Logical Composition up to Special Relativity Theory

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Abstract

Science theory is composed from definitions, logic and facts. Facts are real and not deniable. And logic is fact. So we need to choose most appropriate expression on logical fact. A composition from definitions, facts and logic to Special Relativity Theory is tried here.

## 1. Introduction

Report [1] has developed already actual logic from definitions. This report is another version of it adding definitions, changing and sorting out logic expressions.

Here definitions of systems, words, marks and others are inherited from [1] as long as these are not redefined.

## 2. Starting point

Starting point was Michelson-Morley experiment. Overview of it is following.

An origin of light is in system A. System B is leaving with velocity $v$ from system A.

An equipment described on fig.1 is in system B.

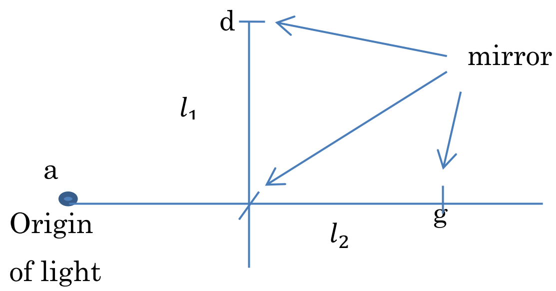


Fig. 1

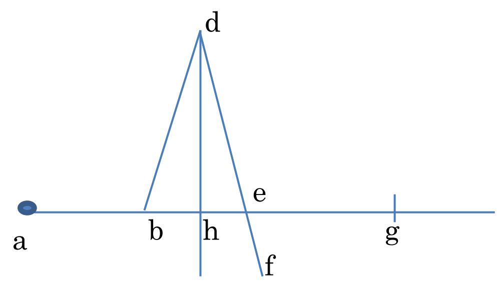


Fig. 2

Light from origin go on following two routes C,D on fig.2.

Route C:a,b,d,e,f

Route D:a,b,g,e,f

Each route has following unique route.

C:b,d,e

D:b,g,e

Following length is calculated.

$$\overline{\mathrm{dh}}:\overline{bh}:\overline{bd} = \sqrt{c^2-v^2}:v:c$$

Then

$$\overline{\mathrm{dh}} = l_1,\ \overline{bh} = \frac{vl_1}{\sqrt{c^2-v^2}},\ \overline{bd} = \frac{cl_1}{\sqrt{c^2-v^2}}$$

Same as above,

$$\overline{de} = \frac{cl_1}{\sqrt{c^2-v^2}},\ \overline{he} = \frac{vl_1}{\sqrt{c^2-v^2}}$$

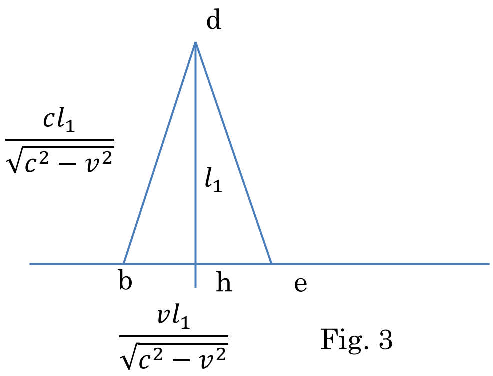


Fig. 3

For each route from b to e, required time for light is calculated. At this steps, required time is same for system A and system B.

For route b,d,e: (view of A system)

Moved distance for $\overline{bd}$; $\frac{cl_1}{\sqrt{c^2-v^2}}$

Required time for $\overline{bd}$; $\frac{l_1}{\sqrt{c^2-v^2}}$

Required time for $\overline{de}$; $\frac{l_1}{\sqrt{c^2-v^2}}$

Then required time for $\overline{bde}$; $\frac{2l_1}{\sqrt{c^2-v^2}}$ (1)

For route b,g,e: (view of B system)

Moved distance for $\overline{bg}$: $l_2$

Speed of light for this route: $c-v$

Required time for $\overline{bg}$; $\frac{l_2}{c-v}$

Moved distance for $\overline{ge}$: $l_2$

Speed of light for this route: $c+v$

Required time for $\overline{ge}$; $\frac{l_2}{c+v}$

Required time for $\overline{bge}$: $\frac{l_2}{c-v}+\frac{l_2}{c+v}=\frac{2cl_2}{c^2-v^2}$ (2)

On (1),(2), required time for the two routes of C and D are different. So when the position of d and g are changed, timing of interference at f should be moving during the changing process.

But such result has not been observed on the experiment. This indicates the calculation to (1),(2) should not reflect some rules of nature.

3. Recalculation with new definition

Definitions

When time t is passed, time moves toward time direction also toward space direction with speed c. (3)

Origin of moving time in space recognizes passed time distance at the moving time reached point by moved distance. (4)

Applying these definitions, required time for light to go along two routes from b to e on the view of system B.(fig.2) is recalculated.

Moved distance for $\overline{bd}$ (on the view of system A); $\frac{cl_1}{\sqrt{c^2-v^2}}$

Required time for $\overline{bd}$ (on the view of system A); $\frac{l_1}{\sqrt{c^2-v^2}}$

Moved distance of system B while this timing; $\frac{vl_1}{\sqrt{c^2-v^2}}$

On above, moved distance of light for system B; $\sqrt{(\frac{cl_1}{\sqrt{c^2-v^2}})^2-\left(\frac{vl_1}{\sqrt{c^2-v^2}}\right)^2}=l_1$

Moved distance of time is same as light: $l_1$

Passed time: $\frac{l_1}{c}$

Velocity of light; c

Same as $\overline{bd}$, moved distance of light and time for $\overline{de}$; $l_1$

Passed time; $\frac{l_1}{c}$

Velocity of light; c

Then passed time for $\overline{bde}$; $\frac{2l_1}{c}$ (5)

Moved distance for $\overline{bg}$ (on the view of system A); $\frac{cl_2}{c-v}$

Required time for $\overline{bg}$ (on the view of system A); $\frac{l_2}{c-v}$

Moved distance of system B while this timing; $\frac{vl_2}{c-v}$

On above, moved distance of light for system B; $\frac{cl_2}{c-v} - \frac{vl_2}{c-v} = l_2$

Moved distance of time is same as light: $l_2$

Passed time: $\frac{l_2}{c}$

Velocity of light; c

Same as $\overline{bg}$, moved distance of light and time for $\overline{ge}$ (on the view of system A); $\frac{cl_2}{c+v}$

Required time for $\overline{ge}$ (on the view of system A); $\frac{l_2}{c+v}$

Moved distance system B while this timing; $\frac{vl_2}{c+v}$

On above, moved distance of light for system B; $\frac{cl_2}{c+v} + \frac{vl_2}{c+v} = l_2$

Moved distance of time is same as light: $l_2$

Passed time: $\frac{l_2}{c}$

Velocity of light; c

Then passed time for $\overline{bge}$; $\frac{2l_2}{c}$ (6)

On (3),

moved distance of light=moved distance of time

passed time of moved time=(moved distance of time)/c

Then

passed time of moved light=(moved distance of light)/c

This means passed time of moved light depend only on its moved distance not on its relative velocity for every inertial system's view.

This also means velocity of light is c for all inertia system's view.

(5) and (6) shows this fact and the result of Michelson-Morley experiment.

4. Lorentz transformation

Objective

Initially origin of $S'$ system (0,0) is at origin of S system (0,0), and $S'$ system is moving with velocity $v$ relatively to S system. [1]

On this situation objective is,

To describe $S'$ system frame of reference on S system frame of reference.

To get formula of relation between S system indication and $S'$ system indication for a time-space point.

Premise

Frame of reference transformation can be done only when about all related dimension, unit for value is same. (7)

Time axis of $S'$ system

According to time $t$ passing, time moves ct along time axis of S system. While this timing, space zero point of $S'$ system moves $x$ along space axis. Then its track could be

$$x = \mathrm{a}ct$$

Possible point it has is $(ct, vt)$. Then

$$vt = \mathrm{a}ct$$

$$a = \frac{v}{c}$$

So its track is

$$x = \frac{v}{c}ct \tag{8}$$

This is space zero line of $S'$ system. Then this is time axis of $S'$ system.

If a space point $x$ of S system is space point $x'$ of $\mathrm{S}'$ system, its relation is

$$x' = \mathrm{x} - \frac{v}{c}ct$$

$x' = \mathrm{x} - vt$ (9)

(Fig. 4)

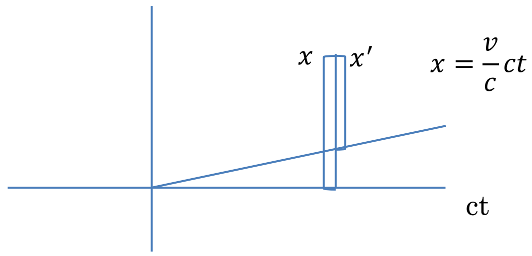


Fig. 4

<u>Space axis of $S'$ system</u>

When moving time reached at point P whose distance is $x$ from origin, moved time distance is $x$ in space for S system origin. Then S system origin recognizes passed time distance $x$ at point P based on (4),

While this timing, $S'$ system origin moves $\frac{v}{c}x$ in space. Then moved distance of moving time for $S'$ system is

$x - \frac{v}{c}x$ (Fig.5)

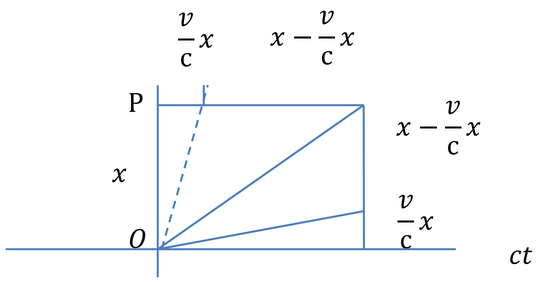


Fig.5

Then $S'$ system origin recognizes passed time distance $x - \frac{v}{c}x$ at point P based on (4),

So $S'$ system frame of reference is advanced $\frac{v}{c}x$ about time dimension than S system frame of reference

So $S'$ system time zero is on the following line

$\mathrm{ct} = \frac{v}{c}x$

$x = \frac{c}{v}ct$ (10)

This is time zero line and space axis of $S'$ system.

Then if a time point $ct$ of S system is time point $ct'$ of S′ system, its relation is

$ct' = \mathrm{ct} - \frac{v}{c}x$ (11)

(Fig. 6)

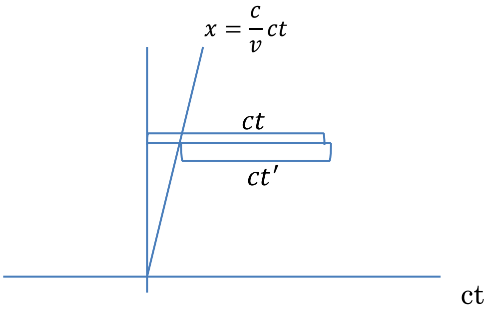


Fig. 6

<u>Scaling</u>

To satisfy a fact got on the experiment that light speed constancy for every inertia systems, (9),(11) are used. As elements to be determined, $\alpha, \beta. \gamma$ are applied. [2]

$$x' = \alpha(\mathrm{x} - vt) \tag{12}$$

$$t' = \beta \mathrm{t} + \gamma x \tag{13}$$

Using these, equations of light speed constancy can be solved.

As result, we get following $\alpha, \beta. \gamma$ determined.

$$\alpha = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{14}$$

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

$$\gamma = \frac{-v}{c^2\sqrt{1-\frac{v^2}{c^2}}} \tag{16}$$

Comparing (11) and (16), $\frac{-v}{c^2}$ is multiplier of $x$ included in (11).

Remained

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{17}$$

is same as $\alpha, \beta$. This is same value as scaling factor derived in [1] considering $S'$ system is oblique system. Applying $\alpha, \beta, \gamma$, (12),(13) become Lorentz transformation formula.

5. Background of definitions

We can accept definitions unconditionally if these have no contradiction with any facts, But these definition should have meaning why these can be definition. Here we approach to such meanings.

About definition (3)

Space is expanding and newly expanded space part has same time as existed space have. This means newly expanded space have no own time counted from zero.

On this fact, we can recognize time moved in space from neighbor space.

About definition (4)

When moving time reached to $x$ toward space direction, time passed $\frac{x}{c}$ or time distance $x$ at $x$. This is because why moving time velocity toward to time and toward to space is same $c$.

## 6. Conclusion

Based on (3),(4), result of Michelson-Morley experiment can be explained. Also (3),(4) introduce light speed constancy for all inertia systems as the result of calculation. Based on (3),(4) and using condition light speed constancy, Lorentz transformation formula can be got. So the definitions could reflect rule of nature.

Reference

[1] viXra:1611.0077

[2] Peter Gabriel Bergmann, *Introduction to the Theory of Relativity*, (Dover Publication, INC 1976),p19