Riemann Hypothesis

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1 Abstract

The Proof involves Analytic Continuation of the

Riemann Zeta function expressed as a Hadamard Product

2 Proof

The Analytic Continuity of Riemann Zeta -function

over

$$0 < Re(s) < 1$$

defined as a Hadamard Product [2] is,

$\zeta(s) = \frac{1}{2} \prod_{\rho}(1 - s/\rho)$

$Let,\ s = \sigma + it$

$and\ \rho = a + ib.$

let;

$$1/2 < \sigma < \eta < 1$$

.

$$\zeta(\sigma + it)$$

$= \prod_\rho [1-(\sigma+it)/(a+ib)]$

$|\zeta(\sigma+it)| = \frac{1}{2}\prod_\rho (\sigma-a)^2+(t-b)^2)^{1/2}/(a^2+b^2)^{1/2})$

$|\zeta(\eta+it)| = \frac{1}{2}\prod[(\eta-a)^2+(t-b)^2]^{1/2}/[(a^2+b^2)^{1/2}]$

$CASE\ 1: 1/2 < a < \sigma < \eta < 1\ and\ t\ is\ fixed$

$(\eta-a) > (\sigma-a) > 0$

$(\eta-a)^2 > (\sigma-a)^2$

$\mid \zeta(\eta+it) \mid > \frac{1}{2}\prod[(\sigma-a)^2+(t-b)^2]^{1/2}.$

$\mid \zeta(\eta+it) \mid > \mid \zeta(\sigma+it) \mid$

$So,\ for\ fixed\ t,$

$\mid \zeta(\sigma+it) \mid\ is\ Strictly\ Monotonically\ Increasing\ for\ 1/2 < \sigma < 1.$

CASE 2:

$$1/2 < \sigma < \eta < 1$$

(a-η) $> (a-\sigma)$.

$(a-\eta)^2 > (a-\sigma)^2.$

$\mid \zeta(\sigma+it) \mid < \mid \zeta(\eta+it)$

$\mid \zeta(\sigma+it) \mid is\ Strictly\ Monotonically\ Increasing.$

$CASE\ 3:\qquad 0 < \sigma < a < \eta < 1/2.$

$(\eta - a) > (a - \sigma)$

$(\eta - a)^2 > (a - \sigma)^2$

$\mid \zeta(\eta + it) \mid > \mid \zeta(\sigma + it) \mid .$

$So, \mid \zeta(\sigma + it) \mid is\ Strictly\ Monotonically\ Increasing\ in\ this\ case.$

$This\ gives\ that\ \zeta(\sigma + it) \neq 0\ \forall \sigma \in (0,1) - [1/2].$

$but,\ by\ Hypothesis,\ \zeta(\sigma + it) = 0, \sigma \in (0,1)$

$Hence,$

$\sigma = 1/2.$

$So,\ the\ real\ part\ of\ all\ the\ non\ trivial\ zeroes\ is\ 1/2.$

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