Riemann Hypothesis

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1 Abstract

The Proof involves Analytic Continuation of the

Riemann Zeta function expressed as a Hadamard Product

Later , since $|\zeta^{*}(\sigma+i\,t)|$ $\mathit{is\ increasing\ for}\ 0<\sigma<1,\ thus,\ we\ get\ a\ proof\ of\ the$

Riemann Hypothesis.

The Analytic Continuity of Riemann Zeta $-$ *function*

over $0 < Re(s) < 1$ *defined as a Hadamard Product* [1] *is,*

$\zeta(s)= \frac{\pi^{s/2}\prod_{\rho}(1-s/\rho)}{2(s-1)\Gamma(1+\frac{s}{2})}$

Let, $s = \sigma+it$

and $\rho = a + ib.$

Let $, \kappa = \eta + it.$

let; $\sigma < \eta.$

$\zeta(\sigma + it)= \frac{\pi^{(\sigma+it)/2}\prod_{\rho}(1-(\sigma+it)/\rho)}{2(\sigma-1+it)\Gamma(1+\frac{\sigma+it}{2})}$

using $\mid \pi^{it/2} \mid = \mid e^{it/2\ ln(\pi)} \mid = 1.$

$|\zeta(\sigma + it)| \leq \frac{\pi^{\sigma/2}\prod_{\rho}(1+(\sigma^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\sigma-1)^2+t^2)|\Gamma(1+\frac{\sigma+it}{2})|}$

$$|\zeta(\sigma+it)| \leq \frac{\pi^{\sigma/2}\prod_{\rho}(1+(\sigma^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\sigma-1)^2+t^2)^{1/2}|\Gamma(1+\frac{\sigma+it}{2})|}$$

$$Since, \sigma < \eta.$$

$$|\zeta(\sigma+it)| < \frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((1-\sigma)^2+t^2)^{1/2}|\Gamma(1+\frac{\sigma+it}{2})|}$$

$$|\zeta(\sigma+it)| < \frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((1-\eta)^2+t^2)^{1/2}|\Gamma(1+\frac{\sigma+it}{2})|}$$

$$|\zeta(\sigma+it)| < \frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\eta-1)^2+t^2)^{1/2}|\Gamma(1+\frac{\sigma+it}{2})|}$$

$$|\zeta(\eta+it)| < \frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\eta-1)^2+t^2)^{1/2}|\Gamma(1+\frac{\eta+it}{2})|}$$

$$|\ \Gamma(1+\frac{\eta+it}{2})\ | = |\int_0^\infty e^{-x}x^{(\eta+it)/2}dx\ |$$

$$|\ \Gamma(1+\frac{\eta+it}{2})\ | \leq \int_0^\infty e^{-x}x^{\eta/2}dx$$

$$1/|\ \Gamma(1+\frac{\eta+it}{2})\ | \geq \int_0^\infty e^{-x}x^{\eta/2}dx$$

$$-\ 1/|\ \Gamma(1+\frac{\eta+it}{2})\ | \leq -\int_0^\infty e^{-x}x^{\eta/2}dx$$

$$So,$$

$$-|\zeta(\eta+it)| \leq -\frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\eta-1)^2+t^2)^{1/2}|\Gamma(1+\frac{\eta+it}{2})|}$$

$$|\zeta(\sigma+it)| - |\zeta(\eta+it)| \leq \frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\eta-1)^2+t^2)^{1/2}|\Gamma(1+\frac{\sigma+it}{2})}\ | - \frac{\pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})}{2((\eta-1)^2+t^2)^{1/2}|\Gamma(1+\frac{\eta+it}{2})|}$$

$$|\zeta(\sigma+it)| - |\zeta(\eta+it)|$$

$$\leq \pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})/2((\eta-1)^2+t^2)^{1/2}[1/|\ \Gamma(1+\frac{\eta+it}{2})\ | - 1/\int_0^\infty e^{-x}x^{\eta/2}dx]$$

$$\leq \pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})$$

$$/2((\eta-1)^2+t^2)^{1/2}[1/|\ \Gamma(1+\frac{\eta+it}{2})\ | \leq \pi^{\eta/2}\prod_{\rho}(1+(\eta^2+t^2)^{1/2}/(a^2+b^2)^{1/2})/2((\eta-1)^2+t^2)^{1/$$

$\leq \phi[\int_0^\infty e^{-x}x^{\sigma/2}(cos(t\ ln\ x/2)^2]^{1/2}dx$

$\leq \phi[\int_0^\infty e^{-x}x^{\sigma/2}(cos(t\ ln\ x/2]dx$

$now,$

$-\int_0^\infty e^{-x}x^{\sigma/2} \leq$

$\int_0^\infty e^{-x}x^{\sigma/2}(cos(t\ ln\ x/2$

$\leq \int_0^\infty e^{-x}x^{\sigma/2}dx$

$1/\int_0^\infty e^{-x}x^{\sigma/2}(cos(t\ ln\ x/2 \leq -1/\int_0^\infty e^{-x}x^{\sigma/2}dx \leq 0$

$|\zeta(\sigma+it)| - |\zeta(\eta+it) \leq 0$

$\sigma < \eta, implies\ |\zeta(\sigma+it)| \leq |\zeta(\eta+it)|$

$thus, |\zeta(\sigma+it)|\ \ is\ Monotonically\ Increasing\ w.r.t.\ \sigma.$

$So, 0 < \sigma \leq 1/2 implies$

$|\zeta(\sigma+it)| \leq |\zeta(1/2+it)|$

$and\ 1/2 \leq \sigma < 1\ , implies$

$|\zeta(1/2+it)| \leq |\zeta(\sigma+it)|$

$So, |\zeta(\sigma+it)| \leq |\zeta(1/2+it)| \leq |\zeta(\sigma+it)|,\ \sigma\ \in (0,1)$

$also, |\zeta(\sigma+it)| = 0\ \sigma \in\ (0,1)$

$hence\ ,$

$|\zeta(1/2+it)| = 0$

$Hence\ all\ the\ zeroes\ lie\ on\ the\ line\ x = 1/2$

2 References:-

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