Calculating the twin paradox using length contraction

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Abstract: I show that the traveling twin ages less because that twin's path is shorter. The reason typically given, that only the traveling twin accelerates, is superfluous.

The simplest asymmetry

A common question about the twin paradox is: what is the asymmetry? The common answer is that only the traveling twin accelerates. There is a simpler answer. In Calculating "Speeding to Andromeda" easier I said:

> The speed required to get [from Earth] to Andromeda while aging 1 year is the speed that length-contracts the distance to Andromeda to that which is traversed in 1 year at that speed. Time dilation and length contraction go hand-in-hand that way.

This is a clue to a deeper understanding of the twin paradox. The gamma factor formula calculates both time dilation and length contraction; they always coexist. Let Bob be the earthbound twin and Sue be the traveling twin. They travel paths relative to each other. Sue's path relative to Bob is shorter as she measures, due to length contraction caused by her movement relative to it. Bob's path relative to Sue isn't length-contracted because he isn't moving relative to it. At any given instant Bob and Sue have the same velocity relative to each other. Traveling less distance at the same velocity (at any given instant) takes less time, so Sue ages less than Bob. Her shorter path is the asymmetry.

Consider: Bob and Sue are in a room together. Sue walks around the room while Bob stands still. The room moves in Sue's frame, so for her the room is length-contracted along her axis of motion. Hence her path relative to Bob is length-contracted, whereas Bob's path relative to Sue isn't length-contracted because the room isn't moving in his frame. For example, when Sue is moving directly toward or away from Bob, the distance that she measures between them, at any given instant, is less than Bob measures.

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The equations below support my assertion, in geometric units. Here is the equation for Sue's aging (the elapsed time on her clock) from The Relativistic Rocket, which also defines all of the variables below:

$$T = \frac{\operatorname{asinh}(at)}{a} = \frac{\operatorname{acosh}(ad + 1)}{a}$$

Here is my alternate equation:

$$T = \sum \frac{\Delta t}{\gamma} = \sum \frac{\Delta d}{v\gamma}$$

This equation returns expected results, proving that Sue's aging can be calculated by summing her time required to traverse each length-contracted (as she measures) segment of her trip relative to Bob, where each segment is small enough that her velocity in that segment negligibly changes. This explains the twin paradox without using acceleration, which agrees with the clock postulate:

> [The ratio of the rates of 2 clocks] depends only on *v*, and does not depend on any derivatives of *v*, such as acceleration.

Here is code to verify my equation using Bob's segmented time. Here is code to verify my equation using Bob's segmented distance. Click the Run button to get the output shown below, or change the inputs that are set near the top. Search The Relativistic Rocket for "Here are some of the times you will age when journeying to a few well known space marks, arriving at low speed". The values listed there, divided by 2, serve as inputs and expected results for testing the code.

The code calculates Bob's and Sue's aging (the elapsed time on their respective clocks) while Sue makes a trip from Earth to the midpoint between Earth and Vega, accelerating at 1 Earth gravity the whole way. The "room" in this case is the Earth / Vega system, which moves only in Sue's frame, not Bob's. The code outputs:

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While Bob ages 14.4 years as predicted by the relativistic rocket
equations, Sue ages:
     3.3 years as predicted by the relativistic rocket equations
     3.3 years as predicted by the numerical method herein
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Sue is distant from Bob and moving relative to him at the end of the scenario. In any given instant her aging can be compared to his effectively in person, by employing observers arrayed along her path and at rest relative to him. To include Sue's deceleration from the midpoint to Vega, so that she's then at rest relative to Bob, which is the scenario given at The Relativistic Rocket, double the outputted values. For the traditional twin paradox where Sue returns to stand next to Bob, quadruple the outputted values.

The derivation of my equation above

$$T = \sum \Delta T$$

From Time Dilation:

$$T = t\sqrt{1 - v^2}$$

From The gamma factor:

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

From basic physics:

$$t = \frac{d}{v}$$

Substituting gives my equation above.