Calculating gravitational time dilation using length contraction

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Abstract: I show that gravitational time dilation is a twin paradox scenario that can be calculated using length contraction.

The Gravitational Redshift of the Sun's Light

Geometric units are used herein. As a prerequisite read Calculating the twin paradox using length contraction, where I show that the traveling twin ages less because that twin's path is shorter due to length contraction. In particular note the "room" there. My analysis compares to Einstein's comment about the gravitational redshift of the Sun's light as observed from Earth:

> For the sun, the displacement towards the red predicted by the theory amounts to about two millionths of the wave-length.

In my analysis there are 2 twin paradoxes, both involving Bob as the stationary twin, hovering a great distance above our Solar System. Sue stays at the Sun's surface (let's pretend she can survive there), and Eve is earthbound. Sue and Eve are analogous to traveling twins; more on this below. One twin paradox involves Bob and Sue; the other involves Bob and Eve. I focus on the twin paradox that involves Bob and Sue. The same logic applies to the other twin paradox involving Bob and Eve.

Some escape velocities are needed. Escape velocity is calculated by:

$$v_e = \sqrt{\frac{2M}{r}}$$

From Selected Physical and Astronomical Constants the mass of the Sun is 1.48 x 10^3 meters, the radius of the Sun is 6.96 x 10^8 meters, and the mean distance of Earth from the Sun is 1.50 x 10^{11} meters. So the Sun's escape velocity at its surface is 2.06 x 10^{-3} *c*, or 618 kilometers per second, and the Sun's escape velocity at the Earth's mean distance from the Sun is 1.40 x 10^{-4} *c*, or 42 km/s.

To visualize how Bob and Sue travel relative to each other, consider the river or waterfall model of black holes. In that model, space falls toward a center of gravitational

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attraction at the escape velocity at each radius. Let Bob drop a stone. At first it negligibly moves relative to him, as the escape velocity is almost zero at his position. Eventually the stone passes Sue at 618 km/s. (To verify this, see Equations for a falling body at "used for large fall distances".)

The "room" in this case is the falling space that's indicated by the stone's behavior. Bob and Sue are in that room together. The room significantly moves only in Sue's frame, as if she's moving across the room while Bob stands still. In that way Bob and Sue travel paths relative to each other. The scenario is analogous to a twin paradox where Bob is earthbound while Sues moves away from Earth at 618 km/s.

Sue's path relative to Bob is shorter as she measures, due to length contraction caused by her movement relative to it. Bob's path relative to Sue isn't length-contracted because he isn't moving relative to it. Bob and Sue have the same velocity relative to each other. Traveling less distance at the same velocity takes less time, so Sue ages less than Bob.

Here is my equation from the other paper to calculate the twin paradox using length contraction, simplified because the velocity in this case, the escape velocity, is constant:

$$T = \frac{t}{\gamma} = \frac{d}{v\gamma} \quad [1]$$

Rearranging:

$$\frac{T}{t} = \frac{1}{\gamma}$$

Therefore the difference in aging between Sue and Eve is calculated by:

$$T_{diff} = \frac{T_{Eve} - T_{Sue}}{T_{Eve}} = 1 - \frac{T_{Sue}}{T_{Eve}} = 1 - \frac{T_{Sue}}{t} \Big/ \frac{T_{Eve}}{t}$$

$$= 1 - \frac{\gamma_{Eve}}{\gamma_{Sue}} = 1 - \sqrt{\frac{1 - (\text{Sun's escape velocity at Sue's radius}/c)^2}{1 - (\text{Sun's escape velocity at Eve's radius}/c)^2}}$$

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The result, inputting 2.06 x 10^{-3} *c* for the escape velocity for Sue, and 1.40 x 10^{-4} *c* for the escape velocity for Eve, is 2.1 x 10^{-6}, or about two millionths, agreeing with Einstein. That is, Sue ages slower than Eve does by about two millionths of a second per second. (The same result is returned when using my new equation for escape velocity, eq. 1 in Solving incompatibility between GR and QM re black holes.)

Einstein's analysis is really between just Sue and Bob, the stationary twin, in which case:

$$T_{diff} = \frac{t - T}{t} = 1 - \frac{T}{t} = 1 - \frac{1}{\gamma}$$

$$= 1 - \sqrt{1 - (\text{Sun's escape velocity at Sue's radius}/c)^2} \quad [2]$$

The result is still the expected 2.1 x 10^{-6}.

Elaborating to directly consider length contraction:

In Bob's frame, in *t* = 1 second on his clock, Sue moves *d* = 2.06 x 10^{-3} light seconds through the room they share, the falling space, and relative to Bob. In Sue's frame that distance is length contracted, so, having the same velocity, she traverses it in less than 1 second, calculated by (from eq. 1):

$$T = \frac{d}{v\gamma}$$

Then:

$$T_{diff} = \frac{t - T}{t} = 1 - \frac{T}{t} = 1 - \frac{d}{tv\gamma}$$

This returns the same as eq. 2, since:

$$\frac{d}{tv} = 1$$

See also Gravitational time dilation at "Here is the proof", and A Non-Mathematical Proof of Gravitational Time Dilation.