The relation between the particle of the mutual energy principle and the wave of Schrödinger equation

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Abstract

This author has replaced the Maxwell equations with the corresponding mutual energy principle, self-energy principle as the axioms in electromagnetic field theory. The advantage of doing this is that it can overcome the difficulty of the Maxwell equations, which conflicts to the energy conservation law. The same conflict also exists in the Schrödinger equation in the quantum mechanics. This author would like to intruded the mutual energy principle to quantum mechanics, but has met the difficulty that there is no advanced solution for the Schrödinger equation. This difficulty is overcome by introducing a negative radius. After this, all the theory about the mutual energy can be extend from the field satisfying Maxwell equations to the field satisfying Schrödinger equation. The Schrödinger equation can also be derived from the corresponding mutual energy principle. However, this doesn't mean both sides are equivalent. The mutual energy principle cannot derive a single solution of Schrödinger equation. The mutual energy principle can only derive a pair solutions of the Schrödinger equations. One is for retarded waves and another is for advanced waves. The retarded wave and the advanced wave must be synchronized. The solutions of the mutual energy principle is in accordance with the theory of the action-at-a-distance and the absorber theory. A action is done always between two objects, for example a source (emitter) and a sink (absorber). The mutual energy principle tell us that a particle is an action and a reaction between the source and the sink. In other hand the wave satisfying Schrödinger equation only need one source or one sink. From the mutual energy principle, it is easy to derive the mutual energy theorem, the mutual energy flow theorem, corresponding Huygens–Fresnel principle. All these will solve the wave-particle duality paradox.

Keyword: photon; electron; retarded wave; advanced wave; time-reversal; absorber; emitter; action-at-a-distance; Maxwell; Poynting, Schrödinger; Dirac; mutual energy;

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I. INTRODUCTION

Maxwell equations have the retarded solution and the advanced solution. Many engineers and scientists do not accept the advanced wave based on the consideration of causality. However, there are scientists and engineers believe the advanced wave is real. Wheeler and Feynman have introduced the absorber theory which involved the advanced wave [1][2]. The absorber theory is based on the action-at-a-distance [8, 20, 23]. J. Cramer further worked on the absorber theory and introduced the transactional interpretation for quantum mechanics[5, 6]. Stephenson offered a good tutorial about the advanced wave [22].

In classical electromagnetic field theory the advanced wave is applied on the Welch's reciprocity theorem [24], Rumsey's reciprocity theorem[19], Zhao's (this author) mutual energy theorem [9, 25, 26]. de Hoop's reciprocity theorem[7]. This author found the above 4 theorems can be seen as one theorem in Fourier domain or in time domain. These theorems have the major difference comparing to Lorentz reciprocity theorem[3, 4]. Lorentz reciprocity theorem is a mathematical theorem, these theorems are an energy theorem.

This author combined the absorber theory and the mutual energy theorem and further introduced the concept that the photon energy is transferred by the mutual energy flow[12–18, 21]. And this author further introduced the mutual energy principle[10] and the self-energy principle[11]. The mutual energy principle tell us that the electromagnetic field or the field for photons all should satisfy the formula of the mutual energy principle. The solution of the mutual energy principle is an retarded wave and an advanced wave. Both waves satisfy the Maxwell equations. The formula of the mutual energy principle require that the both waves must be synchronized. The mutual energy flow is the energy flow of the particle. The self-energy principle for photon tells us that the self-energy are returned by the time-reversal waves. There are two time reversal waves, one corresponding to the retarded wave and another one is corresponding to the advanced wave. The energy flow of the two time-reversal waves offset the the self-energy flows. Hence, the self-energy flows do not contribute to any energy transfers of the particle. However, the retarded wave and the advanced wave combined together can build the mutual energy flow, which survived and which can transfer the energy from point $\boldsymbol{a}$ to point $\boldsymbol{b}$. Here $\boldsymbol{a}$ is the source of the energy flow, $\boldsymbol{b}$ is the sink of the energy flow. It can be proven that the shape of the mutual energy flow is thin in the two ends $\boldsymbol{a}$ and $\boldsymbol{b}$, it is thick in the middle between $\boldsymbol{a}$ and $\boldsymbol{b}$. Hence, the mutual energy flow looks like a wave in the middle between $\boldsymbol{a}$ and $\boldsymbol{b}$ and the mutual energy flow looks like a particle at the ends of $\boldsymbol{a}$ and $\boldsymbol{b}$. Hence, photon is not wave or particle but the mutual energy flow.

In this article the concept of the mutual energy principle, self-energy principle, mutual energy theorem, mutual energy flow theorem is extended to the system of the quantum mechanics, where Schrödinger equation is applied instead of Maxwell equations.

It is often said that the Schrödinger equation only has the solution of the retarded wave. However, this author found that if we allow the distance of a point to the origin has a negative value, the Schrödinger equation can have the advanced solution. After we obtained the advanced wave solution of Schrödinger equation, all theory of mutual energy can be extended from the electromagnetic fields to quantum mechanics. This all will be show in this article.

II. RETARDED AND ADVANCED WAVE SOLUTIONS FOR THE SCHRÖDINGER EQUATION

We assume that the quantum for example electron runs in the empty space from point $\boldsymbol{a}$ to $\boldsymbol{b}$. This electron must satisfy the Schrödinger equation,

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2+V(\boldsymbol{r},t)\right]\Psi(\boldsymbol{r},t) \tag{1}$$

where $i=\sqrt{-1}$. $\Psi(\boldsymbol{r},t)$ is the wave function. In the free space the amplitude of the wave function will decrease with distance, hence we have

$$\Psi(\boldsymbol{r},t)=\frac{1}{r}\exp(-(j\omega t-kr)) \tag{2}$$

where we assume that if the wave is retarded wave, then

$$r=+||\boldsymbol{x}-\boldsymbol{x}'|| \tag{3}$$

if the wave is advanced wave,

$$r=-||\boldsymbol{x}-\boldsymbol{x}'|| \tag{4}$$

$\boldsymbol{x}$ is the field point. $\boldsymbol{x}'$ is the source point. Eq.(2) is correct at least at the place of far field where r is not too small. Eq.(2) can be a retarded wave. Hence normally we speak that

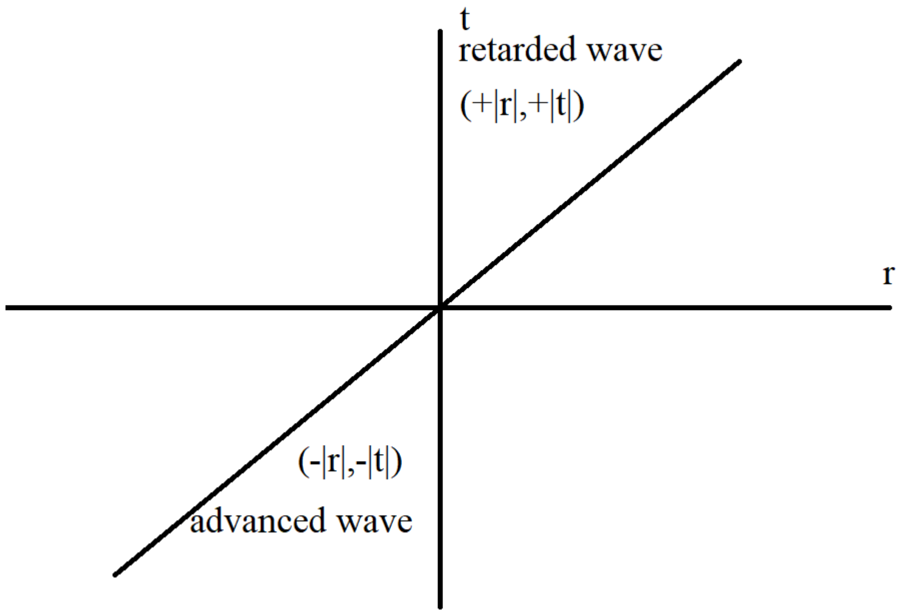

Figure 1. The region of the retarded wave and the advanced wave. The distance r can be negative. For an advanced wave the distance r is negative.

the Schrödinger equation only has the solution of the retarded wave. However, if we allow the the distance r has the negative values4. The Eq.(2) can also be applied to the advanced wave.

For example when t take a negative value: $t = -|t|$, r also take a negative value for example $r = -|r|$, this is a place for an advanced wave. When we allow the distance r has negative value, Schrödinger can have the advanced wave solution. See Figure 1.

III. MUTUAL ENERGY PRINCIPLE FOR SCHRÖDINGER EQUATION

A. Operator of the Schrödinger equation

The Schrödinger equation can be written as,

$$L\Psi(\boldsymbol{r},t) = 0 \tag{5}$$

where, L referred as Schrödinger operator and is defined as,

$$L \equiv \left[-\frac{\hbar^2}{2i\mu}\nabla^2 + \frac{1}{i}V(\boldsymbol{r},t)\right]\Psi(\boldsymbol{r},t) - \hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) \tag{6}$$

We assume the Schrödinger equation also has the source or sink,

$$L\Psi(\boldsymbol{r},t)=S \tag{7}$$

where S is the source or sink. The source of Schrödinger equation can be at very small region for example, the source of an electron is inside an atom. Some reader perhaps will argue how you know there is a source and sink for Schrödinger equation? This author doesn't know whether or not there are source or sink for Schrödinger equation. However it always possible to have artificial source or sink based on Huygens principle. That means the source will be just a place the wave goes to outside from an atomic system to the empty space. The sink just a place the wave goes from empty space to the inside of an atomic system.

B. A mathematics formula for the operator L

Define the inner product in the volume V,

$$(\Psi_b,\Psi_a)_V=\iiint\limits_V \Psi_b^*\Psi_a dV \tag{8}$$

A mathematical formula can be proved as following,

$$(\Psi_b(\boldsymbol{r},t),L\Psi_a(\boldsymbol{r},t))_V+(L\Psi_b(\boldsymbol{r},t),\Psi_a(\boldsymbol{r},t))_V$$

$$=-\oiint\limits_\Gamma \boldsymbol{J}_{ab}\cdot\hat{n}\Gamma dt-\frac{\partial}{\partial t}\iiint\limits_V udVdt \tag{9}$$

where L is defined at Eq.(6). V is any volume in the space. Γ is the boundary surface of the volume V. And the mutual energy flow intensity is defined as,

$$\boldsymbol{J}_{ab}\equiv\frac{\hbar}{2\mu i}(\Psi_b^*\nabla\Psi_a-\nabla\Psi_b^*\Psi_a) \tag{10}$$

The mutual energy intensity is defined as,

$$u\equiv\hbar\Psi_b^*\Psi_a \tag{11}$$

See the Appendix VIII for details of the proof. Eq.(9) is only a mathematical formula instead of a physic formula, because Schrödinger equation has not applied on it which is the physical formula.

C. Deriving the mutual energy principle from Schrödinger equation

The above Eq.(9) is a mathematical formula, now let us consider the equation of physics, which is Schrödinger equation, assume in the two point $\boldsymbol{a}$ and $\boldsymbol{b}$ has the sources S_a and S_b we have the Schrödinger equations,

$$L\Psi_a(\boldsymbol{r},t) = S_a \tag{12}$$

$$L\Psi_b(\boldsymbol{r},t) = S_b \tag{13}$$

In the above Schrödinger equation we have add two sources, when we speak a source, it can be also a sink. We assume S_a is a source, it sends out the energy of the particle and S_b is a sink, it receives the energy of the particle.

Substitute the above two equations to Eq.(9) we can obtain the mutual energy principle corresponding Schrödinger equation,

$$-\oiint_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n}\Gamma - \frac{\partial}{\partial t} \iiint_{V} u dV = (\Psi_b(\boldsymbol{r},t), S_a)_V + (S_b, \Psi_a(\boldsymbol{r},t))_V \tag{14}$$

This means, that if we have the Schrödinger equation, we can derived the corresponding mutual energy principle Eq.(14). In the above formula the energy flow to the inside of the volume is $-\oiint_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n}\Gamma$ which is equal to the energy increase inside the volume $\frac{\partial}{\partial t}\iiint_V udV$ and consumed energy of the two source S_a and S_b: $(\Psi_b(\boldsymbol{r},t), S_a)_V + (S_b, \Psi_a(\boldsymbol{r},t))_V$.

D. Deriving Schrödinger equation from the mutual energy principle

In other hand, if we have the above mutual energy principle Eq.(14), comparing it with the mathematical formula Eq.(9) we can obtained a pair of Schrödinger equations Eq.(12,13), hence, Schrödinger equations can be derived from the mutual energy principle. The mutual energy principle can be derived from Schrödinger equation and the Schrödinger equation can be derived from the mutual energy principle, does this means that the mutual energy principle and Schrödinger equation are equivalent? This will be discussed in the following section.

IV. THE DIFFERENCE BETWEEN SCHRÖDINGER EQUATION AND THE MUTUAL ENERGY PRINCIPLE

It should notice that, even we can derive from the Schrödinger equation to the corresponding mutual energy principle and we can derive the Schrödinger equation from the corresponding mutual energy principle, but this does not mean that the Schrödinger equation is equivalent to the mutual energy principle. This can be seen by the following reasons.

1. The mutual energy principle needs at least a pair Schrödinger equations exist simultaneously. That means a pair Schrödinger equations must be synchronized. Since if we have known that,

$$\Psi_b(\boldsymbol{r}, t) \equiv 0, \qquad S_b \equiv 0 \tag{15}$$

for example, substituting this to the mutual energy principle Eq(14), we can obtain

$$\Psi_a(\boldsymbol{r}, t) = anything < \infty \tag{16}$$

Is a solution of the mutual energy principle. However, this is not an accepted solution. This means that if we started from mutual energy principle, we cannot get a solution which satisfies only one Schrödinger equation. We can obtained only a solution with a pair of Schrödinger equations. One solution of the Schrödinger equations is not an accepted solution for the mutual energy principle.

In other hand, if we apply the Schrödinger equation as axiom, any the solution which satisfy only one Schrödinger equation is an accept solution.

The concept of a pair solutions, it is suitable to the concept of "action-at-a-distance". In the theory action-at-a-distance [8, 20, 23], an action need at least two points an source (emitter) and an sink (absorber). Only a source without sink can not be accepted. Only a sink without a source can also not be accepted. A particle is an action which needs a source and a sink. The source give the sink a action, a sink will give the source a re-action. The action and the re-action is look like a particle. The particle is the action and reaction between a source and a sink.

2. This author believe that the mutual energy principle can be applied as a correct axiom for quantum mechanics. The Schrödinger equation has the same problem same as

the Maxwell equations. The self-energy flow of Schrödinger equation $\boldsymbol{J}_{aa}$, $\boldsymbol{J}_{bb}$

$$\oiint_{\Gamma} \boldsymbol{J}_{aa} \cdot \hat{n}\Gamma dt \neq 0, \qquad \oiint_{\Gamma} \boldsymbol{J}_{bb} \cdot \hat{n}\Gamma dt \neq 0 \tag{17}$$

where Γ can be a surface of infinite big sphere and where $\boldsymbol{J}_{aa} \equiv \frac{\hbar}{2\mu i}(\Psi_a^* \nabla \Psi_a - \nabla \Psi_a^* \Psi_a)$ will go outside the universe, that is not possible. It is same for $\boldsymbol{J}_{bb} \equiv \frac{\hbar}{2\mu i}(\Psi_b^* \nabla \Psi_b - \nabla \Psi_b^* \Psi_b)$.

The mutual energy flow $\boldsymbol{J}_{ab}$, in other hand, can not go to the outside of our universe:

$$\oiint_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n}\Gamma dt = 0 \tag{18}$$

Where $\boldsymbol{J}_{ab}$ is related a source and a sink. Ψ_b, Ψ_a are two waves. Here, we assume that one of them is the retarded wave and one of them is the advanced wave. The two waves reach the infinite sphere Γ one is at a future time and another one is at a past time. Hence, the two waves cannot nonzero at the sphere simultaneously. Hence, we have Eq.(18). In other side if the two waves are all retarded wave or all advanced wave, it is not possible to obtain Eq.(18). Hence, starting from mutual energy principle as axiom, it automatically require the existence of the advanced wave and the advanced wave has to be synchronized with the retarded wave.

3. From mutual energy principle it is easy to obtained the mutual energy theorem and mutual energy flow theorem which can be used to prove that the mutual energy flow looks like a particle in the place of two ends and looks like wave in the middle between the two ends. This will solve the wave-particle paradox. This will be discussed in the next section.

V. THE MUTUAL ENERGY AND THE MUTUAL ENERGY FLOW THEOREMS

A. The mutual energy theorem

Assume $U = \iiint_V u dV dt$. We can assume $U(t=\infty) = U(t=-\infty)$. $U(t=-\infty)$ is the energy before the particle emitted at the source $\boldsymbol{a}$, $U(t=\infty)$ is the energy after the particle has reached $\boldsymbol{b}$. Hence, $U(t=\infty)$ should equal to $U(t=-\infty)$,hence we have,

$$\int_{t=\infty}^{\infty} \frac{\partial}{\partial t} \iiint_V u dV dt = \int_{t=\infty}^{\infty} \frac{\partial}{\partial t} U dt$$
$$= U(t=\infty) - U(t=-\infty) = 0 \tag{19}$$

Substituting Eq.(18) to the mutual energy principle Eq.(14) we have

$$-\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n}\Gamma = \int_{t=\infty}^{\infty} ((\Psi_b(\boldsymbol{r},t), S_a)_V + (S_b, \Psi_a(\boldsymbol{r},t))_V)dt \tag{20}$$

Considering Eq.(18) we have

$$\int_{t=\infty}^{\infty} ((\Psi_b(\boldsymbol{r},t), S_a)_V + (S_b, \Psi_a(\boldsymbol{r},t))_V)dt = 0 \tag{21}$$

where V is the whole space. Hence, we can obtained the mutual energy theorem:

$$-\int_{t=\infty}^{\infty} (\Psi_b(\boldsymbol{r},t), S_a)_{V_a} dt = \int_{t=\infty}^{\infty} (S_b, \Psi_a(\boldsymbol{r},t))_{V_b} dt \tag{22}$$

V_a and V_b is the place where S_a or S_b are not zero. This can be rewritten as,

$$-(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a} = (S_b, \Psi_a(\boldsymbol{r},t))_{TV_b} \tag{23}$$

where $(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a} = \int_{t=\infty}^{\infty} (\Psi_b(\boldsymbol{r},t), S_a)_{V_a} dt$. $(S_b, \Psi_a(\boldsymbol{r},t))_{TV_b} = \int_{t=\infty}^{\infty} (S_b, \Psi_a(\boldsymbol{r},t))_{V_b} dt$. The subscript $T = [-\infty, \infty]$ express the inner product also includes a integral with time $\int_{t=\infty}^{\infty} \cdot dt$. $-(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a}$ is the produced energy of the source S_a . $(S_b, \Psi_a(\boldsymbol{r},t))_{TV_b}$ is the received energy of the sink S_b. This energy is the energy of the particle. It is the energy from the source (emitter) S_a moving to the sink (absorber) S_b.

The word "mutual" can be drop out. The mutual energy theorem is actually the energy conservation law for the particle. The energy of particle can only emitted from $\boldsymbol{a}$ and received by $\boldsymbol{b}$. It is clear there are no any other energy go from $\boldsymbol{a}$ to $\boldsymbol{b}$. If the energy of the particle has moved from $\boldsymbol{a}$ to $\boldsymbol{b}$, there should have energy flow between $\boldsymbol{a}$ and $\boldsymbol{b}$. This will be discussed in next subsection. We will also discuss the self-energy does not contributed to the energy flow of the particle in next section.

B. The mutual energy flow theorem

We assume V_a and V_b are inside V. V_a is the region include the source S_a. V_b is the region include the sink V_b. $-(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a}$ is the energy send by the source S_a. $(S_b, \Psi_a(\boldsymbol{r},t))_{TV_b}$ is the energy received by the sink S_b. We can also obtain the mutual energy flow theorem,

$$-(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a} = (\Psi_b, \Psi_a)_{T\Gamma_{ab}} = (S_b, \Psi_a(\boldsymbol{r},t))_{TV_b} \tag{24}$$

where,

$$(\Psi_b, \Psi_a)_{T\Gamma_{ab}} = \int\limits_{t=\infty}^{\infty} \oiint\limits_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma dt$$

$$= \int\limits_{t=\infty}^{\infty} \oiint\limits_{\Gamma_{ab}} \frac{\hbar}{2\mu i}(\Psi_b^* \nabla \Psi_a - \nabla \Psi_b^* \Psi_a) \cdot \hat{n} \Gamma dt \tag{25}$$

where Γ_{ab} is a closed surface surround $\boldsymbol{a}$, or a closed surface surround $\boldsymbol{b}$. It also can be a infinite plane between $\boldsymbol{a}$ and $\boldsymbol{b}$. The normalized vector $\hat{n}$ is the norm vector of the surface. The direction is from $\boldsymbol{a}$ to $\boldsymbol{b}$. Γ_{ab} can be seen as an arbitrary surface separating V_a and V_b. We have written Eq.(25) as an inner product form. It can be proven that it is really an inner product and satisfy the inner product 3 condition:

I. Conjugate symmetry,

$$(\Psi_b, \Psi_a)_{T\Gamma_{ab}} = (\Psi_a, \Psi_b)^*_{T\Gamma_{ab}} \tag{26}$$

II. Linearity,

$$(\Psi_b, k\Psi_a)_{T\Gamma_{ab}} = k(\Psi_b, \Psi_a)_{T\Gamma_{ab}} \tag{27}$$

$$(\Psi_b, \Psi_{a1} + \Psi_{a2})_{T\Gamma_{ab}} = (\Psi_b, \Psi_{a1})_{T\Gamma_{ab}} + (\Psi_b, \Psi_{a2})_{T\Gamma_{ab}} \tag{28}$$

III. Positive defined,

$$(\Psi, \Psi)_{T\Gamma_{ab}} \geq 0 \tag{29}$$

$$(\Psi, \Psi)_{T\Gamma_{ab}} = 0 \quad iff \quad \Psi = 0 \tag{30}$$

Where iff means if and only if. $\boldsymbol{J}_{ab}$ is the energy flow intensity. $Q = \oiint_{\Gamma} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma$ is the energy flow. $Energy = \int_{t=\infty}^{\infty} Q dt$ is the all energy from the source $\boldsymbol{a}$ to the sink $\boldsymbol{b}$.

Proof: In the Eq.(20) V can be arbitrarily chosen. We separate V as two parts, V_a and V_b. Γ_{ab} is the boundary between V_a and V_b. We choose V as V_a there is only one source S_a. In this case, Eq.(20) become,

$$-\int\limits_{t=\infty}^{\infty} \oiint\limits_{\Gamma_a} \boldsymbol{J}_{ab} \cdot \hat{n}_{ab} d\Gamma = \int\limits_{t=\infty}^{\infty} ((\Psi_b(\boldsymbol{r}, t), S_a)_{V_a}) dt \tag{31}$$

In the same way, if V is chosen as V_b we have,

$$-\int\limits_{t=\infty}^{\infty} \oiint\limits_{\Gamma_b} \boldsymbol{J}_{ab} \cdot \hat{n}_{ba} d\Gamma = \int\limits_{t=\infty}^{\infty} ((S_b, \Psi_a(\boldsymbol{r}, t))_{V_b}) dt \tag{32}$$

Adjust the direction $\hat{n}_{ba}$ in the above formula from $\boldsymbol{b} \longrightarrow \boldsymbol{a}$ to $\boldsymbol{a} \longrightarrow \boldsymbol{b}$. Γ_{ba} changed to Γ_{ab}, considering $\oiint_{\Gamma_b} \cdot \hat{n}_{ba} d\Gamma = - \oiint_{\Gamma_b} \cdot \hat{n}_{ab} d\Gamma$, substituting Eq.(31,32) to the mutual energy theorem Eq.(22) we obtained the mutual energy flow theorem,

$$-(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a}$$
$$= \int_{t=\infty}^{\infty} \oiint_{\Gamma_a} \boldsymbol{J}_{ab} \cdot \hat{n}_{ab} d\Gamma = \int_{t=\infty}^{\infty} \oiint_{\Gamma_b} \boldsymbol{J}_{ab} \cdot \hat{n}_{ba} d\Gamma$$
$$= (S_b, \Psi_a(\boldsymbol{r},t))_{TV_b} \tag{33}$$

This can be easily extended,

$$-(\Psi_b(\boldsymbol{r},t), S_a)_{TV_a}$$
$$= \int_{t=\infty}^{\infty} \oiint_{\Gamma_a} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma = \int_{t=\infty}^{\infty} \oiint_{\Gamma_{ab}} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma = \int_{t=\infty}^{\infty} \oiint_{\Gamma_b} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma$$
$$= (S_b, \Psi_a(\boldsymbol{r},t))_{TV_b} \tag{34}$$

where Γ_{ab} is any surface between Γ_a and Γ_b. Now we have $\hat{n} = \hat{n}_{ab}$ the normal vector of the surface. The direction is from a to b. Considering,

$$(\Psi_b(\boldsymbol{r},t), \Psi_a(\boldsymbol{r},t))_{T\Gamma_{ab}} = \int_{t=\infty}^{\infty} \oiint_{\Gamma_{ab}} \boldsymbol{J}_{ab} \cdot \hat{n} d\Gamma \tag{35}$$

We have Eq.(24).

The mutual energy flow theorem is stronger than the mutual energy theorem, in next section we will explain that the self-energy flow do not contribute to the energy transfer of the particle. Hence, the mutual energy flow is the only energy flow. Hence, the word "mutual" can be dropped out. It is not only the mutual energy flow theorem but the energy flow theorem.

VI. SELF ENERGY PRINCIPLE

If we assume the mutual energy principle is the axiom of the quantum mechanics, we still obtained two Schrödinger equations. Hence, Schrödinger equations should also satisfied. If Schrödinger equation is satisfied, we still obtained Eq.(17). There is still the energy going to the outside of our universe. This energy is not received by anything. The problem of the Schrödinger equations still not solved.

One possibility is the wave has collapsed. That means the retarded wave has collapsed to the sink $\boldsymbol{b}$. The advanced wave has collapsed to the source $\boldsymbol{a}$. However, there has lot of sinks for example $\boldsymbol{b}'$ in the space, it is difficult the retarded wave just collapsed the particular sink $\boldsymbol{b}$ (which is the target of the mutual energy flow). If the retarded wave collapsed, there should be another kind of particle, which is contributed by the self-energy flow. It is same to the advanced wave which needs to collapsed to a source $\boldsymbol{a}'$. It is difficult for the advanced wave just collapse to the place $\boldsymbol{a}$ (the mutual energy flow started from). Hence, there should be another particle corresponding to the advanced wave which collapsed to $\boldsymbol{a}'$. We only seen one kind of particle.

Wave collapse should be a process of physics. But this process has not been described by any formula. Hence, the theory of wave function collapse is difficult to be accepted. Since we have the mutual energy flow theorem which can transfer the energy of the particle, the wave is not necessary to be collapsed.

This author do not think the there is an energy go to outside of our universe. Since there is the mutual energy flow can transferred the energy from the source to sink, it is not necessary for the self energy flow to transfer the energy.

If this author believe the concept the wave collapse cannot be accept. Here the wave collapse is collapse to it's goal. For example the retarded wave sent from the source $\boldsymbol{a}$ is collapsed to the sink $\boldsymbol{b}$. If wave doesn't collapse where it goes? This author thought it returns. That means that the retarded wave returns to the source. The advanced wave returns to the sink. The return process can be described by a time-reversal process which should satisfies, the time-reverse Schrödinger equation. The time-reverse Schrödinger equation can be obtained by substitute $-t$ to t in the Schrödinger equation, which is,

$$L_{re} \equiv \left[-\frac{\hbar^2}{2i\mu}\nabla^2 + \frac{1}{i}V(\boldsymbol{r},t) \right] \Psi_{re}(\boldsymbol{r},t) - \hbar\frac{\partial}{\partial(-t)}\Psi_{re}(\boldsymbol{r},t) \tag{36}$$

$$L_{re}\Psi_{re}(\boldsymbol{r},t) = 0 \tag{37}$$

The solution of the time-reverse Schrödinger equation is written as $\Psi_{re}(\boldsymbol{r},t)$. It is clear that,

$$\Psi_{re}(\boldsymbol{r},t)=\Psi^*(\boldsymbol{r},t) \tag{38}$$

where $\Psi(\boldsymbol{r},t)$ is the normal wave satisfied the Schrödinger equation. Hence, the time-reverse wave is the complex conjugate of the of the normal wave. Here the normal wave includes the retarded wave and the advanced wave.

The energy flow of the normal wave can be canceled by the time-reversal wave, i.e.,

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{aa}\cdot\hat{n}d\Gamma dt+\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{aare}\cdot\hat{n}d\Gamma dt=0 \tag{39}$$

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{bb}\cdot\hat{n}d\Gamma dt+\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{bbre}\cdot\hat{n}d\Gamma dt=0 \tag{40}$$

Here $\boldsymbol{J}_{aare}$ is the self energy flow intensity of the time-reverse wave corresponding the retarded wave. $\boldsymbol{J}_{bbre}$ is the self-energy flow of the time-reverse wave corresponding the advanced wave. Eq.(39,40) are referred as self-energy principle. The self-energy principle tells us, there exists the time-reverse wave which can cancel the self-energy flow. The self-energy flow of the retarded wave is canceled by the energy flow of the corresponding time-reversal wave of the retarded wave. The self-energy flow of the advanced wave is canceled by the energy flow of the corresponding time-reversal wave of the advanced wave. Here when we speak the cancel we mean the energy flow is canceled, the retarded wave and the corresponding time-reversal wave are not cancel each other. The advanced wave and the corresponding time-reverse wave are all existent.

It is possible that the time-reversal wave also has the corresponding mutual energy principle, and hence, the mutual energy flow theorem and mutual energy flow theorem. There is the problem that the time-reversal mutual energy flow can also cancel the normal mutual energy flow that means,

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{ab}\cdot\hat{n}d\Gamma dt+\int_{t=\infty}^{\infty} \oiint_{\Gamma} \boldsymbol{J}_{abre}\cdot\hat{n}d\Gamma dt=0 \tag{41}$$

where $\boldsymbol{J}_{ab}$ is the mutual energy flow form $\boldsymbol{a}$ to $\boldsymbol{b}$. $\boldsymbol{J}_{abre}$ is the time-reversal mutual energy flow from $\boldsymbol{b}$ returns to $\boldsymbol{a}$. If the mutual energy flow is canceled by the energy flow of the corresponding time-reversal wave. There is no energy flow can transfer the energy for a particle. The particle cannot move form $\boldsymbol{a}$ to $\boldsymbol{b}$. This author assume that if the sink receive an energy of a whole particle, it will not allow the energy return from $\boldsymbol{b}$ to $\boldsymbol{a}$. If the sink received only a half or partial energy of a particle it will allow the energy to return from $\boldsymbol{b}$ to $\boldsymbol{a}$. This is the reason why we cannot obtained a partial particle. The self-energy principle tell us there is no energy flow go outside to our universe. Self-energy principle is applied to make the theory selfconsistent. The mutual energy principle is applied to the calculation of the quantum mechanics.

It should be noticed if the mutual energy flow of time-reversal wave existent, it can also be an anti-particle for example the positron (a positive electron) for electron.

After we have the self-energy principle, both the Schrödinger equation and the mutual energy principle can be applied as the axioms of quantum mechanics. If we apply Schrödinger equation as the axiom we need two Schrödinger equations one is for the retarded wave one is for the advanced wave. We need to explain to the reader the advanced wave exists same as the retarded wave. If we apply the mutual energy principle as the axiom of quantum mechanics, we only need one mutual energy principle. The advanced wave and retarded wave can all be derived from the mutual energy principle. Hence, use the mutual energy principle as axiom it is better than the Schrödinger equation.

Since the self-energy principle is only this author's hypothesize, it still needs to be proven. For example the self-energy flow goes to the infinite big sphere perhaps go back from another world or another universe. It go back by a wormhole but not the time-reverse wave. That is also possible.

But mutual energy principle can be proved from Schrödinger equation, it should be correct. We need only compare the mutual energy principle and the Schrödinger equation which is more close to the nature. If we do not derive Schrödinger equations from the The mutual energy principle, the mutual energy principle will has no problem. All the problem of the mutual energy principle comes only through the Schrödinger equations (Here we speak about the problem of Schrödinger equation, which is the self energy flow of Schrödinger equation goes to outside of our universe.). Hence, even without the self-energy principle we should also apply the mutual energy principle as the axioms. Most reader perhaps would like to accept the problem of the Schrödinger equations than to accept the self-energy principle. The problem of Schrödinger equations is the self-energy flow go out our universe, the problem of the self-energy principle is introduction of a new kind of field that needs to be tested. The self-energy principle introduced two time-reverse field, even without proof, it make the theory selfconsistent. Hence it still very valuable.

The formula of the mutual energy principle is similar for the electromagnetic field and quantum mechanics, even the Maxwell equations and the Schrödinger equation are not similar to each other. They all can be written as,

$$-(S_f, \xi_i^f) = (\xi_f^\Gamma, \xi_i^\Gamma) = (\xi_f^i, S_i)$$

where (S_f, ξ_i^f) and (ξ_f^i, S_i) is a inner product with 3D volume integral and 1D time integral. $(\xi_f^\Gamma, \xi_i^\Gamma)$ is a inner product with 2D surface integral and 1D time integral. i is at the initial place which is the source of the particle. f is the final place which is the sink place of the particle. Γ is any 2D surface between i and f. S_f is the sink intensity, S_i is the source intensity. ξ_i^f is the field at the place f emitted from i, ξ_f^Γ is the field at Γ emitted from f. It is similar to ξ_f^Γ and ξ_f^i.

Hence, the mutual energy flow should be more general than Schrödinger equation or Maxwell equation. It is also possible we have the mutual energy principle corresponding to hadron, graviton and so on.

VII. HAMILTON AND MOMENTUM OPERATORS

In quantum mechanics when we often speak the energy and momentum of the particle. We define the average of the Hamilton operator and say that it is the energy of the particle. We define average of momentum operator and say that it is the momentum of the particle. This author do not satisfy that kind of definition without a proof and think the other reader will also very confused for that. In the following a better proof will be given.

A. Average of the Hamilton operator is the energy of the particle

Considering the particle is at the orbit, the wave is static and stable, hence, $\hbar\frac{\partial}{\partial t}$ can be omit. Hence, we have

$$L \equiv \left[-\frac{\hbar^2}{2i\mu}\nabla^2 + \frac{1}{i}V(\boldsymbol{r},t)\right] - \hbar\frac{\partial}{\partial t}$$

$$= \frac{1}{i}H \tag{42}$$

where H is the Hamilton,

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(\boldsymbol{r}, t) \tag{43}$$

Hence, we have,

$$(\Psi_b(\boldsymbol{r}, t), L\Psi_a(\boldsymbol{r}, t))_{TV_a} = (\Psi_b(\boldsymbol{r}, t), \frac{1}{i}H\Psi_a(\boldsymbol{r}, t))_{TV_a} \tag{44}$$

According the mutual energy flow theorem Eq.(24), we have

$$|(\Psi_b(\boldsymbol{r}, t), H\Psi_a(\boldsymbol{r}, t))_{TV_a}| = |(\Psi_b(\boldsymbol{r}, t), L\Psi_a(\boldsymbol{r}, t))_{TV_a}| = |(\Psi_b, \Psi_a)_{T\Gamma_{ab}}| \tag{45}$$

In case the electron is on the orbit, we know that the advanced wave and the retarded wave is same, this is similar to the electromagnetic field wave inside a wave guide, i.e.,

$$\Psi_b(\boldsymbol{r}, t) = \Psi_a(\boldsymbol{r}, t) = \Psi(\boldsymbol{r}, t) \tag{46}$$

V_a can be written as V, we have,

$$|(\Psi(\boldsymbol{r}, t), H\Psi(\boldsymbol{r}, t))_{TV}| = |(\Psi(\boldsymbol{r}, t), L\Psi(\boldsymbol{r}, t))_{TV}| = |(\Psi, \Psi)_{T\Gamma}| \tag{47}$$

In the case the wave is stable, the subscript T can be drop out, because it means a integral with time. Hence, we have,

$$|(\Psi(\boldsymbol{r}, t), H\Psi(\boldsymbol{r}, t))_V| = |(\Psi(\boldsymbol{r}, t), L\Psi(\boldsymbol{r}, t))_V| = |(\Psi, \Psi)_\Gamma| \tag{48}$$

where $|(\Psi, \Psi)_\Gamma| = |\oiint_\Gamma \boldsymbol{J} \cdot \hat{n} d\Gamma|$ is the absolute value of the total energy of the energy flow of the particle, which is related to the average of the Hamilton $|(\Psi(\boldsymbol{r}, t), H\Psi(\boldsymbol{r}, t))_V|$.

This is the reason in quantum mechanics we can use the average of Hamilton as the energy of the particle! This tell us the average of the Hamilton operator is not just an average energy of the particle but it is the tall energy of the energy flow of the particle. Eq.(47, 48) offers more meaningful result to explain the Hamilton operator.

When we speak about the $\boldsymbol{J}_{ab}$ is energy flow intensity, it can be also mass flow intensity. Since mass and energy is equivalent according the relative theory. Actually it is normalized mass flow intensity. In the text book of quantum mechanics, it is referred as the probability flow intensity. Since this author do not accept the probability interpretation of Copenhagen school, when we speak the probability flow intensity, it can be understand as the normalized mass flow intensity.

B. Average of the momentum operator is the momentum of the particle

In our new quantum mechanics (using the mutual energy principle and self-energy principle as axioms), I hope that the momentum can be defined as,

$$\boldsymbol{p} = \int_V \rho_\mu(x)\boldsymbol{v}dV \tag{49}$$

Where ρ_μ is mass intensity. $\boldsymbol{J}_\mu = \rho_\mu(x)\boldsymbol{v}$ is mass flow intensity, it is the momentum for the unit volume.

$$\mu = \int_V \rho_\mu dV$$
$$= \mu \int_V \rho dV \tag{50}$$

where ρ is the normalized mass intensity or the probability intensity. We have,

$$\rho_\mu = \mu\rho \tag{51}$$

Hence,

$$\boldsymbol{p} \equiv \mu \int_V \rho \boldsymbol{v} dV = \mu \int_V \boldsymbol{J} dV \tag{52}$$

where $\boldsymbol{J} = \boldsymbol{\rho v}$ is normalized mass flow intensity (or the probability flow intensity). It should be notice that this author do not support the concept of probability and the probability flow intensity. Hence, here when we speak the probability intensity it just a normalized mass intensity, the probability flow intensity is just a normalized mass flow intensity.

C. Quasi-plane wave

The wave the electron inside its orbiter can be referred as quasi-lane wave. In this situation we have,

$$\boldsymbol{J} = \boldsymbol{J}_{ab} = \frac{\hbar}{2\mu i}(\nabla\Psi_A\Psi_B^* - \Psi_A\nabla\Psi_B^*)$$

$$= \frac{\hbar}{2\mu i}(\nabla\Psi_A\Psi_B^* - \Psi_A\nabla\Psi_B^*)$$
$$= \frac{1}{2\mu}((\frac{\hbar}{i}\nabla\Psi_A)\Psi_B^* + \Psi_A(\frac{\hbar}{i}\nabla\Psi_B)^*)$$

$$= \frac{1}{\mu}[\frac{1}{2}(\Psi_B^*(\hat{\boldsymbol{p}}\Psi_A) + (\hat{\boldsymbol{p}}\Psi_B)^*\Psi_A)] \tag{53}$$

where $\hat{\boldsymbol{p}} = \frac{\hbar}{i}\nabla$.

Hence the momentum of the particle is,

$$\boldsymbol{p} \equiv \mu \int_V \boldsymbol{J} dV = \int_V \frac{1}{2}(\Psi_B^*(\hat{\boldsymbol{p}}\Psi_A) + (\hat{\boldsymbol{p}}\Psi_B)^*\Psi_A) dV \tag{54}$$

In the case for example the wave is in a wave guide or an electron is on its orbit inside an atom, the wave can be seen as quasi-plane wave. In the case of quasi-plane wave, the retarded wave and the advanced wave are the same, each has the half value of the total field. Hence, we can take away the subscript of the wave function. In this case we have

$$\Psi_A = \Psi_B = \Psi \tag{55}$$

Here Ψ_A is the retarded wave send from point A which is an emitter. Ψ_B is the advanced wave send from B which is an absorber. Ψ is either Ψ_A or Ψ_B.

$$\boldsymbol{J} = \frac{1}{2\mu}(\Psi^*(\hat{\boldsymbol{p}}\Psi) + (\hat{\boldsymbol{p}}\Psi)^*\Psi) = \frac{1}{\mu}\Re[\Psi^*\hat{\boldsymbol{p}}\Psi] \tag{56}$$

where $\Re$ is take the real part for a complex number. This example tell us even we started from the probability flow intensity $\boldsymbol{J}$

$$\boldsymbol{p} \equiv \mu \int_V \boldsymbol{J} dV = \Re(\int_V (\Psi^*\hat{\boldsymbol{p}}\Psi) dV) \tag{57}$$

Hence, $\int_V (\Psi^*\hat{\boldsymbol{p}}\Psi) dV$ can be seen as complex momentum.

In quantum mechanics, the average of momentum of the particle is defined as,

$$< \boldsymbol{p} > \equiv \int_V (\Psi^*\hat{\boldsymbol{p}}\Psi) dV \tag{58}$$

It is noticed that $< \boldsymbol{p} >$ is not only the average of the momentum of the particle, it is the a complex value of the momentum of the particle. Take the real part it is the momentum of the particle, i.e.,

$$\boldsymbol{p} = \Re(< \boldsymbol{p} >) = \Re(\int_V (\Psi^*\hat{\boldsymbol{p}}\Psi) dV) \tag{59}$$

VIII. CONCLUSION

This author notice the fact that the mutual energy principle can be derived from the Schrödinger equation and Schrödinger equation can also be derived from mutual energy principle, however, this two formula as axioms are not equivalent. The mutual energy principle can only derive a pair of Schrödinger equations one corresponding to the retarded wave and one corresponding the advanced wave. The two wave have to be synchronized. This solution is the action and reaction pair solution. In other side the Schrödinger equation can obtained the retarded wave as the solution.

After this author introduced the the self-energy principle, the solution of the Schrödinger equation looks like a probability wave, because there are time-reverse wave which cancel the energy flow of the self-energy flow of the Schrödinger equation. Self-energy flow principle guarantees that the mutual energy flow are actually the energy flow. The word "mutual" can be dropped.

This author has introduced the mutual energy principle and self -energy principle for photon and electromagnetic fields. In this theory, the photon is nothing else but the mutual energy flow. This author would like to bring the same theory to quantum mechanics, but there was a difficulty because the Schrödinger equation has no advanced wave solution. In this article the negative distance is introduced which make the advanced wave solution for Schrödinger equation become possible.

After this the mutual energy principle, self-energy principle, mutual energy theory, mutual flow theorem is introduced to quantum mechanics. The mutual energy flow is consist of the retarded wave and the advanced wave which satisfy the Schrödinger equation. The energy flow of the particle is proved as the mutual energy flow.

In the end the author also proved that the energy of the particle is the average of the Hamilton operator. The particle's momentum is the real part of the average of the momentum operator.

The mutual energy flow theorem will be applied to introduce the Huygens principle which will further introduce a updated path integral which will be discussed in a separated article.

APPENDIX

proof:

$$(\Psi_b(\boldsymbol{r},t), L\Psi_a(\boldsymbol{r},t))_V + (L\Psi_b(\boldsymbol{r},t), \Psi_a(\boldsymbol{r},t))_V$$

$$= \int\limits_{t=\infty}^{\infty} \iiint\limits_V (\Psi_b^*(-\frac{\hbar}{2\mu i}\nabla^2 + \frac{1}{i}V(\boldsymbol{r},t) - \hbar\frac{\partial}{\partial t})\Psi_a$$

$$+((-\frac{\hbar}{2\mu i}\nabla^2 + \frac{1}{i}V(\boldsymbol{r},t) - \hbar\frac{\partial}{\partial t})\Psi_b)^*\Psi_a)dVdt$$

$$= \int\limits_{t=\infty}^{\infty} \iiint\limits_V (\Psi_b^*(-\frac{\hbar}{2\mu i}\nabla^2\Psi_a) + (-\frac{\hbar}{2\mu i}\nabla^2)^*\Psi_b^*\Psi_a$$

$$+\Psi_b^*\frac{1}{i}V(\boldsymbol{r},t)\Psi_a + \Psi_b^*(\frac{1}{i})^*V(\boldsymbol{r},t)\Psi_a$$

$$\Psi_b^*(-\hbar)(\frac{\partial}{\partial t}\Psi_a) + (-\hbar)(\frac{\partial}{\partial t}\Psi_b^*)\Psi_a)dVdt$$

$$= \int\limits_{t=\infty}^{\infty} \iiint\limits_V ((-\frac{\hbar}{2\mu i})(\Psi_b^*(\nabla^2\Psi_a) - (\nabla^2\Psi_b^*)\Psi_a)$$

$$+\frac{1}{i}(\Psi_b^*V(\boldsymbol{r},t)\Psi_a - \Psi_b^*V(\boldsymbol{r},t)\Psi_a)$$

$$+(-\hbar)(\Psi_b^*\frac{\partial}{\partial t}\Psi_a + \frac{\partial}{\partial t}\Psi_b^*\Psi_a))dVdt \tag{60}$$

The first term inside are,

$$(-\frac{\hbar}{2\mu i})(\Psi_b^*\nabla^2\Psi_a - \nabla^2\Psi_b^*\Psi_a)$$

$$= (-\frac{\hbar}{2\mu i})(\Psi_b^*\nabla^2\Psi_a + \nabla\Psi_b^*\cdot\nabla\Psi_a - \nabla\Psi_b^*\cdot\nabla\Psi_a + \nabla^2\Psi_b^*\Psi_a)$$

$$= -\nabla\cdot(\frac{\hbar}{2\mu i}(\Psi_b^*\nabla\Psi_a - \nabla\Psi_b^*\Psi_a))$$

$$-\boldsymbol{J}_{ab} \tag{61}$$

The second term,

$$\frac{1}{i}(\Psi_b^*V(\boldsymbol{r},t)\Psi_a - \Psi_b^*V(\boldsymbol{r},t)\Psi_a) = 0 \tag{62}$$

The third term,

$$(-\hbar)(\Psi_b^*\frac{\partial}{\partial t}\Psi_a + \frac{\partial}{\partial t}\Psi_b^*\Psi_a)$$

$$= -\frac{\partial}{\partial t}(\hbar \Psi_b^* \Psi_a) = -u \tag{63}$$

Proof finished.

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