Derive the Huygens Principle through the mutual energy flow theorem

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Abstract

Absorber theory published in 1945 and 1949 by Wheeler and Feynman. In the electromagnetic field theory, W. J. Welch introduced the reciprocity theorem in 1960. V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula in 1963. In early of 1987 Shuang-ren Zhao (this author) introduced the mutual energy theorem in frequency domain. In the end of 1987. Adrianus T. de Hoop introduced the time-domain cross-correlation reciprocity theorem. All these theories can be seen as a same theorem in different domain: Fourier domain or in time domain. After 30 years silence on this topic, finally, this author has introduced the mutual energy principle and self-energy principle which updated the Maxwell's electromagnetic field theory and Schrödinger's quantum mechanics. According to the theory of mutual energy principle, the energy of all particles are transferred through the mutual energy flows. The mutual energy flow are inner product of the retarded wave and the advanced wave. The mutual energy flow satisfies the mutual energy flow theorem. The retarded wave is the action the emitter gives to the absorber. The advanced wave is the reaction the absorber gives to the emitter. In this article this author will derive the Huygens principle from the mutual energy flow theorem. The bra, ket and the unit operator of the quantum mechanics will be applied to the inner space defined on 2D surface instead of the 3D volume.

Keywords: Poynting; Maxwell; Schrödinger; Self-energy; Mutual energy; Mutual energy flow; Reciprocity theorem; Radiation; Newton's third law; Action; Reaction; Advanced wave; Photon; Electron; Wave and particle duality; Huygens Principle;

1 Introduction

1.1 Action at a distance and the absorber theory

The theory about advanced wave became most interesting work for this author. This author noticed the absorber theory of Wheeler and Feynman[1, 2]. The absorber theory is based on the action-at-a-distance [11, 13, 6]. In the absorber theory, any current source will send half retarded wave and half advanced wave.

For a source we only notice the source sends the retarded wave, we did not notice it also sends the advanced wave. Some one will argue that if in the same time the source sends the retarded wave, it also sends the advanced wave, the source loss the energy from the retarded wave and acquire the energy from the advanced wave, and hence, it doesn't send any energy out. However we all know that the source can send the energy out. This means the absorber theory also has some thing which is not self-consistence. This is also the reason that the absorber theory has not been widely accept. But any way the absorber theory accept the advanced wave as a real wave instead of some virtual wave. This author is inspired by this a lot. The transactional interpretation of John Cramer has introduced the advance wave to the whole quantum mechanics [3, 4]. This author has accepted the advanced wave as a real wave instead of a virtual wave. This all tell this author for sure the mutual energy theorem is a real energy theorem. Stephenson offered a good tutorial about the advanced wave [12].

1.2 The mutual energy theorems

W.J. Welch introduced a reciprocity theorem in arbitrary time-domain [14] in 1960 (this will be referred as Welch's reciprocity theorem in this article). In 1963 V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula[10], (this will be referred as Rumsey's reciprocity theorem). In early of 1987 Shuang-ren Zhao (this author) has introduced the concept of mutual energy and the mutual energy theorem [7] (this will be referred as Zhao's mutual energy theorem). In the end of 1987 Adrianus T. de Hoop introduced the time domain cross-correlation reciprocity theorem[5], (this will be referred as Hoop's reciprocity theorem). Welch's reciprocity theorem is a special case of the Hoop's reciprocity theorem.

Assume there are two current sources $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. $\boldsymbol{J}_1$ is the current of a transmitting antenna. $\boldsymbol{J}_2$ is the current of a receiving antenna. The field of $\boldsymbol{J}_1$ is described as $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$. The field of the current $\boldsymbol{J}_2$ is $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$. Assume $\boldsymbol{J}_2$ has a some distance with $\boldsymbol{J}_1$. Hoop's reciprocity theorem can be written as,

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{J}_1(t)\cdot\boldsymbol{E}_2(t+\tau)dV = \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t+\tau)dV \qquad (1)$$

if $\tau=0$, we have,

$$-\int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{J}_1(t)\cdot\boldsymbol{E}_2(t)dV = \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dV \qquad (2)$$

This is Welch's reciprocity theorem. The Fourier transform of Hoop's reciprocity theorem can be written as,

$$-\iiint_{V_1} \boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2(\omega)^* dV = \iiint_{V_2} \boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)^* dV \tag{3}$$

Where "*" is the complex conjugate operator. In this article if the variable t is applied in a formula, it is in time-domain. If ω is applied, it is in Fourier frequency domain. This is the Rumsey's reciprocity theorem and is also Zhao's mutual energy theorem. Hence this 4 theorems can be seen as one theorem in different domain: time-domain and Fourier domain.

Shuang-ren Zhao noticed this theorem is an energy theorem, hence the two fields in the formula must all physic waves. The other author referred the theorem as reciprocity theorem, as a reciprocity theorem, it can be a mathematical theorem. One of the two fields can be virtual instead of real. If it is virtual even it is advance wave, that can be easily accepted. If the two fields are all real as the mutual energy theorem required, we must first accept the advanced wave. The advanced wave are not obey the traditional causality consideration.

2 Mutual energy principle

2.1 Mutual energy principle

If there are N charges, the mutual energy principle can be widen to

$$\begin{aligned}
&-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}\,d\Gamma dt\\
&=\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt\\
&\quad+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt
\end{aligned}\tag{4}$$

If it is two charges, the above formula becomes,

$$\begin{aligned}
&-\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}\,d\Gamma dt\\
&=\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_1\frac{\partial}{\partial t}\boldsymbol{E}_2+\epsilon\boldsymbol{E}_2\frac{\partial}{\partial t}\boldsymbol{E}_1+\mu\boldsymbol{H}_1\frac{\partial}{\partial t}\boldsymbol{H}_2+\mu\boldsymbol{H}_2\frac{\partial}{\partial t}\boldsymbol{H}_1)dVdt\\
&\quad+\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dVdt
\end{aligned}\tag{5}$$

The above formulas are referred as the mutual energy principle. The mutual energy principle can be derived from Maxwell equations,

$$\begin{cases} -\frac{\partial}{\partial t}(\epsilon \boldsymbol{E})+\nabla\times\boldsymbol{H}=\boldsymbol{J} \\ -\nabla\times\boldsymbol{E}-\frac{\partial}{\partial t}(\mu\boldsymbol{H})=0 \end{cases} \tag{6}$$

Maxwell equations can also be derived from the Mutual energy principle. However the mutual energy principle is not equivalent to the Maxwell equations. Because the solution of the mutual energy principle require there must be two wave solution which satisfy the Maxwell equations. The two waves one must be retarded wave, another must be advanced wave. The two wave must be synchronized. One wave satisfy Maxwell equations is not a solution of the mutual energy principle, but it is a solution of the Maxwell equations.

$Solution\ set\ Maxwell\ equations \supset Solutions\ set\ Mutual\ energy\ principle$

One wave satisfy the Maxwell equation is a probability wave. That still not a real physics wave. This author has introduced also the self-energy principle, that principle says that there is a time-reversal waves which can cancel the energy of the original waves [8]. The self-energy principle will role out some solution of Maxwell equations, the solution with the retarded wave alone and the solution with the advanced wave alone. This kind solution of Maxwell equation is not a real physic solution. The retarded wave or the advance wave alone are canceled by the corresponding time-reversal waves. After the cancellation the waves do not carry any energy. The mutual energy flow is built from the retarded wave and the advanced wave which doesn't cancel. The energy is transferred by the mutual energy flow. In the electromagnetic field theory, the mutual energy flow is the photon. The photon's energy is transferred by the mutual energy flow. The mutual energy flow satisfied the mutual energy flow theorem which can be derived from the mutual energy principle[8]. The following is the mutual energy flow theorem,

$$-\int_{t=-\infty}^{\infty}\iiint_{V}\boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dVdt$$

$$=\int_{t=-\infty}^{\infty}\oiint_{\Gamma}(E_1(t)\times H_2(t)+E_2(t)\times H_1(t))\cdot\hat{n}d\Gamma dt$$

$$=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dVdt \tag{7}$$

In the above formula, $-\int_{t=-\infty}^{\infty}\iiint_V \boldsymbol{E}_2(t)\cdot\boldsymbol{J}_1(t)dVdt$ is the the energy of the source $\boldsymbol{J}_1(t)$ offers to the system, the system include a source $\boldsymbol{J}_1(t)$ and a sink $\boldsymbol{J}_2(t)$. $\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dVdt$ is the received energy by the sink $\boldsymbol{J}_2(t)$. $S_{12}=(E_1(t)\times H_2(t)+E_2(t)\times H_1(t)$ is the energy flow intensity. $Q=\oiint_\Gamma(E_1(t)\times H_2(t)+E_2(t)\times H_1(t))\cdot\hat{n}d\Gamma$is the energy flow go through the surface

Γ. $Eenergy = \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$ is the whole energy go through the surface Γ. Here Γ is any surface between the volume V_1 and V_2. Source $\boldsymbol{J}_1(t)$ is inside the volume V_1. The sink $\boldsymbol{J}_2(t)$ is inside the volume V_2.

This theorem tell us, the source sends the retarded wave $\boldsymbol{E}_1$ to the sink, cause the current $\boldsymbol{J}_2$ on the sink. The energy received by the sink is $\int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt$. This energy is equal to the sucked energy on the source by the advanced wave produced by the current $\boldsymbol{J}_2$ of the sink, which is $-\int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$. The energy is transferred in the space through the mutual energy flow $Eenergy = \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$.

This concept has been widened to whole quantum mechanics. Hence all particles are mutual energy flows.

The sink give a reaction force to the source, this force looks like the recoil force to the emitter. The source will offers the sink an action force, this will looks like the photon offers the sink some momentum. The action force is retarded and the reaction force is advanced.

2.2 Inner product of electromagnetic fields

Shuang-ren Zhao has define the inner product for electromagnetic fields[7]. Assume $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ we have inner product,

$$(\xi_1, \xi_2) = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{8}$$

Γ is closed surface. Shuang-ren find that this formula satisfy inner product 3 conditions[7],

(I) Conjugate symmetry:

$$(\xi_1, \xi_2) = (\xi_2, \xi_1)^* \tag{9}$$

(II) Linearity:

$$(a\xi_1' + b\xi"_1, \xi_2) = a(\xi_1', \xi_2) + b(\xi"_1, \xi_2) \tag{10}$$

(III) Positive-definiteness:

$$(\xi, \xi) > 0 \tag{11}$$

$$(\xi, \xi) = 0 \Rightarrow \xi = 0 \tag{12}$$

"$\Rightarrow$" means "can derive",

$$-(J_1, \xi_2)_{V_1} = (\xi_1, \xi_2) = (\xi_1, J_2)_{V_2} \tag{13}$$

where

$$(J_1, \xi_2)_{V_1} = \iiint_{V} \boldsymbol{E}_2(\omega) \cdot \boldsymbol{J}_1(\omega) dV \tag{14}$$

$$(\xi_1, J_2)_{V_2} = \iiint\limits_{V_2} \boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2(\omega) dV \tag{15}$$

$$(\xi_1, \xi_2) = (\xi_1, \xi_2)_\Gamma = \oint\!\!\!\oint\limits_{\Gamma} (\boldsymbol{E}_1(\omega) \times \boldsymbol{H}_2^*(\omega) + \boldsymbol{E}_2^*(\omega) \times \boldsymbol{H}_1(\omega)) \cdot \hat{n} d\Gamma \tag{16}$$

It is clear that the integral at V_1 and V_2 Eq.(14,15) are also the inner product. Whether or not Eq.(16) is a inner product is not clear, but shuang-ren Zhao has discovered that is a inner product. It is found that Γ does not need to be written, since it can be proved that Γ can be taken at any surface between the 2 volumes: V_1 and V_2.

If we work in time-domain

$$(J_1, \xi_2)_{V_1} = \int\limits_{t=-\infty}^{\infty} dt \iiint\limits_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV \tag{17}$$

$$(\xi_1, J_2)_{V_2} = \int\limits_{t=-\infty}^{\infty} dt \iiint\limits_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV \tag{18}$$

$$(\xi_1, \xi_2) = (\xi_1, \xi_2)_\Gamma$$

$$= \int\limits_{t=-\infty}^{\infty} dt \oint\!\!\!\oint\limits_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma \tag{19}$$

2.3 Inner product of quantum mechanics

Shuang-ren Zhao found that the mutual energy flow theorem can be also written as [7, 15]

The mutual energy flow theorem is also suitable to quantum mechanics, where the wave satisfies Schrödinger equation instead of Maxwell equations.

In quantum mechanics, the 3d volume inner product is defined as,

$$(\psi, \phi) = \int \psi^*(x) \cdot \phi(x) dV \tag{20}$$

Assume a is the source, b is the sink. The particle's energy intensity can be written as,

$$\rho_{ab} = \Psi_a \Psi_b^* \tag{21}$$

Where a is the source place, b is the sink place, the particle is the energy flow from the source to the sink. Here the energy flow is also referred as mutual energy flow.

The particle's energy flow intensity can be written as,

$$\boldsymbol{J}_{ab} = \frac{\hbar}{2\mu i} (\nabla \Psi_a \Psi_b^* - \Psi_a \nabla \Psi_b^*) \tag{22}$$

The energy flow can be used to define the surface inner product as,

$$(\Psi_b, \Psi_a)_\Gamma \equiv \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma dt$$

$$= \int\limits_{-\infty}^{\infty} \oiint\limits_{\Gamma} \frac{\hbar}{2\mu i} (\nabla \Psi_a \Psi_b^* - \Psi_a \nabla \Psi_b^*) \cdot \hat{n} d\Gamma dt \tag{23}$$

In quantum mechanics the mutual energy flow theorem can be written as,

$$(\Psi_b, \Psi_a)_b = (\Psi_b, \Psi_a)_\Gamma = (\Psi_b, \Psi_a)_a \tag{24}$$

where a is the source and also the surface enclosed the a. b is the sink and also the surface enclosed the sink. $\Gamma$is any surface between a and b.

3 Huygens principle

3.1 Huygens sources

We have know that there is the Newton's third law that says that the action and the reaction are equal and are in opposite direction.

$$\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) \cdot \hat{n} = \boldsymbol{E}_1(t) \cdot \boldsymbol{H}_2(t) \times \hat{n} = \boldsymbol{E}_1(t) \cdot J_{h2}(t) \tag{25}$$

$$\boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t) \cdot \hat{n} = \hat{n} \cdot \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t) = \hat{n} \times \boldsymbol{E}_2(t) \cdot \boldsymbol{H}_1(t) = K_{h2} \cdot \boldsymbol{H}_1(t) \tag{26}$$

In the above we have defined the Huygens source,

$$\sigma_2 = \begin{cases} J_{h2}(t) = \boldsymbol{H}_2(t) \times \hat{n} \\ K_{h2}(t) = \hat{n} \times \boldsymbol{E}_2(t) \end{cases} \tag{27}$$

The inner product Eq(19) becomes,

$$(\xi_1, \xi_2) = \int\limits_{t=-\infty}^{\infty} dt \oiint\limits_{\Gamma} (\boldsymbol{E}_1(t) \cdot J_{h2}(t) + \boldsymbol{H}_1(t) \cdot \boldsymbol{J}_{h2}(t)) d\Gamma \tag{28}$$

$$\hat{n} \cdot \boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) = \hat{n} \times \boldsymbol{E}_1(t) \cdot \boldsymbol{H}_2(t) = K_{h1}(t) \cdot \boldsymbol{H}_2(t) \tag{29}$$

$$\boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t) \cdot \hat{n} = \boldsymbol{E}_2(t) \cdot \boldsymbol{H}_1(t) \times \hat{n} = \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_{h1} \tag{30}$$

The Huygens source is,

$$\sigma_1 = \begin{cases} J_{h1}(t) = \boldsymbol{H}_1(t) \times \hat{n} \\ K_{h1}(t) = \hat{n} \times \boldsymbol{E}_1(t) \end{cases} \tag{31}$$

The inner product Eq(19) becomes,

$$(\xi_2, \xi_1) = \int\limits_{t=-\infty}^{\infty} dt \oiint\limits_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma$$

$$= \int\limits_{t=-\infty}^{\infty} dt \oiint\limits_{\Gamma} (\boldsymbol{H}_2(t) \cdot K_{h1}(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_{h1}(t)) d\Gamma \tag{32}$$

In the above, we have choose ξ_1 as the retarded wave, ξ_2 as the advanced wave.

$$(\xi_2, \xi_1) = (\xi_2, \sigma_1) = (\sigma_2, \xi_1) \tag{33}$$

where,

$$(\xi_2, \sigma_1) \equiv \int\limits_{t=-\infty}^{\infty} dt \oiint\limits_{\Gamma} (\boldsymbol{H}_2(t) \cdot K_{h1}(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_{h1}(t)) d\Gamma \tag{34}$$

$$(\xi_1, \sigma_2) \equiv \int\limits_{t=-\infty}^{\infty} dt \oiint\limits_{\Gamma} (\boldsymbol{E}_1(t) \cdot J_{h2}(t) + \boldsymbol{H}_1(t) \cdot \boldsymbol{J}_{h2}(t)) d\Gamma \tag{35}$$

From the above we can see the field ξ_1 and ξ_2 can be replaced as the corresponding Huygens sources σ_1 and σ_2.

It should be clear that the only advantage we using Huygens source to replace the original fields is Eq.(34,34) looks more like a inner product. It use the dot product "$\cdot$". However, if we know Eq.(19) is also a good inner product. We do not need to convert the fields to the corresponding Huygens source. Here, we need only to know to convert the field to its corresponding Huygens source is possible.

3.2 Generalized Huygens principle

In the quantum mechanics there is the bra $\langle\xi|$ and the ket $|\xi\rangle$, which can be used as the definition for the inner product, i.e.,

$$\langle\xi_1|\xi_2\rangle \equiv \langle\xi_1||\xi_2\rangle \equiv (\xi_1, \xi_2)$$

$$\equiv \int\limits_{t=-\infty}^{\infty} dt \oiint\limits_{\Gamma} (\boldsymbol{E}_1(t) \times \boldsymbol{H}_2(t) + \boldsymbol{E}_2(t) \times \boldsymbol{H}_1(t)) \cdot \hat{n} d\Gamma \tag{36}$$

where

$$\xi_1 \equiv [\boldsymbol{E}_1, \boldsymbol{H}_1] \tag{37}$$

$$\xi_2 \equiv [\boldsymbol{E}_2, \boldsymbol{H}_2] \tag{38}$$

The mutual energy flow theorem Eq.(13) can be written as,

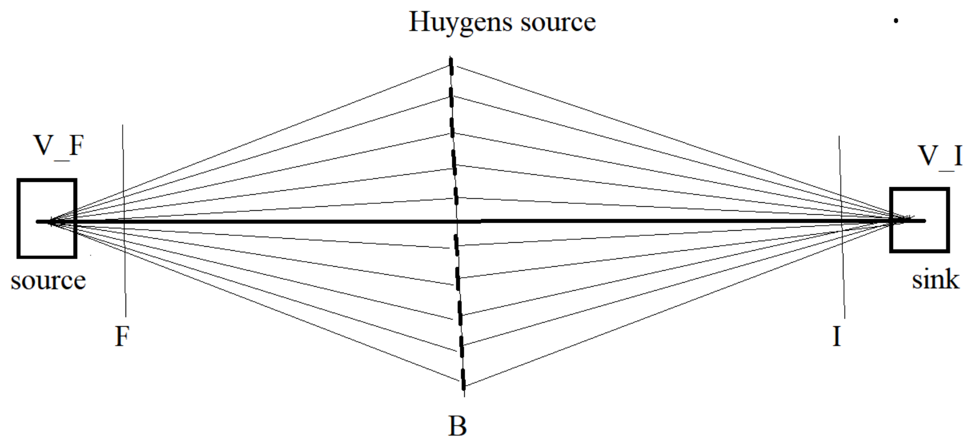

Figure 1: The source is inside the volume V_F, the sink is inside of the volume V_I. F is the surface of the volume V_F, I is the surface of the volume V_I. B is the surface between the source and the sink. .

$$\langle \xi_F | \xi_I^F \rangle = \langle \xi_F^B | \xi_I^B \rangle = \langle \xi_F^I | \xi_I \rangle \tag{39}$$

ξ_F^B is the field at B produced by the source at F. The subscript F is used to express the final point or the place of the sink. The superscript B is the place of the field. Here B is the surface between the I and F. I is the initial source place. The mutual energy flow theorem and the mutual energy theorem together can be rewritten as,

$$\begin{aligned} & -\int_{t=-\infty}^{\infty} \iiint_{V_I} \boldsymbol{J}_I(t) \cdot \boldsymbol{E}_F^I(t) dV \\ & = \langle \xi_F | \xi_I^F \rangle = \langle \xi_F^B | \xi_I^B \rangle = \langle \xi_F^I || \xi_I \rangle \\ & = \int_{t=-\infty}^{\infty} \iiint_{V_F} \boldsymbol{E}_I^F(t) \cdot \boldsymbol{J}_F(t) dV \end{aligned} \tag{40}$$

The details, can be found in Figure 1. In the above ξ_I^F is field at F, which is produced by the source I. ξ_F is the field at F produced by the source at F. $\langle \xi_F | \xi_I^F \rangle$ is the inner product at the surface F. $\langle \xi_F^B | \xi_I^B \rangle$ is a inner product at the surface B. $\langle \xi_F^I | \xi_I \rangle$ is a inner product at the surface I. We use F to express the surface of the volume V_F. I is the surface of the volume V_I. B is any surface between I and F. We assume V_I is the source.

The above mutual energy flow theorem is also suitable to quantum mechanics, Eq.(24) can be written as

$$\langle \Psi_b | \Psi_a^b \rangle = \langle \Psi_b^c | \Psi_a^c \rangle = \langle \Psi_b^a | \Psi_a \rangle \tag{41}$$

$\langle \Psi_b^c | \Psi_a^c \rangle$ is the inner produce on the surface c.$\langle \Psi_b | \Psi_a^b \rangle$ is the inner product on the surface b $\langle \Psi_b^a | \Psi_a \rangle$ is the inner product on the surface a.

In the following discussion we only use electromagnetic field as example, the result is also suitable to the quantum mechanics.

We have known that in quantum mechanics there is,

$$\sum_i |q_i\rangle\langle q_i| = 1 \tag{42}$$

In our situation, our inner product is at the surface B. That is a integral on the surface B and, hence, we can rewritten the above formula as,

$$\sum_i |\xi_{Bi}\rangle\langle \xi_{Bi}| = 1 \tag{43}$$

Substitute the above formula to the mutual energy flow theorem Eq.(40), the mutual energy flow theorem can be written as,

$$\langle \xi_F | \xi_I^F \rangle = \sum_i \langle \xi_F^B | \xi_{Bi} \rangle \langle \xi_{Bi} | \xi_I^B \rangle = \langle \xi_F^I | \xi_I \rangle \tag{44}$$

Considering the mutual energy flow theorem about the surface B and F we have,

$$\langle \xi_F^B | \xi_{Bi} \rangle = \langle \xi_F | \xi_{Bi}^F \rangle = \langle \xi_F | G_B^F | \xi_{Bi} \rangle \tag{45}$$

In the above $\langle \xi_F^B | \xi_{Bi} \rangle$is a inner product at the surface B. $\langle \xi_F | \xi_{Bi}^F \rangle$ is a inner product at F. The first equal sign is because the mutual energy flow theorem, the inner product can be moved from the place of B to the place F. In the second equal sign, we have considered the definition of G_B^F, hence, $|\xi_{Bi}^F\rangle = G_B^F |\xi_{Bi}\rangle$. Where G_B^F is the gain from $|\xi_{Bi}\rangle$to $|\xi_{Bi}^F\rangle$. G_B^F is a operator, or matrix. Hence, we have,

$$\begin{aligned}
\langle \xi_F | \xi_I^F \rangle &= \sum_i \langle \xi_F | \xi_{Bi}^F \rangle \langle \xi_{Bi} | \xi_I^B \rangle \\
&= \sum_i \langle \xi_F | G_B^F | \xi_{Bi} \rangle \langle \xi_{Bi} | \xi_I^B \rangle \\
&= \sum_i \langle \xi_F | G_{Bi}^F | \xi_{Bi} \rangle \langle \xi_{Bi} | G_I^{Bi} | \xi_I \rangle \\
&= \langle \xi_F^I | \xi_I \rangle
\end{aligned} \tag{46}$$

The above formula actually has problem, G_B^F is still the gain operator, but it actually is the gain operator from $|\xi_{Bi}\rangle$ to $|\xi_I^F\rangle$. And hence can be written as

G^F_{Bi}. G^F_{Bi} actually is the matrix element. In the above, insert the unit operator $\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1$,

$$AB = \sum_j A|\xi_j\rangle\langle\xi_j|B$$

$$= \sum_j A^k_j|\xi_j\rangle\langle\xi_j|B^j_i$$

$$= \sum_j A^k_j B^j_i \tag{47}$$

The two matrix AB after insert the unit operator become the matrix element expression. Hence, Eq(46) can be rewritten as,

$$\langle\xi_F|\xi^F_I\rangle = \sum_i \langle\xi_F|G^F_{Bi}|\xi_{Bi}\rangle\langle\xi_{Bi}|G^{Bi}_I|\xi_I\rangle = \langle\xi^I_F||\xi_I\rangle \tag{48}$$

This is the Huygens principle for the retarded wave. In the surface B, $|\xi_{Bi}\rangle$ is the unit Huygens source, $\langle\xi_{Bi}|\xi^{Bi}_I\rangle = \langle\xi_{Bi}|G^{Bi}_I|\xi_I\rangle$ is the value of the Huygens source, $|\xi_{Bi}\rangle\langle\xi_{Bi}||\xi^{Bi}_I\rangle$is the Huygens source. $\langle\xi_F|G^F_B|\xi_{Bi}\rangle\langle\xi_{Bi}|\xi^{Bi}_I\rangle$ is the contribution of the Huygens source to the surface F. $\sum_i\langle\xi_F|G^F_B|\xi_{Bi}\rangle\langle\xi_{Bi}|\xi^B_I\rangle$ is all contributions of the Huygens sources to the surface F.

From the above Eq.(46), we still have also,

$$\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1 \tag{49}$$

It should be noticed that, in Eq.(44), the unit operator Eq.(43) is inserted to the inner product $\langle\xi^B_F|\xi^B_I\rangle$. In Eq.(46) to obtained the unit operator Eq.(49) we have to apply the mutual energy flow theorem $\langle\xi^B_F|\xi_{Bi}\rangle = \langle\xi_F|\xi^F_{Bi}\rangle$.

In other side,

$$\langle\xi^I_F| \equiv \langle\xi_F|T^I_F| \equiv \langle\xi_F|T^B_F T^I_B|$$

Where T^I_F is the gain to the left vector $\langle\xi_F|$, Hence,

$$\langle\xi^I_F|\xi_I\rangle = \langle\xi_F|T^B_F T^I_B|\xi_I\rangle$$

Insert $\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1$ to the above we obtain,

$$\langle\xi^I_F|\xi_I\rangle = \sum_i \langle\xi_F|T^B_F|\xi_{Bi}\rangle\langle\xi_{Bi}|T^I_B|\xi_I\rangle$$

$$= \sum_i \langle\xi_F|T^{Bi}_F|\xi_{Bi}\rangle\langle\xi_{Bi}|T^I_{Bi}|\xi_I\rangle = \langle\xi_F|\xi^F_I\rangle$$

This is the Huygens principle for the advanced wave.

3.3 Application of the Huygens principle to more surfaces

The received energy at the final point F is

$$\langle \xi_F || \xi_I^F \rangle \tag{50}$$

Substitute the following 3 formulas

$$\sum_i |\xi_{Ai}\rangle\langle\xi_{Ai}| = 1 \tag{51}$$

$$\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1 \tag{52}$$

$$\sum_i |\xi_{Ci}\rangle\langle\xi_{Ci}| = 1 \tag{53}$$

to the above formula we have,

$$\begin{aligned}
\langle \xi_F | \xi_I^F \rangle &= \langle \xi_F | G_I^F \xi_I \rangle \\
&= \langle \xi_F G_C^F G_B^C G_A^B G_I^A \xi_I \rangle \\
&= \sum_{kji} \langle \xi_F | G_C^F | \xi_{Ck} \rangle \langle \xi_{Ck} | G_B^C | \xi_{Bj} \rangle \langle \xi_{Bj} | G_A^B | \xi_{Ai} \rangle \langle \xi_{Ai} | G_I^A | \xi_I \rangle \\
&= \sum_{kji} \langle \xi_F | G_{Ck}^F | \xi_{Ck} \rangle \langle \xi_{Ck} | G_{Bj}^{Ck} | \xi_{Bj} \rangle \langle \xi_{Bj} | G_{Ai}^{Bj} | \xi_{Ai} \rangle \langle \xi_{Ai} | G_I^{Ai} | \xi_I \rangle
\end{aligned} \tag{54}$$

In the above, we have considered that,

$$\xi_I^F = G_I^F \xi_I$$

where G_I^F is the gain from ξ_I to ξ_I^F, it is clear that we have,

$$G_I^F = G_C^F G_B^C G_A^B G_I^A$$

There is also the similarly formula for the advanced wave.

4 Conclusion

The unit operator,

$$\sum_i |q_i\rangle\langle q_i| = 1 \tag{55}$$

is usually work on the 3D volume integral, now we have widened it to a series of surface integral. The surface integral is 2D. In the surface integrals, the unit operator is consist of two parts bra and ket. The bra part $\langle \xi_{Bi}^B|$ is used to receive the energy. The ket part $|\xi_{Bi}^F\rangle$ is used to send the energy to next surface.

$$\sum_i |\xi_{Bi}\rangle\langle\xi_{Bi}| = 1 \tag{56}$$

This is exactly Huygens principle tell us.

We also obtained the Huygens principle for the advanced wave, which have, the right vector $|\xi_{Bi}^B\rangle$ is used to receive the energy.

This article we have generalized the Huygens principle, hence, the Huygens principle does not only work for the retarded wave, it also works for the advanced wave. Originally, the unit operator originally only works on the inner space of 3D integral, now it can work on any 2D surface between the source and the sink. This Huygens principle has applied in the updated path integral: streamline integral [9]. In this article we have further simplified the derivation of the Huygens principle.

It is worth to mention that (1) the Huygens principle is applied to inner product on the 2D surface. (2) To derive the Huygens principle, the mutual energy flow theorem are applied two times. Hence the mutual energy flow theorem are the basis of the Huygens principle. (3) The Huygens principle can be applied both the retarded wave and the advanced wave. (4) Huygens principle can be applied for the updated path integral which is streamline integral.

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