5) + 160))) + 1936 - 4(3) * (((12(31) + 2(41) + (9) + 168)))) \right)^{1/15}}$$

**Input:**

$$\sqrt{6 \left( (16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168)) + 233 + 144 + 89 + 34 + 21 + 8 \right)^{(1/15)}}$$

**Result:**

$$\sqrt{6} \sqrt[30]{1729}$$

**Decimal approximation:**

3.140524060167724424861180128206628582126936767544347593022...

3.140524060...  $\approx \pi$

$$\left( (16 + 1 + 8(-3) + 64(17) + 16(187) + (1 + 16) - 4(9)(9) + 2(5) * ((4(33) + 3(5) + 160))) + 1936 - 4(3) * (((12(31) + 2(41) + (9) + 168)))) \right)^{1/14}$$

**Input:**

$$(16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168)) + 233 + 144 + 89 + 55 + 34 + 21 + 8$$

**Result:**

1784

1784 result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

We have also that:

$$\left( (16 + 1 + 8(-3) + 64(17) + 16(187) + (1 + 16) - 4(9)(9) + 2(5) * ((4(33) + 3(5) + 160))) + 1936 - 4(3) * (((12(31) + 2(41) + (9) + 168)))) \right)^{1/14}$$

**Input:**

$$(16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{(1/14)}$$

**Result:**

$$2^{2/7} \sqrt[14]{3} \sqrt[7]{5}$$

**Decimal approximation:**

1.659363441249059998468894975666886829865439502419041316767...

1.65936344... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

And:

$$\left(\left(\left(\left(\left(16+1+8(-3)+64(17)+16(187)+(1+16)-4(9)(9)+2(5)*\left(\left(4(33)+3(5)+160\right)\right)\right)+1936-4(3)*\left(\left(12(31)+2(41)+(9)+168\right)\right)\right)\right)\right)^{\left(4\pi/185\right)}$$

**Input:**

$$(16 + 1 + 8 \times (-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{\left(4\pi/185\right)}$$

**Exact result:**

$$2^{\left(16\pi/185\right)} \times 3^{\left(4\pi/185\right)} \times 5^{\left(8\pi/185\right)}$$

**Decimal approximation:**

1.618666856192106955432768780240541038186971636705943653845...

1.618666856... This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternative representations:**

$$(16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{\left(4\pi/185\right)} = 1200^{\left(720^\circ/185\right)}$$

$$(16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{\left(4\pi/185\right)} = 1200^{-4/185 i \log(-1)}$$

$$(16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{\left(4\pi/185\right)} = 1200^{4/185 \cos^{-1}(-1)}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

**Series representations:**

$$(16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{\left(4\pi/185\right)} = 2^{64/185 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \times 3^{16/185 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \times 5^{32/185 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$



$$\begin{aligned}
& (16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + \\
& \quad 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{(4\pi)/185} = \\
& 2^{16/185} \sum_{k=0}^{\infty} \left( 4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k}) \right) / (1+2k) \times \\
& 3^{4/185} \sum_{k=0}^{\infty} \left( 4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k}) \right) / (1+2k) \times \\
& 5^{8/185} \sum_{k=0}^{\infty} \left( 4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k}) \right) / (1+2k)
\end{aligned}$$

$$\begin{aligned}
& (16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + \\
& \quad 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{(4\pi)/185} = \\
& 2^{16/185} \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k) + 2/(1+4k) + 1/(3+4k)) \times \\
& 3^{4/185} \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k) + 2/(1+4k) + 1/(3+4k)) \times \\
& 5^{8/185} \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k) + 2/(1+4k) + 1/(3+4k))
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& (16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + \\
& \quad 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{(4\pi)/185} = \\
& 2^{64/185} \int_0^1 \sqrt{1-t^2} dt \times 3^{16/185} \int_0^1 \sqrt{1-t^2} dt \times 5^{32/185} \int_0^1 \sqrt{1-t^2} dt
\end{aligned}$$

$$\begin{aligned}
& (16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + \\
& \quad 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{(4\pi)/185} = \\
& 2^{32/185} \int_0^{\infty} 1/(1+t^2) dt \times 3^{8/185} \int_0^{\infty} 1/(1+t^2) dt \times 5^{16/185} \int_0^{\infty} 1/(1+t^2) dt
\end{aligned}$$

$$\begin{aligned}
& (16 + 1 + 8(-3) + 64 \times 17 + 16 \times 187 + (1 + 16) - 4 \times 9 \times 9 + \\
& \quad 2 \times 5 (4 \times 33 + 3 \times 5 + 160) + 1936 - 4 \times 3 (12 \times 31 + 2 \times 41 + 9 + 168))^{(4\pi)/185} = \\
& 2^{32/185} \int_0^1 1/\sqrt{1-t^2} dt \times 3^{8/185} \int_0^1 1/\sqrt{1-t^2} dt \times 5^{16/185} \int_0^1 1/\sqrt{1-t^2} dt
\end{aligned}$$

We have also:

If  $P = \frac{\varphi(q)}{\varphi(q^2)} \frac{\varphi(q^8)}{\varphi(q^{16})}$  and  $Q = \frac{\varphi(q)}{\varphi(q^2)} \frac{\varphi(q^{16})}{\varphi(q^8)}$ , then

$$\begin{aligned}
 & 16Q^4 + \frac{1}{Q^4} + 8 \left( 8Q^3 - \frac{11}{Q^3} \right) + 64 \left( 4Q^2 + \frac{13}{Q^2} \right) + 16 \left( 74Q + \frac{113}{Q} \right) \\
 & + \left( P^4 + \frac{16}{P^4} \right) - 4 \left( P^3 + \frac{8}{P^3} \right) \left[ \left( 2Q + \frac{1}{Q} \right) + 6 \right] + 2 \left( P^2 + \frac{4}{P^2} \right) \\
 & \times \left[ 4 \left( 14Q + \frac{19}{Q} \right) + 3 \left( 4Q^2 + \frac{1}{Q^2} \right) + 160 \right] + 1936 - 4 \left( P + \frac{2}{P} \right) \\
 & \times \left[ 12 \left( 10Q + \frac{21}{Q} \right) + 2 \left( 20Q^2 + \frac{21}{Q^2} \right) + \left( 8Q^3 + \frac{1}{Q^3} \right) + 168 \right] = 0.
 \end{aligned} \tag{44}$$

For  $P = 2$  and  $Q = 1/2$ , we obtain:

$$\begin{aligned}
 & 16 \cdot \frac{1}{16} + 16 + 8(8 \cdot \frac{1}{8} - 88) + 64(4 \cdot \frac{1}{4} + 13 \cdot 4) + 16(74 \cdot \frac{1}{2} + 226) + (16 + \frac{16}{16}) - \\
 & 4(8 + \frac{8}{8})[(2 \cdot \frac{1}{2} + 2)] + 2(4 + \frac{4}{4}) * [(((4(14 \cdot \frac{1}{2} + 38) + 3(4 \cdot \frac{1}{4} + 4) + 160)))] + 1936 - \\
 & 4(2 + \frac{2}{2}) * [(((12(10 \cdot \frac{1}{2} + 42) + 2(20 \cdot \frac{1}{4} + 84) + (8 \cdot \frac{1}{8} + 8) + 168)))]
 \end{aligned}$$

**Input:**

$$\begin{aligned}
 & 16 \times \frac{1}{16} + 16 + 8 \left( 8 \times \frac{1}{8} - 88 \right) + 64 \left( 4 \times \frac{1}{4} + 13 \times 4 \right) + 16 \left( 74 \times \frac{1}{2} + 226 \right) + \left( 16 + \frac{16}{16} \right) - \\
 & 4 \left( 8 + \frac{8}{8} \right) \left( 2 \times \frac{1}{2} + 2 \right) + 2 \left( 4 + \frac{4}{4} \right) \left[ \left( 4 \left( 14 \times \frac{1}{2} + 38 \right) + 3 \left( 4 \times \frac{1}{4} + 4 \right) + 160 \right) \right] + \\
 & 1936 - 4 \left( 2 + \frac{2}{2} \right) \left[ \left( 12 \left( 10 \times \frac{1}{2} + 42 \right) + 2 \left( 20 \times \frac{1}{4} + 84 \right) + \left( 8 \times \frac{1}{8} + 8 \right) + 168 \right) \right]
 \end{aligned}$$

**Exact result:**

1288

1288

We note that:

$$\begin{aligned}
 & -55 + 16 \cdot \frac{1}{16} + 16 + 8(8 \cdot \frac{1}{8} - 88) + 64(4 \cdot \frac{1}{4} + 13 \cdot 4) + 16(74 \cdot \frac{1}{2} + 226) + (16 + \frac{16}{16}) - \\
 & 4(8 + \frac{8}{8})[(2 \cdot \frac{1}{2} + 2)] + 2(4 + \frac{4}{4}) * [(((4(14 \cdot \frac{1}{2} + 38) + 3(4 \cdot \frac{1}{4} + 4) + 160)))] + 1936 - \\
 & 4(2 + \frac{2}{2}) * [(((12(10 \cdot \frac{1}{2} + 42) + 2(20 \cdot \frac{1}{4} + 84) + (8 \cdot \frac{1}{8} + 8) + 168)))]
 \end{aligned}$$

**Input:**

$$\begin{aligned}
& -55 + 16 \times \frac{1}{16} + 16 + 8 \left( 8 \times \frac{1}{8} - 88 \right) + 64 \left( 4 \times \frac{1}{4} + 13 \times 4 \right) + 16 \left( 74 \times \frac{1}{2} + 226 \right) + \\
& \left( 16 + \frac{16}{16} \right) - 4 \left( 8 + \frac{8}{8} \right) \left( 2 \times \frac{1}{2} + 2 \right) + 2 \left( 4 + \frac{4}{4} \right) \left( 4 \left( 14 \times \frac{1}{2} + 38 \right) + 3 \left( 4 \times \frac{1}{4} + 4 \right) + 160 \right) + \\
& 1936 - 4 \left( 2 + \frac{2}{2} \right) \left( 12 \left( 10 \times \frac{1}{2} + 42 \right) + 2 \left( 20 \times \frac{1}{4} + 84 \right) + \left( 8 \times \frac{1}{8} + 8 \right) + 168 \right)
\end{aligned}$$

**Exact result:**

1233

1233 result practically equal to the rest mass of Delta baryon 1232

We have that:

$$\begin{aligned}
& \text{If } P = \frac{\psi(q)\psi(q^{17})}{q^{9/2}\psi(q^3)\psi(q^{51})} \text{ and } Q = \frac{\psi(q)\psi(q^{51})}{q^{-4}\psi(q^3)\psi(q^{17})}, \text{ then} \\
& Q^9 - \frac{1}{Q^9} + 34 \left( Q^8 + \frac{1}{Q^8} \right) + 272 \left( Q^7 - \frac{1}{Q^7} \right) + 238 \left( Q^6 + \frac{1}{Q^6} \right) \\
& - 595 \left( Q^5 - \frac{1}{Q^5} \right) - 510 \left( Q^4 + \frac{1}{Q^4} \right) + 16303 \left( Q^3 - \frac{1}{Q^3} \right) \\
& - 5202 \left( Q^2 + \frac{1}{Q^2} \right) - 26911 \left( Q - \frac{1}{Q} \right) + \left( P^8 + \frac{3^8}{P^8} \right) + 20230 = \\
& 17 \left\{ \left( P^2 + \frac{3^2}{P^2} \right) \left[ 7 \left( Q^6 + \frac{1}{Q^6} \right) + 28 \left( Q^5 - \frac{1}{Q^5} \right) + 34 \left( Q^4 + \frac{1}{Q^4} \right) \right. \right. \quad (56) \\
& \left. \left. - 168 \left( Q^3 - \frac{1}{Q^3} \right) + 160 \left( Q^2 + \frac{1}{Q^2} \right) + 378 \left( Q - \frac{1}{Q} \right) - 210 \right] \right. \\
& \left. - \left( P^4 + \frac{3^4}{P^4} \right) \left[ 5 \left( Q^4 + \frac{1}{Q^4} \right) + 21 \left( Q^3 - \frac{1}{Q^3} \right) - 35 \left( Q^2 + \frac{1}{Q^2} \right) \right. \right. \\
& \left. \left. - 14 \left( Q - \frac{1}{Q} \right) - 30 \right] - \left( P^6 + \frac{3^6}{P^6} \right) \left[ \left( Q^2 + \frac{1}{Q^2} \right) - 2 \left( Q - \frac{1}{Q} \right) - 2 \right] \right\}.
\end{aligned}$$

From:

$$\begin{aligned}
& Q^9 - \frac{1}{Q^9} + 34 \left( Q^8 + \frac{1}{Q^8} \right) + 272 \left( Q^7 - \frac{1}{Q^7} \right) + 238 \left( Q^6 + \frac{1}{Q^6} \right) \\
& - 595 \left( Q^5 - \frac{1}{Q^5} \right) - 510 \left( Q^4 + \frac{1}{Q^4} \right) + 16303 \left( Q^3 - \frac{1}{Q^3} \right) \\
& - 5202 \left( Q^2 + \frac{1}{Q^2} \right) - 26911 \left( Q - \frac{1}{Q} \right) + \left( P^8 + \frac{3^8}{P^8} \right) + 20230 -
\end{aligned}$$

We obtain, for P and Q = 1:

15912

$\ln 15912$

**Input:**

$\log(15912)$

$\log(x)$  is the natural logarithm

**Decimal approximation:**

9.674828820533808127345208897658026253914952476072216569547...

9.67482882.... result very near to the mean of the following black hole entropies:

$$9,934 + 9,3664 = 19,3004 / 2 = 9.6502$$

**Property:**

$\log(15912)$  is a transcendental number

**Alternate forms:**

$$3 \log(2) + 2 \log(3) + \log(221)$$

$$3 \log(2) + 2 \log(3) + \log(13) + \log(17)$$

**Alternative representations:**

$$\log(15912) = \log_e(15912)$$

$$\log(15912) = \log(a) \log_a(15912)$$

$$\log(15912) = -\text{Li}_1(-15911)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Integral representations:**

$$\log(15912) = \int_1^{15912} \frac{1}{t} dt$$

$$\log(15912) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{15911^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$$1/6 (\ln 15912)$$

**Input:**

$$\frac{1}{6} \log(15912)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\log(15912)}{6}$$

**Decimal approximation:**

1.612471470088968021224201482943004375652492079345369428257...

1.61247147... result that is a golden number

**Property:**

$\frac{\log(15912)}{6}$  is a transcendental number

**Alternate forms:**

$$\frac{1}{6} (3 \log(2) + 2 \log(3) + \log(13) + \log(17))$$

$$\frac{\log(2)}{2} + \frac{\log(3)}{3} + \frac{\log(221)}{6}$$

$$\frac{\log(2)}{2} + \frac{\log(3)}{3} + \frac{\log(13)}{6} + \frac{\log(17)}{6}$$

**Alternative representations:**

$$\frac{\log(15912)}{6} = \frac{\log_e(15912)}{6}$$

$$\frac{\log(15912)}{6} = \frac{1}{6} \log(a) \log_a(15912)$$

$$\frac{\log(15912)}{6} = -\frac{\text{Li}_1(-15911)}{6}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\frac{\log(15912)}{6} = \frac{\log(15911)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{15911}\right)^k}{k}$$

$$\frac{\log(15912)}{6} = \frac{1}{3} i\pi \left\lfloor \frac{\arg(15912-x)}{2\pi} \right\rfloor + \frac{\log(x)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (15912-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\begin{aligned} \frac{\log(15912)}{6} &= \frac{1}{6} \left\lfloor \frac{\arg(15912-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \frac{\log(z_0)}{6} + \\ &\frac{1}{6} \left\lfloor \frac{\arg(15912-z_0)}{2\pi} \right\rfloor \log(z_0) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k (15912-z_0)^k z_0^{-k}}{k} \end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\frac{\log(15912)}{6} = \frac{1}{6} \int_1^{15912} \frac{1}{t} dt$$

$$\frac{\log(15912)}{6} = -\frac{i}{12\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{15911^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

We have that:

If  $X = \frac{\varphi(q)\varphi(q^9)}{\varphi(q^4)\varphi(q^{36})}$  and  $Y = \frac{\varphi(q)\varphi(q^{36})}{\varphi(q^4)\varphi(q^9)}$ , then

$$\begin{aligned}
 & Y^6 + \frac{1}{Y^6} - 908 \left[ Y^5 + \frac{1}{Y^5} \right] - 83582 \left[ Y^4 + \frac{1}{Y^4} \right] - 1369692 \left[ Y^3 + \frac{1}{Y^3} \right] \\
 & - 3 \left[ Y^2 + \frac{1}{Y^2} \right] \left\{ 2657883 + 96832 \left[ X^2 + \frac{16}{X^2} \right] \right\} - 24 \left[ Y + \frac{1}{Y} \right] \\
 & \times \left\{ 892353 + 289628 \left[ X + \frac{4}{X} \right] \right\} + 17323008 \left[ \sqrt{Y} + \frac{1}{\sqrt{Y}} \right] \left[ \sqrt{X} + \frac{2}{\sqrt{X}} \right] \\
 & + 1831776 \left[ \sqrt{Y^3} + \frac{1}{\sqrt{Y^3}} \right] \left[ \sqrt{X^3} + \frac{8}{\sqrt{X^3}} \right] + 21504 \left[ \sqrt{Y^5} + \frac{1}{\sqrt{Y^5}} \right] \\
 & \times \left[ \sqrt{X^5} + \frac{32}{\sqrt{X^5}} \right] - 29469924 = 128 \left\{ 2 \left[ X^4 + \frac{256}{X^4} \right] \right. \\
 & \left. + 420 \left[ X^3 + \frac{64}{X^3} \right] + 9987 \left[ X^2 + \frac{16}{X^2} \right] + 75426 \left[ X + \frac{4}{X} \right] \right\}.
 \end{aligned}$$

(63)

For  $X = 1$ , we have that:

$$128[(((2((1+256)+420(1+64)+9987(1+16)+75426(1+4)))))]$$

**Input:**

$$128 (2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75426 (1 + 4)))$$

**Result:**

147063296

147063296

**Scientific notation:**

$$1.47063296 \times 10^8$$

$$\ln (((((128[(((2((1+256)+420(1+64)+9987(1+16)+75426(1+4)))))]))))))$$

**Input:**

$$\log(128 (2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75426 (1 + 4))))$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log(147063296)$$

**Decimal approximation:**

18.80637363710189031561760589527905935677554565758749412375...

18.80637... result very near to the black hole entropy 18.7328

**Property:**

log(147063296) is a transcendental number

**Alternate form:**

$$9 \log(2) + \log(287233)$$

**Alternative representations:**

$$\log(128 \times 2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75\,426 (1 + 4))) = \log_e(147\,063\,296)$$

$$\log(128 \times 2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75\,426 (1 + 4))) = \log(a) \log_a(147\,063\,296)$$

$$\log(128 \times 2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75\,426 (1 + 4))) = -\text{Li}_1(-147\,063\,295)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Integral representations:**

$$\log(128 \times 2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75\,426 (1 + 4))) = \int_1^{147063296} \frac{1}{t} dt$$

$$\log(128 \times 2 ((1 + 256) + 420 (1 + 64) + 9987 (1 + 16) + 75\,426 (1 + 4))) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{147063295^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$$128[(((2((16+256/16)+420(8+64/8))+9987(4+16/4)+75426(2+4/2))))]$$

**Input:**



$$128 \left( 2 \left( \left( 16 + \frac{256}{16} \right) + 420 \left( 8 + \frac{64}{8} \right) + 9987 \left( 4 + \frac{16}{4} \right) + 75\,426 \left( 2 + \frac{4}{2} \right) \right) \right)$$

**Exact result:**

99418112

99418112

**Scientific notation:**

$9.9418112 \times 10^7$

For  $X = 2$ , we obtain:

$$\ln(\ln(\ln(\ln(\ln(128[\ln(\ln(\ln(\ln(2((16+256/16)+420(8+64/8)+9987(4+16/4)+75426(2+4/2)))))]))))))$$

**Input:**

$$\log\left(128 \left( 2 \left( \left( 16 + \frac{256}{16} \right) + 420 \left( 8 + \frac{64}{8} \right) + 9987 \left( 4 + \frac{16}{4} \right) + 75\,426 \left( 2 + \frac{4}{2} \right) \right) \right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$\log(99\,418\,112)$

**Decimal approximation:**

18.41484486830765719891057298739967984148449020633102186319...

**18.41484...** result very near to the black hole entropy 18.2773

**Property:**

$\log(99\,418\,112)$  is a transcendental number

**Alternate forms:**

$$16 \log(2) + \log(1517)$$

$$16 \log(2) + \log(37) + \log(41)$$

**Alternative representations:**

$$\log\left(128 \times 2 \left( \left( 16 + \frac{256}{16} \right) + 420 \left( 8 + \frac{64}{8} \right) + 9987 \left( 4 + \frac{16}{4} \right) + 75\,426 \left( 2 + \frac{4}{2} \right) \right) \right) = \log_e\left(256 \left( 301\,720 + 9987 \left( 4 + \frac{16}{4} \right) + 420 \left( 8 + \frac{64}{8} \right) + \frac{256}{16} \right) \right)$$

$$\log\left(128 \times 2 \left( \left(16 + \frac{256}{16}\right) + 420 \left(8 + \frac{64}{8}\right) + 9987 \left(4 + \frac{16}{4}\right) + 75\,426 \left(2 + \frac{4}{2}\right) \right)\right) = \log(a) \log_a \left( 256 \left( 301\,720 + 9987 \left(4 + \frac{16}{4}\right) + 420 \left(8 + \frac{64}{8}\right) + \frac{256}{16} \right) \right)$$

$$\log\left(128 \times 2 \left( \left(16 + \frac{256}{16}\right) + 420 \left(8 + \frac{64}{8}\right) + 9987 \left(4 + \frac{16}{4}\right) + 75\,426 \left(2 + \frac{4}{2}\right) \right)\right) = -\text{Li}_1 \left( 1 - 256 \left( 301\,720 + 9987 \left(4 + \frac{16}{4}\right) + 420 \left(8 + \frac{64}{8}\right) + \frac{256}{16} \right) \right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Integral representations:

$$\log\left(128 \times 2 \left( \left(16 + \frac{256}{16}\right) + 420 \left(8 + \frac{64}{8}\right) + 9987 \left(4 + \frac{16}{4}\right) + 75\,426 \left(2 + \frac{4}{2}\right) \right)\right) = \int_1^{\infty} \frac{418\,112}{t} dt$$

$$\log\left(128 \times 2 \left( \left(16 + \frac{256}{16}\right) + 420 \left(8 + \frac{64}{8}\right) + 9987 \left(4 + \frac{16}{4}\right) + 75\,426 \left(2 + \frac{4}{2}\right) \right)\right) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{99\,418\,111^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

Now, we have that:

*If  $X = \frac{\varphi(q)\varphi(q^2)}{\varphi(q^4)\varphi(q^8)}$  and  $Y = \frac{\varphi(q)\varphi(q^8)}{\varphi(q^4)\varphi(q^2)}$ , then*

$$X^2 - 2X^3Y + X^4Y^2 - 4XY + 4Y^2 - 4XY^3 + 4X^2Y^2 - 2X^3Y^3 + 2X^2Y^4 = 0. \tag{58}$$

For  $X = Y = 1$ , from the (58), we obtain:

$$1 - 2 + 1 - 4 + 4 - 4 + 4 - 2 + 2 = 0 \text{ (supersymmetric condition } \rightarrow 0)$$

For  $X = Y = 2$ , we obtain:

$$2^2 - 2 \cdot 2^3 \cdot 2 + 2^4 \cdot 2^2 - 4 \cdot 2 \cdot 2 + 4 \cdot 2^2 - 4 \cdot 2 \cdot 2^3 + 4 \cdot 2^2 \cdot 2^2 - 2 \cdot 2^3 \cdot 2^3 + 2 \cdot 2^2 \cdot 2^4$$

**Input:**

$$2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4$$

**Result:**

36

36

And:

$$48 \cdot (2^2 - 2 \cdot 2^3 \cdot 2 + 2^4 \cdot 2^2 - 4 \cdot 2 \cdot 2 + 4 \cdot 2^2 - 4 \cdot 2 \cdot 2^3 + 4 \cdot 2^2 \cdot 2^2 - 2 \cdot 2^3 \cdot 2^3 + 2 \cdot 2^2 \cdot 2^4)$$

**Input:**

$$48 (2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4)$$

**Result:**

1728

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$[48 \cdot (2^2 - 2 \cdot 2^3 \cdot 2 + 2^4 \cdot 2^2 - 4 \cdot 2 \cdot 2 + 4 \cdot 2^2 - 4 \cdot 2 \cdot 2^3 + 4 \cdot 2^2 \cdot 2^2 - 2 \cdot 2^3 \cdot 2^3 + 2 \cdot 2^2 \cdot 2^4)]^{1/15}$$

**Input:**

$$(48 (2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4))^{(1/15)}$$

**Result:**

$$2^{2/5} \sqrt[5]{3}$$

**Decimal approximation:**

1.643751829517225762308497936230979517383492589945475200411...

$$1.6437518... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$55 + [48 * (2^2 - 2 * 2^3 * 2 + 2^4 * 2^2 - 4 * 2 * 2 + 4 * 2^2 - 4 * 2 * 2^3 + 4 * 2^2 * 2^2 - 2 * 2^3 * 2^3 + 2 * 2^2 * 2^4)]$$

**Input:**

$$55 + 48 (2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4)$$

**Result:**

1783

1783 result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

We note that:

$$\ln^2 (2^2 - 2 * 2^3 * 2 + 2^4 * 2^2 - 4 * 2 * 2 + 4 * 2^2 - 4 * 2 * 2^3 + 4 * 2^2 * 2^2 - 2 * 2^3 * 2^3 + 2 * 2^2 * 2^4)$$

**Input:**

$$\log^2 (2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4)$$

log(x) is the natural logarithm

**Exact result:**

$$\log^2(36)$$

**Decimal approximation:**

12.84160798227360550147650833539615462653447817971397330954...

12.8416079... result near to the value of black hole entropy 12,5664

**Property:**

$\log^2(36)$  is a transcendental number

•

**Alternate forms:**

$4 \log^2(6)$

•

$(2 \log(2) + 2 \log(3))^2$

**Alternative representations:**

$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = \log_e^2(-156 + 12 \times 2^4)$

•

$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = (\log(a) \log_a(-156 + 12 \times 2^4))^2$

•

$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = (-\text{Li}_1(157 - 12 \times 2^4))^2$

$\log_b(x)$  is the base-  $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = \left( \log(35) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{35}\right)^k}{k} \right)^2$

•

$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = \left( 2i\pi \left[ \frac{\arg(36-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (36-x)^k x^{-k}}{k} \right)^2$  for  $x < 0$

•

$$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = \left( \log(z_0) + \left\lfloor \frac{\arg(36 - z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (36 - z_0)^k z_0^{-k}}{k} \right)^2$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = \left( \int_1^{36} \frac{1}{t} dt \right)^2$$

$$\log^2(2^2 - 2 \times 2^3 \times 2 + 2^4 \times 2^2 - 4 \times 2 \times 2 + 4 \times 2^2 - 4 \times 2 \times 2^3 + 4 \times 2^2 \times 2^2 - 2 \times 2^3 \times 2^3 + 2 \times 2^2 \times 2^4) = - \frac{\left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{35^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{4\pi^2} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

Now, for  $X = Y = 1$ , from (59), we obtain:

$$\text{If } X = \frac{\varphi(q)\varphi(q^3)}{\varphi(q^4)\varphi(q^{12})} \text{ and } Y = \frac{\varphi(q)\varphi(q^{12})}{\varphi(q^4)\varphi(q^3)}, \text{ then}$$

$$Y^2 + \frac{1}{Y^2} - 12 \left( Y + \frac{1}{Y} \right) + 12 \left( \sqrt{X} + \frac{2}{\sqrt{X}} \right) \left( \sqrt{Y} + \frac{1}{\sqrt{Y}} \right) = 4 \left( X + \frac{4}{X} \right) + 30. \quad (59)$$

$1 + 1 - 12(1+1) + 12(1+2)(1+1)$  and

$4(1+4)+30$

We obtain:

$$1 + 1 - 12(1+1) + 12(1+2)(1+1)$$

**Input:**

$$1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)$$

**Result:**

50

50

And:

$$4(1+4) + 30$$

**Input:**

$$4(1 + 4) + 30$$

**Result:**

50

50

Note that:

$$[1 + 1 - 12(1+1) + 12(1+2)(1+1)]^{1/8}$$

**Input:**

$$\sqrt[8]{1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)}$$

**Result:**

$$\sqrt[8]{2} \sqrt[4]{5}$$

**Decimal approximation:**

1.630689408953309633325441386540290377805497589955618125005...

1.6306894... result that is a golden number

$$\ln^2 [1 + 1 - 12(1+1) + 12(1+2)(1+1)]$$

**Input:**

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1))$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log^2(50)$$

### Decimal approximation:

15.30392399499906448681657226084668020224925553986996573637...

15.30392399... result near to the black hole entropy 15.6730

### Property:

$\log^2(50)$  is a transcendental number

•

### Alternate form:

$$(\log(2) + 2 \log(5))^2$$

### Alternative representations:

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = \log_e^2(50)$$

•

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = (\log(a) \log_a(50))^2$$

•

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = (-\text{Li}_1(-49))^2$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = \left( \log(49) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{49}\right)^k}{k} \right)^2$$

•

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = \left( 2i\pi \left[ \frac{\arg(50 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (50 - x)^k x^{-k}}{k} \right)^2 \text{ for } x < 0$$

•

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = \left( \log(z_0) + \left[ \frac{\arg(50 - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (50 - z_0)^k z_0^{-k}}{k} \right)^2$$

$\arg(z)$  is the complex argument



[x] is the floor function

**Integral representations:**

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = \left( \int_1^{50} \frac{1}{t} dt \right)^2$$

$$\log^2(1 + 1 - 12(1 + 1) + 12(1 + 2)(1 + 1)) = - \frac{\left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{49^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{4\pi^2} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

From:

*If  $X = \frac{\varphi(q)\varphi(q^4)}{\varphi(q^4)\varphi(q^{16})}$  and  $Y = \frac{\varphi(q)\varphi(q^{16})}{\varphi(q^4)\varphi(q^4)}$ , then*

$$X^4Y^2 + 6X^3Y + X^2 + 16Y^2 + 24XY - 4\sqrt{XY} (X^3Y + X^2 + 2X + 8Y) = 0. \tag{60}$$

For  $X = Y = 1$ , we obtain:

$$1 + 6 + 1 + 16 + 24 - 4(1+1+2+8)$$

**Input:**

$$1 + 6 + 1 + 16 + 24 - 4(1 + 1 + 2 + 8)$$

**Result:**

0  
 0 (supersymmetric condition  $\rightarrow$  0)

For  $X = Y = 2$ , we obtain:

$$2^4 \cdot 2^2 + 6 \cdot 2^3 \cdot 2 + 2^2 + 16 \cdot 2^2 + 24 \cdot 2 \cdot 2 - 4 \cdot (\sqrt{4}) \cdot (2^3 \cdot 2 + 2^2 + 2 \cdot 2 + 8 \cdot 2)$$

**Input:**

$$2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)$$

**Result:**

4

4

$$9 \ln \left( (2^4 \cdot 2^2 + 6 \cdot 2^3 \cdot 2 + 2^2 + 16 \cdot 2^2 + 24 \cdot 2 \cdot 2 - 4 \sqrt{4}) (2^3 \cdot 2 + 2^2 + 2 \cdot 2 + 8 \cdot 2) \right)$$

**Input:**

$$9 \log \left( 2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2) \right)$$

log(x) is the natural logarithm

**Exact result:**

$$9 \log(4)$$

**Decimal approximation:**

12.47664925007901556951017818624717822535900241848459457417...

12.47664... result very near to the value of black hole entropy 12,5664

**Property:**

9 log(4) is a transcendental number

•

**Alternate form:**

$$18 \log(2)$$

**Alternative representations:**

• More

$$9 \log \left( 2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2) \right) = 9 \log_e \left( 260 + 4 \times 2^4 - 160 \sqrt{4} \right)$$

•

$$9 \log \left( 2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2) \right) = 9 \log(a) \log_a \left( 260 + 4 \times 2^4 - 160 \sqrt{4} \right)$$

•

$$9 \log \left( 2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2) \right) = -9 \operatorname{Li}_1 \left( -259 - 4 \times 2^4 + 160 \sqrt{4} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$9 \log\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4\sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) =$$

$$9 \log(3) - 9 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^k}{k}$$

- 

$$9 \log\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4\sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) =$$

$$18 i \pi \left\lfloor \frac{\arg(4-x)}{2\pi} \right\rfloor + 9 \log(x) - 9 \sum_{k=1}^{\infty} \frac{(-1)^k (4-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

- 

$$9 \log\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4\sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) =$$

$$9 \left\lfloor \frac{\arg(4-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 9 \log(z_0) + 9 \left\lfloor \frac{\arg(4-z_0)}{2\pi} \right\rfloor \log(z_0) - 9 \sum_{k=1}^{\infty} \frac{(-1)^k (4-z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$9 \log\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4\sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) =$$

$$9 \int_1^4 \frac{1}{t} dt$$

- 

$$9 \log\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4\sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) =$$

$$-\frac{9i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{3^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$$34+16*2 * \exp(((2^4*2^2+6*2^3*2+2^2+16*2^2+24*2*2-4*(\text{sqrt}(4))(2^3*2+2^2+2*2+8*2))))$$

**Input:**

$$34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right)$$

**Exact result:**

$$34 + 32 e^4$$

**Decimal approximation:**

1781.140801060615650499528358491548108889303585235650199226...

1781.1408... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

**Property:**

$34 + 32 e^4$  is a transcendental number

•

**Alternate form:**

$$2(17 + 16 e^4)$$

**Series representations:**

$$34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) = 34 + 32 \sum_{k=0}^{\infty} \frac{4^k}{k!}$$

•

$$34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) = 34 + 32 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^4$$

•

$$34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right) = 34 + \frac{32}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}\right)^4}$$

$n!$  is the factorial function

$$\left(\left(\left(\left(34+16 \cdot 2 \cdot \exp\left(\left(2^4 \cdot 2^2+6 \cdot 2^3 \cdot 2+2^2+16 \cdot 2^2+24 \cdot 2 \cdot 2-4 \cdot \sqrt{4}\right)\left(2^3 \cdot 2+2^2+2 \cdot 2+8 \cdot 2\right)\right)\right)\right)\right)\right)^{1/15}$$

**Input:**

$$\left(34+16 \times 2 \exp\left(2^4 \times 2^2+6 \times 2^3 \times 2+2^2+16 \times 2^2+24 \times 2 \times 2-4 \sqrt{4}\left(2^3 \times 2+2^2+2 \times 2+8 \times 2\right)\right)\right)^{(1 / 15)}$$

**Exact result:**

$$\sqrt[15]{34+32 e^4}$$

**Decimal approximation:**

1.647074398815548255993322448035971780412937233384321777791...

$$1.64707439 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Property:**

$\sqrt[15]{34+32 e^4}$  is a transcendental number

•

**Alternate form:**

$$\sqrt[15]{2\left(17+16 e^4\right)}$$

**All 15th roots of  $34+32 e^4$ :**

$$\sqrt[15]{34+32 e^4} e^{0} \approx 1.64707 \text{ (real, principal root)}$$

•

$$\sqrt[15]{34+32 e^4} e^{(2 i \pi) / 15} \approx 1.5047+0.6699 i$$

•

$$\sqrt[15]{34+32 e^4} e^{(4 i \pi) / 15} \approx 1.1021+1.2240 i$$

•

$$\sqrt[15]{34+32 e^4} e^{(2 i \pi) / 5} \approx 0.5090+1.5665 i$$

•

$$\sqrt[15]{34+32 e^4} e^{(8 i \pi) / 15} \approx -0.17217+1.63805 i$$

**Series representations:**

$$\left(34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right)\right)^{(1/15)} = \sqrt[15]{34 + 32 \sum_{k=0}^{\infty} \frac{4^k}{k!}}$$

- $$\left(34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right)\right)^{(1/15)} = \sqrt[15]{34 + 32 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^4}$$

- $$\left(34 + 16 \times 2 \exp\left(2^4 \times 2^2 + 6 \times 2^3 \times 2 + 2^2 + 16 \times 2^2 + 24 \times 2 \times 2 - 4 \sqrt{4} (2^3 \times 2 + 2^2 + 2 \times 2 + 8 \times 2)\right)\right)^{(1/15)} = \sqrt[15]{34 + \frac{32}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}\right)^4}}$$

$n!$  is the factorial function

- ### Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$  is the gamma function

$\text{Re}(z)$  is the real part of  $z$

$\arg(z)$  is the complex argument

$|z|$  is the absolute value of  $z$

$i$  is the imaginary unit

If  $P := \frac{\psi(-q)\psi(-q^{11})}{q^6\psi(-q^5)\psi(-q^{55})}$  and  $Q := \frac{\psi(-q)\psi(-q^{55})}{q^{-5}\psi(-q^5)\psi(-q^{11})}$ , then

$$\begin{aligned}
 & Q^6 + \frac{1}{Q^6} - 33 \left( Q^5 + \frac{1}{Q^5} \right) - 99 \left( Q^4 + \frac{1}{Q^4} \right) - 1529 \left( Q^3 + \frac{1}{Q^3} \right) \\
 & - 1683 \left( Q^2 + \frac{1}{Q^2} \right) - 8800 \left( Q + \frac{1}{Q} \right) - 6534 - \left( P^5 + \frac{5^5}{P^5} \right) \\
 & - 11 \left\{ \left( P^4 + \frac{5^4}{P^4} \right) \left( Q + \frac{1}{Q} \right) + \left( P^3 + \frac{5^3}{P^3} \right) \left[ 11 + 4 \left( Q^2 + \frac{1}{Q^2} \right) \right] \right. \\
 & + \left( P^2 + \frac{5^2}{P^2} \right) \left[ 18 + 56 \left( Q + \frac{1}{Q} \right) + 3 \left( Q^2 + \frac{1}{Q^2} \right) + 8 \left( Q^3 + \frac{1}{Q^3} \right) \right] \\
 & + \left( P + \frac{5}{P} \right) \left[ 324 + 126 \left( Q + \frac{1}{Q} \right) + 160 \left( Q^2 + \frac{1}{Q^2} \right) + 18 \left( Q^3 + \frac{1}{Q^3} \right) \right. \\
 & \left. \left. + 9 \left( Q^4 + \frac{1}{Q^4} \right) \right] + \left( P^3 + \frac{5^3}{P^3} \right) \left[ 11 + 4 \left( Q^2 + \frac{1}{Q^2} \right) \right] \right\} = 0.
 \end{aligned} \tag{70}$$

((1+1-66-198-1529\*2-1683\*2-17600-6534-(1+5^5)-  
 11((((((((((((((1+625)\*2+(1+125)))))))))  
 ((11+8)))+(1+25)(18+112+6+16)+6(324+252+320+36+18)+(1+125)(11+8)))))))))))))

**Input:**

1 + 1 - 66 - 198 - 1529 × 2 - 1683 × 2 - 17600 - 6534 - (1 + 5<sup>5</sup>) -  
 11 (((1 + 625) × 2 + (1 + 125)) (11 + 8) + (1 + 25) (18 + 112 + 6 + 16) +  
 6 (324 + 252 + 320 + 36 + 18) + (1 + 125) (11 + 8))

**Result:**

-454454  
 -454454

-454454\*0 = 0 → 0 or -454454/0 = ∞ (complex infinity = **supersymmetric condition** → ∞)

If we take only the result -454454 we obtain:

1.61803398^2+ ln(-(((1+1-66-198-1529\*2-1683\*2-17600-6534-(1+5^5)-  
 11((((((((((((((1+625)\*2+(1+125)))))))))  
 ((11+8)))+(1+25)(18+112+6+16)+6(324+252+320+36+18)+(1+125)(11+8)))))))))))))

Where 1.61803398... is the golden ratio

**Input interpretation:**

$$1.61803398^2 + \log(-(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5)) - 11(((1 + 625) \times 2 + (1 + 125))(11 + 8) + (1 + 25)(18 + 112 + 6 + 16) + 6(324 + 252 + 320 + 36 + 18) + (1 + 125)(11 + 8))))$$

$\log(x)$  is the natural logarithm

**Result:**

15.6448859...

15.6448... result practically equal to the black hole entropy 15.6730

**Alternative representations:**

$$1.61803^2 + \log(-(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5)) - 11(((1 + 625) \times 2 + (1 + 125))(11 + 8) + (1 + 25)(18 + 112 + 6 + 16) + 6(324 + 252 + 320 + 36 + 18) + (1 + 125)(11 + 8)))) = \log_e(451329 + 5^5) + 1.61803^2$$

- $$1.61803^2 + \log(-(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5)) - 11(((1 + 625) \times 2 + (1 + 125))(11 + 8) + (1 + 25)(18 + 112 + 6 + 16) + 6(324 + 252 + 320 + 36 + 18) + (1 + 125)(11 + 8)))) = \log(a) \log_a(451329 + 5^5) + 1.61803^2$$

- $$1.61803^2 + \log(-(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5)) - 11(((1 + 625) \times 2 + (1 + 125))(11 + 8) + (1 + 25)(18 + 112 + 6 + 16) + 6(324 + 252 + 320 + 36 + 18) + (1 + 125)(11 + 8)))) = -\text{Li}_1(-451328 - 5^5) + 1.61803^2$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$1.61803^2 + \log(-(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5)) - 11(((1 + 625) \times 2 + (1 + 125))(11 + 8) + (1 + 25)(18 + 112 + 6 + 16) + 6(324 + 252 + 320 + 36 + 18) + (1 + 125)(11 + 8)))) = 2.61803 + \log(454453) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{454453}\right)^k}{k}$$

-



$$1.61803^2 + \log\left(-\left(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5) - 11 \left( (1 + 625) 2 + (1 + 125) \right) (11 + 8) + (1 + 25) (18 + 112 + 6 + 16) + 6 (324 + 252 + 320 + 36 + 18) + (1 + 125) (11 + 8) \right)\right) =$$

$$2.61803 + 2 i \pi \left\lfloor \frac{\arg(454454 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (454454 - x)^k x^{-k}}{k}$$

for  
 $x < 0$

$$1.61803^2 + \log\left(-\left(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5) - 11 \left( (1 + 625) 2 + (1 + 125) \right) (11 + 8) + (1 + 25) (18 + 112 + 6 + 16) + 6 (324 + 252 + 320 + 36 + 18) + (1 + 125) (11 + 8) \right)\right) =$$

$$2.61803 + \left\lfloor \frac{\arg(454454 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) +$$

$$\left\lfloor \frac{\arg(454454 - z_0)}{2 \pi} \right\rfloor \log(z_0) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k (454454 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$1.61803^2 + \log\left(-\left(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5) - 11 \left( (1 + 625) 2 + (1 + 125) \right) (11 + 8) + (1 + 25) (18 + 112 + 6 + 16) + 6 (324 + 252 + 320 + 36 + 18) + (1 + 125) (11 + 8) \right)\right) = 2.61803 + \int_1^{454454} \frac{1}{t} dt$$

$$1.61803^2 + \log\left(-\left(1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5) - 11 \left( (1 + 625) 2 + (1 + 125) \right) (11 + 8) + (1 + 25) (18 + 112 + 6 + 16) + 6 (324 + 252 + 320 + 36 + 18) + (1 + 125) (11 + 8) \right)\right) =$$

$$2.61803 + \frac{1}{2 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{454453^{-s} \Gamma(-s)^2 \Gamma(1 + s)}{\Gamma(1 - s)} ds \text{ for}$$

$$-1 < \gamma < 0$$

$$\frac{1}{16^2}(-((1+1-66-198-1529*2-1683*2-17600-6534-(1+5^5)-11((((((((((((((1+625)*2+(1+125))))))))))))))((11+8)))+(1+25)(18+112+6+16)+6(324+252+320+36+18)+(1+125)(11+8))))))))))$$

**Input:**

$$\frac{1}{16^2}(- (1 + 1 - 66 - 198 - 1529 \times 2 - 1683 \times 2 - 17600 - 6534 - (1 + 5^5) - 11 ((( (1 + 625) \times 2 + (1 + 125)) (11 + 8) + (1 + 25) (18 + 112 + 6 + 16) + 6 (324 + 252 + 320 + 36 + 18) + (1 + 125) (11 + 8))))))$$

**Exact result:**

$$\frac{227227}{128}$$

**Decimal form:**

$$1775.2109375$$

1775.2109... result in the range of the mass of candidate “glueball”  $f_0(1710)$  and the hypothetical mass of Gluino (“glueball” =  $1760 \pm 15$  MeV; gluino = 1785.16 GeV).

Now, we have that:

$$\begin{aligned} \text{If } P &:= \frac{\varphi(q)\varphi(q^4)}{\varphi(q^5)\varphi(q^{20})} \text{ and } Q := \frac{\varphi(q)\varphi(q^{20})}{\varphi(q^5)\varphi(q^4)}, \text{ then} \\ Q^4 + \frac{1}{Q^4} - 112 \left( Q^3 + \frac{1}{Q^3} \right) + 1440 \left( Q^2 + \frac{1}{Q^2} \right) - 3184 \left( Q + \frac{1}{Q} \right) + 7316 \\ &= 8 \left( P + \frac{1}{P} \right) \left[ 22 \left( Q^2 + \frac{1}{Q^2} \right) - 31 \left( Q + \frac{1}{Q} \right) + 170 \right] - 2 \left( P^2 + \frac{5^2}{P^2} \right) \\ &\times \left[ 3 \left( Q^2 + \frac{1}{Q^2} \right) + 24 \left( Q + \frac{1}{Q} \right) + 64 \right] + 4 \left( P^3 + \frac{5^3}{P^3} \right) \left[ \left( Q + \frac{1}{Q} \right) + 4 \right]. \end{aligned} \tag{73}$$

For  $X = Y = 1$ , we obtain:

$$1+1-112(1+1)+1440(1+1)-3184(1+1)+7316$$

$$8*2(22*2-31*2+170)-2(1+25)*(3*2+24*2+64)+4(1+125)(1+1+4)$$

$$1+1-112(1+1)+1440(1+1)-3184(1+1)+7316$$

**Input:**

$$1 + 1 - 112 (1 + 1) + 1440 (1 + 1) - 3184 (1 + 1) + 7316$$

**Result:**

$$3606$$

$$3606$$

$$8*2(22*2-31*2+170)-2(1+25)*(3*2+24*2+64)+4(1+125)(1+1+4)$$

**Input:**

$$8 \times 2 (22 \times 2 - 31 \times 2 + 170) - 2 (1 + 25) (3 \times 2 + 24 \times 2 + 64) + 4 (1 + 125) (1 + 1 + 4)$$

**Result:**

$$-680$$

$$-680$$

3606 = -680 ; thence

$$(((((((((((16(44-62+170)-2*(26)*(6+48+64)+4*126*6))))/3606))))))$$

**Input:**

$$\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}$$

**Exact result:**

$$-\frac{340}{1803}$$

**Decimal approximation:**

$$-0.18857459789240155296727676095396561286744315030504714364...$$

$$-0.1885745978924...$$

$$2 \exp(((((((((((16(44-62+170)-2*(26)*(6+48+64)+4*126*6))))/3606))))))$$

**Input:**

$$2 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right)$$

**Exact result:**

$$\frac{2}{e^{340/1803}}$$

**Decimal approximation:**

1.656277447461892529037520343846480063201508336177366775283...

1.65627744... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

**Property:**

$\frac{2}{e^{340/1803}}$  is a transcendental number

**Series representations:**

$$2 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{2}{\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{340/1803}}$$

$$\bullet \quad 2 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{2}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{340/1803}}$$

$$\bullet \quad 2 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{2}{\left(\frac{\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}\right)^{340/1803}}$$

$n!$  is the factorial function

$$12 \exp(((((((((((16(44-62+170)-2*(26)*(6+48+64)+4*126*6))))/3606))))))$$

**Input:**

$$12 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right)$$

**Exact result:**

$$\frac{12}{e^{340/1803}}$$

**Decimal approximation:**

9.937664684771355174225122063078880379209050017064200651699...

9.93766468 result practically equal to the black hole entropy 9.9340

**Property:**

$\frac{12}{e^{340/1803}}$  is a transcendental number

**Series representations:**

$$12 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{12}{\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{340/1803}}$$

- $$12 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{12}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{340/1803}}$$

- $$12 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{12}{\left(\frac{\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}\right)^{340/1803}}$$

$n!$  is the factorial function

$$21 \exp(((((((((((16(44-62+170)-2*(26)*(6+48+64)+4*126*6)))))/3606))))))$$

**Input:**

$$21 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right)$$

**Exact result:**

$$\frac{21}{e^{340/1803}}$$

**Decimal approximation:**

17.39091319834987155489396361038804066361583752986235114047...

17.390913198... result near to the black hole entropy 17.5764

**Property:**

$\frac{21}{e^{340/1803}}$  is a transcendental number

**Series representations:**

$$21 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{21}{\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{340/1803}}$$

- $$21 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{21}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{340/1803}}$$

- $$21 \exp\left(\frac{16(44 - 62 + 170) - 2 \times 26(6 + 48 + 64) + 4 \times 126 \times 6}{3606}\right) = \frac{21}{\left(\frac{\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}\right)^{340/1803}}$$

n! is the factorial function

$$55 + (((((1+1-112(1+1)+1440(1+1)-3184(1+1)+7316)))) + (((((8*2(22*2-31*2+170)-2(1+25)*(3*2+24*2+64)+4(1+125)(1+1+4)))))))$$

**Input:**

$$55 + ((1 + 1 - 112(1 + 1) + 1440(1 + 1) - 3184(1 + 1) + 7316) + (8 \times 2(22 \times 2 - 31 \times 2 + 170) - 2(1 + 25)(3 \times 2 + 24 \times 2 + 64) + 4(1 + 125)(1 + 1 + 4)))$$

**Result:**

2981



$$\frac{1}{625} \left( 625 \sqrt[16]{2926} - 18 \right)$$

• **Minimal polynomial:**

$$\begin{aligned}
& 542\,101\,086\,242\,752\,217\,003\,726\,400\,434\,970\,855\,712\,890\,625\,x^{16} + \\
& 249\,800\,180\,540\,660\,221\,595\,317\,125\,320\,434\,570\,312\,500\,000\,x^{15} + \\
& 53\,956\,838\,996\,782\,607\,864\,588\,499\,069\,213\,867\,187\,500\,000\,x^{14} + \\
& 7\,251\,799\,161\,167\,582\,497\,000\,694\,274\,902\,343\,750\,000\,000\,x^{13} + \\
& 678\,768\,401\,485\,285\,721\,719\,264\,984\,130\,859\,375\,000\,000\,x^{12} + \\
& 46\,916\,471\,910\,662\,949\,085\,235\,595\,703\,125\,000\,000\,000\,x^{11} + \\
& 2\,477\,189\,716\,883\,003\,711\,700\,439\,453\,125\,000\,000\,000\,x^{10} + \\
& 101\,918\,662\,637\,472\,152\,709\,960\,937\,500\,000\,000\,000\,x^9 + \\
& 3\,302\,164\,669\,454\,097\,747\,802\,734\,375\,000\,000\,000\,x^8 + \\
& 84\,535\,415\,538\,024\,902\,343\,750\,000\,000\,000\,000\,x^7 + \\
& 1\,704\,233\,977\,246\,582\,031\,250\,000\,000\,000\,000\,x^6 + \\
& 26\,771\,966\,478\,928\,125\,000\,000\,000\,000\,000\,x^5 + \\
& 321\,263\,597\,747\,137\,500\,000\,000\,000\,000\,x^4 + \\
& 2\,846\,889\,727\,728\,480\,000\,000\,000\,000\,x^3 + \\
& 17\,569\,376\,605\,410\,048\,000\,000\,000\,x^2 + 67\,466\,406\,164\,774\,584\,320\,000\,x - \\
& 1586\,187\,778\,346\,292\,986\,952\,903\,447\,551\,285\,192\,719\,323\,716\,974
\end{aligned}$$

• Now, we have that:

$$\text{If } P = \frac{\phi(q)\phi(q^{11})}{\phi(q^5)\phi(q^{55})} \text{ and } Q = \frac{\phi(q)\phi(q^{55})}{\phi(q^5)\phi(q^{11})}, \text{ then}$$

$$\begin{aligned}
& Q^6 + \frac{1}{Q^6} + 33 \left( Q^5 + \frac{1}{Q^5} \right) - 99 \left( Q^4 + \frac{1}{Q^4} \right) + 1529 \left( Q^3 + \frac{1}{Q^3} \right) \\
& - 1683 \left( Q^2 + \frac{1}{Q^2} \right) + 8800 \left( Q + \frac{1}{Q} \right) = 6534 + \left( P^5 + \frac{5^5}{P^5} \right) \\
& - 11 \left\{ \left( P^4 + \frac{5^4}{P^4} \right) \left( Q + \frac{1}{Q} \right) - \left( P^3 + \frac{5^3}{P^3} \right) \left[ 11 + 4 \left( Q^2 + \frac{1}{Q^2} \right) \right] \right. \\
& - \left( P^2 + \frac{5^2}{P^2} \right) \left[ 18 - 56 \left( Q + \frac{1}{Q} \right) + 3 \left( Q^2 + \frac{1}{Q^2} \right) - 8 \left( Q^3 + \frac{1}{Q^3} \right) \right] \\
& - \left( P + \frac{5}{P} \right) \left[ 324 - 126 \left( Q + \frac{1}{Q} \right) + 160 \left( Q^2 + \frac{1}{Q^2} \right) - 18 \left( Q^3 + \frac{1}{Q^3} \right) \right. \\
& \left. \left. + 9 \left( Q^4 + \frac{1}{Q^4} \right) \right] - \left( P^3 + \frac{5^3}{P^3} \right) \left[ 11 + 4 \left( Q^2 + \frac{1}{Q^2} \right) \right] \right\}. \tag{76}
\end{aligned}$$

For  $P = Q = 1$ , we obtain:



$$1+1+33*2-99*2+1529*2-1683*2+8800*2$$

$$6534+(1+3125)-11[(((1+625)2-(1+125)(11+8)-(1+25)(18-112+5-8)-(1+5)(324-252+320-36+18)-(1+125)(11+8)))]$$

$$1+1+33*2-99*2+1529*2-1683*2+8800*2$$

**Input:**

$$1 + 1 + 33 \times 2 - 99 \times 2 + 1529 \times 2 - 1683 \times 2 + 8800 \times 2$$

**Result:**

$$17162$$

$$17162$$

$$6534+(1+3125)-11[(((1+625)2-(1+125)(11+8)-(1+25)(18-112+5-8)-(1+5)(324-252+320-36+18)-(1+125)(11+8)))]$$

**Input:**

$$6534 + (1 + 3125) - 11 ((1 + 625) \times 2 - (1 + 125) (11 + 8) - (1 + 25) (18 - 112 + 5 - 8) - (1 + 5) (324 - 252 + 320 - 36 + 18) - (1 + 125) (11 + 8))$$

**Result:**

$$45498$$

$$45498$$

Thence:

$$17162x = 45498;$$

And:

$$1/17162(((((((6534+(1+3125)-11[(((1+625)2-(1+125)(11+8)-(1+25)(18-112+5-8)-(1+5)(324-252+320-36+18)-(1+125)(11+8)))])))))))))$$

**Input:**

$$\frac{1}{17162} (6534 + (1 + 3125) - 11 ((1 + 625) \times 2 - (1 + 125) (11 + 8) - (1 + 25) (18 - 112 + 5 - 8) - (1 + 5) (324 - 252 + 320 - 36 + 18) - (1 + 125) (11 + 8)))$$

**Exact result:**

$$\frac{22749}{8581}$$

**Decimal approximation:**

2.651089616594802470574525113623120848385969001281901876238...

2.6510896....

$$-0.0089-0.0013+\left(\frac{1}{17162}\left(6534+(1+3125)-11\left[\left(\left(1+625\right)^2-\left(1+125\right)\left(11+8\right)-\left(1+25\right)\left(18-112+5-8\right)-\left(1+5\right)\left(324-252+320-36+18\right)-\left(1+125\right)\left(11+8\right)\right]\right)\right)\right)^{1/2}$$

**Input:**

$$-0.0089 - 0.0013 + \sqrt{\left(\frac{1}{17162} (6534 + (1 + 3125) - 11 ((1 + 625) \times 2 - (1 + 125) (11 + 8) - (1 + 25) (18 - 112 + 5 - 8) - (1 + 5) (324 - 252 + 320 - 36 + 18) - (1 + 125) (11 + 8)))\right)}$$

**Result:**

1.6180167...

1.6180167...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

$$1.61803398+\ln\left(\frac{1}{17162}\left(6534+(1+3125)-11\left[\left(\left(1+625\right)^2-\left(1+125\right)\left(11+8\right)-\left(1+25\right)\left(18-112+5-8\right)-\left(1+5\right)\left(324-252+320-36+18\right)-\left(1+125\right)\left(11+8\right)\right]\right)\right)\right)$$

**Input interpretation:**

$$1.61803398 + \log(-17162 + (6534 + (1 + 3125) - 11 ((1 + 625) \times 2 - (1 + 125) (11 + 8) - (1 + 25) (18 - 112 + 5 - 8) - (1 + 5) (324 - 252 + 320 - 36 + 18) - (1 + 125) (11 + 8))))$$

log(x) is the natural logarithm

**Result:**

11.86992234...

11.86992234... result practically equal to the black hole entropy 11.8458

## Alternative representations:

$$1.61803 + \log(-17162 + (6534 + (1 + 3125) - 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = 1.61803 + \log_e(28336)$$

$$1.61803 + \log(-17162 + (6534 + (1 + 3125) - 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = 1.61803 + \log(a) \log_a(28336)$$

$$1.61803 + \log(-17162 + (6534 + (1 + 3125) - 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = 1.61803 - \text{Li}_1(-28335)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

## Series representations:

$$1.61803 + \log(-17162 + (6534 + (1 + 3125) - 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = 1.61803 + \log(28335) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{28335}\right)^k}{k}$$

$$1.61803 + \log(-17162 + (6534 + (1 + 3125) - 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = 1.61803 + 2i\pi \left[ \frac{\arg(28336 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (28336 - x)^k x^{-k}}{k}$$

for  
 $x < 0$

$$\begin{aligned}
& 1.61803 + \log(-17162 + (6534 + (1 + 3125) - \\
& \quad 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - \\
& \quad (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = \\
& 1.61803 + \left\lfloor \frac{\arg(28336 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \\
& \left\lfloor \frac{\arg(28336 - z_0)}{2\pi} \right\rfloor \log(z_0) - \\
& \sum_{k=1}^{\infty} \frac{(-1)^k (28336 - z_0)^k z_0^{-k}}{k}
\end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$\begin{aligned}
& 1.61803 + \\
& \log(-17162 + (6534 + (1 + 3125) - 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25) \\
& \quad (18 - 112 + 5 - 8) - (1 + 5)(324 - 252 + 320 - 36 + 18) - \\
& \quad (1 + 125)(11 + 8)))) = 1.61803 + \int_1^{28336} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& 1.61803 + \log(-17162 + (6534 + (1 + 3125) - \\
& \quad 11((1 + 625)2 - (1 + 125)(11 + 8) - (1 + 25)(18 - 112 + 5 - 8) - \\
& \quad (1 + 5)(324 - 252 + 320 - 36 + 18) - (1 + 125)(11 + 8)))) = \\
& 1.61803 + \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{28335^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for} \\
& -1 < \gamma < 0
\end{aligned}$$

$\Gamma(x)$  is the gamma function

From:

Quantum Black Holes, Localization & Mock Modular Forms

ATISH DABHOLKAR

CNRS - University of Paris VI

VII Regional Meeting in String Theory - 19 June 2013

Now, we have that:

## Ramanujan's example

In Ramanujan's famous last letter to Hardy in 1920, he gives 17 examples of mock theta functions, without giving any complete definition of this term. A typical example (Ramanujan's second mock theta function of "order 7" — a notion that he also does not define) is

$$\begin{aligned}\mathcal{F}_7(\tau) &= -q^{-25/168} \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1-q^n) \cdots (1-q^{2n-1})} \\ &= -q^{143/168} (1 + q + q^2 + 2q^3 + \cdots) .\end{aligned}$$

Hints of modularity such as Cardy behavior of Fourier coefficients but not quite modular! Despite much work, this fascinating 'hidden' modular symmetry remained mysterious until the thesis of [Zwegers \[2005\]](#).

A mock modular form  $h(\tau)$  of weight  $k$  is the first of the pair  $(h, g)$

- 1  $g(\tau)$  is a modular form of weight  $2 - k$ ,
- 2 the sum  $\hat{h} = h + g^*$ , of  $h$  is modular with weight  $k$  with

$$g^*(\tau, \bar{\tau}) = \left(\frac{i}{2\pi}\right)^{k-1} \int_{-\bar{\tau}}^{\infty} (z + \tau)^{-k} \overline{g(-\bar{z})} dz .$$

Then  $g$  is called the *shadow* of  $h$  and  $\hat{h}$  is called *modular completion* of  $h$  which obeys a 'holomorphic anomaly' equation

$$(4\pi\tau_2)^k \frac{\partial \hat{h}(\tau)}{\partial \bar{\tau}} = -2\pi i \overline{g(\tau)} .$$

Ramanujan never specified the shadow which was part of the mystery.

*"My dream is that I will live to see the day when our young physicists, struggling to bring the predictions of superstring theory into correspondence with the facts of nature, will be led to enlarge their analytic machinery to include not only theta-functions but mock theta-functions... But before this can happen, the purely mathematical exploration of the mock-modular forms and their mock-symmetries must be carried a great deal further."*

*Freeman Dyson (1987 Ramanujan Centenary Conference)*

We will encounter mock modular forms naturally while dealing with **quantum black holes and holography in situations with wall-crossing.**

$$\begin{aligned} \mathcal{F}_7(\tau) &= -q^{-25/168} \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1-q^n) \cdots (1-q^{2n-1})} \\ &= -q^{143/168} (1 + q + q^2 + 2q^3 + \cdots) . \end{aligned}$$

(Ramanujan's second mock theta function of "order 7" - a notion that he also does not define)

$-q^{(-0.1488)} \sum ((q^{(n^2)})/(1-q^n)((1-q^{(2n-1)})), n = 1 \text{ to infinity}$

For  $q = 0.60653$  and  $n = -0.1488$ , we obtain:

$-0.60653^{(-0.1488)} * ((0.60653^{(-0.1488^2)}) * 1/(((1-0.60653^{(-0.1488)})*(1-0.60653^{(-1.2976))}))$

**Input interpretation:**

$$- \frac{0.60653^{-0.1488^2} \times \frac{1}{1 - \frac{1}{1 - \frac{0.60653^{1.2976}}{0.60653^{0.1488}}}}}{0.60653^{0.1488}}$$

**Result:**

-0.549067...

-0.549067...

$$3 * -(((((-0.60653^{(-0.1488)}) * ((0.60653^{(-0.1488^2)}) * 1/(((1-0.60653^{(-0.1488)}) * (1-0.60653^{(-1.2976))}))))))$$

**Input interpretation:**

$$\frac{3 \times (-1) \left( \left( 0.60653^{-0.1488^2} \times \frac{1}{\frac{1 - \frac{1}{1 - \frac{0.60653^{1.2976}}{0.60653^{0.1488}}}}{0.60653^{0.1488}}}} \right) \right)}{0.60653^{0.1488}}$$

**Result:**

1.647200820546231482040946562274819090577227454175911992902...

$$1.64720082... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

For n = 2, we obtain:

$$-0.60653^{(-0.1488)} * ((0.60653^4) * 1/(((1-0.60653^2*(1-0.60653^3))))$$

**Input:**

$$-\frac{0.60653^4 \times \frac{1}{1-0.60653^2(1-0.60653^3)}}{0.60653^{0.1488}}$$

**Result:**

-0.204126...

-0.204126...

$$-8 * ((((-0.60653^{(-0.1488)}) * ((0.60653^4) * 1/(((1-0.60653^2*(1-0.60653^3))))))))$$

**Input:**

$$-\frac{8 \left( - \left( 0.60653^4 \times \frac{1}{1-0.60653^2(1-0.60653^3)} \right) \right)}{0.60653^{0.1488}}$$

**Result:**

1.63300...

1.63300... result that is a golden number

Or:



$$(-1/3) / ((((-0.60653^{(-0.1488)} * ((0.60653^4) * 1/(((1-0.60653^2*(1-0.60653^3))))))))$$

**Input:**

$$\frac{1}{3 \left( - \left( \frac{0.60653^4 \times \frac{1}{1 - 0.60653^2 (1 - 0.60653^3)}}{0.60653^{0.1488}} \right) \right)}$$

**Result:**

1.63298...

1.63298... result that is a golden number

From the result -0.204126..., we obtain:

$$8 \operatorname{colog} - ((((-0.60653^{(-0.1488)} * ((0.60653^4) * 1/(((1-0.60653^2*(1-0.60653^3))))))))$$

**Input:**

$$8 \left( - \log \left( - \frac{\left( 0.60653^4 \times \frac{1}{1 - 0.60653^2 (1 - 0.60653^3)} \right)}{0.60653^{0.1488}} \right) \right)$$

$\log(x)$  is the natural logarithm

**Result:**

12.7122...

12.7122... result very near to the black hole entropy 12.5664

**Alternative representations:**

$$8(-1) \log \left( - \frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))} \right) = -8 \log_e \left( \frac{0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))} \right)$$

$$8(-1) \log \left( - \frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))} \right) = -8 \log(a) \log_a \left( \frac{0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))} \right)$$

$$8(-1) \log\left(-\frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))}\right) =$$

$$8 \operatorname{Li}_1\left(1 - \frac{0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\operatorname{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$8(-1) \log\left(-\frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))}\right) = 8 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.795874)^k}{k}$$

$$8(-1) \log\left(-\frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))}\right) =$$

$$-16 i \pi \left\lfloor \frac{\arg(0.204126 - x)}{2 \pi} \right\rfloor - 8 \log(x) + 8 \sum_{k=1}^{\infty} \frac{(-1)^k (0.204126 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$8(-1) \log\left(-\frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))}\right) =$$

$$-8 \left\lfloor \frac{\arg(0.204126 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 8 \log(z_0) -$$

$$8 \left\lfloor \frac{\arg(0.204126 - z_0)}{2 \pi} \right\rfloor \log(z_0) + 8 \sum_{k=1}^{\infty} \frac{(-1)^k (0.204126 - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representation:

$$8(-1) \log\left(-\frac{-0.60653^4}{0.60653^{0.1488} (1 - 0.60653^2 (1 - 0.60653^3))}\right) = -8 \int_1^{0.204126} \frac{1}{t} dt$$

$$(ii) \quad \frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)} + \dots$$

For instance when  $q = e^{-t}$  and  $t \rightarrow 0$

When  $q = -e^{-t}$  and  $t \rightarrow 0$

If we take  $q = e^{-t}$  for  $t = 0.5$ , we obtain:  $q = e^{-0.5} = 0,6065306597126334236\dots$

$q = e^{0.5} = 1,6487212707001281468486507878142$  or

$1/q = 1/e^{-0.5} = 1,6487212707001281468486507878142$

Now let's develop the above formula directly with the parameters provided by Ramanujan. We obtain the following interesting results:

$$-0.60653/(1+0.60653) - 0.60653^4/(((1+0.60653^2)(1+0.60653^3))) - 0.60653^9/(((1+0.60653^3)(1+0.60653^4)(1+0.60653^5)))$$

**Input:**

$$\frac{\frac{0.60653}{1+0.60653} - \frac{0.60653^4}{(1+0.60653^2)(1+0.60653^3)} - \frac{0.60653^9}{(1+0.60653^3)(1+0.60653^4)(1+0.60653^5)}}{0.60653^9}$$

**Result:**

-0.46582222623447136636580377333045672125872851787626590863...

-0.465822...

$$-(0.55+0.21+0.01) / (-0.46582222623447136636580377333045672125872851787626590863))))))$$

**Input interpretation:**

$$\frac{-(0.55 + 0.21 + 0.01)}{-0.46582222623447136636580377333045672125872851787626590863}$$

**Result:**

1.652991112563231182633112566373934524808676757442302329030...

1.65299111... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

$(13+2) \exp(-0.46582222623447136636580377333045672125872851787626590863)$

**Input interpretation:**

$(13 + 2) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Result:**

23.899855845176015752737469922654492089477170533858921145...

**23.8998558...** result practically equal to the black hole entropy 23.9078

$13 \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Input interpretation:**

$13 \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Result:**

20.713208399152546985705807266300559810880214462677731659...

**20.7132...** result very near to the black hole entropy 20.5520

$8 \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Input interpretation:**

$8 \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Result:**

12.746589784093875068126650625415729114387824284724757944...

**12.7465...** result very near to the black hole entropy 12.5664

$(8+2) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Input interpretation:**

$(8 + 2) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$

**Result:**

15.933237230117343835158313281769661392984780355905947430...

15.93323... result very near to the black hole entropy 15.8174

$$(8+3) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$$

**Input interpretation:**

$$(8 + 3) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$$

**Result:**

17.526560953129078218674144609946627532283258391496542173...

17.5265... result very near to the black hole entropy 17.5764

$$(2*8) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$$

**Input interpretation:**

$$(2 \times 8) \exp(-(-0.46582222623447136636580377333045672125872851787626590863))$$

**Result:**

25.493179568187750136253301250831458228775648569449515888...

25.493179... result very near to the black hole entropy 25.1327

For the following value 23.8998558, considered as entropy, we obtain:

$$\text{Mass} = 4.555136e-8 \text{ kg}$$

$$\text{Radius} = 6.763701e-35 \text{ m}$$

$$\text{Temperature} = 2.694109e+30 \text{ K}$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}[\text{[[[1/((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(4.555136e-8)* \text{sqrt}[-(((2.694109e+30 * 4*Pi*(6.763701e-35)^3-(6.763701e-35)^2)))] / ((6.67*10^-11)))]]]]]]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.555136 \times 10^{-8}}\right) \sqrt{-\frac{2.694109 \times 10^{30} \times 4 \pi (6.763701 \times 10^{-35})^3 - (6.763701 \times 10^{-35})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.618249196705554406508987110241162708382294892231060572383...

1.61824919...

From::

**The Mordell integral, quantum modular forms, and mock Jacobi forms**

Bobbie Chern<sup>1\*</sup> and Robert C Rhoades<sup>2</sup> - Chern and Rhoades *Research in Number Theory* (2015) 1:1 DOI 10.1007/s40993-015-0002-x

It is explained how the Mordell integral

$$\int_{\mathbb{R}} \frac{e^{\pi i \tau x^2 - 2\pi z x}}{\cosh(\pi x)} dx$$

unifies the mock theta functions, partial (or false) theta functions, and some of Zagier’s quantum modular forms. As an application, we exploit the connections between  $q$ -hypergeometric series and mock and partial theta functions to obtain finite evaluations of the Mordell integral for rational choices of  $\tau$  and  $z$ .

The Mordell integral is

$$h(z; \tau) := \int_{\mathbb{R}} \frac{e^{\pi i \tau w^2 - 2\pi z w}}{\cosh(\pi w)} dw = 2 \int_0^{\infty} e^{\pi i \tau w^2} \frac{\cosh(2\pi z w)}{\cosh(\pi w)} dw.$$

### 1.1 Zagier's quantum modular forms

In a 1997 Max Planck lecture Zagier credits Kontsevich with introducing the function

$$F(q) := \sum_{n=0}^{\infty} (q; q)_n = 1 + (1-q) + (1-q)(1-q^2) + (1-q)(1-q^2)(1-q^3) + \dots, \quad (1.4)$$

where  $(x; q)_n := \prod_{j=0}^{n-1} (1-xq^j)$ . This function, called "Kontsevich's strange function", exists only when  $q$  is a root of unity. With  $\zeta_k := e^{2\pi i/k}$ , Kontsevich conjectured the following elegant asymptotic expansion

$$F(\zeta_k) \sim e^{-\frac{\pi i}{12}(k-3+\frac{1}{k})} k^{\frac{3}{2}} + \sum_{n=0}^{\infty} \frac{b_n}{n!} \left(-\frac{2\pi i}{k}\right)^n \text{ as } k \rightarrow \infty \quad (1.5)$$

for some constants  $b_n$ . Zagier [38] proved this asymptotic, with explicit  $b_n$ . Moreover, he proved that this function satisfies the "identity"

$$F(q) = -\frac{1}{2} \sum_{n=1}^{\infty} n \left(\frac{12}{n}\right) q^{\frac{n^2-1}{24}}. \quad (1.6)$$

Neither side of this identity makes sense simultaneously. Indeed, the right hand side converges in the unit disk  $|q| < 1$ , but nowhere on the unit circle. The identity means that  $F(q)$  at roots of unity agrees with the radial limit of the right hand side.

From

$$F(q) = -\frac{1}{2} \sum_{n=1}^{\infty} n \left(\frac{12}{n}\right) q^{\frac{n^2-1}{24}}.$$

we obtain for  $q = 0.26$ :

$$-1/2 * -\sum_{n=1}^{\infty} n(12/n) 0.26^{((n^2-1)/24)}, n=1 \text{ to infinity}$$

**Input interpretation:**

$$-\frac{1}{2} \times (-1) \sum_{n=1}^{\infty} n \times \frac{12}{n} \times 0.26^{1/24(n^2-1)}$$

**Result:**

20.5669

20.5669 result practically equal to the black hole entropy 20.5520

For  $q = 0.165$ , we obtain:



$-1/2 * -\sum n(12/n) (0.144+0.021)^{((n^2-1)/24)}$ , n=1 to infinity

**Input interpretation:**

$$-\frac{1}{2} \times (-1) \sum_{n=1}^{\infty} n \times \frac{12}{n} (0.144 + 0.021)^{1/24(n^2-1)}$$

**Result:**

17.6856

17.6856 result very near to the black hole entropy 17.7715

For q = 0.377, we obtain:

$-1/2 * -\sum n(12/n) 0.377^{((n^2-1)/24)}$ , n=1 to infinity

**Input interpretation:**

$$-\frac{1}{2} \times (-1) \sum_{n=1}^{\infty} n \times \frac{12}{n} \times 0.377^{1/24(n^2-1)}$$

**Result:**

24.3443

24.3443 result very near to the black hole entropy 24.4233

From this result, we obtain also:

$[-144/2 * -1/2 \sum n(12/n) 0.377^{((n^2-1)/24)}$ , n=1 to infinity]<sup>1/15</sup>

**Input interpretation:**

$$\sqrt[15]{\frac{1}{2} \left(-\frac{144}{2}\right) \times (-1) \sum_{n=1}^{\infty} n \times \frac{12}{n} \times 0.377^{1/24(n^2-1)}}$$

**Result:**

1.64531

$1.64531 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

And:

$-27/10^3 + [-144/2 * -1/2 \sum n(12/n) 0.377^{((n^2-1)/24)}$ , n=1 to infinity]<sup>1/15</sup>

**Input interpretation:**



$$-\frac{27}{10^3} + 15 \sqrt{\frac{1}{2} \left(-\frac{144}{2}\right) \times (-1) \sum_{n=1}^{\infty} n \times \frac{12}{n} \times 0.377^{1/24(n^2-1)}}$$

**Result:**

1.61831

1.61831

This result is a very good approximation to the value of the golden ratio 1,618033988749...

For the following value 20.5669, considered as entropy, we obtain:

Mass = 4.225598e-8 kg

Radius = 6.274385e-35 m

Temperature = 2.904212e+30 K

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[[[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2)))\*1/(4.225598e-8)\* sqrt[[-(((2.904212e+30 \* 4\*Pi\*(6.274385e-35)^3-(6.274385e-35)^2)))] / ((6.67\*10^-11))))]]]]]]]

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.225598 \times 10^{-8}} \right)} \sqrt{\frac{2.904212 \times 10^{30} \times 4 \pi (6.274385 \times 10^{-35})^3 - (6.274385 \times 10^{-35})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249321850445590329777464278919068613001919750462477867...

1.61824932...

Now, we have that:

To establish the claim we use Theorem 1.4 with

$$\int_0^\infty \frac{e^{-\pi i \frac{k}{2} x^2}}{\cosh(\pi x)} dx = \sqrt{\frac{2}{k}} e^{-\frac{\pi i}{4}} \int_0^\infty \frac{e^{\frac{2\pi i}{k} x^2}}{\cosh(\pi x)} dx = \sqrt{\frac{2}{k}} e^{-\frac{\pi i}{4}} \sum_{n=0}^\infty \frac{1}{n!} \left(\frac{2\pi i}{k}\right)^n \int_0^\infty \frac{x^{2n}}{\cosh(\pi x)} dx.$$

The first equation follows from (1) of [34] or Proposition 1.2 (5) of [42]. The second result follows from Taylor expanding the exponential in the integral and using  $\int_0^\infty \frac{x^{2n}}{\cosh(\pi x)} dx = \frac{(-1)^n E_{2n}}{2^{2n+1}}$  (see, for instance, [13]).  $\square$

For  $k = 1$  and  $n = 2$  in the integral, we obtain:

$$\sqrt{2} \exp(-\pi i/4) * \sum (1/n!) (2\pi i)^n, n= 0 \text{ to infinity} * \int_0^\infty \frac{x^4}{\cosh(\pi x)} dx$$

**Indefinite integral:**

$$\begin{aligned} & \sqrt{2} \exp\left(-\frac{\pi i}{4}\right) \sum_{n=0}^\infty \frac{(2\pi i)^n}{n!} \int \frac{x^4}{\cosh(\pi x)} dx = \\ & \frac{1}{40 \sqrt{2} \pi^5} \sqrt[4]{-1} \left( -320 \pi^3 x^3 \operatorname{Li}_2(-i e^{\pi x}) - 80 \pi^3 (4x^3 + 3x + i) \operatorname{Li}_2(-i e^{-\pi x}) - \right. \\ & 960 \pi^2 x^2 \operatorname{Li}_3(-i e^{-\pi x}) + 960 \pi^2 x^2 \operatorname{Li}_3(-i e^{\pi x}) - 240 \pi^3 x \operatorname{Li}_2(i e^{\pi x}) - \\ & 1920 \pi x \operatorname{Li}_4(-i e^{-\pi x}) - 1920 \pi x \operatorname{Li}_4(-i e^{\pi x}) - 80 i \pi^3 \operatorname{Li}_2(i e^{\pi x}) - \\ & 240 \pi^2 \operatorname{Li}_3(-i e^{-\pi x}) + 240 \pi^2 \operatorname{Li}_3(i e^{\pi x}) - 1920 \operatorname{Li}_5(-i e^{-\pi x}) + \\ & 1920 \operatorname{Li}_5(-i e^{\pi x}) + 16 \pi^5 x^5 + 80 \pi^4 x^4 \log(1 + i e^{-\pi x}) - \\ & 80 \pi^4 x^4 \log(1 + i e^{\pi x}) + 40 \pi^5 x^3 + 40 i \pi^5 x^2 + 120 \pi^4 x^2 \log(1 + i e^{-\pi x}) - \\ & 120 \pi^4 x^2 \log(1 - i e^{\pi x}) - 15 \pi^5 x + 80 i \pi^4 x \log(1 + i e^{-\pi x}) - \\ & 80 i \pi^4 x \log(1 - i e^{\pi x}) - 15 \pi^4 \log(1 + i e^{-\pi x}) + 10 \pi^4 \log(1 - i e^{\pi x}) + \\ & \left. 5 \pi^4 \log(1 + i e^{\pi x}) - 5 \pi^4 \log\left(\tan\left(\frac{1}{4}(\pi + 2i\pi x)\right)\right) + 10 i \pi^5 \right) + \text{constant} \end{aligned}$$

(assuming a complex-valued logarithm)

$n!$  is the factorial function  
 $\cosh(x)$  is the hyperbolic cosine function  
 $\log(x)$  is the natural logarithm  
 $\operatorname{Li}_n(x)$  is the polylogarithm function  
 $i$  is the imaginary unit

**Series expansion of the integral at  $x = 0$ :**

$$\left(\frac{1}{24} + \frac{i}{24}\right) \left( \frac{72 (\operatorname{Li}_3(i) - \operatorname{Li}_3(-i))}{\pi^3} + 4i + \frac{3 (\log(1 - i) - \log(1 + i))}{\pi} \right) + O(x^4)$$

(Taylor series)

Now for  $n = 2$  in the integral:

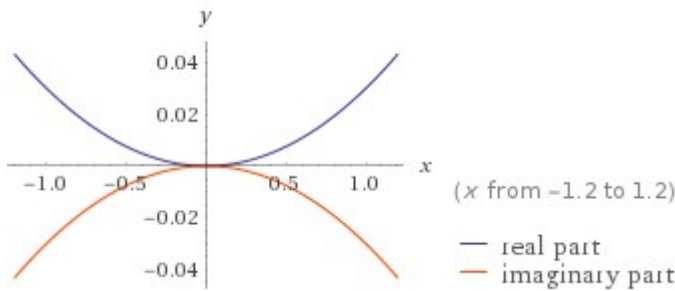
$$\sqrt{2} \exp(-\pi i/4) * \sum (1/n!) (2\pi i)^n, n= 0 \text{ to infinity} * \int_0^\infty \frac{x^4}{\cosh(\pi x)} dx$$

**Indefinite integral:**

$$\sqrt{2} \exp\left(-\frac{\pi i}{4}\right) \left(\sum_{n=0}^{\infty} \frac{(2\pi i)^n}{n!}\right) \int \frac{2^4 x}{\cosh(\pi 2)} dx = 8 \sqrt{2} e^{-(i\pi)/4} x^2 \operatorname{sech}(2\pi) + \text{constant}$$

$n!$  is the factorial function  
 $\cosh(x)$  is the hyperbolic cosine function  
 $\operatorname{sech}(x)$  is the hyperbolic secant function  
 $i$  is the imaginary unit

**Plot:**



**Alternate forms:**

$$\frac{8 \sqrt{2} x^2}{e^{(i\pi)/4} \sinh^2(\pi) + e^{(i\pi)/4} \cosh^2(\pi)}$$

- $$-\frac{8 (-1)^{3/4} \sqrt{2} x^2}{(\cosh(\pi) - i \sinh(\pi)) (\cosh(\pi) + i \sinh(\pi))}$$

- $$-8 (-1)^{3/4} \sqrt{2} x^2 \operatorname{sech}(2\pi)$$

$\sinh(x)$  is the hyperbolic sine function

**Alternate form assuming x is real:**

$$\frac{(16 - 16 i) x^2 \cosh(2\pi)}{1 + \cosh(4\pi)}$$

Thence, for  $x = 2$  and  $i^2$ , we obtain:

$$8 \sqrt{2} e^{-(i^2 \pi)/4} 2^2 \operatorname{sech}(2\pi)$$

**Input:**

$$8 \sqrt{2} e^{-1/4(i^2 \pi)} \times 2^2 \operatorname{sech}(2\pi)$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$32 \sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi)$$

**Decimal approximation:**

0.370710458093732542951389948588538309687994521267656955336...

(using the principal branch of the logarithm for complex exponentiation)

0.37071045...

**Property:**

$32 \sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi)$  is a transcendental number

**Alternate forms:**

$$\frac{32 \sqrt{2} e^{\pi/4}}{\cosh(2\pi)}$$

$$\frac{64 \sqrt{2} e^{(9\pi)/4}}{1 + e^{4\pi}}$$

•

$$\frac{64 \sqrt{2} e^{\pi/4} \cosh(2\pi)}{1 + \cosh(4\pi)}$$

**Alternative representations:**

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi) = \frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}$$

•

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi) = \frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}$$

•

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi) = \frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}$$

**Series representations:**

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi) = 64 \sqrt{2} e^{-(7\pi)/4} \sum_{k=0}^{\infty} e^{(-4+i)k\pi}$$

•

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi) = -64\sqrt{2} e^{\pi/4} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \text{ for } q = e^{2\pi}$$

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi) = \frac{128\sqrt{2} e^{\pi/4} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2}}{\pi}$$

[More information »](#)

**Integral representation:**

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi) = \frac{64\sqrt{2} e^{\pi/4}}{\pi} \int_0^{\infty} \frac{t^{4i}}{1+t^2} dt$$

Now, in conclusion, we obtain:

$$1/\sqrt{((((8 \sqrt{2}) e^{-(i^2 * \pi)/4} 2^2 \operatorname{sech}(2 \pi))))}$$

**Input:**

$$\frac{1}{\sqrt{8\sqrt{2} e^{-1/4(i^2 \pi)} \times 2^2 \operatorname{sech}(2\pi)}}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$\frac{e^{-\pi/8}}{4 \times 2^{3/4} \sqrt{\operatorname{sech}(2\pi)}}$$

**Decimal approximation:**

1.642413783106222633829092096996853168042332483317176059351...

(using the principal branch of the logarithm for complex exponentiation)

$$1.6424137... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

**Alternate forms:**

$$\frac{e^{-\pi/8} \sqrt{\cosh(2\pi)}}{4 \times 2^{3/4}}$$

$$\frac{e^{-(9\pi)/8} \sqrt{1 + e^{4\pi}}}{8 \sqrt[4]{2}}$$

$$\frac{e^{-\pi/8} \sqrt{e^{-2\pi} + e^{2\pi}}}{8 \sqrt[4]{2}}$$

$\cosh(x)$  is the hyperbolic cosine function

### Alternative representations:

$$\frac{1}{\sqrt{8 \sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi)}} = \frac{1}{\sqrt{\frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}}$$

$$\frac{1}{\sqrt{8 \sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi)}} = \frac{1}{\sqrt{\frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}}$$

$$\frac{1}{\sqrt{8 \sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi)}} = \frac{1}{\sqrt{\frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}}$$

### Series representations:

$$\frac{1}{\sqrt{8 \sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi)}} = \frac{e^{(7\pi)/8}}{8 \sqrt[4]{2} \sqrt{\sum_{k=0}^{\infty} e^{(-4+i)k\pi}}$$

$$\frac{1}{\sqrt{8 \sqrt{2} e^{-1/4(i^2 \pi)} 2^2 \operatorname{sech}(2\pi)}} = \frac{e^{-\pi/8} \sqrt{\pi}}{8 \times 2^{3/4} \sqrt{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2}}$$

$$\frac{1}{\sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)}} = -\frac{e^{-\pi/8} \sqrt{-\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}}}{8\sqrt[4]{2} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k}} \text{ for } q = e^{2\pi}$$

**Integral representation:**

$$\frac{1}{\sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)}} = \frac{e^{-\pi/8} \sqrt{\pi}}{8\sqrt[4]{2} \sqrt{\int_0^{\infty} \frac{t^4 i}{1+t^2} dt}}$$

And:

$$-(8+2) * \ln((((8 \operatorname{sqrt}(2) e^{-(i^2 * \pi)/4} 2^2 \operatorname{sech}(2 \pi))))))$$

**Input:**

$$-(8 + 2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} \times 2^2 \operatorname{sech}(2\pi)\right)$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

**Exact result:**

$$-10 \log\left(32\sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi)\right)$$

**Decimal approximation:**

9.923339574787691008755378866198952579570524555126047156610...

(using the principal branch of the logarithm for complex exponentiation)

9.92333... result practically equal to the black hole entropy 9.9340

**Alternate forms:**

$$-\frac{5}{2} (\pi + \log(4) + 4 \log(32 \operatorname{sech}(2\pi)))$$

$$-\frac{45\pi}{2} - 65 \log(2) + 10 \log(1 + e^{4\pi})$$

$$-10 \left( \frac{\pi}{4} + \frac{\log(2)}{2} + \log(32) + \log(\operatorname{sech}(2\pi)) \right)$$

### Alternative representations:

$$-(8+2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)\right) = -10 \log_e\left(32 e^{-(\pi i^2)/4} \operatorname{sech}(2\pi) \sqrt{2}\right)$$

- $$-(8+2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)\right) = -10 \log\left(\frac{32 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}\right)$$
- $$-(8+2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)\right) = -10 \log(a) \log_a\left(32 e^{-(\pi i^2)/4} \operatorname{sech}(2\pi) \sqrt{2}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

### Series representation:

$$-(8+2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)\right) = 10 \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 32\sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi))^k}{k}$$

### Integral representations:

$$-(8+2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)\right) = -10 \int_1^{32\sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi)} \frac{1}{t} dt$$

- $$-(8+2) \log\left(8\sqrt{2} e^{-1/4(i^2\pi)} 2^2 \operatorname{sech}(2\pi)\right) = -10 \log\left(\frac{64\sqrt{2} e^{\pi/4}}{\pi} \int_0^{\infty} \frac{t^{4i}}{1+t^2} dt\right)$$

For  $x = 13$ , we obtain:

$$8 \sqrt{2} e^{-(i^2\pi)/4} 13^2 \operatorname{sech}(2\pi)$$

### Input:

$$8\sqrt{2} e^{-1/4(i^2\pi)} \times 13^2 \operatorname{sech}(2\pi)$$



$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$1352 \sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi)$$

**Decimal approximation:**

$$15.66251685446019993969622532786574358431776852355850636295\dots$$

(using the principal branch of the logarithm for complex exponentiation)

15.6625168... result practically equal to the black hole entropy 15.6730, directly obtained from the integral result for  $x = 13$

**Property:**

$1352 \sqrt{2} e^{\pi/4} \operatorname{sech}(2\pi)$  is a transcendental number

**Alternate forms:**

$$\frac{1352 \sqrt{2} e^{\pi/4}}{\cosh(2\pi)}$$

$$\frac{2704 \sqrt{2} e^{(\ominus\pi)/4}}{1 + e^{4\pi}}$$

$$\frac{2704 \sqrt{2} e^{\pi/4} \cosh(2\pi)}{1 + \cosh(4\pi)}$$

$\cosh(x)$  is the hyperbolic cosine function

**Alternative representations:**

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi) = \frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}$$

•

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi) = \frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}$$

•

$$8 \sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi) = \frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}$$

**Series representations:**

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi) = 2704\sqrt{2} e^{-(7\pi)/4} \sum_{k=0}^{\infty} e^{(-4+i)k\pi}$$

•

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi) = -2704\sqrt{2} e^{\pi/4} \sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \text{ for } q = e^{2\pi}$$

•

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi) = \frac{5408\sqrt{2} e^{\pi/4} \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2}}{\pi}$$

**Integral representation:**

$$8\sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi) = \frac{2704\sqrt{2} e^{\pi/4}}{\pi} \int_0^{\infty} \frac{t^{4i}}{1+t^2} dt$$

For the result 15.6625168 considered as entropy, we obtain:

Mass = 3.687521e-8

Radius = 5.475421e-35

Temperature = 3.327990e+30

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.687521 \times 10^{-8}} \right)} \right] \right] \right] \right] \sqrt{\left[ \left[ \left[ \left[ \left[ \frac{3.327990 \times 10^{30} \times 4 \pi (5.475421 \times 10^{-35})^3 - (5.475421 \times 10^{-35})^2}{6.67 \times 10^{-11}} \right] \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.687521 \times 10^{-8}} \right) \sqrt{\left( \frac{3.327990 \times 10^{30} \times 4 \pi (5.475421 \times 10^{-35})^3 - (5.475421 \times 10^{-35})^2}{6.67 \times 10^{-11}} \right)} \right)}$$

**Result:**

1.618249266803150984475808847466374872323364709332183940569...

1.6182492...

$$-(34-3)/(10^3) + (((8 \sqrt{2} e^{-(i^2 \pi)/4} 13^2 \operatorname{sech}(2 \pi))))^{1/(11/2)}$$

**Input:**

$$-\frac{34-3}{10^3} + \sqrt[11]{8 \sqrt{2} e^{-1/4(i^2 \pi)} \times 13^2 \operatorname{sech}(2 \pi)}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$2^{7/11} \times 13^{4/11} e^{\pi/22} \operatorname{sech}^{2/11}(2 \pi) - \frac{31}{1000}$$

**Decimal approximation:**

1.618102138220003688477282191961811155372890138924155120028...

(using the principal branch of the logarithm for complex exponentiation)

1.6181021...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Alternate forms:**

$$-\frac{31}{1000} + \frac{2^{7/11} \times 13^{4/11} e^{\pi/22}}{\cosh^{2/11}(2 \pi)}$$

$$\frac{1000 \times 2^{7/11} \times 13^{4/11} e^{\pi/22} \operatorname{sech}^{2/11}(2 \pi) - 31}{1000}$$

$$\frac{2^{9/11} \times 13^{4/11} e^{\pi/22}}{(e^{-2 \pi} + e^{2 \pi})^{2/11}} - \frac{31}{1000}$$

$\cosh(x)$  is the hyperbolic cosine function

### Alternative representations:

$$-\frac{34-3}{10^3} + \frac{11}{2} \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -\frac{31}{10^3} + \frac{11}{2} \sqrt{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}}$$

•

$$-\frac{34-3}{10^3} + \frac{11}{2} \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -\frac{31}{10^3} + \frac{11}{2} \sqrt{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}}$$

•

$$-\frac{34-3}{10^3} + \frac{11}{2} \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -\frac{31}{10^3} + \frac{11}{2} \sqrt{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}}$$

### Series representations:

$$-\frac{34-3}{10^3} + \frac{11}{2} \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -\frac{31}{1000} + 2^{9/11} \times 13^{4/11} e^{-(7\pi)/22} \left( \sum_{k=0}^{\infty} e^{(-4+i)k\pi} \right)^{2/11}$$

•

$$-\frac{34-3}{10^3} + \frac{11}{2} \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -\frac{31}{1000} + 2^{9/11} \times 13^{4/11} e^{\pi/22} \left( -\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^{2/11} \quad \text{for } q = e^{2\pi}$$

•

$$-\frac{34-3}{10^3} + \frac{11}{2} \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -\frac{31}{1000} + \frac{2 \times 13^{4/11} e^{\pi/22} \left( \sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2} \right)^{2/11}}{\pi^{2/11}}$$

### Integral representation:

$$-\frac{34-3}{10^3} + \sqrt[11]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} =$$

$$-\frac{31}{1000} + \frac{2^{9/11} \times 13^{4/11} e^{\pi/22} \left(\int_0^\infty \frac{t^4 i}{1+t^2} dt\right)^{2/11}}{\pi^{2/11}}$$

$$-5 + 10^3 \left( (8 \sqrt{2} e^{-(i^2 \pi)/4} 13^2 \operatorname{sech}(2 \pi)) \right)^{1/5}$$

**Input:**

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} \times 13^2 \operatorname{sech}(2\pi)}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)} - 5$$

**Decimal approximation:**

1728.693457470946712924664685224022059998779661184635766934...

(using the principal branch of the logarithm for complex exponentiation)

1728.6934...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate forms:**

$$-5 + \frac{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{\cosh(2\pi)}}$$

$$5 \left( 200 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)} - 1 \right)$$

•

$$\frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{e^{-2\pi} + e^{2\pi}}} - 5$$

$\cosh(x)$  is the hyperbolic cosine function

### Alternative representations:

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}}$$

•

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}}$$

•

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} = -5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}}$$

### Series representations:

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} =$$

$$-5 + 1000 \times 2^{9/10} \times 13^{2/5} e^{-(7\pi)/20} \sqrt[5]{\sum_{k=0}^{\infty} e^{(-4+i)k\pi}}$$

•

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} =$$

$$-5 + 1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{-\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}} \text{ for } q = e^{2\pi}$$

•

$$-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)} =$$

$$-5 + \frac{2000 \sqrt[10]{2} 13^{2/5} e^{\pi/20} \sqrt[5]{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2}}}{\sqrt[5]{\pi}}$$

**Integral representation:**

$$-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2 \pi)} =$$

$$-5 + \frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\int_0^\infty \frac{t^4 i}{1+t^2} dt}}{\sqrt[5]{\pi}}$$

$$(34+13)+10^3(((8 \operatorname{sqrt}(2) e^{-(i^2 * \pi)/4} 13^2 \operatorname{sech}(2 \pi))))^{1/5}$$

**Input:**

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} \times 13^2 \operatorname{sech}(2 \pi)}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$47 + 1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2 \pi)}$$

**Decimal approximation:**

1780.693457470946712924664685224022059998779661184635766934...

(using the principal branch of the logarithm for complex exponentiation)

1780.6934... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

**Alternate forms:**

$$47 + \frac{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{\cosh(2 \pi)}}$$

$$47 + \frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{e^{-2 \pi} + e^{2 \pi}}}$$

•

$$47 + 1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\frac{\cosh(2\pi)}{1 + \cosh(4\pi)}}$$

$\cosh(x)$  is the hyperbolic cosine function

### Alternative representations:

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)} = 47 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}}$$

•

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)} = 47 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}}$$

•

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)} = 47 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}}$$

### Series representations:

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)} = 47 + 1000 \times 2^{9/10} \times 13^{2/5} e^{-(7\pi)/20} \sqrt[5]{\sum_{k=0}^{\infty} e^{(-4+i)k\pi}}$$

•

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)} = 47 + 1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{-\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}} \text{ for } q = e^{2\pi}$$

•

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)} = 47 + \frac{2000 \sqrt[10]{2} 13^{2/5} e^{\pi/20} \sqrt[5]{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2}}}{\sqrt[5]{\pi}}$$



**Integral representation:**

$$(34 + 13) + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2 \pi)} =$$

$$47 + \frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\int_0^\infty \frac{t^4 i}{1+t^2} dt}}{\sqrt[5]{\pi}}$$

$$\left(\left(\left(\left(\left(-5 + 10^3 \left[\left(\left(\left(\left(8 \sqrt{2} e^{-(i^2 \pi)/4} 13^2 \operatorname{sech}(2 \pi)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/5}\right)^{1/15}$$

**Input:**

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} \times 13^2 \operatorname{sech}(2 \pi)}}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$\sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2 \pi)} - 5}$$

**Decimal approximation:**

1.643795797826281244060780172517256678599052387546336431985...

(using the principal branch of the logarithm for complex exponentiation)

$$1.6437957... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

**Alternate forms:**

$$\sqrt[15]{-5 + \frac{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{\cosh(2 \pi)}}}$$

$$\sqrt[15]{5 \left(200 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2 \pi)} - 1\right)}$$

•

$$\sqrt[15]{\frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{e^{-2\pi} + e^{2\pi}}}} - 5$$

$\cosh(x)$  is the hyperbolic cosine function

**All 15th roots of  $1000 \cdot 2^{7/10} \cdot 13^{2/5} \cdot e^{\pi/20} \cdot \operatorname{sech}(2\pi)^{1/5} - 5$ :**

$$e^0 \sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)}} - 5 \approx 1.64380 \quad (\text{real, principal root})$$

•

$$e^{(2i\pi)/15} \sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)}} - 5 \approx 1.5017 + 0.6686i$$

•

$$e^{(4i\pi)/15} \sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)}} - 5 \approx 1.0999 + 1.2216i$$

•

$$e^{(2i\pi)/5} \sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)}} - 5 \approx 0.5080 + 1.5633i$$

•

$$e^{(8i\pi)/15} \sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)}} - 5 \approx -0.17182 + 1.63479i$$

**Alternative representations:**

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} = \sqrt[15]{-5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}}}$$

•

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} = \sqrt[15]{-5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}}}$$

•

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} = \sqrt[15]{-5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}}}$$

**Series representations:**

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2 \pi)}} =$$

$$\sqrt[15]{-5 + 1000 \times 2^{9/10} \times 13^{2/5} e^{-(7\pi)/20} \sqrt[5]{\sum_{k=0}^{\infty} (-1)^k e^{-4k\pi}}}$$

•

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2 \pi)}} =$$

$$\sqrt[15]{-5 + 1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{-\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}} \quad \text{for } q = e^{2\pi}}$$

•

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2 \pi)}} =$$

$$\sqrt[15]{-5 + \frac{2000 \sqrt[10]{2} 13^{2/5} e^{\pi/20} \sqrt[5]{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{17+4k+4k^2}}}{\sqrt[5]{\pi}}}}$$

### Integral representation:

$$\sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2 \pi)}} =$$

$$\sqrt[15]{-5 + \frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\int_0^{\infty} \frac{t^4 i}{1+t^2} dt}}{\sqrt[5]{\pi}}}}$$

$$-(16^2)/(10^4) + (((((((((-5+10^3[(((8 \sqrt{2}) e^{-(i^2 * \pi)/4}) 13^2 \operatorname{sech}(2 \pi)))))))]^{1/5}))))))^{1/15}$$

### Input:

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} \times 13^2 \operatorname{sech}(2 \pi)}}$$

$\operatorname{sech}(x)$  is the hyperbolic secant function

$i$  is the imaginary unit

**Exact result:**

$$\sqrt[15]{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20}} \sqrt[5]{\operatorname{sech}(2\pi)} - 5 - \frac{16}{625}$$

**Decimal approximation:**

1.618195797826281244060780172517256678599052387546336431985...

(using the principal branch of the logarithm for complex exponentiation)

1.6181957...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Alternate forms:**

$$-\frac{16}{625} + \sqrt[15]{-5 + \frac{1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{\cosh(2\pi)}}}$$

$$\sqrt[15]{\frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20}}{\sqrt[5]{e^{-2\pi} + e^{2\pi}}}} - 5 - \frac{16}{625}$$

- $$\frac{1}{625} \left( 625 \sqrt[15]{5 \left( 200 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\operatorname{sech}(2\pi)} - 1 \right)} - 16 \right)$$

$\cosh(x)$  is the hyperbolic cosine function

**Alternative representations:**

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8 \sqrt{2} e^{-1/4(i^2 \pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cosh(2\pi)}}}$$

-

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(-2i\pi)}}}$$

•

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{\frac{8 \times 13^2 e^{-(\pi i^2)/4} \sqrt{2}}{\cos(2i\pi)}}}$$

### Series representations:

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16}{625} + \sqrt[15]{-5 + 1000 \times 2^{9/10} \times 13^{2/5} e^{-(7\pi)/20} \sqrt[5]{\sum_{k=0}^{\infty} (-1)^k e^{-4k\pi}}}$$

•

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16}{625} + \sqrt[15]{-5 + 1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{-\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}} \text{ for } q = e^{2\pi}}$$

•

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt[5]{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16}{625} + \sqrt[15]{-5 + 1000 \times 2^{7/10} \times 13^{2/5} e^{\pi/20} \sqrt[5]{\pi} \sqrt[5]{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{4\pi^2 + (\frac{1}{2} + k)^2 \pi^2}}}$$

### Integral representation:

$$-\frac{16^2}{10^4} + \sqrt[15]{-5 + 10^3 \sqrt{8\sqrt{2} e^{-1/4(i^2\pi)} 13^2 \operatorname{sech}(2\pi)}} =$$

$$-\frac{16}{625} + \sqrt[15]{-5 + \frac{1000 \times 2^{9/10} \times 13^{2/5} e^{\pi/20} \sqrt{\int_0^\infty \frac{t^4}{1+t^2} dt}}{\sqrt[5]{\pi}}}$$

Now, we have that:

The most significant applications of mock theta functions to the theory of partitions come from the study of Dyson's rank (see the works of Bringmann and Ono [4,5]). An integer partition of  $n$  is a sequence  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_\ell > 0$  such that  $\lambda_1 + \lambda_2 + \dots + \lambda_\ell = n$ . The numbers  $\lambda_j$  are the parts of the partition. Dyson's rank [10] of a partition is the largest part minus the number of parts. For example,  $5 \geq 4 \geq 4 \geq 3 \geq 3 \geq 2 \geq 1 \geq 1 \geq 1 \geq 1$  is a partition of 25 with rank  $5 - 10 = -5$ . Let  $N(m, n)$  denote the number of integer partitions of  $n$  with rank  $m$ . Define

$$R(u; \tau) := 1 + \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} N(m, n) w^m q^n = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(wq, q)_n (w^{-1}q; q)_n}, \quad (1.2)$$

where  $w = e^{2\pi i u}$ ,  $q = e^{2\pi i \tau}$  with  $\tau \in \mathbb{H} = \{x + iy : x, y \in \mathbb{R}, y > 0\}$ ,  $(a; q)_n := \prod_{j=0}^{n-1} (1 - aq^j)$  for positive integers  $n$ , and  $(a; q)_0 := 1$ . Zwegers's thesis [42] shows that when  $z \in \mathbb{Q} + \tau\mathbb{Q}$ , the function  $R(z; \tau)$  may be viewed as a mock theta function (see [39]).

The next theorem shows that the partial theta function

$$\psi(u; \tau) := \sum_{n=0}^{\infty} \left(\frac{12}{n}\right) e^{-\pi i n u} q^{-\frac{n^2}{24}} \quad (1.3)$$

defined for  $\tau \in \mathbb{H}^-$ , has a modular transformation property that mirrors that for  $R(u; \tau)$ . Here,  $(\cdot)$  is the Kronecker symbol. The function is a "partial theta function" because it is the sum over half of the integer lattice.

**Theorem 1.1.** Define  $\tilde{R}(u; \tau) = \frac{iq^{\frac{1}{24}}}{2 \sin(\pi u)} R(u; \tau)$ . For  $\tau \in \mathbb{H} = \{x + iy : x, y \in \mathbb{R}, y > 0\}$  and  $u \in \mathbb{C} \setminus (\mathbb{Z} + \tau\mathbb{Z})$

$$\tilde{R}(u; \tau) - \frac{e^{\frac{3\pi i u^2}{\tau}}}{\sqrt{i\tau}} \tilde{R}\left(\frac{u}{\tau}; -\frac{1}{\tau}\right) = \frac{1}{2} q^{-\frac{1}{6}} \sum_{\pm} \pm e^{-2\pi i u} h(3u \pm \tau; 3\tau).$$

On the other hand, for  $\tau \in (0, \frac{1}{2})$  and  $u \in \mathbb{H}^- = \{x + iy : x, y \in \mathbb{R}, y < 0\}$  and  $\operatorname{Re}(u) < \frac{1}{6}$

$$\psi(u; \tau) - \frac{e^{\frac{3\pi i u^2}{\tau}}}{\sqrt{i\tau}} \psi\left(\frac{u}{\tau}; -\frac{1}{\tau}\right) = \frac{1}{2} q^{-\frac{1}{6}} \sum_{\pm} \pm e^{+2\pi i u} h(3u \pm \tau; 3\tau).$$

In this section the Mordell integral is represented in terms of the partial theta function. The result is used to deduce the second part of Theorem 1.1. The following result, slightly reworded, appears in Ramanujan's article [34]. In particular, see (19) and (20) on page 62 of [34].

**Theorem 2.1.** *If  $\tau$  is a positive real number and  $z \in \mathbb{H}^- = \{x + iy : x, y \in \mathbb{R}, y < 0\}$  with  $|\operatorname{Re}(z)| < \frac{1}{2}$ , we have*

$$\frac{1}{2}h(z; \tau) = \eta(z; \tau) + e^{\frac{\pi iz^2}{\tau}} \sqrt{\frac{i}{\tau}} \eta\left(\frac{z}{\tau}; -\frac{1}{\tau}\right)$$

where

$$\eta(z; \tau) := \sum_{n=0}^{\infty} \binom{-4}{n} e^{-\pi inz} e^{-\pi i \tau \frac{n^2}{4}}.$$

of second part of Theorem 1.1.

We consider

$$\frac{1}{2} \sum_{\pm} \pm e^{\mp 2\pi i u} h(3u \pm \tau; 3\tau) = \sum_{\pm} \pm e^{\mp 2\pi i u} \eta(3u \pm \tau; 3\tau) + \frac{\sqrt{i} e^{\frac{3\pi i u^2}{\tau}} q^{\frac{1}{6}}}{\sqrt{3\tau}} \sum_{\pm} \pm \eta\left(\frac{u}{\tau} \pm \frac{1}{3}; -\frac{1}{3\tau}\right).$$

First, we have

$$\begin{aligned}
& \sum_{\pm} \pm e^{\mp 2\pi i u} \eta(3u \pm \tau; 3\tau) \\
&= e^{-2\pi i u} \sum_{n=0}^{\infty} \left(\frac{-4}{n}\right) e^{-3\pi i n u - \pi i n \tau - \frac{3\pi i n^2 \tau}{4}} - e^{2\pi i u} \sum_{n=0}^{\infty} \left(\frac{-4}{n}\right) e^{-3\pi i n u + \pi i n \tau - \frac{3\pi i n^2 \tau}{4}} \\
&= \sum_{n=0}^{\infty} \left(\frac{-4}{n}\right) e^{-\pi i(3n+2) - \frac{\pi i \tau}{12}((3n)^2 + 12n)} - \sum_{n=0}^{\infty} \left(\frac{-4}{n}\right) e^{-\pi i(3n-2) - \frac{\pi i \tau}{12}((3n)^2 - 12n)} \\
&= -q^{\frac{1}{6}} \sum_{n=0}^{\infty} \chi(n) e^{-\pi i u n} q^{-\frac{n^2}{24}} \\
&= -q^{\frac{1}{6}} \psi(u; \tau)
\end{aligned}$$

where

$$\chi(n) := \begin{cases} -1 & n \equiv 2 \pmod{3} \text{ and } \frac{n-2}{3} \equiv 1 \pmod{4} \\ -1 & n \equiv 1 \pmod{3} \text{ and } \frac{n+2}{3} \equiv 3 \pmod{4} \\ 1 & n \equiv 2 \pmod{3} \text{ and } \frac{n-2}{3} \equiv 3 \pmod{4} \\ 1 & n \equiv 1 \pmod{3} \text{ and } \frac{n+2}{3} \equiv 1 \pmod{4} \\ 0 & \text{else} \end{cases} .$$

The final line follows from  $\chi(n) = \left(\frac{12}{n}\right)$ .

On the other hand

$$\begin{aligned}
\sum_{\pm} \eta\left(\frac{u}{\tau} \pm \frac{1}{3}; -\frac{1}{3\tau}\right) &= \sum_{n=0}^{\infty} \left(\frac{-4}{n}\right) e^{-\pi i n \frac{u}{\tau} + \frac{\pi i n^2}{12\tau}} \sum_{\pm} \pm e^{\mp \frac{\pi i n}{3}} \\
&= -2i \sum_{n=0}^{\infty} \left(\frac{-4}{n}\right) \sin\left(\frac{\pi n}{3}\right) e^{-\pi i n \frac{u}{\tau}} q_1^{-\frac{n^2}{24}} \\
&= -\sqrt{3}i \sum_{n=0}^{\infty} \left(\frac{12}{n}\right) e^{-\pi i n \frac{u}{\tau}} q_1^{-\frac{n^2}{24}} \\
&= -i\sqrt{3} \psi\left(\frac{u}{\tau}; -\frac{1}{\tau}\right).
\end{aligned}$$

since  $\left(-\frac{4}{n}\right) \sin\left(\frac{\pi n}{3}\right) = \frac{\sqrt{3}}{2} \left(\frac{12}{n}\right)$ . Hence

$$\frac{1}{2} \sum_{\pm} \pm e^{\mp 2\pi i u} \eta(3u \pm \tau; 3\tau) = -q^{\frac{1}{6}} \left( \psi(u; \tau) - \frac{e^{\frac{3\pi i u^2}{\tau}}}{\sqrt{i\tau}} \psi\left(\frac{u}{\tau}; -\frac{1}{\tau}\right) \right).$$

Rewriting the sum of Mordell integrals gives the second claim of the theorem.  $\square$

From:



$$= -2i \sum_{n=0}^{\infty} \left( \frac{-4}{n} \right) \sin \left( \frac{\pi n}{3} \right) e^{-\pi i n \frac{u}{\tau}} q_1^{-\frac{n^2}{24}}$$

$$\text{since } \left( -\frac{4}{n} \right) \sin \left( \frac{\pi n}{3} \right) = \frac{\sqrt{3}}{2} \left( \frac{12}{n} \right)$$

For  $n = 2$ ,  $i u \tau = 1$ ;  $i \tau = 1$  and  $q = e^{2\pi}$ , we obtain:

$$-2 * i * ((((((\text{sqrt}(3))/2))) * 6)))) * e^{(-2\text{Pi})} * (((e^{(2\text{Pi}))})^{(-1/6)})$$

**Input:**

$$-2i \left( \frac{\sqrt{3}}{2} \times 6 \right) e^{-2\pi} (e^{2\pi})^{-1/6}$$

$i$  is the imaginary unit

**Exact result:**

$$-6i\sqrt{3} e^{-(7\pi)/3}$$

**Decimal approximation:**

$$-0.00681031268156340658039251962414008068481149791504539717... i$$

•

**Property:**

$$-6i\sqrt{3} e^{-(7\pi)/3} \text{ is a transcendental number}$$

•

**Polar coordinates:**

$$r \approx 0.00681031 \text{ (radius), } \theta = -90^\circ \text{ (angle)}$$

$$0.00681031$$

**Series representations:**

$$\frac{1}{2} \left( -2 \times 6i \left( e^{-2\pi} (e^{2\pi})^{-1/6} \right) \right) \sqrt{3} = -6 e^{-4\pi} (e^{2\pi})^{5/6} i \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}$$

•

$$\frac{1}{2} \left( -2 \times 6i \left( e^{-2\pi} (e^{2\pi})^{-1/6} \right) \right) \sqrt{3} = -6 e^{-4\pi} (e^{2\pi})^{5/6} i \sqrt{2} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -\frac{1}{2} \right)_k}{k!}$$

•

$$\frac{1}{2} \left( -2 \times 6i \left( e^{-2\pi} (e^{2\pi})^{-1/6} \right) \right) \sqrt{3} = - \frac{3 e^{-4\pi} (e^{2\pi})^{5/6} i \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$1 / (((((((((-2 * i * (((((((((\sqrt{3}) / 2)))) * 6)))) * e^{(-2\pi)} * (((e^{(2\pi)})^{(-1/6)}))))))))))$$

**Input:**

$$\frac{1}{(-2i \left(\frac{\sqrt{3}}{2} \times 6\right)) e^{-2\pi} (e^{2\pi})^{-1/6}}$$

$i$  is the imaginary unit

**Exact result:**

$$\frac{i e^{(7\pi)/3}}{6 \sqrt{3}}$$

**Decimal approximation:**

- More digits  
146.8361361303069301268067978130044126238625344455468149241...  $i$

**Property:**

$$\frac{i e^{(7\pi)/3}}{6 \sqrt{3}}$$

is a transcendental number

**Polar coordinates:**

$$r \approx 146.836 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$146.836$$

**Series representations:**

$$\frac{1}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} - 6))} = - \frac{(e^{2\pi})^{7/6}}{6 i \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{1}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} - 6))} = - \frac{(e^{2\pi})^{7/6}}{6 i \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-\frac{1}{2}}{k} \binom{-\frac{1}{2}}{k}}{k!}}$$

$$\frac{1}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} - 6))} = - \frac{(e^{2\pi})^{7/6} \sqrt{\pi}}{3 i \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$12 / \left( \left( \left( \left( \left( \left( \left( \left( -2i \left( \frac{\sqrt{3}}{2} \times 6 \right) \right) \right) \right) \right) \right) \right) \right) \right) \cdot e^{-2\pi} \cdot \left( \left( \left( e^{2\pi} \right) \right)^{-1/6} \right) \right)$$

**Input:**

$$\frac{12}{\left( -2i \left( \frac{\sqrt{3}}{2} \times 6 \right) \right) e^{-2\pi} \left( e^{2\pi} \right)^{-1/6}}$$

$i$  is the imaginary unit

**Exact result:**

$$\frac{2i e^{(7\pi)/3}}{\sqrt{3}}$$

**Decimal approximation:**

1762.033633563683161521681573756052951486350413346561779089...  $i$

1762.03... result in the range of the mass of candidate “glueball”  $f_0(1710)$  (“glueball” =  $1760 \pm 15$  MeV).

**Property:**

$\frac{2i e^{(7\pi)/3}}{\sqrt{3}}$  is a transcendental number

•

**Polar coordinates:**

$r \approx 1762.03$  (radius),  $\theta = 90^\circ$  (angle)

**Series representations:**

$$\frac{12}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} \ 6))} = -\frac{2 (e^{2\pi})^{7/6}}{i \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

•

$$\frac{12}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} \ 6))} = -\frac{2 (e^{2\pi})^{7/6}}{i \sqrt{2} \sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!}}$$

•

$$\frac{12}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} \ 6))} = -\frac{4 (e^{2\pi})^{7/6} \sqrt{\pi}}{i \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$$\left(\frac{12}{\left(\frac{1}{2} \left(e^{-2\pi} \left(e^{2\pi}\right)^{-1/6}\right) \left(-2\right) \left(i \left(\frac{\sqrt{3}}{2} \times 6\right)\right)\right)}\right)^{1/15} e^{-2\pi} \left(e^{2\pi}\right)^{-1/6}$$

**Input:**

$$\sqrt[15]{\frac{12}{\left(-2 i \left(\frac{\sqrt{3}}{2} \times 6\right)\right) e^{-2\pi} \left(e^{2\pi}\right)^{-1/6}}}$$

$i$  is the imaginary unit

**Exact result:**

$$\sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} e^{(7\pi)/45}$$

**Decimal approximation:**

1.6368741688310321924780947434368952037497857927717604417... +  
 0.17204240774112542160226724431870324317130633528194921453...  $i$

• **Property:**

$\sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} e^{(7\pi)/45}$  is a transcendental number

• **Polar coordinates:**

$r \approx 1.64589$  (radius),  $\theta = 6^\circ$  (angle)

$$1.64589 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Alternate forms:**

$e^{(7\pi)/45}$  root of  $3x^{30} + 4$  near  $x = 1.0041 + 0.105536i$

• 
$$\frac{\sqrt[15]{2} e^{(7\pi)/45} \cos\left(\frac{\pi}{30}\right)}{\sqrt[30]{3}} + \frac{i \sqrt[15]{2} e^{(7\pi)/45} \sin\left(\frac{\pi}{30}\right)}{\sqrt[30]{3}}$$

**All 15th roots of  $(2i e^{(7\pi)/3})/\sqrt{3}$ :**

• 
$$\frac{\sqrt[15]{2} e^{(7\pi)/45} e^{(i\pi)/30}}{\sqrt[30]{3}} \approx 1.63687 + 0.17204i \text{ (principal root)}$$

• 
$$\frac{\sqrt[15]{2} e^{(7\pi)/45} e^{(i\pi)/6}}{\sqrt[30]{3}} \approx 1.42538 + 0.82295i$$

• 
$$\frac{\sqrt[15]{2} e^{(7\pi)/45} e^{(3i\pi)/10}}{\sqrt[30]{3}} \approx 0.9674 + 1.3316i$$

• 
$$\frac{\sqrt[15]{2} e^{(7\pi)/45} e^{(13i\pi)/30}}{\sqrt[30]{3}} \approx 0.3422 + 1.6099i$$

• 
$$\frac{\sqrt[15]{2} e^{(7\pi)/45} e^{(17i\pi)/30}}{\sqrt[30]{3}} \approx -0.3422 + 1.6099i$$

• **Series representations:**

$$\sqrt[15]{\frac{12}{\frac{1}{2}(e^{-2\pi}(e^{2\pi})^{-1/6})(-2)(i(\sqrt{3}6))}} = \sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} e^{28/45 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

•

$$\sqrt[15]{\frac{12}{\frac{1}{2}(e^{-2\pi}(e^{2\pi})^{-1/6})(-2)(i(\sqrt{3}6))}} = \sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{(7\pi)/45}$$

•

$$\sqrt[15]{\frac{12}{\frac{1}{2}(e^{-2\pi}(e^{2\pi})^{-1/6})(-2)(i(\sqrt{3}6))}} = \sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{(7\pi)/45}$$

$n!$  is the factorial function

**Integral representations:**

$$\sqrt[15]{\frac{12}{\frac{1}{2}(e^{-2\pi}(e^{2\pi})^{-1/6})(-2)(i(\sqrt{3}6))}} = \sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} e^{28/45} \int_0^1 \sqrt{1-t^2} dt$$

•

$$\sqrt[15]{\frac{12}{\frac{1}{2}(e^{-2\pi}(e^{2\pi})^{-1/6})(-2)(i(\sqrt{3}6))}} = \sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} e^{14/45} \int_0^1 1/\sqrt{1-t^2} dt$$

•

$$\sqrt[15]{\frac{12}{\frac{1}{2}(e^{-2\pi}(e^{2\pi})^{-1/6})(-2)(i(\sqrt{3}6))}} = \sqrt[30]{-\frac{1}{3}} \sqrt[15]{2} e^{14/45} \int_0^{\infty} 1/(1+t^2) dt$$

•

$$0.61803398 + \left( \frac{1}{\left( \frac{-2i \left( \frac{\sqrt{3}}{2} \times 6 \right)}{e^{-2\pi} (e^{2\pi})^{-1/6}} \right)} \right)^{1/2} e^{-2\pi}$$

**Input interpretation:**

$$0.61803398 + \sqrt{\frac{1}{\left( -2i \left( \frac{\sqrt{3}}{2} \times 6 \right) \right) e^{-2\pi} (e^{2\pi})^{-1/6}}}$$

$i$  is the imaginary unit

**Result:**

$$9.18646838... + 8.56843440... i$$

**Polar coordinates:**

$$r = 12.5622 \text{ (radius), } \theta = 43.0064^\circ \text{ (angle)}$$

12.5622 result practically equal to the black hole entropy 12.5664

**Series representations:**

$$0.618034 + \sqrt{\frac{1}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} 6))}} =$$

$$0.618034 + 0.408248 \sqrt{-\frac{(e^{2\pi})^{7/6}}{i \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}}$$

•

$$0.618034 + \sqrt{\frac{1}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} 6))}} =$$

$$0.618034 + 0.408248 \sqrt{-\frac{(e^{2\pi})^{7/6}}{i \sqrt{2} \sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!}}}$$

•

$$0.618034 + \sqrt{\frac{1}{\frac{1}{2} (e^{-2\pi} (e^{2\pi})^{-1/6}) (-2) (i(\sqrt{3} 6))}} =$$

$$0.618034 + 0.57735 \sqrt{-\frac{(e^{2\pi})^{7/6} \sqrt{\pi}}{i \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

# SOME NEW EXPLICIT VALUES FOR RAMANUJAN'S CLASS INVARIANTS

M. S. MAHADEVA NAIKA

We have that:

In this section, we establish several new explicit evaluations of the class invariant  $G_n$  for odd values of  $n$  using the Ramanujan's modular equations of different degrees.

**Theorem 3.1.** *We have*

$$(3.1) \quad G_{95} = \left(\frac{\sqrt{5}+1}{2}\right)^{1/2} \left(\frac{1+\sqrt{2\sqrt{5}-1}}{\sqrt{2}}\right)^{1/2} \left(\sqrt{\frac{218+105\sqrt{5}+\sqrt{102475+45850\sqrt{5}}}{8}} + \sqrt{\frac{210+105\sqrt{5}+\sqrt{102475+45850\sqrt{5}}}{8}}\right)^{1/6}$$

$$(3.2) \quad G_{19/5} = \left(\frac{\sqrt{5}+1}{2}\right)^{1/2} \left(\frac{1+\sqrt{2\sqrt{5}-1}}{\sqrt{2}}\right)^{1/2} \left(\sqrt{\frac{218+105\sqrt{5}+\sqrt{102475+45850\sqrt{5}}}{8}} - \sqrt{\frac{210+105\sqrt{5}+\sqrt{102475+45850\sqrt{5}}}{8}}\right)^{1/6}$$

(sqrt((((1/8\*(210+105sqrt(5))+((sqrt(102475+45580sqrt(5)))))))

**Input:**

$$\sqrt{\frac{1}{8} \left( 210 + 105\sqrt{5} + \sqrt{102475 + 45580\sqrt{5}} \right)}$$

**Result:**

$$\frac{1}{2} \sqrt{\frac{1}{2} \left( 210 + 105\sqrt{5} + \sqrt{102475 + 45580\sqrt{5}} \right)}$$

**Decimal approximation:**

10.58824682298135184158476531507733232897583876153086091814...

10.58824682298135184158476531507733232897583876153086091814+((((sqrt(((1/8\*(218+105sqrt(5))+((sqrt(102475+45580sqrt(5)))))))



**Input interpretation:**

10.58824682298135184158476531507733232897583876153086091814 +

$$\sqrt{\frac{1}{8} \left( 218 + 105 \sqrt{5} + \sqrt{102475 + 45580 \sqrt{5}} \right)}$$

**Result:**

21.22361098150336695980644780179075994682558655803292568104...

$$(21.22361098150336695980644780179075994682558655803292568104)^{1/6}$$

**Input interpretation:**

$\sqrt[6]{21.22361098150336695980644780179075994682558655803292568104}$

**Result:**

1.663935724552899421892612967615746549023249644080225899876...

1.663935... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$(((\sqrt{5}+1)/2))^{0.5} * (((1+\sqrt{2\sqrt{5}-1})/(\sqrt{2})))^{0.5}$$

$$(21.22361098150336695980644780179075994682558655803292568104)^{1/6}$$

**Input interpretation:**

$$\sqrt{\frac{1}{2}(\sqrt{5} + 1)} \left( \sqrt{\frac{1 + \sqrt{2\sqrt{5} - 1}}{\sqrt{2}}} \right) \sqrt[6]{21.22361098150336695980644780179075994682558655803292568104}$$

**Result:**

3.011697596713666179769133877213060156296516322549462225073...

3.0116975...

$$1+1/((((((((((1/2*(((sqrt(5)+1)/2))^{0.5} * (((1+sqrt(2sqrt(5)-1)/sqrt(2)))^{0.5} (21.2236109815033669)^{1/6}))))))))))))))$$

**Input interpretation:**

$$1 + \frac{1}{\frac{1}{2} \left( \sqrt{\frac{1}{2} (\sqrt{5} + 1)} \left( \sqrt{\frac{1 + \sqrt{2\sqrt{5} - 1}}{\sqrt{2}}} \sqrt[6]{21.2236109815033669} \right) \right)}$$

**Result:**

1.664077297196896422...

1.6640772... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

And:

$$\exp((((((((((\sqrt{5}+1)/2)))^0.5 * (((1+\sqrt{2\sqrt{5}-1})/(\sqrt{2})))^0.5 (21.2236109815033669)^{1/6}))))))))))$$

**Input interpretation:**

$$\exp \left( \sqrt{\frac{1}{2} (\sqrt{5} + 1)} \left( \sqrt{\frac{1 + \sqrt{2\sqrt{5} - 1}}{\sqrt{2}}} \sqrt[6]{21.2236109815033669} \right) \right)$$

**Result:**

20.3218689976970667...

20.32186... result near to the black hole entropy 20.5520

$$8((((((((((\sqrt{5}+1)/2)))^0.5 * (((1+\sqrt{2\sqrt{5}-1})/(\sqrt{2})))^0.5 (21.2236109815033669)^{1/6}))))))))))$$

**Input interpretation:**

$$8 \left( \sqrt{\frac{1}{2} (\sqrt{5} + 1)} \left( \sqrt{\frac{1 + \sqrt{2\sqrt{5} - 1}}{\sqrt{2}}} \sqrt[6]{21.2236109815033669} \right) \right)$$

**Result:**

24.09358077370932943...

24.09358... result very near to the black hole entropy 24.2477

From 20.32186 as entropy, we obtain:

Mass = 4.200350e-8 (equivalent to  $2.356224 \times 10^{19}$  GeV, **practically near to the mean value  $1.962 * 10^{19}$  of DM particle that has a Planck scale mass:  $m \approx 10^{19}$  GeV (Planck mass =  $1,2209 \times 10^{19}$  GeV/c<sup>2</sup> = 21,76  $\mu$ g Wikipedia) and is very nearly to the result of the following Ramanujan mock theta function:  $\chi(\mathbf{q}) = 1.962364415$ )**)

Radius = 6.236896e-35

Temperature = 2.921669e+30

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}\left[\left[\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2}\right) \times \frac{1}{4.200350 \times 10^{-8}}}\right]} \times \sqrt{\left[\frac{2.921669 \times 10^{30} \times 4 \pi (6.236896 \times 10^{-35})^3 - (6.236896 \times 10^{-35})^2}{6.67 \times 10^{-11}}\right]}\right] / \left(\frac{6.67 \times 10^{-11}}{1}\right)\right]\right]$$

**Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.200350 \times 10^{-8}}\right)\right) \times \sqrt{\frac{2.921669 \times 10^{30} \times 4 \pi (6.236896 \times 10^{-35})^3 - (6.236896 \times 10^{-35})^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618249253491065545007799946812616150878153219107481818267...

1.6182492...

And, for 1.897512108 as Ramanujan mock theta function, we obtain:

$$\text{sqrt}\left[\left[\left[\frac{1}{\left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2}\right) \times \frac{1}{4.200350 \times 10^{-8}}}\right]} \times \sqrt{\left[\frac{2.921669 \times 10^{30} \times 4 \pi (6.236896 \times 10^{-35})^3 - (6.236896 \times 10^{-35})^2}{6.67 \times 10^{-11}}\right]}\right] / \left(\frac{6.67 \times 10^{-11}}{1}\right)\right]\right]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.200350 \times 10^{-8}}\right) \sqrt{-\frac{2.921669 \times 10^{30} \times 4 \pi (6.236896 \times 10^{-35})^3 - (6.236896 \times 10^{-35})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.645670815522307375014611054851623524543068015538530014593...

$$1.64567... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From:

$$(4.17) \quad N = \sqrt{\frac{140451 + 81090\sqrt{3} - 69\sqrt{8286609 + 4784310\sqrt{3}}}{2}} + \sqrt{\frac{140449 + 81090\sqrt{3} - 69\sqrt{8286609 + 4784310\sqrt{3}}}{2}}$$

We obtain:

1.0000035599604134601464847119723

We note that, from the inverse of result, we obtain:

$$\ln(\left(\left(\left(\frac{1}{1.00000355996041346}\right)\right)\right))$$

**Input interpretation:**

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.00000355996041346}\right)}\right)$$

$\log(x)$  is the natural logarithm

**Result:**

12.545762912954...

12.54576... result very near to the black hole entropy 12.5663

**Alternative representations:**

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = \log_e\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right)$$

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = \log(a) \log_a\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right)$$

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = -\text{Li}_1\left(1 - \frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = \log\left(-1 - \frac{1}{\log(0.9999964400522598130291)}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 - \frac{1}{\log(0.9999964400522598130291)}\right)^k}{k}$$

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = 2i\pi \left\lfloor \frac{\arg\left(-x - \frac{1}{\log(0.9999964400522598130291)}\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x - \frac{1}{\log(0.9999964400522598130291)}\right)^k}{k} \quad \text{for } x < 0$$

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = \left\lfloor \frac{\arg\left(-\frac{1}{\log(0.9999964400522598130291)} - z_0\right)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg\left(-\frac{1}{\log(0.9999964400522598130291)} - z_0\right)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{1}{\log(0.9999964400522598130291)} - z_0\right)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

**Integral representations:**

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = \int_1^{-\frac{1}{\log(0.9999964400522598130291)}} \frac{1}{t} dt$$

$$\log\left(-\frac{1}{\log\left(\frac{1}{1.000003559960413460000}\right)}\right) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left(-1 - \frac{1}{\log(0.9999964400522598130291)}\right)^{-s}}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

Now, we have that:

**Theorem 4.1.** *We have*

$$(4.1) \quad g_{57} = 2^{-1/8} \left(\frac{3\sqrt{19}-13}{\sqrt{2}}\right)^{1/12} (2+\sqrt{3})^{1/4} \left(\sqrt{\frac{170+39\sqrt{19}}{2}} + \sqrt{\frac{168+39\sqrt{19}}{2}}\right)^{1/4} \\ \left(\sqrt{\frac{339+78\sqrt{19}-45\sqrt{113+26\sqrt{19}}}{2}} + \sqrt{\frac{337+78\sqrt{19}-45\sqrt{113+26\sqrt{19}}}{2}}\right)^{1/4}$$

$$(0,91700404320467123174354159479414 * 0,78437161389013532079588337771744*1,3899106635241477179115488119922 * 2,2589318177595206222038904975762*1,0003685316911066866003881134365) = \\ = 2.2591400259551219341077739935978$$

$$1+((\ln(2.2591400259551219341077739935978)))^2$$

**Input interpretation:**

$$1 + \log^2(2.2591400259551219341077739935978)$$

$\log(x)$  is the natural logarithm

**Result:**

$$1.6641992810973847054689759980764...$$

1.664199... is very near to the 14th root of the following Ramanujan's class invariant  
 $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

**Alternative representations:**

$$1 + \log^2(2.25914002595512193410777399359780000) = 1 + \log_e^2(2.25914002595512193410777399359780000)$$

$$1 + \log^2(2.25914002595512193410777399359780000) = 1 + (\log(a) \log_a(2.25914002595512193410777399359780000))^2$$

$$1 + \log^2(2.25914002595512193410777399359780000) = 1 + (-\text{Li}_1(-1.25914002595512193410777399359780000))^2$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$1 + \log^2(2.25914002595512193410777399359780000) = 1 + \left( \log(1.25914002595512193410777399359780000) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-0.23042896885796648722259902564393857k}}{k} \right)^2$$

$$1 + \log^2(2.25914002595512193410777399359780000) = 1 + \left( 2i\pi \left\lfloor \frac{\arg(2.25914002595512193410777399359780000 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2.25914002595512193410777399359780000 - x)^k x^{-k}}{k} \right)^2$$

for  $x < 0$

$$1 + \log^2(2.25914002595512193410777399359780000) =$$

$$1 + \left( \log(z_0) + \left\lfloor \frac{\arg(2.25914002595512193410777399359780000 - z_0)}{2\pi} \right\rfloor \right.$$

$$\left. \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2.25914002595512193410777399359780000 - z_0)^k z_0^{-k}}{k} \right)^2$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$1 + \log^2(2.25914002595512193410777399359780000) =$$

$$1 + \left( \int_1^{2.25914002595512193410777399359780000} \frac{1}{t} dt \right)^2$$

$$1 + \log^2(2.25914002595512193410777399359780000) =$$

$$1 + \frac{\left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-0.23042896885796648722259902564393857s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^2}{4i^2 \pi^2} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

$$8 * (2.2591400259551219341077739935978)$$

### Input interpretation:

$$8 \times 2.2591400259551219341077739935978$$

### Result:

$$18.0731202076409754728621919487824$$

18.07312... result very near to the black hole entropy 18.0524

With an entropy equal to the result 18.07312 that we have obtained from the ln of the value of expression, by the Hawking radiation calculator, we have the following black hole parameters:



Mass = 3.961141e-8 (equivalent to 2.222038×10<sup>19</sup> GeV, **practically near to the mean value 1.962 \* 10<sup>19</sup> of DM particle that has a Planck scale mass: m ≈ 10<sup>19</sup> GeV (Planck mass = 1,2209 × 10<sup>19</sup> GeV/c<sup>2</sup> = 21,76 μg Wikipedia) and is very nearly to the result of the following Ramanujan mock theta function:  $\chi(\mathbf{q}) = 1.962364415$ )**)

Radius = 5.881706e-35

Temperature = 3.098105e+30

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))) \* 1/(3.961141e-8) \* sqrt[-((3.098105e+30 \* 4\*Pi\*(5.881706e-35)^3 - (5.881706e-35)^2))])))) / ((6.67\*10^-11)))]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.961141 \times 10^{-8}} \sqrt{-\frac{3.098105 \times 10^{30} \times 4 \pi (5.881706 \times 10^{-35})^3 - (5.881706 \times 10^{-35})^2}{6.67 \times 10^{-11}}} \right)} \right)$$

**Result:**

1.618249274997892019504587237680764169194850285735542064077...

1.6182492...

We have that:

**Theorem 4.2.** *We have*

$$(4.11) \quad g_{177} = 2^{-1/8} \left( \frac{3\sqrt{59} + 23}{\sqrt{2}} \right)^{1/6} \left( \frac{\sqrt{3} - 1}{\sqrt{2}} \right)^{3/4} \left( \sqrt{\frac{70226 + 40545\sqrt{3}}{2}} + \sqrt{\frac{70224 + 40545\sqrt{3}}{2}} \right)^{1/4} \\ \left( \sqrt{\frac{140451 + 81090\sqrt{3} - 69\sqrt{8286609 + 4784310\sqrt{3}}}{2}} + \sqrt{\frac{140449 + 81090\sqrt{3} - 69\sqrt{8286609 + 4784310\sqrt{3}}}{2}} \right)^{1/4}$$

0.91700404320467123174354159479414 \* 1.7869351424559208571495550416157

$$0.61026669566895201864910886043457 * 4.798100649682333646698531102299 * 1.0000008899889152439277927665906 =$$

**Input interpretation:**

$$0.91700404320467123174354159479414 \times 1.7869351424559208571495550416157 \times 0.61026669566895201864910886043457 \times 4.798100649682333646698531102299 \times 1.0000008899889152439277927665906$$

**Result:**

$$4.798101717247500639972682852539393492684632365633000040389... \\ 4.798101717247500639972682852539393492684632365633000040389$$

$$(4.798101717247500639972682852539393492684632365633000040389)^5 - 812$$

**Input interpretation:**

$$4.798101717247500639972682852539393492684632365633000040389^5 - 812$$

**Result:**

$$1731.005226314068792602977636603629658557174712405515442944... \\ 1731.0052...$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$(4.798101717247500639972682852539393492684632365633000040389) * \sqrt{26}$$

**Input interpretation:**

$$4.798101717247500639972682852539393492684632365633000040389 \sqrt{26}$$

**Result:**

$$24.46561428444805632461738753281995797105904539978275887912... \\ 24.46561428... \text{ result practically equal to black hole entropy } 24.4233$$

From this entropy 24.46561428, we obtain:

$$\text{Mass} = 4.608736e-8$$

$$\text{Radius} = 6.843288e-35$$

Temperature =  $2.662776e+30$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}[1/\text{sqrt}[1/(((4*1.962364415e+19)/(5*0.0864055^2)) * 1/(4.608736e-8) * \text{sqrt}[-((2.662776e+30 * 4*\text{Pi}*(6.843288e-35)^3 - (6.843288e-35)^2)) / ((6.67*10^-11)))]]]]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.608736 \times 10^{-8}}\right) \sqrt{-\frac{2.662776 \times 10^{30} \times 4\pi (6.843288 \times 10^{-35})^3 - (6.843288 \times 10^{-35})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.618249320144158002340722774115529964322392175785035433397...

1.6182493...

## Appendix A

Mathematical connection between 728,  $\zeta(2) = \pi^2/6$  and  $\phi = 1.61803398...$

We have the following interesting expression that link 728,  $\zeta(2)$  and  $\phi$ :  
 $\zeta(2) - ((\text{sqrt}728)/(10^3))$

**Input:**

$$\zeta(2) - \frac{\sqrt{728}}{10^3}$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\frac{\pi^2}{6} - \frac{\sqrt{\frac{91}{2}}}{250}$$

**Decimal approximation:**

1.617952591721762353541314555423034099274039165006363415026...

1.6179525...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Property:**

$-\frac{\sqrt{\frac{91}{2}}}{250} + \frac{\pi^2}{6}$  is a transcendental number

**Alternate forms:**

$$\frac{250 \pi^2 - 3 \sqrt{182}}{1500}$$

•

$$\frac{1}{750} \left( 125 \pi^2 - 3 \sqrt{\frac{91}{2}} \right)$$

**Alternative representations:**

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = -\frac{\sqrt{728}}{10^3} + \zeta(2, 1)$$

•

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = S_{1,1}(1) - \frac{\sqrt{728}}{10^3}$$

•

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = -\frac{\text{Li}_2(-1)}{\frac{1}{2}} - \frac{\sqrt{728}}{10^3}$$

$\zeta(s, \alpha)$  is the generalized Riemann zeta function

$S_{n,p}(x)$  is the Nielsen generalized polylogarithm function

$\text{Li}_n(x)$  is the polylogarithm function

**Series representations:**

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = \frac{1000 \sum_{k=1}^{\infty} \frac{1}{k^2} - \sqrt{727} \sum_{k=0}^{\infty} 727^{-k} \binom{\frac{1}{2}}{k}}{1000}$$

•

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = \frac{1000 \sum_{k=1}^{\infty} \frac{1}{k^2} - \exp\left(i\pi \left\lfloor \frac{\text{arg}(728-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (728-x)^k x^{-k} \binom{-\frac{1}{2}}{k}}{k!}}{1000}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

•

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = \frac{1000 \exp\left(\sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{(p_k)^{-2j}}{j}\right) - \exp\left(i\pi \left\lfloor \frac{\text{arg}(728-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (728-x)^k x^{-k} \binom{-\frac{1}{2}}{k}}{k!}}{1000}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$\binom{n}{m}$  is the binomial coefficient

$\text{arg}(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

$p_n$  is the  $n^{\text{th}}$  prime number

### Integral representations:

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = \frac{1}{\Gamma(2)} \int_0^{\infty} \frac{t}{-1+e^t} dt - \frac{\sqrt{728}}{1000}$$

•

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = \frac{2}{3\Gamma(2)} \int_0^{\infty} t \operatorname{csch}(t) dt - \frac{\sqrt{728}}{1000}$$

•

$$\zeta(2) - \frac{\sqrt{728}}{10^3} = \frac{2}{\Gamma(3)} \int_0^\infty t^2 \operatorname{csch}^2(t) dt - \frac{\sqrt{728}}{1000}$$

$\Gamma(x)$  is the gamma function

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

## Entropies from $\pi$

$8\pi$

### Input:

$8\pi$

### Decimal approximation:

25.13274122871834590770114706623602307357735519500084656779...

25.1327... equal to the black hole entropy 25.1327

### Conversion from radians to degrees:

$1440^\circ$

### Property:

$8\pi$  is a transcendental number

## Application example of Ramanujan-Nardelli mock formula on the particle Delta baryon rest mass, considered as a quantum black hole

Now, from the Delta baryon rest mass 1232, we obtain:

$$[[[[[(1232)]]]]]^{1/14}$$

### Input:

$$\sqrt[14]{1232}$$

### Result:

$$2^{2/7} \sqrt[14]{77}$$

**Decimal approximation:**

1.662485659149614509161733052004467065715390440093706908333...

1.6624856... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

From the formula of  $\zeta(2) = \pi^2/6 = 1.64493$ , we obtain about  $2\pi$ . Indeed:

$$2(((((((6*(((1232)))))))))^{1/14}))))))^{1/2}$$

**Input:**

$$2\sqrt{6^{14}\sqrt[14]{1232}}$$

**Result:**

$$2 \times 2^{9/14} \sqrt{3} \sqrt[28]{77}$$

**Decimal approximation:**

6.316617434956049973673897320907890669067683682030624571693...

6.3166174...

From the ratio with  $2\pi$ , we obtain:

$$6.316617434956049973673897320907890669067683682030624571693/(2\pi)$$

**Input interpretation:**

6.316617434956049973673897320907890669067683682030624571693

$$2\pi$$

**Result:**

1.005320888393704021174092136477898180118967136778065504853...

1.0053208... this is the radius

**Alternative representations:**

$$\frac{6.3166174349560499736738973209078906690676836820306245716930000}{2\pi} = \frac{6.3166174349560499736738973209078906690676836820306245716930000}{2i \log(-1)}$$

$$\frac{6.3166174349560499736738973209078906690676836820306245716930000}{2\pi} = \frac{6.3166174349560499736738973209078906690676836820306245716930000}{2 \cos^{-1}(-1)}$$





$$\frac{6.3166174349560499736738973209078906690676836820306245716930000}{0.78957717936950624670923716511348633363346046025382807146162500} = \frac{2\pi}{\int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{6.3166174349560499736738973209078906690676836820306245716930000}{1.5791543587390124934184743302269726672669209205076561429232500} = \frac{2\pi}{\int_0^\infty \frac{\sin(t)}{t} dt}$$

From the above calculated radius 1.005321, inserting this value in the Hawking black hole radiation calculator, we obtain:

$$\text{Mass} = 6.770515e+26$$

$$\text{Radius} = 1.005321$$

$$\text{Temperature} = 0.0001812570$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}[\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.770515 \times 10^{26}} \times \sqrt{-\frac{0.0001812570 \times 4 \pi \times 1.005321^3 - 1.005321^2}{6.67 \times 10^{-11}}}}]$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.770515 \times 10^{26}} \times \sqrt{-\frac{0.0001812570 \times 4 \pi \times 1.005321^3 - 1.005321^2}{6.67 \times 10^{-11}}}}}$$

**Result:**

1.618249201774669727129593653709932216503767446288407609302...

1.6182492... that is a very good approximation to the value of golden ratio

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**Three-dimensional AdS gravity and extremal CFTs at  $c = 8m$**

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**Table 1**

$m$	$L_0$	$d$	$S$	$S_{BH}$
3	1	196883	12.1904	12.5664
	2	21296876	16.8741	17.7715
	3	842609326	20.5520	21.7656
4	2/3	139503	11.8458	11.8477
	5/3	69193488	18.0524	18.7328
	8/3	6928824200	22.6589	23.6954
5	1/3	20619	9.9340	9.3664
	4/3	86645620	18.2773	18.7328
	7/3	24157197490	23.9078	24.7812
6	1	42987519	17.5764	17.7715
	2	40448921875	24.4233	25.1327
	3	8463511703277	29.7668	30.7812
7	2/3	7402775	15.8174	15.6730
	5/3	33934039437	24.2477	24.7812
	8/3	16953652012291	30.4615	31.3460
8	1/3	278511	12.5372	11.8477
	4/3	13996384631	23.3621	23.6954
	7/3	19400406113385	30.5963	31.3460

**Table 1:** Degeneracies, microscopic entropies and semiclassical entropies for the first few values of  $m$  and  $L_0$ .

## Conclusion

In this paper, in addition to the connections obtained between Ramanujan mock theta functions, class invariants and black hole entropy values, in turn connected with golden ratio and  $\zeta(2)$ , we highlight the following results:

(supersymmetric condition  $\rightarrow \infty$ )

(supersymmetric condition  $\rightarrow 0$ )

From our point of view, these results could indicate that Ramanujan had already guessed, albeit in a non-detailed manner, the supersymmetry that is hidden in zero (zero point energy) and in infinity (absolute = supersymmetric infinite-dimensional toroidal space = absence of entropy).

From:

<https://www.cittanuova.it/ramanujanhardy-e-il-piacere-di-scoprire/>

“Indeed Ramanujan elaborated a theory of reality around Zero (representing the Absolute Reality) and the Infinite (the multiple manifestations of that reality): their mathematical product represented all the numbers, each of which corresponded to individual acts of creation. In short, even if his english friends didn't understand him very much, for him "the numbers and their mathematical relationships let us understand how everything was in harmony in the universe" ”

## References

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