Evidence universal gravitation in evidence theory

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Abstract

Since the introduction of the law of universal gravitation, it has been widely used in the field of natural sciences and theoretical exploration. In other disciplines, based on the law of universal gravitation, some scholars have proposed universal gravitation search algorithms, swarm intelligence optimization algorithms and fuzzy control. However, there is no research to apply the law of universal gravitation to the field of evidence theory. In this paper, we present for the first time the concept of evidence universal gravitation. In the evidence universal gravitation formula we define the evidence gravitation parameter and the evidence quality generation algorithm. The evidence universal gravitation formula satisfies some basic properties. This paper gives some numerical examples to further illustrate the significance of the gravitational evidence. In addition, because conflict management is an open question, the measurement of conflict has not been reasonably resolved.

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In this paper, we apply the evidence universal gravitation to conflict processing, and illustrate its wide applicability through the comparison of numerical examples.
Keywords: Information fusion, Evidence gravitation, Evidence universal gravitation, Evidence universal gravitation formula, Evidence gravitation parameter, Evidence theory, Conflict management.

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1. Introduction

Dempster-Shafer evidence theory was produced in the 1960s[1, 2]. In the 1970s and 1980s, Dempster-Shafer evidence theory was introduced into the field of artificial intelligence, and many theoretical and applied research appeared. As an uncertainty reasoning method, evidence theory provides a powerful tool for the representation and fusion of decision-level uncertain information. Therefore, it has been widely used in information fusion, pattern recognition and decision analysis.

The law of universal gravitation was first proposed by Newton in the book "Mathematical Principles of Natural Philosophy" published in 1687[3]. The law of universal gravitation belongs to the law of the natural sciences. It shows that any two objects in nature are attracted to each other. The magnitude of gravity is proportional to the product of the mass of the two objects, and inversely proportional to the square of their distance. Subsequently, the law of universal gravitation has been widely developed and applied in the field of natural science. In recent years, some scholars have proposed a gravitational population optimization algorithm, which has attracted the attention of many scholars. After that, more and more scholars tried to simulate it into relevant scientific research fields. Therefore, derivative research based on gravitation has been further developed.

We think that, as in the field of natural science, in the field of information fusion, each independent evidence is also a qualitative point of visualization. There is also a force between any two different pieces of evidence, which we define as evidence gravitation.

In this paper, we combine the universal gravitation with the evidence theory and propose a formula that can represent the gravitational gravity of the evidence theory, that is, the evidence universal gravitation. We derive the intrinsic meaning and nature of gravitation into the field of information fusion. As with gravitation in the natural sciences, we abstract the evidence obtained by sensors into a qualitative mass. We use the ETEG algorithm to generate quality for each independent source of evidence, and the evidence distance in the evidence theory to characterize the gravitational distance between two independent evidences. In addition, in the evidence universal gravitation formula, We use the eidence gravity parameter G_e to identify the different identification frameworks of the system in which the eidence gravitation is located. The specific expression of the evidence universal gravitation is as follows: In a system in which the evidence universal gravitation operates and under the identified identification framework, the gravitation of evidence is proportional to the product of the evidence quality of two different evidences, and inversely proportional to the square of their distance.

The organizational structure of this paper is as follows. The 2 section introduces some background knowledge of the work done. In the 3 section, the evidence universal gravitation formula is presented and some basic properties it satisfies are discussed. The 3 section gives examples of numerical values to further illustrate the significance of the evidence universal gravitation. In addition, we apply the evidence universal gravitation to conflict processing, and illustrate its practicability and application through numerical comparison. Section 5 summarizes the content of the paper and looks forward to future work.

2. Preliminaries

In this section, we briefly introduce some basic concepts, including the Dempster-Shafer evidence theory, the evidence distance , metric space and the universal gravitation formula.

2.1. Dempster-Shafer evidence theory

Dempster-shafer's evidence theory(DST)[1] was first proposed by Dempster and then further developed by his student Shafer. Dempster-shafer evidence theory is widely used in many fields, including information fusion, pattern recognition and decision support, because of its excellent ability to deal with uncertain information and information fusion, etc. In the Dempster-Shafer evidence theory, a set of exhaustive and mutually exclusive sets of elements is called the identification framework, denoted by Θ. 2^{Θ} is called the Θ power set.

Definition1. Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, power set 2^{Θ} of identification framework can be expressed as

$$2^{\Theta} = \{\emptyset, \{\theta_1\}, \cdots \{\theta_N\}, \{\theta_1, \theta_2\}, \cdots \{\theta_1, \theta_2, \cdots, \theta_i\}, \cdots, \Theta\} \tag{1}$$

Definition2. A basic probability assignment (BPA) is a mapping of m : $2^{\Theta} \rightarrow [0, 1]$, which satisfies the following two attributes.

$$\begin{cases} m\left(\emptyset\right) = 0 \\ \sum\limits_{\theta \subseteq \Theta} m\left(\theta\right) = 1 \end{cases} \tag{2}$$

m(A) is BPA of A, which accurately reflects the extent to which A is supported.

2.2. Dempster's combination rule

The Dempster's combination rule has been widely used to combine multiple independent evidence, and its definition is as follows.

Definition3. Suppose the two evidence functions m_1 and m_2 are on the identification framework Θ, and then the Dempster's combination rule can be defined as follows($\oplus$ represents the orthogonal summation operation):

$$\left[m_1 \oplus m_2\right]\left(\theta\right) = \begin{cases} 0 & \theta = \emptyset \\ \frac{\sum\limits_{A_1 \cap A_2 = \theta} m_1(A_1)m_2(A_2)}{1-k} & \theta \neq \emptyset \end{cases} \tag{3}$$

Where, the conflict coefficient K is defined as follows:

$$k = \sum_{A_1 \cap A_2 = \emptyset} m_1(A_1)m_2(A_2) \tag{4}$$

2.3. Evidence distance

Definition4[4]. Hypothesis m_1 and m_2 are two arbitrary functions on the recognition frame, and the distance between m_1 and m_2 is defined as follows

$$d(m_1, m_2) = \sqrt{\frac{1}{2} \sum_{\emptyset \neq A_1 \subseteq \Theta, \emptyset \neq A_2 \subseteq \Theta} \frac{|A_1 \cap A_2|}{|A_1 \cup A_2|} (m_1(A_1) - m_2(A_1))(m_1(A_2) - m_2(A_2))} \tag{5}$$

It satisfies some of the following property

$$d(m, m) = 0 \tag{6}$$

2.4. Metric space

Because the purpose of this article is to establish universal gravitation in the theory of evidence, the proposed gravitational formula is based on the distance of evidence. Let us introduce some basic concepts about metric space and distance.

Definition5[4]. A metric distance defined on the set $\Re$ is a function.
d satisfies the following conditions:

$$\begin{cases} \delta \times \delta \to \Re \\ (A, B) \to d(A, B) \end{cases}$$

that meets the following requirements for any of A and B in $\Re$:

(i) ***Nonnegativity:*** *d(A,B)*$\geqslant$ *0*

(ii) ***Nondegeneracy:*** *d(A,B)= 0*$\Leftrightarrow$ *A=B*

(iii) ***Symmetry:*** *d(A,B)=d(B,A)*

(iv) ***Triangle inequality:*** *d(A,B)*$\leq$ *d(A,C)+d(C,B) and any* $C \in \delta$

The distance defined in Definition 4 satisfies the above constraints.

2.5. Universal gravitation formula

The law of universal gravitation was published by Newton in 1687 in The Mathematical Principles of Natural Philosophy. The law of universal gravitation is expressed as follows: Any two objects in nature are attracted to each other, and the magnitude of gravity is proportional to the mass product of the two objects, and inversely proportional to the square of their distance.
The universal gravitation formula is as follows

$$F = G\frac{M_1 M_2}{R^2} \tag{7}$$

Where F is the magnitude of the gravitational force that the object is subjected to, G is the universal gravitational constant, M_1 and M_2 are the masses that attract each other, and R is the distance between the two objects.

3. Proposed formula of evidence universal gravitation

In this section, we present the evidence universal gravitation formula and introduce what it means. In addition, we will detail the process of generating evidence quality using our proposed ETEG method.

3.1. Propose a new evidence universal gravitation formula

Suppose m_1 and m_2 represent two separate independent and different evidences from the sensor, and Θ represents the identification framework.

The evidence universal gravitation formula is defined as follows.

$$F_{BPA} = G_e \frac{M_{m_1} M_{m_2}}{d^2} \tag{8}$$

Where, the value of G_e is defined as follows

$$G_e = 10^{-\delta|\Theta|} \tag{9}$$

The restrictions for δ are as follows

$$0 \leqslant \delta \leqslant 1$$

In the above formulas, M_{m_1} indicates the quality of the evidence m_1, and M_{m_2} indicates the quality of the evidence m_2. d represents the distance between evidence from two independent sources. G_e represents the evidence gravitation parameter used to distinguish between different identification frameworks. δ is an adjustable amount and we constrain it to be between 0 and 1. When the universal gravitational formula is applied to the same system. The value can be adjusted within the specified range according to the evidence universal gravitation display effect. It is important to note that in a deterministic system, the value of δ in all the evidence universal gravitation formulas must be consistent.

The gravitational evidence F_{BPA} represents the gravitation between two different evidences m_1 and m_2. It can be seen from Eq.8 that the evidence universal gravitation is proportional to the product of two different evidences qualities and inversely proportional to the distance between the two evidences. In a system in which the evidence universal gravitation operates, the evidence gravitation parameter G_e is directly proportional to the evidence universal gravitation, which is related to the number of identification frame elements.

Some of the properties of gravitational evidence are discussed later. In the following section, we will present the algorithm for generating evidence quality.

3.2. Proposed the quality of evidence generated by the ETEG method

In order to determine the quality of each piece of evidence, we propose the ETEG method. In this section, we describe it with a model.

Assumption. *The identification framework* Θ=$\{A_1, A_2, A_3\}$*, and there are two different pieces of evidence on the identification framework* Θ*. On the identification framework, there are two sets of BPAs, and the values of BPA are defined as follows:*

$$m_1 : m_1(A_1) = B_{1m_1}, m_1(A_2) = B_{2m_1}, m_1(A_3) = B_{3m_1}$$

$$m_2 : m_2(A_1) = B_{1m_2}, m_2(A_2) = B_{2m_2}, m_2(A_3) = B_{3m_2}$$

Step 1: Determine the assigned value for each BPA.

We will sort the elements on the identification framework in the order given in the assumption. For any proposition in two pieces of evidence, if an element in the frame is identified, we mark it as 1 in the given order, and the element that does not appear as 0.

Therefore, each BPA in the evidence m_1 can be expressed as follows:

$$m_1(A_1) \rightarrow 100, m_1(A_2) \rightarrow 010, m_1(A_3) \rightarrow 001$$

Similarly, each BPA in m_2 can also be expressed as follows:

$$m_1(A_1) \rightarrow 100, m_1(A_2) \rightarrow 010, m_1(A_3) \rightarrow 001$$

Step 2: Convert all BPA values of each piece of evidence in step 2 from binary to corresponding decimal number.

The transformation result of m_1 is shown below:

$$m_1(A_1) \rightarrow 4, m_1(A_2) \rightarrow 2, m_1(A_3) \rightarrow 1$$

Similarly, the transformation result of m_2 is shown below:

$$m_1(A_1) \rightarrow 4, m_1(A_2) \rightarrow 2, m_1(A_3) \rightarrow 1$$

Step 3: Quality of evidence is generated.

The quality of the evidence m_1 is as follows

$$M_{m_1} = \frac{4 * B_{1m_1} + 2 * B_{2m_1} + 1 * B_{3m_1}}{n} = \frac{4 * B_{1m_1} + 2 * B_{2m_1} + 1 * B_{3m_1}}{3}$$

Similarly, the quality of the evidence m_2 is as follows

$$M_{m_1} = \frac{4 * B_{1m_2} + 2 * B_{2m_2} + 1 * B_{3m_2}}{n} = \frac{4 * B_{1m_2} + 2 * B_{2m_2} + 1 * B_{3m_2}}{3}$$

Where n is the number of supporting propositions in which the BPA value in the evidence is not 0.

3.3. The evidence universal gravitation satisfies several basic properties

In this section, we discuss and demonstrate some of the properties that the evidence universal gravitation formula satisfies.

Definition6.Assume m_1 and m_2 are two different pieces of evidence on the same identification frame Θ. Some of the properties they have are defined as follows.

PROPOSITION 1. ***Non-negative.***

Proof: From Eq.(5), Eq.(8) and Eq.(9) it can be known that since any BPA on the identification framework is positive, the evidence distance $d > 0$. From Eq.(9), we can see that $G_e > 0$. As can be seen from the previous introduction, the evidence quality generated by ETEG method is also non-negative. In conclusion, the evidence of universal gravitation F_{BPA} is non-negative.

PROPOSITION 2. ***Symmetry.***

Proof: By Definition 5, we can know that d(A, B) = d(B, A), that is, the evidence distance has symmetry. It can be seen from Eq.(8) that Evidence universal gravitation has nothing to do with the order of the quality of two different evidences. Therefore, the evidence universal gravitation formula also has symmetry.

PROPOSITION 3. ***Unbounded.***

Proof: From Eq.(8), we can see that the evidence universal gravitation F_{BPA} is proportional to the product of two different evidence qualities, and the number of identification frames is inversely proportional. From Definition 4, the evidence distance d is a measure of the difference in evidence, and its value is between 0 and 1. When the identification framework is infinitely large, the value of the evidence quality will increase exponentially from the generation of evidence quality. Even if the evidence gravitation parameter G_e weakens it, with the infinite expansion of the identification framework element, the gravitational value of the evidence will be close to infinity.

4. Numerical examples

In this section, we explain the characteristics of the evidence universal gravitation F_{BPA} by a numerical example.

Example1. Suppose there are 20 elements in the recognition framework Θ, such as $\Theta = \{1, 2, 3, \cdots, 20\}$. The BPA values of two different evidences m_1 and m_2 are defined as follows:

$$m_1 : m_1(2, 3, 4) = 0.05, m_1(7) = 0.05, m_1(\Theta) = 0.1, m_1(A) = 0.8$$
$$m_2 : m_2(1, 2, 3, 4, 5) = 1$$

Here, we define that A is not a constant set, it can be changed based on the identification framework. Its value increases by an order of magnitude from the beginning of 1 to the end of 20. Then, we can clearly see the evidence distance, the quality of evidence and the trend of the gravitational function of evidence. Among them, the change data is recorded in Table 1. The trend of the square of the evidence distance is shown in Figure 1. The evidence universal gravitation change trend is recorded in Figure 2.

Table 1: Evidence gravitational in example 1

A	G_e	M_{m_1}	M_{m_2}	d	F_{BPA}
$\{1\}$	10^{-10}	136,908.775	1,015,808	0.6175	22.5226
$\{1,2\}$	10^{-10}	189,337.575	1,015,808	0.4714	40.7982
$\{1,2,3\}$	10^{-10}	215,551.975	1,015,808	0.3255	67.2748
$\{1,2,3,4\}$	10^{-10}	228,659.175	1,015,808	0.1795	129.3848
$\{1,2,3,4,5\}$	10^{-10}	235,212.775	1,015,808	0.0175	1365.1000
$\{1,2,\cdots,6\}$	10^{-10}	238,489.575	1,015,808	0.1509	160.5915
$\{1,2,\cdots,7\}$	10^{-10}	240,127.975	1,015,808	0.2529	96.4475
$\{1,2,\cdots,8\}$	10^{-10}	240,947.175	1,015,808	0.3255	75.2007
$\{1,2,\cdots,9\}$	10^{-10}	241,356.775	1,015,808	0.3828	64.0488
$\{1,2,\cdots,10\}$	10^{-10}	241,561.575	1,015,808	0.4295	57.1251
$\{1,2,\cdots,11\}$	10^{-10}	241,663.975	1,015,808	0.4684	52.4087
$\{1,2,\cdots,12\}$	10^{-10}	241,715.175	1,015,808	0.5015	48.9557
$\{1,2,\cdots,13\}$	10^{-10}	241,740.775	1,015,808	0.5301	46.3212
$\{1,2,\cdots,14\}$	10^{-10}	241,753.575	1,015,808	0.5552	44.2339
$\{1,2,\cdots,15\}$	10^{-10}	241,759.975	1,015,808	0.5774	42.5288
$\{1,2,\cdots,16\}$	10^{-10}	241,763.175	1,015,808	0.5975	41.1001
$\{1,2,\cdots,17\}$	10^{-10}	241,764.775	1,015,808	0.6156	39.8940
$\{1,2,\cdots,18\}$	10^{-10}	241,765.575	1,015,808	0.6322	38.8475
$\{1,2,\cdots,19\}$	10^{-10}	241,765.975	1,015,808	0.6474	37.9356
$\{1,2,\cdots,20\}$	10^{-10}	241,766.175	1,015,808	0.6615	37.1283

[1] The value of adjustable amount δ is 1/2.

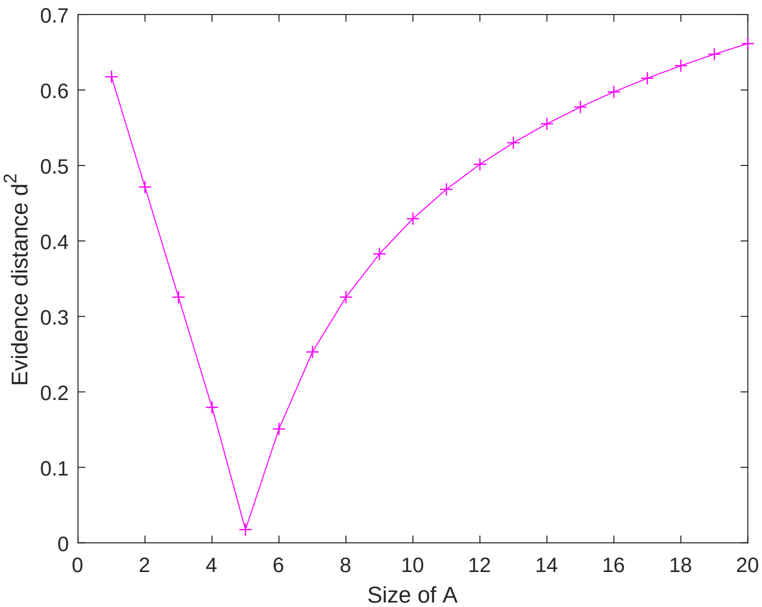

Figure 1: Variation trend of evidence distance d^2

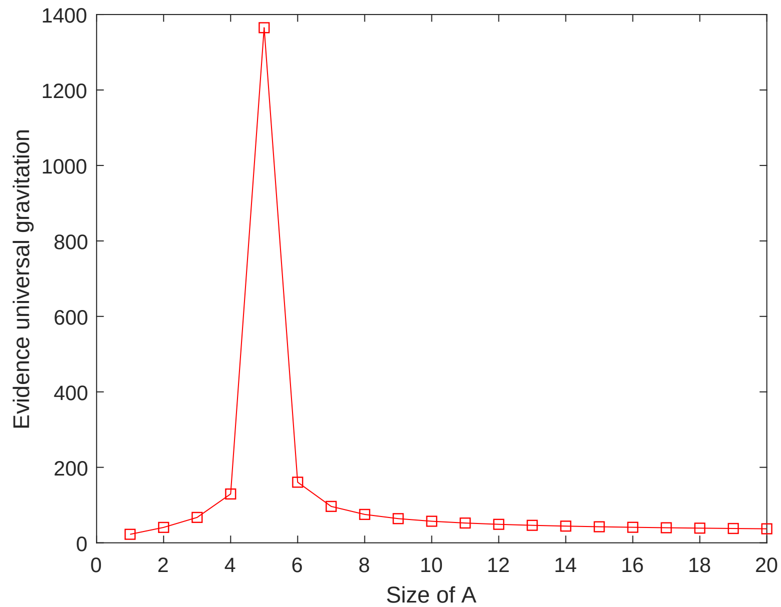

Figure 2: Variation trend of evidence universal gravitation

As can be seen from Table 1, as the number of elements in set A increases, the evidence quality of m_1 remains monotonically increasing. Since the mass of the evidence m_2 remains the same, the product of the mass of the evidence m_1 and m_2 also remains monotonically increasing.

Figure 1 and Figure 2 show that the square of the evidence distance has the opposite trend of the evidence universal gravitation. When set A approaches set $\{1, 2, 3, 4, 5\}$, the value of the evidence distance and the square of the evidence distance tend to be the lowest. Conversely, when the value of set A deviates from the set $\{1, 2, 3, 4, 5\}$, the value of the evidence universal gravitation increases.

In summary, As mentioned earlier, In a system in which the evidence universal gravitation operates, when the size of the identification framework is determined, the evidence universal gravitation is proportional to the quality of the evidence and inversely proportional to the square of the evidence distance.

5. Application in conflict management

In conflict management, how to describe the degree of similarity between evidence is a crucial issue. In this section, we use the evidence universal gravitation to describe the degree of similarity in evidence. The example in Example 1 is compared with the work done by the predecessors to show the applicability of the evidence gravitation formula in conflict management (the system in which the evidence universal gravitation is the same as in Example 1).

Table 2: Comparison of conflicts between different parameter values

A	k_r[5]	d_{BPA}	k	$CWAC$	DisSim	Pl_C	F^*_{BPA}
$\{1\}$	0.7348	0.7858	0.05	0.0393	0.3710	0.05	0.0225
$\{1,2\}$	0.5483	0.6866	0.05	0.0343	0.4855	0.05	0.0408
$\{1,2,3\}$	0.3690	0.5705	0.05	0.0285	0.3974	0.05	0.0673
$\{1,2,3,4\}$	0.1964	0.4237	0.05	0.0212	0.3644	0.05	0.1294
$\{1,2,3,4,5\}$	0.0094	0.1323	0.05	0.0066	0.3375	0.00	1.3651
$\{1,2,\cdots,6\}$	0.1639	0.3884	0.05	0.0195	0.4188	0.05	0.1606
$\{1,2,\cdots,7\}$	0.2808	0.5029	0.05	0.0251	0.6000	0.05	0.0965
$\{1,2,\cdots,8\}$	0.3637	0.5705	0.05	0.0285	0.6497	0.05	0.0752
$\{1,2,\cdots,9\}$	0.4288	0.6187	0.05	0.0309	0.6884	0.05	0.0640
$\{1,2,\cdots,10\}$	0.4770	0.6554	0.05	0.0328	0.7194	0.05	0.0571
$\{1,2,\cdots,11\}$	0.5202	0.6844	0.05	0.0342	0.7448	0.05	0.0524
$\{1,2,\cdots,12\}$	0.5565	0.7082	0.05	0.0354	0.7660	0.05	0.0490
$\{1,2,\cdots,13\}$	0.5872	0.7281	0.05	0.0364	0.7839	0.05	0.0463
$\{1,2,\cdots,14\}$	0.6137	0.7451	0.05	0.0372	0.7992	0.05	0.0442
$\{1,2,\cdots,15\}$	0.6367	0.7599	0.05	0.0380	0.8126	0.05	0.0425
$\{1,2,\cdots,16\}$	0.6569	0.7730	0.05	0.0386	0.8242	0.05	0.0411
$\{1,2,\cdots,17\}$	0.6748	0.7846	0.05	0.0392	0.8345	0.05	0.0399
$\{1,2,\cdots,18\}$	0.6907	0.7951	0.05	0.0397	0.8438	0.05	0.0388
$\{1,2,\cdots,19\}$	0.7050	0.8046	0.05	0.0402	0.8519	0.05	0.0379
$\{1,2,\cdots,20\}$	0.7178	0.8133	0.05	0.0407	0.8389	0.05	0.0371

[1] The value of adjustable amount δ is $1/2$.

[2] In order to show the evidence of universal gravitation more intuitively, we reduce it, which does not affect its properties. F^*_{BPA} is the value of F_{BPA} after processing.

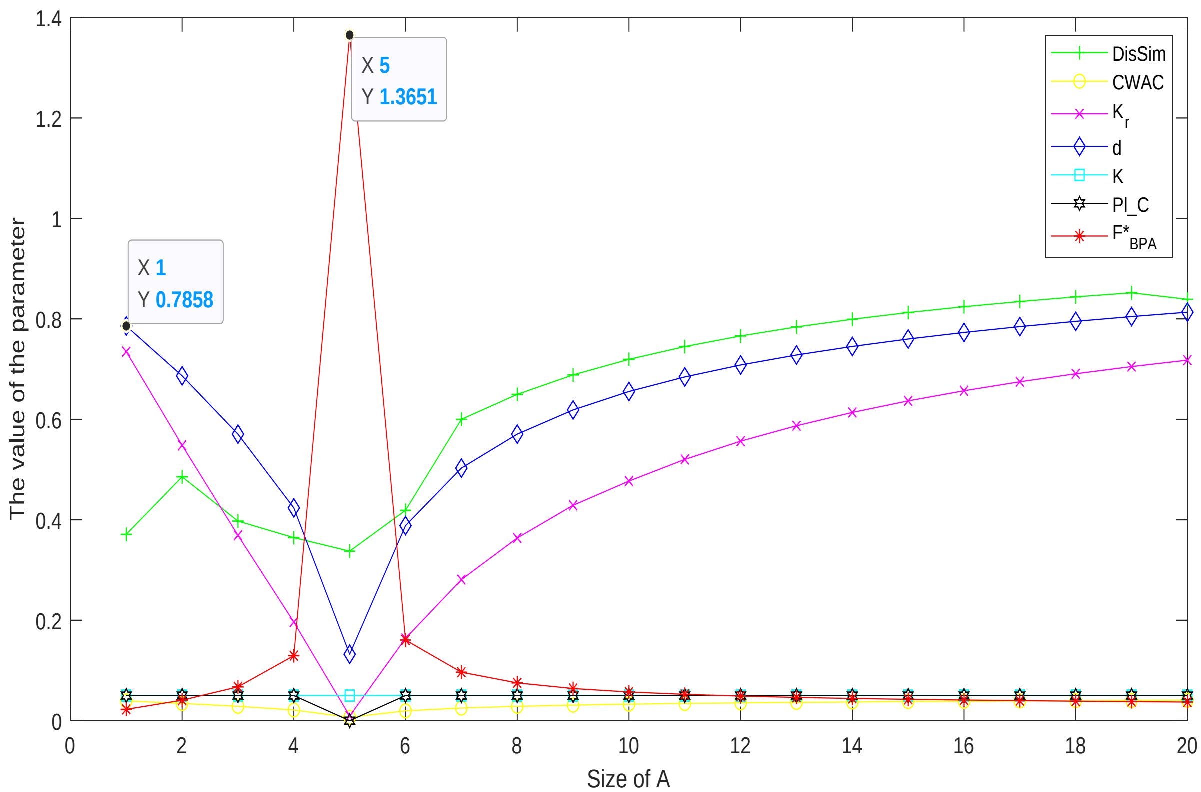

Figure 3: Comparison of conflicts between different parameter values

As shown in Figure 3, the methods of d and K_r show a consistent trend for conflict measurement. When the size of set A approaches 5, the measurement value of conflict is the lowest. When the size of set A deviates from 5, the conflicting measurements increase. DisSim's method is not monotonic until the size of set A is 5, so it is not an effective way to measure conflicts. The conflict value of CWAC method is always kept at a low level, and it is insensitive to the conflict change. The classical conflict coefficient k is always maintained at 0.5, which cannot distinguish the variation of evidence m_1. The method of Pl_C takes A value of 0 when set A is 5, which is unacceptable[5]. The change curve of the evidence universal gravitation shows the opposite trend to d and K_r, just as the evidence universal gravitation formula expresses, its value is inversely proportional to the square of the distance of the evidence.

In summary, it can be seen that the universal gravitation of evidence is a good measure of conflict of evidence.

6. Conclusion

In this paper, we present a new concept of the evidence universal gravitation in the evidence theory system. Like the law of universal gravitation in the natural sciences, we believe that the evidence provided by the sensor also has a force, which we call the evidence gravitation. In the formula of universal gravitation of evidence, we define the evidence gravitation parameter to distinguish different recognition frameworks, and use ETEG method to generate evidence quality.

The expression of the evidence universal gravitation formula is as follows: In a system in which the evidence universal gravitation operates and under the identified identification framework, the gravitation of evidence is proportional to the product of the masses of two different kinds of evidence and inversely proportional to the square of the evidence distance. In addition, it satisfies some basic properties. The work done in this paper is the cornerstone of our follow-up work. We consider the introduction of evidence speed and evidence acceleration in the later work for the more complete establishment of evidence gravitation theory. In the future, we plan to apply the theory of evidence universal gravitation to evidence anti-monitoring, interference interception and measurement of transmission media.

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Conflict of interest

The authors declare that they have no conflict of interest.

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