Riemann Hypothesis

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1 Abstract

The Proof involves Analytic Continuation of the Riemann Zeta function expressed as a functional equation.

Further as all the conditions of Rolle's Theorem are satisfied, specific calculations produce the result.

Till date millions of non trivial zeroes of the Riemann Zeta function has been found on the critical line.

THE RIEMANN HYPOTHESIS (1859): states that the Riemann Zeta Function,

$$\zeta(s) =$$

$$\sum_{n=1}^{\infty} 1/n^s$$

$$has\ non\ trivial\ zeroes\ for\ Re(s)\ = 1/2.$$

2 Proof:

The Analytic Continuity of Riemann Zeta-function over

$$0 < Re(s) < 1$$

is defined as,

$$\zeta^*(s)$$

$=$ (s(s-1)$\pi^{-s/2}\Gamma(s/2)\zeta(s))/2$.

$\zeta^*(0)= 0$

$\zeta^*(1)= 0$

$thus,\ \ \zeta^*(0)=\zeta^*(1)$

$Also, \zeta^*\ being\ Analytic\ is\ Continuous\ on\ [0,1]\ and\ Differentiable\ on\ (0,1)$

$So,\ By\ Rolle's\ Theorem\ ,there\ exists\ a\ s_0 \in (0,1)\ such\ that,$

$\mid \frac{d}{ds}\ \ \zeta^*(s_0)|= 0$

$\mid \frac{2s_0-1}{2}\pi^{-s_0/2}\Gamma(s_0/2)\zeta(s_0 \mid = 0$

$\mid (2s_0-1) \mid\mid \zeta(s_0) \mid = 0$

$\mid (2s_0-1) \mid = 0\ \ or \mid \zeta(s_0) \mid = 0$

$(2\sigma_0 - 1)^2 = 0, \sigma = Re(s)$

$implies,\ \ \ \sigma_0 = 1/2 or \zeta(1/2) = 0$

$But,\ \ \ \zeta(1/2) = \sum_{n=1}^{\infty} 1/n^{1/2} = \infty \neq 0.$

$Therefore,\ \ \ \sigma = 1/2.$

$So,\ Real\ part\ = 1/2.$

$Let,\ imaginary\ part\ = t.$
$so,\ we\ have\ s = 1/2 + it$

$then,$

$\mid 2(1/2+it) - 1\zeta(1/2+it) \mid = 0$

$\zeta(1/2+it) = 0$

$Hence\ all\ the\ zeroes\ lie\ on\ the\ line\ x = 1/2\ and\ are\ infinitely\ many.$

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