

**On the Polchinski's equation concerning the exact renormalization group.
Mathematical connections with some sectors of Ramanujan mathematics, String
Theory and Particle Physics**

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Abstract

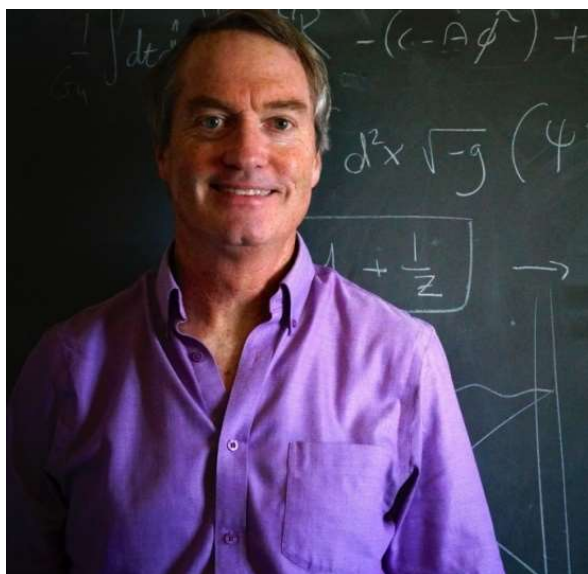
In the present research thesis, we have obtained various and interesting new possible mathematical connections concerning the exact renormalization group and some sectors of Ramanujan mathematics, String Theory and Particle Physics

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This work was dedicated to the memory of the theoretical physicist Joseph Polchinski (1954-2018)

From:

<https://www.news.ucsb.edu/2014/014133/joseph-polchinski-named-59th-annual-faculty-research-lecturer>



From: <http://ramanujan.sirinudi.org/>

Polchinski equation

From:

Exact Renormalization Group Equations.

An Introductory Review.

C. Bagnuls* and C. Bervillier†

C. E. Saclay, F91191 Gif-sur-Yvette Cedex, France

February 1, 2008

2.5.2 Polchinski's equation

With a view to study field theory, Polchinski [12] has derived his own smooth cutoff version of the ERGE (see also section 2.10.2). A general ultraviolet (UV) cutoff function $K(p^2/\Lambda^2)$ is introduced (we momentarily restore the dimensions) with the property that it vanishes rapidly when $p > \Lambda$. (Several kinds of explicit functions K may be chosen, the sharp cutoff would be introduced with the Heaviside step function $K(x) = \Theta(1 - x)$.) The Euclidean action reads:

$$S[\phi] \equiv \frac{1}{2} \int_p \phi_p \phi_{-p} p^2 K^{-1}(p^2/\Lambda^2) + S_{\text{int}}[\phi] \quad (32)$$

Compared to [12], the “mass” term has been incorporated into $S_{\text{int}}[\phi]$, this does not corrupt in any way the eventual analysis of the massive theory because the RG naturally generates quadratic terms in ϕ and the massive or massless character of the (field) theory is not defined at the level of eq. (32) but a posteriori in the process of defining the continuum limit (modern conception of the renormalization of field theory, see sections 2.10.1 and 3.4.1).

Polchinski's ERGE is obtained from the requirement that the coarsening step (step 1 of section 2.3) leaves the generating functional $Z[J]$ [eq. (4)] invariant. The difficulty of dealing with an external source is circumvented by imposing that $J(p) = 0$ for $p > \Lambda$. As in [1], the derivation relies upon the writing of an (ad hoc) expression under the form of a complete derivative with respect to the field in such a way as to impose $dZ[J]/d\Lambda = 0$. The original form of Polchinski's equation accounts only for the step 1 and reads (for more details on this derivation, see for example [36, 37]):

$$\Lambda \frac{dS_{\text{int}}}{d\Lambda} = \frac{1}{2} \int_p p^{-2} \Lambda \frac{dK}{d\Lambda} \left(\frac{\delta S_{\text{int}}}{\delta \phi_p} \frac{\delta S_{\text{int}}}{\delta \phi_{-p}} - \frac{\delta^2 S_{\text{int}}}{\delta \phi_p \delta \phi_{-p}} \right) \quad (33)$$

then, considering the rescaling (step 2) and the complete action, the Polchinski ERGE is (see for example [38]):

$$\dot{S} = \mathcal{G}_{\text{dil}} S - \int_p K'(p^2) \left(\frac{\delta^2 S}{\delta \phi_p \delta \phi_{-p}} - \frac{\delta S}{\delta \phi_p} \frac{\delta S}{\delta \phi_{-p}} + \frac{2p^2}{K(p^2)} \phi_p \frac{\delta S}{\delta \phi_p} \right) \quad (34)$$

in which all quantities are dimensionless and $K'(p^2)$ stands for $dK(p^2)/dp^2$.

Let us mention that one easily arrives at eq. (33) using the observation that the two following functionals:

$$Z[J] = \int \mathcal{D}\phi \exp \left\{ -\frac{1}{2} \phi \cdot \Lambda^{-1} \cdot \phi - S[\phi] + J \cdot \phi \right\} \quad (35)$$

and

$$Z'[J] = \int \mathcal{D}\phi \exp \left\{ -\frac{1}{2} \phi_1 \cdot \Delta_1^{-1} \cdot \phi_1 - \frac{1}{2} \phi_2 \cdot \Delta_2^{-1} \cdot \phi_2 - S[\phi_1 + \phi_2] + J \cdot (\phi_1 + \phi_2) \right\} \quad (36)$$

are equivalent (up to a multiplicative factor) provided that $\Delta = \Delta_1 + \Delta_2$ and $\phi = \phi_1 + \phi_2$ (see appendix¹⁰ 10 of [5] and also [39]).

2.6.2 Smooth cutoff version

We adopt notations which are close to the writing of (32) and we consider the Wilson effective action with an “additive” [43] IR cutoff Λ such that:

$$S_\Lambda[\phi] \equiv \frac{1}{2} \int_p \phi_p \phi_{-p} C^{-1}(p, \Lambda) + S_{\Lambda_0}[\phi] \quad (38)$$

in which $C(p, \Lambda)$ is an additive infrared cutoff function which is small for $p < \Lambda$ (tending to zero as $p \rightarrow 0$) and $p^2 C(p, \Lambda)$ should be large for $p > \Lambda$ [22]. Due to the additive character of the cutoff function, $S_{\Lambda_0}[\phi]$ is the entire action (involving the kinetic term contrary to eq. (32) and to [33] where the cutoff function was chosen multiplicative). In this section, because C is naturally dimensioned¹⁴ [contrary to K in (32)], all the dimensions are implicitly restored in order to keep the same writing as in the original papers. The ultra-violet regularization is provided by Λ_0 and needs not to be introduced explicitly (see [33] and below). The Legendre transformation is defined as:

$$\Gamma[\Phi] + \frac{1}{2} \int_p \Phi_p \Phi_{-p} C^{-1}(p, \Lambda) = -W[J] + J \cdot \Phi$$

in which $W[J]$ and Φ are defined as usual [see eq (7)] from (38).

Then the ERGE reads:

$$\dot{\Gamma} = \mathcal{G}_{\text{dil}} \Gamma + \frac{1}{2} \int_p \frac{1}{C} \Lambda \frac{\partial C}{\partial \Lambda} (1 + C \Gamma_{p,-p})^{-1} \quad (39)$$

3.2.3 Critical exponents in three dimensions

Once the fixed point has been located, the first idea that generally occurs to someone is to calculate the critical exponents. There is only one exponent to calculate (e.g. ν) since $\eta = 0$. The other exponents are deduced from ν by the scaling relations (e.g. $\gamma - 2\nu$)²⁹. The best way to calculate the exponents is to linearize the flow equation in the vicinity of the fixed point and to look at the eigenvalue problem. One obtains as in the case of the Gaussian fixed point a linear second order differential equation. For example with the Wilson (or Polchinski) version (59), setting $V(\varphi, t) = V^* + e^{\lambda t} v(\varphi)$, one obtains the eigenvalue equation:

$$v'' + \left[\left(1 - \frac{d}{2}\right) \varphi - 2V'' \right] v' + (d - \lambda) v = 0 \quad (73)$$

As Morris explains in the case of eq. (65) [22], “(\dots) again one expects solutions to (73) labelled by two parameters, however by linearity one can choose $v(0) = 1$ (arbitrary normalization of the eigenvectors) and by symmetry $v'(0) = 0$ (or by asymmetry and linearity: $v(0) = 0$ and $v'(0) = 1$). Thus the solutions are unique, given λ . Now for large φ , $v(\varphi)$ is generically a superposition³⁰ of $v_1 \sim \varphi^{2(d-\lambda)/(d+2)}$ and of $v_2 \sim \exp(\frac{d+2}{4}\varphi^2)$. Requiring zero coefficient for the latter restricts the allowed values of λ to a discrete set”.

The reason for which the exponential must be eliminated is the same as previously mentioned in section 24 to discard nonpolynomial forms of the potential.

For $d = 3$, the Wilson-Fisher fixed point possesses just one positive eigenvalue λ_1 corresponding to the correlation length exponent ($\nu = 1/\lambda_1$) and infinitely many negative eigenvalues. In the symmetric case, the less negative λ_2 corresponds to the first correction-to-scaling exponent $\omega = -\lambda_2$ while λ_3 provides us with the second $\omega_2 = -\lambda_3$ and so on. In the asymmetric case, which is generally not considered (see however [38]), one may also associate the first negative eigenvalue λ_1^{as} to the first non-symmetric correction-to-scaling exponent $\omega_5 = -\lambda_1^{\text{as}}$ (the subscript “5” refers to the ϕ^5 interaction term in the action responsible for this kind of correction, see [58]).

The smooth cutoff Legendre version and the derivative expansion By choosing a power-law cutoff function $\tilde{C}(q^2) = q^{2k}$ in eq. (39), one is sure that the derivative expansion will preserve the reparametrization invariance [33, 22] and that the exponent η will be unambiguously defined.

Let us expand the Legendre (effective) action $\Gamma[\Phi]$ as follows:

$$\Gamma[\Phi] = \int d^d x \left\{ U(\varphi, t) + \frac{1}{2} Z(\varphi, t) (\partial_\mu \Phi)^2 \right\}$$

in which φ is independent on x .

For $d = 3$ and $k = 1$, the first order of the derivative expansion yields (after a long but straightforward computation) the following two coupled equations for U and Z [22]:

$$\begin{aligned} \dot{U} &= -\frac{1 - \eta/4}{\sqrt{Z}\sqrt{U'' + 2\sqrt{Z}}} + 3U - \frac{1}{2}(1 + \eta)\varphi U' \\ \dot{Z} &= -\frac{1}{2}(1 + \eta)\varphi Z' - \eta Z + \left(1 - \frac{\eta}{4}\right) \left\{ \frac{1}{48} \frac{24ZZ'' - 19(Z')^2}{Z^{3/2}(U'' + 2\sqrt{Z})^{3/2}} \right. \\ &\quad \left. - \frac{1}{48} \frac{58U'''Z'\sqrt{Z} + 57(Z')^2 + (Z''')^2Z}{Z(U'' + 2\sqrt{Z})^{5/2}} + \frac{5}{12} \frac{(U''')^2Z + 2U'''Z'\sqrt{Z} + (Z')^2}{\sqrt{Z}(U'' + 2\sqrt{Z})^{7/2}} \right\} \end{aligned} \quad (87)$$

As expected, the search for a non trivial fixed point solution for these equations (a solution which is nonsingular up to $\varphi \rightarrow \infty$) produces a unique solution with an unambiguously defined η [22]:

$$\eta = 0.05393 \quad (88)$$

The linearization about this fixed point yields the eigenvalues:

$$\nu = 0.6181 \quad (89)$$

$$\omega = 0.8975 \quad (90)$$

From:

hep-ph 9403340 - CERN-TH.7203/94 - SHEP 93/94-16

Derivative Expansion of the Exact Renormalization Group

Tim R. Morris - CERN TH-Division - CH-1211 Geneva 23

allowed values of λ to a discrete set. We found just one positive eigenvalue, which yields the correlation length critical exponent[1] through $\nu = 1/\lambda$, and determined the exponent of the first correction to scaling $\omega = -\lambda$, where λ is the least negative eigenvalue. The corresponding solutions $v(\varphi)$ are displayed in figs.3

Approx [?]	η	ν	ω
$O(\partial^0)$	0	.6604	.6285
$O(\partial^2)$.05393	.6181	.8975
Worlds Best	.035(3)	.631(2)	.80(4)

We have that, for $1/\lambda = \nu = \frac{1}{0.6181} = 1,6178611875101116324219381977026$

Now, form eq. (73):

$$v'' + \left[\left(1 - \frac{d}{2}\right) \varphi - 2V^*t \right] v' + (d - \lambda) v = 0$$

we obtain:

$$\text{Pi}^{\left(\left(\left(2(3-1.61786118751)/5\right)\right)\right)}$$

Input interpretation:
 $\pi^{2 \times (3 - 1.61786118751) / 5}$
[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.88300751760...

1.8830075... = v_1

$$\exp(5/4 * \text{Pi}^2)$$

Input:
 $\exp\left(\frac{5}{4} \pi^2\right)$
[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Exact result:

$e^{(5\pi^2)/4}$

Decimal approximation:

More digits

227978.2483385697197729890597740242410899825545161180622366...

[Open code](#)

227978.248... = v_2

Thence, from:

$$v'' + \left[\left(1 - \frac{d}{2} \right) \varphi - 2V^{*'} \right] v' + (d - \lambda) v = 0$$

we obtain, for

$v'' = \exp(5/4 * \pi^2)$, $v' = 1.88300751760$, $d = 3$, $\lambda = 1.617861187510$, $V^{*' = 1}$ and $v = 1$:

$$\exp(5/4 * \pi^2) + (((1-3/2)*\pi-2))) * 1.88300751760 + (3-1.6178611875101)$$

Input interpretation:

$$\exp\left(\frac{5}{4} \pi^2\right) + \left(\left(1 - \frac{3}{2}\right)\pi - 2\right) \times 1.88300751760 + (3 - 1.6178611875101)$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

227972.90664105504...

227972.906...

From this result of eigenvalue equation, we obtain:

$$1 / ((((((\exp(5/4 * \pi^2) + (((1-3/2)*\pi-2))) * 1.88300751760 + (3-1.6178611875101))))))))))$$

Input interpretation:

$$\frac{1}{\left(\exp\left(\frac{5}{4} \pi^2\right) + \left(\left(1 - \frac{3}{2}\right)\pi - 2\right)\right) \times 1.88300751760 + (3 - 1.6178611875101)}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

2.32948510438... × 10⁻⁶

2.32948510438 * 10⁻⁶

Then, performing the 27th root, we obtain:

$$\left(\frac{1}{\left(\left(\left(\left(\left(\exp(5/4 * \pi^2) + (((1-3/2)*\pi-2))) * 1.88300751760 + (3-1.6178611875101)\right)\right)\right)\right)\right)\right)} \right)^{1/27}$$

Input interpretation:

$$\sqrt[27]{\frac{1}{\left(\exp\left(\frac{5}{4}\pi^2\right) + \left(1 - \frac{3}{2}\right)\pi - 2\right) \times 1.88300751760 + (3 - 1.6178611875101)}}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.6185573869314...

0.6185573...

And:

$$1/\left(\left(\left(\left(\left(\left(\left(1 / \left(\left(\left(\exp(5/4*\pi^2) + \left((1-3/2)*\pi-2\right)\right) * 1.88300751760 + (3-1.6178611875101)\right)\right)\right)\right)\right)\right)\right)^{1/27}\right)$$

Input interpretation:

$$\sqrt[27]{\frac{1}{\left(\exp\left(\frac{5}{4}\pi^2\right) + \left(1 - \frac{3}{2}\right)\pi - 2\right) \times 1.88300751760 + (3 - 1.6178611875101)}}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.616664873991...

1.6166648...

The result 0.6185573... is very near to the following eigenvalue, above mentioned:

The linearization about this fixed point yields the eigenvalues:

$$\nu = 0.6181 \tag{89}$$

$$\omega = 0.8975 \tag{90}$$

precisely to the $\nu = 0.6181$. We want to highlight that $\lambda = \frac{1}{\nu} = \frac{1}{0.6181} = 1.61786118751 \dots$, result very near to the golden ratio 1.61803398...!

We have also:

$$-233 + 10^4 \left(\left(\left(\left(\left(\left(1 / \left(\left(\left(\exp(5/4*\pi^2) + \left((1-3/2)*\pi-2\right)\right) * 1.88300751760 + (3-1.6178611875101)\right)\right)\right)\right)\right)\right)\right)^{1/8}$$

Input interpretation:

$$-233 + 10^4 \sqrt[8]{\frac{1}{\left(\exp\left(\frac{5}{4}\pi^2\right) + \left(1 - \frac{3}{2}\right)\pi - 2\right) \times 1.88300751760 + (3 - 1.6178611875101)}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1743.548576509...

1743.54857... result very near to the mass of candidate “glueball” f_0 (1710) scalar meson

Now, we have that:

With a view to study field theory, Polchinski [12] has derived his own smooth cutoff version of the ERGE (see also section 2.10.2). A general ultraviolet (UV) cutoff function $K(p^2/\Lambda^2)$ is introduced (we momentarily restore the dimensions) with the property that it vanishes rapidly when $p > \Lambda$. (Several kinds of explicit functions K may be chosen, the sharp cutoff would be introduced with the Heaviside step function $K(x) = \Theta(1 - x)$.) The Euclidean action reads:

$$S[\phi] \equiv \frac{1}{2} \int_p \phi_p \phi_{-p} p^2 K^{-1}(p^2/\Lambda^2) + S_{\text{int}}[\phi]$$

For $\phi = 5$, $p = 0.6181$, $\Lambda = 1$, $L = -1$, $d = 3$, $x = \pi$, $\lambda > 0 = 1.61786\dots$, $S_{\text{int}}[\phi] > 0 = 3/2$ and $K = 0.988136\dots$, i.e.

$$H(x) \approx \frac{1}{2} + \frac{1}{2} \tanh kx = \frac{1}{1 + e^{-2kx}},$$

$$= 0.988136\dots$$

we obtain:

1/2 integrate $[25 - 0.6181^2 * (1/0.998136) * (0.6181^2) + 3/2]x$

Input interpretation:

$$\frac{1}{2} \int \left(25 - 0.6181^2 \times \frac{1}{0.998136} \times 0.6181^2 + \frac{3}{2} \right) x dx$$

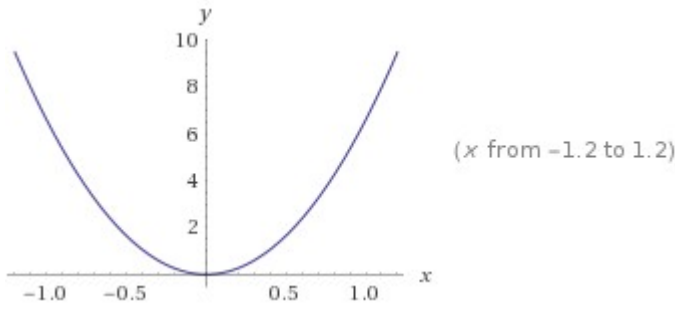
[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

6.58844 x^2

Plot:



and for $x = 1$, we obtain: 6.58844

Furthermore, we have that:

$$(6.58844 \times 1^2)^{\tan(1/4)}$$

Input interpretation:

$$(6.58844 \times 1^2)^{\tan(1/4)}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

1.618339077156827626582885446303228682684186726203018135705...

1.6183390771568... a very good approximation to the value of golden ratio

1.61803398...

Series representations:

- More

$$(6.58844 \times 1^2)^{\tan(1/4)} = 6.58844^{i \sum_{k=-\infty}^{\infty} (-1)^k e^{(i k)/2} \operatorname{sgn}(k)}$$

[Open code](#)

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$$(6.58844 \times 1^2)^{\tan(1/4)} = 6.58844^{i+2i \sum_{k=1}^{\infty} (-1)^k q^{2k}} \text{ for } q = e^{i/4}$$

[Open code](#)

$$(6.58844 \times 1^2)^{\tan(1/4)} = 6.58844^{2 \times \sum_{k=1}^{\infty} 1 / \left(-\frac{1}{4} + (1-2k)^2 \pi^2 \right)}$$

[Open code](#)

- $\operatorname{sgn}(x)$ is the sign of x
- i is the imaginary unit

Integral representations:

$$(6.58844 \times 1^2)^{\tan(1/4)} = 6.58844^{b^{1/4} \operatorname{sech}^2(t) dt}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$(6.58844 \times 1^2)^{\tan(1/4)} = 6.58844^{2/\pi \int_0^{\infty} (-1+t^2)^{-1/4} dt}$$

[Open code](#)

- $\sec(x)$ is the secant function

And:

$$1 / (6.58844 \times 1^2)^{\tan(1/4)}$$

Input interpretation:

$$\frac{1}{(6.58844 \times 1^2)^{\tan(1/4)}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

0.617917477316833936036951407946939865955650162088712739717...

0.6179174773... result practically equal to the previous eigenvalue 0.6181

Series representations:

More

$$\frac{1}{(6.58844 \times 1^2)^{\tan(1/4)}} = 6.58844^{-i \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k} \right)} \text{ for } q = e^{i/4}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{(6.58844 \times 1^2)^{\tan(1/4)}} = 6.58844^{-i \sum_{k=-\infty}^{\infty} (-1)^k e^{(i k)/2} \operatorname{sgn}(k)}$$

[Open code](#)

$$\frac{1}{(6.58844 \times 1^2)^{\tan(1/4)}} = 6.58844^{-2 \times \sum_{k=1}^{\infty} 1 / \left(-\frac{1}{4} + (1-2k)^2 \pi^2 \right)}$$

[Open code](#)

Integral representations:

$$\frac{1}{(6.58844 \times 1^2)^{\tan(1/4)}} = e^{-1.88532 \int_0^{1/4} \sec^2(t) dt}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{(6.58844 \times 1^2)^{\tan(1/4)}} = 6.58844^{-2/\pi} \int_0^{\infty} \frac{(-1 + \sqrt{t})^{-2/\pi}}{(-1+t^2)} dt$$

For $x = (27^2 - 21^2 - 5^2) = 263$

6.58844 $(27^2 - 21^2 - 5^2)$

Input interpretation:

6.58844 $(27^2 - 21^2 - 5^2)$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1732.75972

1732.75972

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

From Wikipedia:

L'equazione di Polchinski coinvolge un cut-off di regolarizzazione ultravioletto liscio. L'idea di base è un miglioramento dell'equazione di Wilson: anziché un cut-off netto, si utilizza un cut-off liscio. Essenzialmente, i contributi dai momenti più grandi di Λ sono pesantemente soppressi. Il cut-off liscio tuttavia consente di ottenere un'equazione differenziale funzionale nella scala Λ . Come nel caso dell'equazione di Wilson si ha un funzionale azione diverso per ogni energia di cut-off Λ . Ciascuna di queste azioni descrive esattamente lo stesso modello, questo significa che le funzioni di partizione devono coincidere esattamente.

In altre parole, per un campo scalare reale

$$Z_\Lambda[J] = \int \mathcal{D}\phi \exp(-S_\Lambda[\phi] + J \cdot \phi) = \int \mathcal{D}\phi \exp\left(-\frac{1}{2}\phi \cdot R_\Lambda \cdot \phi - S_{\text{int}\Lambda}[\phi] + J \cdot \phi\right) \quad (\text{a})$$

e Z_Λ è realmente indipendente da Λ . Qui stata utilizzato la notazione di deWitt. Inoltre è stata separato l'azione nuda S_Λ nella parte cinetica quadratica e nella parte interagente $S_{\text{int}\Lambda}$. Questa separazione non è banale. Infatti la parte "interagente" può contenere termini cinetici quadratici. Anzi, se la funzione d'onda viene rinormalizzata, li contiene sicuramente. Questo può essere ridotto introducendo un riscaldamento dei campi. R_Λ è una funzione del momento p e il secondo termine nell'esponente è

$$\frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \bar{\phi}^*(p) R_\Lambda(p) \bar{\phi}(p) \quad (b)$$

quando viene espanso. Quando $p \ll \Lambda$, $R_\Lambda(p)/p^2$ è essenzialmente uguale a 1. Quando $p \gg \Lambda$, $R_\Lambda(p)/p^2$ diventa molto grande e tende all'infinito. $R_\Lambda(p)/p^2$ è sempre maggiore o uguale a 1 ed è liscio. Sostanzialmente, il suo effetto è di lasciare le fluttuazioni con momento minore di Λ inalterati, ma sopprime fortemente i contributi dalle fluttuazioni con momento maggiore di Λ . Questo è un netto miglioramento rispetto all'approccio di Wilson.

Ricordiamo che:

Nella teoria quantistica dei campi un **campo scalare** è associato a particelle di spin 0, come i mesoni. Il campo scalare può avere valori reali o complessi. Campi scalari complessi rappresentano particelle cariche. Un esempio di campo scalare è quello relativo all'equazione di Klein-Gordon

In fisica teorica, il **cutoff** (o **cut-off**) oppure **valore di taglio** è un valore di soglia, massimo o minimo, associato ad una grandezza fisica, quale energia, impulso o lunghezza, e tale per cui oggetti con valori di queste grandezze fisiche superiori o inferiori al cut-off vengono ignorati. È di solito rappresentata all'interno di una determinata scala di energia o di lunghezze, come unità di Planck.

From (a), with the above result and $J \cdot \phi = 5$, we obtain:

Integrate $[\exp(-6.58844+5)]x$

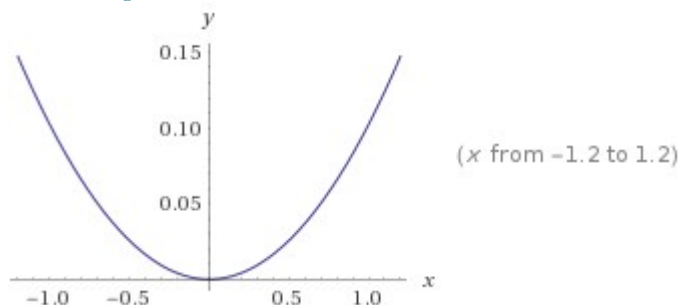
Indefinite integral:

$$\int \exp(-6.58844 + 5) x dx = 0.102122 x^2 + \text{constant}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Plot of the integral:



For $x = 10^2$, we obtain:

$$0.102122 (10^2)^2$$

Input interpretation:
 $0.102122 (10^2)^2$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

1021.22

[Open code](#)

1021.22 a result that is very near to the value of rest mass of Phi meson 1019.445 \approx 1020

For $x^2 = ((27*4+1)e^2)/(27+21+2) = 16.108142295668\dots$; $x = 4.0134950225\dots$, we obtain:

$$0.102122 ((27*4+1)e^2)/(27+21+2)$$

Input interpretation:

$$0.102122 \times \frac{(27 \times 4 + 1) e^2}{27 + 21 + 2}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

Fewer digits

More digits

1.644995707518290980261392054620873266416768406997052740298...

$$1.6449957\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$\frac{0.102122 ((27 \times 4 + 1) e^2)}{27 + 21 + 2} = 0.222626 \sum_{k=0}^{\infty} \frac{2^k}{k!}$$

[Open code](#)

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$$\frac{0.102122 ((27 \times 4 + 1) e^2)}{27 + 21 + 2} = 0.222626 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^2$$

[Open code](#)

$$\frac{0.102122 ((27 \times 4 + 1) e^2)}{27 + 21 + 2} = \frac{0.222626}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^2}$$

[Open code](#)

• $n!$ is the factorial function

And we note that:

$$\sqrt{\left(\left(\left(\left(6 \cdot \left(\frac{0.102122 \cdot (27 \cdot 4 + 1) e^2}{27 + 21 + 2}\right)\right)\right)\right)\right)}$$

Input interpretation:

$$\sqrt{6 \times \frac{0.102122 (27 \times 4 + 1) e^2}{27 + 21 + 2}}$$

[Open code](#)

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Result:

• Fewer digits

• More digits

3.141651515542382030598033095438593527379282018058705566768...

3.1416515155...

$$2\sqrt{\left(\left(\left(\left(6 \cdot \left(\frac{0.102122 \cdot (27 \cdot 4 + 1) e^2}{27 + 21 + 2}\right)\right)\right)\right)\right)}$$

Input interpretation:

$$2\sqrt{6 \times \frac{0.102122 (27 \times 4 + 1) e^2}{27 + 21 + 2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• Fewer digits

• More digits

6.283303031084764061196066190877187054758564036117411133536...

6.283303031... $\approx 2\pi$

Series representations:

• More

$$2\sqrt{\frac{6 (0.102122 (27 \times 4 + 1) e^2)}{27 + 21 + 2}} = 2\sqrt{-1 + 1.33576 e^2} \sum_{k=0}^{\infty} (-1 + 1.33576 e^2)^{-k} \binom{\frac{1}{2}}{k}$$

[Open code](#)

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$$2\sqrt{\frac{6 (0.102122 (27 \times 4 + 1) e^2)}{27 + 21 + 2}} = 2\sqrt{-1 + 1.33576 e^2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 1.33576 e^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$2 \sqrt{\frac{6(0.102122((27 \times 4 + 1)e^2))}{27 + 21 + 2}} = 2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.33576 e^2 - z_0)^k z_0^{-k}}{k!}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

Open code

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- \mathbb{R} is the set of real numbers

With regard the integral (b), we have, for

$$p \ll \Lambda, R_\Lambda(p)/p^2 \text{ equal to } 1, p = 0.6181 \text{ and } d = 3$$

$$\frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \bar{\phi}^*(p) R_\Lambda(p) \bar{\phi}(p)$$

$$1/2 \text{ integrate } [0.6181/((2\text{Pi})^3) * 5 * 0.6181 * 0.38204761 * 5 * 0.6181]d^3x$$

After a bit calculus, we have:

$$1/2 \text{ integrate } ((((((0.6181))/((2\text{Pi})^3)))) * 3.6490094076678025))))x$$

Input interpretation:

$$\frac{1}{2} \int \left(\frac{0.6181}{(2\pi)^3} \times 3.6490094076678025 \right) x dx$$

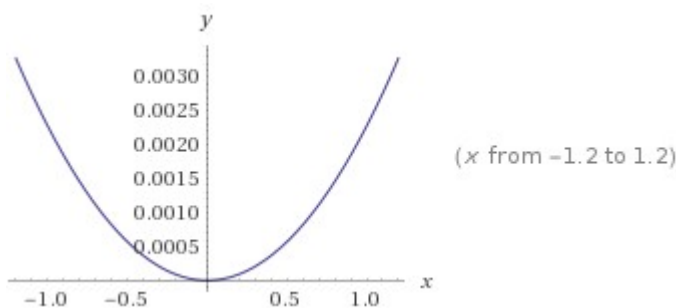
Open code

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Result:

$$0.00227318 x^2$$

Plot:



$$0.00227318$$

For $x = 27$, we obtain:

$$0.00227318 \times 27^2$$

Input interpretation:

$$0.00227318 \times 27^2$$

[Open code](#)

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Result:

$$1.65714822$$

[Open code](#)

1.65714822.. is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

This result (less 1) is very near to the value 0.6557:

It is however interesting to notice that the LPA estimates are not unique but depend on the equation studied. Hence with the Legendre version (65) it comes [22]:

$$\nu = 0.6604, \quad \omega = 0.6285$$

which is closer to the "best" values. And the closest to the "best" are obtained from the Wilson (or Polchinski) version [38, 30, 34]:

$$\nu = 0.6496, \quad \omega = 0.6557$$

Note that $27^2 = 729 = 9^3$ (see Ramanujan's sum of cubes $6^3 + 8^3 = 9^3 - 1^3 = 728$)

Furthermore, we have that:

$$((0.00227318) (10^3)^2) + 13$$

Input interpretation:

$$0.00227318 (10^3)^2 + 13$$

[Open code](#)

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Result:

$$2286.18$$

[Open code](#)

2286.18 result very near to the rest mass of charmed Lambda baryon 2286.46

We have also that:

$$0.00227318 (5^2 \times 35)^2$$

Input interpretation:

$$0.00227318 (5^2 \times 35)^2$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

1740.4034375

[Open code](#)

1740.4034 result very near to the mass of the $f_0(1710)$ scalar meson candidate “glueball”

For $x = (27^2 + 12^2) = 873$

0.00227318 $(27^2 + 12^2)^2$

Input interpretation:

0.00227318 $(27^2 + 12^2)^2$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

1732.45640022

[Open code](#)

1732.4564... very near to the mass of $f_0(1710)$ scalar meson candidate “glueball”

Now, we have that:

$$\dot{V} = -\frac{1}{\sqrt{2+V''}} - \frac{N-1}{\sqrt{2+V'/\varphi}} + 3V - \frac{1}{2}\varphi V' \quad (66)$$

For

$-1/(\sqrt{2+21}) - 12/(\sqrt{2+13/5}) + 24 - (13*5)/2$

Input:

$$-\frac{1}{\sqrt{2+21}} - \frac{12}{\sqrt{2+\frac{13}{5}}} + 24 - \frac{13 \times 5}{2}$$

[Open code](#)

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Result:

$$-\frac{17}{2} - 12\sqrt{\frac{5}{23}} - \frac{1}{\sqrt{23}}$$

Decimal approximation:

More digits

• $-14.3035432634989573564942327882614830600469238829785904358...$

-14.3035...

We have also:

$$233+55+ (((-10^2(((((-1/(\sqrt{2+21})) - 12 /(\sqrt{2+13/5})) + 24 - (13*5)/2))))))$$

Input:

$$288 - 100 \left(-\frac{17}{2} - 12 \sqrt{\frac{5}{23}} - \frac{1}{\sqrt{23}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

1718.354326349895735649423278826148306004692388297859043582...

[Open code](#)

1718.3543...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

And:

$$\text{integrate } (((((-1/(\sqrt{2+21})) - 12 /(\sqrt{2+13/5})) + 24 - (13*5)/2))))x$$

Indefinite integral:

Approximate form

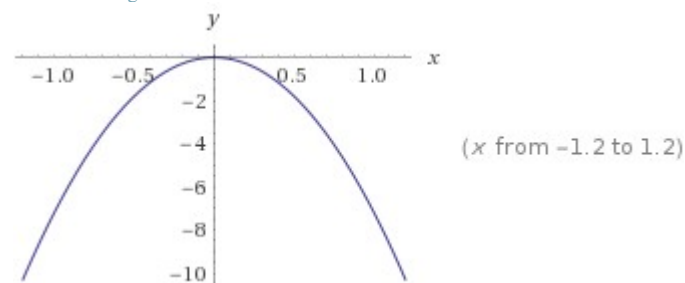
Step-by-step solution

$$\int \left(-\frac{1}{\sqrt{2+21}} - \frac{12}{\sqrt{2+\frac{13}{5}}} + 24 - \frac{13 \times 5}{2} \right) x dx =$$
$$-\frac{x^2}{2\sqrt{23}} - 6\sqrt{\frac{5}{23}}x^2 - \frac{17x^2}{4} + \text{constant}$$

[Open code](#)

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Plot of the integral:



That is equal to

Input:

$$\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)x^2$$

[Open code](#)

For $x^2 = -(2\pi^2)$, we obtain:

$$(-17/4 - 6 \text{sqrt}(5/23) - 1/(2 \text{sqrt}(23))) * -(2\pi^2)$$

Input:

$$\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right) \times (-1) (2\pi^2)$$

[Open code](#)

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Result:

$$-2\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)\pi^2$$

Decimal approximation:

More digits

141.1703135446013570550324606516903231443579000304602382480...

[Open code](#)

Property:

$$-2\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)\pi^2 \text{ is a transcendental number}$$

Series representations:

$$\left(\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-1)\right)2\pi^2 = \left(\pi^2 \left(2 + 17\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (23 - z_0)^k z_0^{-k}}{k!} + 24\sqrt{z_0}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(\frac{5}{23} - z_0\right)^{k_1} (23 - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}\right)\right) \Bigg/ \left(2\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (23 - z_0)^k z_0^{-k}}{k!}\right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

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$$\left(\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-1) \right) 2\pi^2 =$$

$$\left(\pi^2 \left(2 + 17 \exp\left(i\pi \left[\frac{\arg(23-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (23-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \right.$$

$$24 \exp\left(i\pi \left[\frac{\arg\left(\frac{5}{23}-x\right)}{2\pi} \right] \right) \exp\left(i\pi \left[\frac{\arg(23-x)}{2\pi} \right] \right) \sqrt{x}^{-2}$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(\frac{5}{23}-x\right)^{k_1} (23-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right) /$$

$$\left(2 \exp\left(i\pi \left[\frac{\arg(23-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (23-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\left(\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-1) \right) 2\pi^2 =$$

$$\left(\pi^2 \left(\frac{1}{z_0} \right)^{-1/2 \lfloor \arg(23-z_0)/(2\pi) \rfloor} z_0^{-1/2-1/2 \lfloor \arg(23-z_0)/(2\pi) \rfloor} \left(2 + 17 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg(23-z_0)/(2\pi) \rfloor} \right. \right.$$

$$z_0^{1/2+1/2 \lfloor \arg(23-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (23-z_0)^k z_0^{-k}}{k!} +$$

$$24 \left(\frac{1}{z_0} \right)^{1/2 \lfloor \arg\left(\frac{5}{23}-z_0\right)/(2\pi) \rfloor + 1/2 \lfloor \arg(23-z_0)/(2\pi) \rfloor}$$

$$z_0^{1+1/2 \lfloor \arg\left(\frac{5}{23}-z_0\right)/(2\pi) \rfloor + 1/2 \lfloor \arg(23-z_0)/(2\pi) \rfloor}$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(\frac{5}{23}-z_0\right)^{k_1} (23-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right) /$$

$$\left(2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (23-z_0)^k z_0^{-k}}{k!} \right)$$

$$(-17/4 - 6 \sqrt{5/23} - 1/(2 \sqrt{23})) * (\text{colog}(196884) 2\pi^2)$$

Input:

$$\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-\log(196884) \times 2\pi^2)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$-2 \left(-\frac{17}{4} - 6 \sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) \pi^2 \log(196884)$$

Decimal approximation:

More digits

1720.918355379215565400874778751762331361574505404965933105...

1720.91835... result very near to the mass of candidate “glueball” $f_0(1710)$ scalar meson

Series representations:

More

$$\left(-\frac{17}{4} - 6 \sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-\log(196884)) (2\pi^2) = \frac{1}{46} (391 + 2\sqrt{23} + 24\sqrt{115}) \pi^2 \left(\log(196884) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{196884}\right)^k}{k} \right)$$

[Open code](#)

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$$\left(-\frac{17}{4} - 6 \sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-\log(196884)) (2\pi^2) = \frac{1}{46} (391 + 2\sqrt{23} + 24\sqrt{115}) \pi^2 \left(2i\pi \left\lfloor \frac{\arg(196884 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (196884 - x)^k x^{-k}}{k} \right) \text{ for } x < 0$$

[Open code](#)

$$\left(-\frac{17}{4} - 6 \sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-\log(196884)) (2\pi^2) = \frac{1}{46} (391 + 2\sqrt{23} + 24\sqrt{115}) \pi^2 \left(\log(z_0) + \left\lfloor \frac{\arg(196884 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (196884 - z_0)^k z_0^{-k}}{k} \right)$$

[Open code](#)

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function

Integral representations:

$$\left(-\frac{17}{4} - 6 \sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}} \right) (-\log(196884)) (2\pi^2) = \frac{1}{46} (391 + 2\sqrt{23} + 24\sqrt{115}) \pi^2 \int_1^{196884} \frac{1}{t} dt$$

[Open code](#)

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$$\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2) = -\frac{1}{92}i(391 + 2\sqrt{23} + 24\sqrt{115})\pi$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{196883^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

[Open code](#)

- $\Gamma(x)$ is the gamma function

$$1/\left(\left(\left(\left(-17/4 - 6\sqrt{5/23} - 1/(2\sqrt{23})\right) * (\text{colog}(196884)2\text{Pi}^2)\right)\right)\right)^{1/16}$$

Input:

$$\frac{1}{\sqrt[16]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884) \times 2\pi^2)}}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{1}{\sqrt[8]{\pi} \sqrt[16]{2 \left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right) \log(196884)}}$$

Decimal approximation:

More digits

0.627718579554455890154269476191009044477545891267891656627...

[Open code](#)

0.627718579...

Series representations:

More

$$\frac{1}{\sqrt[16]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)}} =$$

$$\frac{\sqrt[16]{\frac{46}{391+2\sqrt{23}+24\sqrt{115}}}}{\sqrt[8]{\pi} \sqrt[16]{\log(196883) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{196883}\right)^k}{k}}}$$

[Open code](#)

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$$\frac{1}{\sqrt[16]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)}} =$$

$$\frac{1}{\sqrt[16]{2\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right)\sqrt[8]{\pi}}}$$

$$\sqrt[16]{2i\pi\left[\frac{\arg(196884-x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (196884-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

Open code

$$\frac{1}{\sqrt[16]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)}} =$$

$$\frac{\sqrt[16]{\frac{46}{391+2\sqrt{23}+24\sqrt{115}}}}{\sqrt[8]{\pi} \sqrt[16]{2i\pi\left[\frac{\arg(196884-x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (196884-x)^k x^{-k}}{k}}} \quad \text{for } x < 0$$

Integral representations:

$$\frac{1}{\sqrt[16]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)}} = \frac{\sqrt[16]{\frac{46}{391+2\sqrt{23}+24\sqrt{115}}}}{\sqrt[8]{\pi} \sqrt[16]{\int_1^{196884} \frac{1}{t} dt}}$$

Open code

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$$\frac{1}{\sqrt[16]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)}} =$$

$$\frac{1}{\sqrt[16]{\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right)\pi} \sqrt[16]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{196883^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}} \quad \text{for } -1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

And:

$$\left(\left(\left(\left(-17/4 - 6 \sqrt{5/23} - 1/(2 \sqrt{23})\right) * (\text{colog}(196884)2\pi^2)\right)\right)\right)^{1/15}$$

Input:

$$\sqrt[15]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884) \times 2\pi^2)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\pi^{2/15} \sqrt[15]{2\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right) \log(196884)}$$

Decimal approximation:

More digits

1.643301876297824293078333357563929424712113685305612560855...

$$1.6433018... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$\begin{aligned} \sqrt[15]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)} = \\ \sqrt[15]{\frac{17}{2} + 12\sqrt{\frac{5}{23}} + \frac{1}{\sqrt{23}}} \pi^{2/15} \sqrt[15]{\log(196884) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{196883}\right)^k}{k}} \end{aligned}$$

[Open code](#)

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$$\begin{aligned} \sqrt[15]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)} = \sqrt[15]{\frac{17}{2} + 12\sqrt{\frac{5}{23}} + \frac{1}{\sqrt{23}}} \pi^{2/15} \\ \sqrt[15]{2i\pi \left[\frac{\arg(196884-x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (196884-x)^k x^{-k}}{k}} \quad \text{for } x < 0 \end{aligned}$$

[Open code](#)

$$\begin{aligned} \sqrt[15]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)} = \sqrt[15]{2\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right)} \\ \pi^{2/15} \sqrt[15]{2i\pi \left[\frac{\arg(196884-x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (196884-x)^k x^{-k}}{k}} \quad \text{for } x < 0 \end{aligned}$$

[Open code](#)

- $\arg(z)$ is the complex argument

- [\[x\]](#) is the floor function
- [More information](#)

Integral representations:

$$\sqrt[15]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)} = \sqrt[15]{\frac{17}{2} + 12\sqrt{\frac{5}{23}} + \frac{1}{\sqrt{23}}} \pi^{2/15} \sqrt[15]{\int_1^{196884} \frac{1}{t} dt}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\sqrt[15]{\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)(-\log(196884))(2\pi^2)} = \sqrt[15]{\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right)\pi} \sqrt[15]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{196883^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0}$$

[Open code](#)

- [Γ\(x\)](#) is the gamma function

Further:

$$\left(\left(\left(\left(\left(7\pi\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)\right) \times (-1)\right)\right)\right)\right) \times (-2\pi^2)\right)^{1/16}$$

Input:

$$\sqrt[16]{7\pi\left(\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)\right) \times (-1)(2\pi^2)}$$

[Open code](#)

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Exact result:

$$\sqrt[16]{14\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right)\pi^{3/16}}$$

Decimal approximation:

More digits

1.652910946240704363615341718046560151138780843792857659600...

[Open code](#)

1.65291094... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Property:

$16\sqrt{14\left(\frac{17}{4} + 6\sqrt{\frac{5}{23}} + \frac{1}{2\sqrt{23}}\right)}\pi^{3/16}$ is a transcendental number

Series representations:

$$16\sqrt{(7\pi)\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)}(-2\pi^2) =$$

$$16\sqrt{\frac{7}{2}}\left(\left(\pi^3\left(2 + 17\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23-z_0)^k z_0^{-k}}{k!} + 24\sqrt{z_0}^{-2}\right.\right.\right.$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(\frac{5}{23}-z_0\right)^{k_1}(23-z_0)^{k_2}z_0^{-k_1-k_2}}{k_1!k_2!}\right)\right)$$

$$\left./\left(\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23-z_0)^k z_0^{-k}}{k!}\right)\right)^{\wedge}$$

(1/16) for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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$$16\sqrt{(7\pi)\left(-\frac{17}{4} - 6\sqrt{\frac{5}{23}} - \frac{1}{2\sqrt{23}}\right)}(-2\pi^2) =$$

$$16\sqrt{\frac{7}{2}}\left(\left(\pi^3\left(2 + 17\exp\left(i\pi\left[\frac{\arg(23-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(23-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} +\right.\right.\right.$$

$$24\exp\left(i\pi\left[\frac{\arg\left(\frac{5}{23}-x\right)}{2\pi}\right]\right)\exp\left(i\pi\left[\frac{\arg(23-x)}{2\pi}\right]\right)\sqrt{x}^{-2}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(\frac{5}{23}-x\right)^{k_1}(23-x)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!k_2!}\right)\right)$$

$$\left./\left(\exp\left(i\pi\left[\frac{\arg(23-x)}{2\pi}\right]\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(23-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^{\wedge}$$

(1/16) for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& 16 \sqrt{(7\pi) \left(-\frac{17}{4} - 6 \sqrt{\frac{5}{23} - \frac{1}{2\sqrt{23}}} \right) (-2\pi^2)} = \\
& 16 \sqrt{\frac{7}{2}} \left(\left(\pi^3 \left(\frac{1}{z_0} \right)^{-1/2 [\operatorname{arg}(23-z_0)/(2\pi)]} z_0^{-1/2-1/2 [\operatorname{arg}(23-z_0)/(2\pi)]} \left(2 + 17 \left(\frac{1}{z_0} \right)^{1/2 [\operatorname{arg}(23-z_0)/(2\pi)]} \right. \right. \right. \\
& \quad \left. \left. \left. z_0^{1/2+1/2 [\operatorname{arg}(23-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (23-z_0)^k z_0^{-k}}{k!} + \right. \right. \right. \\
& \quad \left. \left. \left. 24 \left(\frac{1}{z_0} \right)^{1/2 [\operatorname{arg}\left(\frac{5}{23}-z_0\right)/(2\pi)]+1/2 [\operatorname{arg}(23-z_0)/(2\pi)]} z_0^{1+1/2 [\operatorname{arg}\left(\frac{5}{23}-z_0\right)/(2\pi)]+1/2 [\operatorname{arg}(23-z_0)/(2\pi)]} \right. \right. \right. \\
& \quad \left. \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} \left(\frac{5}{23}-z_0\right)^{k_1} (23-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right) \right) \right) \\
& \quad \left. \left. \left. / \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (23-z_0)^k z_0^{-k}}{k!} \right) \right) \right)^{\wedge} (1/16)
\end{aligned}$$

Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(\alpha) \text{ and } |\operatorname{arg}(z)| < \pi)$$

[Open code](#)

- $\Gamma(x)$ is the gamma function
- $\operatorname{Re}(z)$ is the real part of z
- $|z|$ is the absolute value of z

The results 0.627718 and 1.652910 less 1, are very near to the following values of ω

It is however interesting to notice that the LPA estimates are not unique but depend on the equation studied. Hence with the Legendre version (65) it comes [22]:

$$\nu = 0.6604, \quad \omega = 0.6285$$

which is closer to the “best” values. And the closest to the “best” are obtained from the Wilson (or Polchinski) version [38, 30, 34]:

$$\nu = 0.6496, \quad \omega = 0.6557$$

From [73]:

Field-Theoretic Techniques in the Study of Critical Phenomena

C. Bagnuls* and C. Bervillier† C. E. A.-Saclay, F91191 Gif-sur-Yvette Cedex, France - January 18, 2018

...at present: $\mu_0 = e^{-1(2,3)}\Lambda_0$ and let us consider it as the new fixed unit of momentum...

...practically occurs in each case, on figure (B) we have artificially translated the “time” scales $l = -\ln(\lambda)$ (vertical dashed lines)....

We have that:

But g is not at all constant it is scale dependent and this is usually expressed via the beta function

$$\beta(g) = \mu \frac{dg}{d\mu}$$

in which μ is some momentum scale of reference and the function $\beta(g)$ is defined relatively to the flow running along the attractive submanifold T_1 (the slowest flow [109, 73] in the critical submanifold S_c). [73]

Following Zumbach [85, 86] one may easily verify that the local potential approximation of the Wilson (or Polchinski) ERGE, written in terms of $\mu(\varphi, t) = \exp(-V(\varphi, t))$ [eq. (68)] may be expressed as a gradient flow:

$$g(\varphi) \dot{\mu} = -\frac{\delta \mathcal{F}[\mu]}{\delta \mu}$$

where

$$g(\varphi) = \exp\left[-\frac{1}{4}(d-2)\varphi^2\right]$$
$$\mathcal{F}[\mu] = \int d\varphi g(\varphi) \left\{ \frac{1}{2}\mu'' + \frac{d}{4}\mu^2(1-2\ln\mu) \right\}$$

For $\phi = 5$, $p = 0.6181$, $\Lambda = 1$, $L = -1$, $d = 3$, $x = \pi$, $\lambda > 0 = 1.61786...$ and $\mu = \lambda$, we obtain:

$$g(\varphi) = \exp\left[-\frac{1}{4}(d-2)\varphi^2\right]$$

is equal to: $\exp(-1/4*25)$

Input:

$$\exp\left(-\frac{25}{4}\right)$$

[Open code](#)

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Exact result:

$$\frac{1}{e^{25/4}}$$

Decimal approximation:

More digits

0.001930454136227709242213511975650732143585407919243576211...

[Open code](#)

0.00193045...

Property:

$\frac{1}{e^{25/4}}$ is a transcendental number

$$\mathcal{F}[\mu] = \int d\varphi g(\varphi) \left\{ \frac{1}{2}\mu'' + \frac{d}{4}\mu^2 (1 - 2\ln \mu) \right\}$$

is equal to: integrate [5 * 0.001930454136227709 * (((((1/2*1.61786118751 + ((1/4*(1.61786118751)^2)))*(1-2ln(1.61786118751)))))))]x

Indefinite integral:

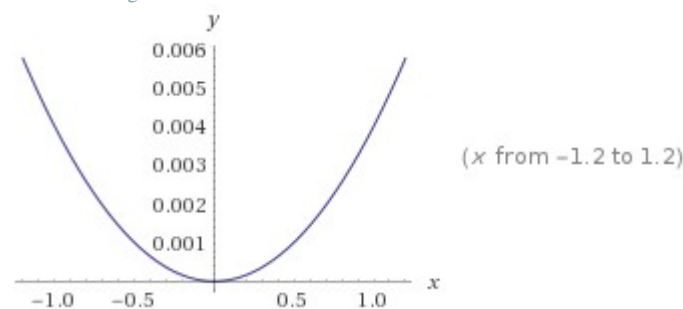
$$\int 5 \times 0.001930454136227709 \left(\left(\frac{1.61786118751}{2} + \frac{1}{4} \times 1.61786118751^2 (1 - 2 \log(1.61786118751)) \right) x \right) dx = 0.0040233519241 x^2 + \text{constant}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Plot of the integral:



For $x = \pi$, we obtain:

$$0.0040233519241 \pi^2$$

Input interpretation:

$$0.0040233519241 \pi^2$$

[Open code](#)

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Result:

- More digits
0.039708891857...
0.039708...

Now, from:

$$g(\varphi) \dot{\mu} = -\frac{\delta \mathcal{F}[\mu]}{\delta \mu}$$

Remembering that

$$-\frac{\delta \mathcal{F}[\mu]}{\delta \mu}$$

Where

$$\mathcal{F}[\mu] = \int d\varphi g(\varphi) \left\{ \frac{1}{2} \mu'' + \frac{d}{4} \mu^2 (1 - 2 \ln \mu) \right\}$$

Is

$$d\varphi g(\varphi) \left\{ \frac{1}{2} \mu'' + \frac{d}{4} \mu^2 (1 - 2 \ln \mu) \right\}$$

with minus sign, we obtain:

$$-[5 * 0.001930454136227709 * (((((1/2 * 1.61786118751 + ((1/4 * (1.61786118751)^2))) * ((1 - 2 \ln(1.61786118751)))))))]$$

Input interpretation:

$$-\left(5 \times 0.001930454136227709 \left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right)\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

- More digits
-0.00804670384815...
-0.0080467...

Series representations:

- More

$$-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) = -0.01412416092405333 - 0.01263228774175189 \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}$$

Open code

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$$-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) = -0.01412416092405333 + 0.0252645754835038 i \pi \left[\frac{\arg(1.617861187510000 - x)}{2 \pi} \right] + 0.01263228774175189 \log(x) - 0.01263228774175189 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \text{ for } x < 0$$

Open code

$$-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) = -0.01412416092405333 + 0.01263228774175189 \left[\frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 0.01263228774175189 \log(z_0) + 0.01263228774175189 \left[\frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right] \log(z_0) - 0.01263228774175189 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k}$$

- $\arg(z)$ is the complex argument
- $[x]$ is the floor function
- i is the imaginary unit

Integral representations:

$$-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) = -0.01412416092405333 + 0.01263228774175189 \int_1^{1.617861187510000} \frac{1}{t} dt$$

Open code

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$$\begin{aligned}
& -5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \right. \right. \\
& \quad \left. \left. \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) = \\
& -0.01412416092405333 + \frac{0.00631614387087594}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{0.481491462433938s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0}
\end{aligned}$$

[Open code](#)

• $\Gamma(x)$ is the gamma function

Now, we have that:

$$\begin{aligned}
& (((((((6.626 * -[5 * 0.001930454136227709 * (((((1/2 * 1.61786118751 + \\
& ((1/4 * (1.61786118751)^2))) * ((1 - 2 \ln(1.61786118751))))))]))]))))
\end{aligned}$$

where 6.626 is the absolute value of Planck constant

Input interpretation:

$$\begin{aligned}
& 6.626 \times (-1) \left(5 \times 0.001930454136227709 \right. \\
& \quad \left. \left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right) \right)
\end{aligned}$$

[Open code](#)

• $\log(x)$ is the natural logarithm

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Result:

• More digits

-0.0533175...

-0.0533175...

Series representations:

• More

$$\begin{aligned}
& (6.626 (-1)) 5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \right. \right. \\
& \quad \left. \left. \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) = \\
& -0.0935867 - 0.0837015 \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}
\end{aligned}$$

[Open code](#)

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$$(6.626 (-1))^5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) =$$

$$-0.0935867 + 0.167403 i \pi \left[\frac{\arg(1.617861187510000 - x)}{2 \pi} \right] + 0.0837015 \log(x) -$$

$$0.0837015 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \text{ for } x < 0$$

Open code

$$(6.626 (-1))^5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) =$$

$$-0.0935867 + 0.0837015 \left[\frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$0.0837015 \log(z_0) + 0.0837015 \left[\frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right] \log(z_0) -$$

$$0.0837015 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$ is the complex argument
- $[x]$ is the floor function
- i is the imaginary unit
- [More information](#)

Integral representations:

$$(6.626 (-1))^5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) =$$

$$-0.0935867 + 0.0837015 \int_1^{1.617861187510000} \frac{1}{t} dt$$

Open code

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$$(6.626 (-1))^5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) =$$

$$-0.0935867 + \frac{0.0418508}{i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{0.481491462433938 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for}$$

$$-1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

$$0.034 / (((((((((6.626 * -[5 * 0.001930454136227709 * (((((1/2 * 1.61786118751 + ((1/4 * (1.61786118751)^2))) * ((1 - 2 \ln(1.61786118751)))))))])))))))))$$

$$0.034 = 34 * 10^{-3}$$

Input interpretation:

$$0.034 / \left((6.626 \times (-1)) \left(5 \times 0.001930454136227709 \left(\left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right) \right) \right) \right)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

More digits

-0.637690...

-0.63769...

Series representations:

More

$$0.034 / \left((6.626 (-1)) 5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \frac{0.406205}{-1.1181 - \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}}$$

Open code

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$$0.034 / \left((6.626 (-1)) 5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \frac{0.203103}{\left(-0.55905 + i \pi \left[\frac{\arg(1.617861187510000 - x)}{2 \pi} \right] + 0.5 \log(x) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \right)} \text{ for } x < 0$$

Open code

$$0.034 / \left((6.626 (-1)) 5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) =$$

$$- \left(0.531616 / \left(1.463299299267817 - 1.308737411025634 \left(\log(z_0) + \left[\frac{\arg(1.617861187510000 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k} \right) \right) \right)$$

Open code

- $\arg(z)$ is the complex argument
 - $[x]$ is the floor function
 - i is the imaginary unit
 - [More information](#)

Integral representation:

$$0.034 / \left((6.626 (-1)) 5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) =$$

$$- \frac{0.3633 i \pi}{i \pi - 0.447187 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{0.481491462433938 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \text{ for } -1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

$$[(((2*3)/(2*89-3)))] / (((((((((6.626*-[5*0.001930454136227709*(((1/2*1.61786118751+(((1/4*(1.61786118751)^2))))*(1-2\ln(1.61786118751)))))))))))))$$

Input interpretation:

$$\frac{2 \times 3}{2 \times 89 - 3} / \left(6.626 \times (-1) \left(5 \times 0.001930454136227709 \left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right) \right) \right)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

More digits

-0.643049...

-0.643049...

Series representations:

More

$$(2 \times 3) / \left(\left(6.626 (-1) \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) \right)$$

$$(2 \times 89 - 3) = \frac{0.409619}{-1.1181 - \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}}$$

[Open code](#)

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$$(2 \times 3) / \left(\left(6.626 (-1) \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) \right)$$

$$(2 \times 89 - 3) = 0.204809 / \left(-0.55905 + i \pi \left[\frac{\arg(1.617861187510000 - x)}{2 \pi} \right] + 0.5 \log(x) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \right) \text{ for } x < 0$$

[Open code](#)

$$(2 \times 3) / \left(\left(6.626 (-1) \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) \right)$$

$$(2 \times 89 - 3) = - \left(0.536083 / \left(1.463299299267817 - 1.308737411025634 \left(\log(z_0) + \left[\frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k} \right) \right) \right)$$

[Open code](#)

- $\arg(z)$ is the complex argument
 - $\lfloor x \rfloor$ is the floor function
 - i is the imaginary unit
 - [More information](#)

Integral representation:

$$(2 \times 3) / \left(\left(6.626 (-1) \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 \right) (1 - 2 \log(1.617861187510000)) \right) \right) (2 \times 89 - 3) \right) =$$

$$-\frac{0.366352 i \pi}{i \pi - 0.447187 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{0.481491462433938 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

The following results $-0.0533175\dots$, $-0.63769\dots$ and $-0.643049\dots$ are very near to $\eta = 0.05393$; $\nu = 0.638$ and $\nu = 0.643$

As expected, the search for a non trivial fixed point solution for these equations (a solution which is nonsingular up to $\varphi \rightarrow \infty$) produces a unique solution with an unambiguously defined η [22]:

$$\eta = 0.05393 \tag{88}$$

Let us first quote, for $d = 3$ and $N = 1$, the results found with the supplementary help of a truncation in powers of the field associated to an expansion around the minimum of the potential [83]: $\nu = 0.638$, $\eta = 0.045$, $\gamma = 1.247$, $\beta = 0.333$ and without truncation in the field dependence [148]: $\nu = 0.643$, $\eta = 0.044$, $\gamma = 1.258$, $\beta = 0.336$, $\delta = 4.75$. In this latter work, the scaled equation of state has been calculated using this pseudo derivative expansion. For more details on this approach see the review by Berges et al [13] in this volume.

Furthermore, from the inverse of primitive function, we obtain the following result:

Input interpretation:

$$(-1) / \left(- \left(5 \times 0.001930454136227709 \left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right) \right) \right)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits
124.274487899...
124.2744...

Series representations:
More

$$(-1) / \left(-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \frac{-1.1181000000000043 - 1.0000000000000000 \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}}{79.1622246455666}$$

[Open code](#)

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$$(-1) / \left(-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = - \left(39.58111232278329 / \left(-0.5590500000000021 + \frac{1.0000000000000000 i \pi \left\lfloor \frac{\arg(1.617861187510000 - x)}{2 \pi} \right\rfloor + 0.5000000000000000 \log(x) - 0.5000000000000000 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \right) \right) \text{ for } x < 0$$

[Open code](#)

$$(-1) / \left(-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \frac{103.60256493366841819}{\left(1.463299299267817 - 1.308737411025634 \left(\log(z_0) + \left\lfloor \frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k} \right) \right)}$$

Integral representation:

$$(-1) / \left(-5 \left(0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) =$$

$$(70.8006659919180 i \pi) / \left(1.0000000000000000 i \pi - 0.4471871925587881 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{0.481491462433938 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

Or:

$$-1.0061571663 / -[5 * 0.001930454136227709 * (((((1/2 * 1.61786118751 + ((1/4 * (1.61786118751)^2))) * ((1 - 2 \ln(1.61786118751)))))))]))$$

Where 1.0061571663 is a Ramanujan mock theta function

Input interpretation:

$$(-1.0061571663) / \left(- \left(5 \times 0.001930454136227709 \left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right) \right) \right)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

More digits

125.03966659...

125.0396...

Series representations:

More

$$(-1.00615716630000) / \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) =$$

$$\frac{79.6496396273873}{-1.118100000000004 - 1.000000000000000 \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}}$$

Open code

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$$\begin{aligned}
& (-1.00615716630000) / \\
& \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times \right. \right. \right. \\
& \quad \left. \left. \left. 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \\
& - \left(39.8248198136936 / \left(-0.5590500000000021 + 1.0000000000000000 \right. \right. \\
& \quad \left. \left. i \pi \left[\frac{\arg(1.617861187510000 - x)}{2 \pi} \right] + \right. \right. \\
& \quad \left. \left. 0.5000000000000000 \log(x) - 0.5000000000000000 \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \right) \right) \text{ for } x < 0
\end{aligned}$$

Open code

$$\begin{aligned}
& (-1.00615716630000) / \\
& \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times \right. \right. \right. \\
& \quad \left. \left. \left. 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \\
& 104.240463155072 / \left(1.463299299267817 - 1.308737411025634 \right. \\
& \quad \left(\log(z_0) + \left[\frac{\arg(1.617861187510000 - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k} \right) \right)
\end{aligned}$$

Integral representation:

$$\begin{aligned}
& (-1.00615716630000) / \\
& \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times \right. \right. \right. \\
& \quad \left. \left. \left. 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) = \\
& (71.2365974665810 i \pi) / \left(1.0000000000000000 i \pi - 0.447187192558788 \right. \\
& \quad \left. \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{0.481491462433938 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0
\end{aligned}$$

Open code

- $\Gamma(x)$ is the gamma function

where 124.2744... and 125.0396... are results very near to the value of Higgs boson mass 125.18

And:

$$-1.0864055 / -[5 * 0.001930454136227709 * (((((1/2 * 1.61786118751 + ((1/4 * (1.61786118751)^2))) * ((1 - 2 \ln(1.61786118751)))))))]))$$

where 1.0864055 derive from a Ramanujan mock theta function

Input interpretation:

$$(-1.0864055) / \left(- \left(5 \times 0.001930454136227709 \left(\frac{1}{2} \times 1.61786118751 + \left(\frac{1}{4} \times 1.61786118751^2 \right) (1 - 2 \log(1.61786118751)) \right) \right) \right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

More digits

135.01249...

135.01249... result very near to the rest mass of Pion meson 134.9766

Series representations:

More

$$\frac{(-1.08641) / \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right)}{-1.1181 - \sum_{k=1}^{\infty} \frac{(-0.617861187510000)^k}{k}} =$$

[Open code](#)

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$$\frac{(-1.08641) / \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right)}{- \left(43.0011 / \left(-0.55905 + i \pi \left[\frac{\arg(1.617861187510000 - x)}{2 \pi} \right] + 0.5 \log(x) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - x)^k x^{-k}}{k} \right) \right)} \text{ for } x < 0$$

[Open code](#)

$$(-1.08641) / \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) =$$

$$112.554 / \left(1.463299299267817 - 1.308737411025634 \left(\log(z_0) + \left\lfloor \frac{\arg(1.617861187510000 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.617861187510000 - z_0)^k z_0^{-k}}{k} \right) \right)$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function
- i is the imaginary unit

Integral representation:

$$(-1.08641) / \left(- \left(5 \times 0.0019304541362277090000 \left(\frac{1.617861187510000}{2} + \frac{1}{4} \times 1.617861187510000^2 (1 - 2 \log(1.617861187510000)) \right) \right) \right) =$$

$$\frac{76.9182 i \pi}{i \pi - 0.447187 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{0.481491462433938 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}$$

for $-1 < \gamma < 0$

Open code

- $\Gamma(x)$ is the gamma function

We have that:

In themselves the estimates of critical exponents in the local potential approximation do not present a great interest except as first order estimates in a systematic expansion see section 4.1. It is however interesting to notice that the LPA estimates are not unique but depend on the equation studied. Hence with the Legendre version (65) it comes [22]:

$$\nu = 0.6604, \quad \omega = 0.6285$$

which is closer to the “best” values. And the closest to the “best” are obtained from the Wilson (or Polchinski) version [38, 30, 34]:

$$\nu = 0.6496, \quad \omega = 0.6557$$

In order to get the best estimates for η , one can adjust z in such a way as to get an almost realized reparametrization invariance [52, 11, 30, 29]. The analysis is not simple [30] due to the additional effects of the two cutoff parameters A and B . Finally estimates of the critical and subcritical exponents are proposed ($d = 3$ and $N = 1$) [30]:

$$\begin{aligned} \eta &= 0.042 \\ \nu &= 0.622 \\ \omega &= 0.754 \end{aligned}$$

It is interesting to compare these estimates with those obtained by Golner in [11] from the Wilson version of the ERGE:

$$\eta = 0.024 \pm 0.007 \tag{85}$$

$$\nu = 0.617 \pm 0.008 \tag{86}$$

For $d = 3$ and $k = 1$, the first order of the derivative expansion yields (after a long but straightforward computation) the following two coupled equations for U and Z [22]:

$$\begin{aligned} \dot{U} &= -\frac{1 - \eta/4}{\sqrt{Z}\sqrt{U'' + 2\sqrt{Z}}} + 3U - \frac{1}{2}(1 + \eta)\varphi U' \\ \dot{Z} &= -\frac{1}{2}(1 + \eta)\varphi Z' - \eta Z + \left(1 - \frac{\eta}{4}\right) \left\{ \frac{1}{48} \frac{24ZZ'' - 19(Z')^2}{Z^{3/2}(U'' + 2\sqrt{Z})^{3/2}} \right. \\ &\quad \left. - \frac{1}{48} \frac{58U'''Z'\sqrt{Z} + 57(Z')^2 + (Z''')^2Z}{Z(U'' + 2\sqrt{Z})^{5/2}} + \frac{5}{12} \frac{(U''')^2Z + 2U'''Z'\sqrt{Z} + (Z')^2}{\sqrt{Z}(U'' + 2\sqrt{Z})^{7/2}} \right\} \end{aligned} \tag{87}$$

As expected, the search for a non trivial fixed point solution for these equations (a solution which is nonsingular up to $\varphi \rightarrow \infty$) produces a unique solution with an unambiguously defined η [22]:

$$\eta = 0.05393 \tag{88}$$

The linearization about this fixed point yields the eigenvalues:

$$\nu = 0.6181 \tag{89}$$

$$\omega = 0.8975 \tag{90}$$

$$\dot{U} = -\frac{1-\eta/4}{\sqrt{Z}\sqrt{U''+2\sqrt{Z}}}+3U-\frac{1}{2}(1+\eta)\varphi U'$$

$$\dot{Z} = -\frac{1}{2}(1+\eta)\varphi Z' - \eta Z + \left(1 - \frac{\eta}{4}\right) \left\{ \frac{1}{48} \frac{24ZZ'' - 19(Z')^2}{Z^{3/2}(U'' + 2\sqrt{Z})^{3/2}} - \frac{1}{48} \frac{58U'''Z'\sqrt{Z} + 57(Z')^2 + (Z''')^2Z}{Z(U'' + 2\sqrt{Z})^{5/2}} + \frac{5}{12} \frac{(U''')^2Z + 2U'''Z'\sqrt{Z} + (Z')^2}{\sqrt{Z}(U'' + 2\sqrt{Z})^{7/2}} \right\} \quad (87)$$

For $\eta = 0.05393$, $Z = 1$, $U = 15625$, we obtain, from (87):

$$\dot{U} = -\frac{1-\eta/4}{\sqrt{Z}\sqrt{U''+2\sqrt{Z}}}+3U-\frac{1}{2}(1+\eta)\varphi U'$$

-((((1-0.05393/4)/((sqrt(15625+2)))))+3*15625-1/2*1.05393*5*15625

Input interpretation:

$$-\left(\frac{1-\frac{0.05393}{4}}{\sqrt{15625+2}}+3\times 15625+\frac{1}{2}\times 5\times 15625\times (-1.05393)\right)$$

[Open code](#)

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Result:

More digits

-5705.87...

-5705.87...

Now:

$$\dot{Z} = -\frac{1}{2}(1+\eta)\varphi Z' - \eta Z + \left(1 - \frac{\eta}{4}\right) \left\{ \frac{1}{48} \frac{24ZZ'' - 19(Z')^2}{Z^{3/2}(U'' + 2\sqrt{Z})^{3/2}} - \frac{1}{48} \frac{58U'''Z'\sqrt{Z} + 57(Z')^2 + (Z''')^2Z}{Z(U'' + 2\sqrt{Z})^{5/2}} + \frac{5}{12} \frac{(U''')^2Z + 2U'''Z'\sqrt{Z} + (Z')^2}{\sqrt{Z}(U'' + 2\sqrt{Z})^{7/2}} \right\} \quad (87)$$

-1/2(1.05393)*5-0.05393+(1-0.05393/4)

Input interpretation:

$$-\frac{1.05393\times 5}{2}-0.05393+\left(1-\frac{0.05393}{4}\right)$$

[Open code](#)

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Result:

-1.7022375

$$(1/48*((24-19)/(15625+2)^{1.5})-1/48*(((58*15625)+57+1))/((15625+2)^{2.5}))+5/12*(((15625^2+2*15625+1)/(15625+2)^{3.5}))$$

Input:

$$\left(\frac{1}{48} \times \frac{24 - 19}{(15625 + 2)^{1.5}} - \frac{1}{48} \times \frac{58 \times 15625 + 57 + 1}{(15625 + 2)^{2.5}} \right) + \frac{5}{12} \times \frac{15625^2 + 2 \times 15625 + 1}{(15625 + 2)^{3.5}}$$

[Open code](#)

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Result:

More digits

-3.51920... × 10⁻⁷

$$-1/2(1.05393)*5-0.05393+(1-0.05393/4)* (-3.5192014186968281302761584884158382199653073021 \times 10^{-7})$$

Input interpretation:

$$-\frac{1.05393 \times 5}{2} - 0.05393 + \left(1 - \frac{0.05393}{4}\right) (-3.5192014186968281302761584884158382199653073021 \times 10^{-7})$$

[Open code](#)

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Result:

More digits

-2.68875534717537855692481450097101815957716811646250463994...
-2.68875534...

From this result, we have:

$$\sqrt{\left(\left(\left(\left(\left(-1/2(1.05393)*5-0.05393+(1-0.05393/4)* (-3.51920141869682813 \times 10^{-7})\right)\right)\right)\right)\right)^{1/2}}$$

Input interpretation:

$$\sqrt{-\frac{1.05393 \times 5}{2} - 0.05393 + \left(1 - \frac{0.05393}{4}\right) (-3.51920141869682813 \times 10^{-7})}$$

[Open code](#)

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Result:

More digits

1.63974... *i*

Polar coordinates:

$r = 1.63974$ (radius), $\theta = 90^\circ$ (angle)

[Open code](#)

$$1.63974 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Then:

$$\frac{\left(\left(\left(\left(\left(1 - \frac{0.05393}{4}\right) / \left(\sqrt{15625 + 2}\right)\right) + 3 \times 15625 - \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)\right)\right)\right)}{-2.6887553471753785569248145}$$

Input interpretation:

$$\frac{-\left(\frac{1 - \frac{0.05393}{4}}{\sqrt{15625 + 2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)}{-2.6887553471753785569248145}$$

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Result:

More digits

2122.12...

2122.12... result very near to the rest mass of strange D meson 2112.3

$$\frac{-2.6887553471753785569248145}{\left(\left(\left(\left(\left(1 - \frac{0.05393}{4}\right) / \left(\sqrt{15625 + 2}\right)\right) + 3 \times 15625 - \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)\right)\right)}$$

Input interpretation:

$$\frac{-2.6887553471753785569248145}{-\left(\frac{1 - \frac{0.05393}{4}}{\sqrt{15625 + 2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)}$$

[Open code](#)

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Result:

More digits

0.000471226...

$$\frac{1}{\left(\left(\left(\left(\left(\left(-2.6887553471753785569248145 \times \left(\left(\left(\left(\left(1 - \frac{0.05393}{4}\right) / \left(\sqrt{15625 + 2}\right)\right) + 3 \times 15625 - \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)\right)\right)\right)\right)\right)\right)\right)}$$

Input interpretation:

$$\frac{1}{\left(2.6887553471753785569248145 \times \left(-\left(\frac{1 - \frac{0.05393}{4}}{\sqrt{15625 + 2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)\right)\right)}$$

[Open code](#)

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Result:

- More digits

0.0000651819...

$$10^4 * 1 / ((((((((-2.6887553471753785569248145 * ((((-(((1-0.05393/4)/(\sqrt{15625+2}))))+3*15625-1/2*1.05393*5*15625)))))))))))))$$

Input interpretation:

$$10^4 \left[- \left[1 / \left(2.6887553471753785569248145 \left(- \left(\frac{1 - \frac{0.05393}{4}}{\sqrt{15625 + 2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393) \right) \right) \right) \right] \right]$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

- More digits

0.651819...

0.651819...

Note that, from the previous expression with result 2122.12, we obtain:

$$(((((((((((((-(((1-0.05393/4)/(\sqrt{15625+2}))))+3*15625-1/2*1.05393*5*15625)))))))))) / -2.6887553471753785569248145))))))^{1/15}$$

Input interpretation:

$$\sqrt[15]{\frac{-\left(\frac{1 - \frac{0.05393}{4}}{\sqrt{15625 + 2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)}{-2.6887553471753785569248145}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

- More digits

1.66642...

1.66642...

And:

$$-1 + ((((((((((((-(((1-0.05393/4)/(\sqrt{15625+2}))))+3*15625-1/2*1.05393*5*15625)))))))) / -2.6887553471753785569248145))))))^{1/15}$$

Input interpretation:

$$-1 + \sqrt[15]{\frac{-\left(\frac{1-\frac{0.05393}{4}}{\sqrt{15625+2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393)\right)}{-2.6887553471753785569248145}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• More digits

0.666421...

0.666421...

We have also:

$$(0.5097073 + 0.544717185) * \left(\frac{1}{\left(\frac{10^4 * 1}{\left(-2.688755347 * \left(\frac{-(1-0.05393/4)}{\sqrt{15625+2}} + 3*15625 - 1/2*1.05393*5*15625 \right) \right)} \right)} \right)$$

Where $0.5097073 + 0.544717185$ is the sum of two Ramanujan mock theta functions

Input interpretation:

$$(0.5097073 + 0.544717185) \times$$

$$\frac{1}{10^4 \left(\frac{1}{2.688755347 \left(\frac{1-\frac{0.05393}{4}}{\sqrt{15625+2}} + 3 \times 15625 + \frac{1}{2} \times 5 \times 15625 \times (-1.05393) \right)} \right)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• Fewer digits

• More digits

1.617664421655985631062168769021278451556391711603050387424...

1.617664421... result that is a very good approximation to the value of golden ratio

1.61803398

From:

2012

Matthew Schwartz

III-9: The renormalization group

Defining the gauge coupling e_R so that $\tilde{V}(p_0^2) = \frac{e_R^2}{p_0^2}$ exactly at the scale p_0 fixes the counterterm δ_3 and lets us write the renormalized potential as

$$\tilde{V}(p^2) = \frac{e_R^2}{p^2} \left\{ 1 + \frac{e_R^2}{2\pi^2} \int_0^1 dx x(1-x) \ln \left(\frac{p^2 x(1-x) - m^2}{p_0^2 x(1-x) - m^2} \right) \right\} \quad (3)$$

which is finite and c - and μ -independent.

The entire functional form of this potential is phenomenologically important, especially at low energies, where we saw it gives the Uehling potential and contributes to the Lamb shift. However, when $p \gg m$, the mass drops out and the potential simplifies to

$$\tilde{V}(p^2) = \frac{e_R^2}{p^2} \left(1 + \frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2} \right) \quad (4)$$

In this limit, we can see clearly the problem of **large logarithms**, which the renormalization group will solve. Normally, one would expect that since the correction is proportional to $\frac{e_R^2}{12\pi^2} \sim 10^{-3}$ higher order terms would be proportional to the square, cube, etc. of this term and therefore would be negligible. However, there exist scales $p^2 \gg p_0^2$ where $\ln \frac{p^2}{p_0^2} > 10^3$ so that this correction is of order 1. When these logarithms are this large, then terms of the form $\left(\frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2} \right)^2$, which would appear at the next order in perturbation theory, will also be order 1 and so perturbation theory breaks down.

$$\tilde{V}(p^2) = \frac{e_R^2}{p^2} \left[1 + \frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2} + \left(\frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2} \right)^2 + \dots \right] = \frac{1}{p^2} \left[\frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2}} \right] \quad (5)$$

We then defined the effective coupling through the potential by $e_{\text{eff}}^2(p^2) \equiv p^2 \tilde{V}(p^2)$. So that

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2}} \quad (6)$$

From (4), for $e_R^2 = 89$; $p^2 = 995328$ and $p_0^2 = 576$, we obtain:

$$89/995328 \left(\left(1 + 89 / (12 \pi^2) \cdot \ln(995328/576) \right) \right)$$

Note that $995328 \div 576 = 1728$, that is the Hardy-Ramanujan number less 1

Input:

$$\frac{89}{995328} \left(1 + \frac{89}{12\pi^2} \log \left(\frac{995328}{576} \right) \right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{89 \left(1 + \frac{89 \log(1728)}{12\pi^2} \right)}{995328}$$

Decimal approximation:

[More digits](#)

0.000590332870928253966412987449888453126669930484617717053...

Series representations:

More

$$\frac{\left(1 + \frac{89 \log\left(\frac{995\,328}{576}\right)}{12 \pi^2}\right) 89}{995\,328} = \frac{89}{995\,328} + \frac{7921 \log(1727)}{11\,943\,936 \pi^2} - \frac{7921 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1727}\right)^k}{k}}{11\,943\,936 \pi^2}$$

Open code

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$$\frac{\left(1 + \frac{89 \log\left(\frac{995\,328}{576}\right)}{12 \pi^2}\right) 89}{995\,328} = \frac{89}{995\,328} + \frac{7921 i \left\lfloor \frac{\arg(1728-x)}{2\pi} \right\rfloor}{5\,971\,968 \pi} + \frac{7921 \log(x)}{11\,943\,936 \pi^2} - \frac{7921 \sum_{k=1}^{\infty} \frac{(-1)^k (1728-x)^k x^{-k}}{k}}{11\,943\,936 \pi^2} \quad \text{for } x < 0$$

Open code

$$\frac{\left(1 + \frac{89 \log\left(\frac{995\,328}{576}\right)}{12 \pi^2}\right) 89}{995\,328} = \frac{89}{995\,328} + \frac{7921 \left\lfloor \frac{\arg(1728-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right)}{11\,943\,936 \pi^2} + \frac{7921 \log(z_0)}{11\,943\,936 \pi^2} + \frac{7921 \left\lfloor \frac{\arg(1728-z_0)}{2\pi} \right\rfloor \log(z_0)}{11\,943\,936 \pi^2} - \frac{7921 \sum_{k=1}^{\infty} \frac{(-1)^k (1728-z_0)^k z_0^{-k}}{k}}{11\,943\,936 \pi^2}$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{\left(1 + \frac{89 \log\left(\frac{995\,328}{576}\right)}{12 \pi^2}\right) 89}{995\,328} = \frac{89}{995\,328} + \frac{7921}{11\,943\,936 \pi^2} \int_1^{1728} \frac{1}{t} dt$$

Open code

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$$\frac{\left(1 + \frac{89 \log\left(\frac{995\,328}{576}\right)}{12 \pi^2}\right) 89}{995\,328} = \frac{89}{995\,328} - \frac{7921 i}{23\,887\,872 \pi^3} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1727^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

Open code

- $\Gamma(x)$ is the gamma function

$$-21 + 1 / (((((89 / 995328) * ((1 + 89 / (12 \pi^2)) * \ln(995328 / 576))))))))$$

Input:

$$-21 + \frac{1}{\frac{89}{995328} \left(1 + \frac{89}{12 \pi^2} \log\left(\frac{995328}{576}\right) \right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{995328}{89 \left(1 + \frac{89 \log(1728)}{12 \pi^2} \right)} - 21$$

Decimal approximation:

More digits

• 1672.959542567188128400977852791157438973127398496873615712...

1672.959... result very near to the rest mass of Omega baryon 1672.45

Series representations:

More

$$-21 + \frac{1}{\frac{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12 \pi^2} \right) 89}{995328}} = -21 + \frac{995328}{89 \left(1 + \frac{\left(\log(1728) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1727}\right)^k}{k} \right)}{12 \pi^2} \right)}$$

[Open code](#)

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$$-21 + \frac{1}{\frac{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12 \pi^2} \right) 89}{995328}} = -21 + \frac{995328}{89 \left(1 + \frac{\left(2 i \pi \left\lfloor \frac{\arg(1728-x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728-x)^k x^{-k}}{k} \right)}{12 \pi^2} \right)} \text{ for } x < 0$$

[Open code](#)

$$-21 + \frac{1}{\frac{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{89}}{995328}} =$$

$$-21 + \frac{995328}{89 \left(1 + \frac{89 \left(\log(z_0) + \left[\frac{\arg(1728-z_0)}{2\pi}\right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728-z_0)^k z_0^{-k}}{k}\right)}{12\pi^2}\right)}$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function
- [More information](#)

Integral representations:

$$-21 + \frac{1}{\frac{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{89}}{995328}} = -21 + \frac{995328}{89 \left(1 + \frac{89}{12\pi^2} \int_1^{1728} \frac{1}{t} dt\right)}$$

Open code

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$$-21 + \frac{1}{\frac{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{89}}{995328}} = -21 + \frac{995328}{89 \left(1 - \frac{89i}{24\pi^3} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1727^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right)}$$

for $-1 < \gamma < 0$

Open code

- $\Gamma(x)$ is the gamma function

From the inverse of the formula, that is:

$$1/\left(\left(\left(\left(\left(89/995328\left(\left(1+89/(12\text{Pi}^2)*\ln(995328/576)\right)\right)\right)\right)\right)\right)\right)$$

Input:

$$\frac{1}{\frac{89}{995328} \left(1 + \frac{89}{12\pi^2} \log\left(\frac{995328}{576}\right)\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{995\,328}{89 \left(1 + \frac{89 \log(1728)}{12 \pi^2} \right)}$$

Decimal approximation:

More digits

1693.959542567188128400977852791157438973127398496873615712...

we obtain:

$$\left(\left(\left(\left(\frac{1}{\left(\frac{89}{995328} \left(1 + \frac{89}{12\pi^2} \ln\left(\frac{995328}{576} \right) \right) \right) \right) \right) \right) \right)^{1/15}$$

Input:

$$\sqrt[15]{\frac{1}{\frac{89}{995328} \left(1 + \frac{89}{12\pi^2} \log\left(\frac{995\,328}{576}\right) \right)}}$$

Open code

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{2^{4/5} \sqrt[3]{3}}{15 \sqrt[15]{89 \left(1 + \frac{89 \log(1728)}{12 \pi^2} \right)}}$$

Decimal approximation:

More digits

1.641573007089980890752645165502519576377658005183564626578...

$$1.641573... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$\sqrt[15]{\frac{1}{\left(\frac{89 \log\left(\frac{995\,328}{576}\right)}{12 \pi^2} \right) 89}} = \frac{2^{4/5} \sqrt[3]{3}}{15 \sqrt[15]{89} \sqrt[15]{1 + \frac{89 \left(\log(1728) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1728}\right)^k}{k} \right)}{12 \pi^2}}}$$

Open code

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$$\sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{80}}} = \frac{2^{4/5} \sqrt[3]{3}}{\sqrt[15]{89} \sqrt[15]{1 + \frac{80 \left(2i\pi \left\lfloor \frac{\arg(1728-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728-x)^k x^{-k}}{k}\right)}{12\pi^2}}} \quad \text{for } x < 0$$

Open code

$$\sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{80}}} = \frac{2^{4/5} \sqrt[3]{3}}{\sqrt[15]{89} \sqrt[15]{1 + \frac{80 \left(\log(z_0) + \left\lfloor \frac{\arg(1728-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728-z_0)^k z_0^{-k}}{k}\right)}{12\pi^2}}} \right.$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function
- [More information](#)

Integral representations:

$$\sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{80}}} = \frac{2^{4/5} \sqrt[3]{3}}{\sqrt[15]{89} \sqrt[15]{1 + \frac{89}{12\pi^2} \int_1^{1728} \frac{1}{t} dt}}$$

Open code

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$$\sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{80}}} = \frac{2 \times 3^{2/5} \sqrt[5]{\pi}}{\sqrt[15]{89} \sqrt[15]{24\pi^3 - 89i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1727^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}} \quad \text{for } -1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

$$1.0061571663 * (((((1 / (((89 / 995328 * ((1 + 89 / (12 \pi^2) * \ln(995328 / 576)))))))))))))^{1/15}$$

Where 1.0061571663 is a Ramanujan mock theta function

Input interpretation:

$$1.0061571663 \sqrt[15]{\frac{1}{\frac{89}{995328} \left(1 + \frac{89}{12 \pi^2} \log\left(\frac{995328}{576}\right)\right)}}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

More digits

1.6516804451...

1.65168... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505} / G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

Series representations:

More

$$1.00615716630000 \sqrt[15]{\frac{1}{\frac{89 \log\left(\frac{995328}{576}\right)}{12 \pi^2} + 1} \frac{89}{995328}} = 1.87314287969017 \sqrt[15]{\frac{1}{1 + \frac{89 \left(\log(1727) - \sum_{k=1}^{\infty} \frac{(-1/1727)^k}{k}\right)}{12 \pi^2}}}}$$

Open code

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$$1.00615716630000 \sqrt[15]{\frac{1}{\frac{89 \log\left(\frac{995328}{576}\right)}{12 \pi^2} + 1} \frac{89}{995328}} = 1.87314287969017 \sqrt[15]{\frac{1}{1 + \frac{89 \left(2i\pi \left\lfloor \frac{\arg(1728-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728-x)^k x^{-k}}{k}\right)}{12 \pi^2}}} \text{ for } x < 0$$

Open code

$$1.00615716630000 \sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{89}}}} = 1.87314287969017$$

$$\sqrt[15]{\frac{1}{1 + \frac{89 \left(\log(z_0) + \left\lfloor \frac{\arg(1728-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728-z_0)^k z_0^{-k}}{k} \right)}{12\pi^2}}}}$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function
- i is the imaginary unit

Integral representations:

$$1.00615716630000 \sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{89}}}} =$$

$$1.87314287969017 \sqrt[15]{1 + \frac{89}{12\pi^2} \int_1^{1728} \frac{1}{t} dt}$$

Open code

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$$1.00615716630000 \sqrt[15]{\frac{1}{\left(1 + \frac{89 \log\left(\frac{995328}{576}\right)}{12\pi^2}\right)^{89}}}} =$$

$$2.31518120733120 \sqrt[15]{\frac{i\pi^3}{24i\pi^3 + 89 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1727^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}} \quad \text{for } -1 < \gamma < 0$$

Open code

- $\Gamma(x)$ is the gamma function

From (6),

$$e_{\text{eff}}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2} \ln \frac{p^2}{p_0^2}}$$

for $e_R^2 = 89$; $p^2 = 995328$ and $p_0^2 = 576$, we obtain:

$$\left(\left(\left(\left(\left(\left(89/\left(\left(\left(1-89/(12\pi^2)\right)*\ln(995328/576)\right)\right)\right)\right)\right)\right)\right)\right)$$

Input:

$$\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}$$

Decimal approximation:

More digits

48.03655953064422077820645457203413540042408603620069957489...

48.03655...

We note that, from the square root of (6), we obtain the following results:

$$\sqrt{\left(\left(\left(\left(\left(\left(89/\left(\left(\left(1-89/(12\pi^2)\right)*\ln(995328/576)\right)\right)\right)\right)\right)\right)\right)\right)}$$

Input:

$$\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}}$$

Decimal approximation:

More digits

6.930841184924397095617531364115586115912006948029315713901...

[Open code](#)

and that:

$$\left(\left(\left(\left(\left(\left(\left(\sqrt{\left(\left(\left(\left(\left(\left(89/\left(\left(\left(1-89/(12\pi^2)\right)*\ln(995328/576)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{(1/(2.01^2))}$$

Where 2.01 is a Hausdorff dimension

Input:

$$2.01^{\times 2} \sqrt{\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

- Fewer digits
- More digits

1.618641731855722081525632220558199072667980927023262977864...

1.61864173... result very near to the value of golden ratio

Series representations:

- More

$$2.01 \times 2 \sqrt{\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}} = \sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \left(\log(1727) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1727}\right)^k}{k}\right)}}^{0.248756}$$

[Open code](#)

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$$2.01 \times 2 \sqrt{\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}} = \left(\sqrt{-1 + \frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}\right)^k \right)^{0.248756}$$

[Open code](#)

$$2.01 \times 2 \sqrt{\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}} = \left(\sqrt{-1 + \frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + \frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^{0.248756}$$

[Open code](#)

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function

- $(a)_n$ is the Pochhammer symbol (rising factorial)
- [More information](#)

Integral representations:

$$2.01 \times 2 \sqrt{\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}} = \sqrt{\frac{89}{1 - \frac{89}{12\pi^2} \int_1^{1728} \frac{1}{t} dt}}^{0.248756}$$

[Open code](#)

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$$2.01 \times 2 \sqrt{\sqrt{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}}} = \sqrt{\frac{2136 i \pi^3}{-89 + 12 \pi^2 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1727^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}^{0.248756}$$

for $-1 < \gamma < 0$

[Open code](#)

- $\Gamma(x)$ is the gamma function
- i is the imaginary unit
- [More information](#)

$$36 * (((((89 / (((1 - 89 / (12 \pi^2)) * \ln(995328 / 576))))))))$$

Input:

$$36 \times \frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{3204}{\left(1 - \frac{89}{12\pi^2}\right) \log(1728)}$$

Decimal approximation:

More digits

1729.316143103191948015432364593228874415267097303225184696...

[Open code](#)

1729.31614...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Note that, we have also:

Input:

$$\frac{\frac{27 \times 3 - 1}{89}}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\frac{80}{89} \left(1 - \frac{89}{12\pi^2}\right) \log(1728)$$

Decimal approximation:

More digits

1.665398204652128963003928378683563962439777616901576369510...

[Open code](#)

1.6653982... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Series representations:

More

$$\frac{\frac{27 \times 3 - 1}{89}}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)} = \frac{80}{89} \left(1 - \frac{89}{12\pi^2}\right) \left(\log(1727) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1727}\right)^k}{k} \right)$$

[Open code](#)

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$$\frac{\frac{27 \times 3 - 1}{89}}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)} = \frac{80}{89} \left(1 - \frac{89}{12\pi^2}\right) \left(2i\pi \left\lfloor \frac{\arg(1728 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728 - x)^k x^{-k}}{k} \right) \text{ for } x < 0$$

[Open code](#)

$$\frac{27 \times 3 - 1}{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}} = \frac{80}{89} \left(1 - \frac{89}{12\pi^2}\right) \left(\log(z_0) + \left\lfloor \frac{\arg(1728 - z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1728 - z_0)^k z_0^{-k}}{k}\right)$$

[Open code](#)

- $\arg(z)$ is the complex argument
 - $\lfloor x \rfloor$ is the floor function
 - [More information](#)

Integral representations:

$$\frac{27 \times 3 - 1}{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}} = \int_1^{1728} \frac{20 \left(12 - \frac{89}{\pi^2}\right)}{267 t} dt$$

[Open code](#)

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$$\frac{27 \times 3 - 1}{\frac{89}{\left(1 - \frac{89}{12\pi^2}\right) \log\left(\frac{995328}{576}\right)}} = -\frac{10 i (-89 + 12\pi^2)}{267 \pi^3} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1727^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

[Open code](#)

- $\Gamma(x)$ is the gamma function

Now, we have that:

$$\gamma_m = -\frac{1}{1 + \delta_m} \left(\frac{2}{e_R} \delta_m\right) \left(-\frac{\varepsilon}{2} e_R\right) = \delta_m \varepsilon = -\frac{3e_R^2}{8\pi^2}$$

for $e_R^2 = 89$, we obtain:

$$-3 \cdot 89 / (8 \cdot \pi^2)$$

Input:

$$-3 \times \frac{89}{8 \pi^2}$$

[Open code](#)

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Result:

- Approximate form
- Step-by-step solution

$$-\frac{267}{8\pi^2}$$

Decimal approximation:

- More digits

-3.38159450406302312193947708462465994843297410468622856772...

[Open code](#)

Property:

$-\frac{267}{8\pi^2}$ is a transcendental number

Series representations:

- More

$$-\frac{3 \times 89}{8\pi^2} = -\frac{267}{128 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2}$$

[Open code](#)

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$$-\frac{3 \times 89}{8\pi^2} = -\frac{267}{128 \left(\sum_{k=0}^{\infty} \frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2}$$

[Open code](#)

$$-\frac{3 \times 89}{8\pi^2} = -\frac{267}{8 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2}$$

Integral representations:

- More

$$-\frac{3 \times 89}{8\pi^2} = -\frac{267}{128 \left(\int_0^1 \sqrt{1-t^2} dt \right)^2}$$

[Open code](#)

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$$-\frac{3 \times 89}{8\pi^2} = -\frac{267}{32 \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^2}$$

[Open code](#)

$$-\frac{3 \times 89}{8 \pi^2} = -\frac{267}{32 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2}$$

[Open code](#)

$$\left(-\left(\left(-3 \times 89 / (8 \times \text{Pi}^2) \right) \right) \right)^{1/(12/5)}$$

Input:

$$\sqrt[5]{\sqrt[12]{-\left(-3 \times \frac{89}{8 \pi^2}\right)}}$$

[Open code](#)

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Exact result:

$$\frac{267^{5/12}}{2 \sqrt[4]{2} \pi^{5/6}}$$

Decimal approximation:

More digits

1.661373595030547187245566519185354004599818364840175255550...

[Open code](#)

1.66137359... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Property:

$$\frac{267^{5/12}}{2 \sqrt[4]{2} \pi^{5/6}} \text{ is a transcendental number}$$

Series representations:

More

$$\sqrt[5]{\sqrt[12]{-\frac{3 \times 89}{8 \pi^2}}} = \frac{267^{5/12}}{4 \times 2^{11/12} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{5/6}}$$

[Open code](#)

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$$\sqrt[5]{\sqrt[12]{-\frac{3 \times 89}{8 \pi^2}}} = \frac{267^{5/12}}{2 \sqrt[4]{2} \left(\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^{5/6}}$$

[Open code](#)

$$\sqrt[5]{\frac{12}{-3 \times 89} \pi^2} = \frac{267^{5/12}}{2 \sqrt[4]{2} \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^{5/6}}$$

Integral representations:

More

$$\sqrt[5]{\frac{12}{-3 \times 89} \pi^2} = \frac{267^{5/12}}{4 \sqrt[12]{2} \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^{5/6}}$$

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$$\sqrt[5]{\frac{12}{-3 \times 89} \pi^2} = \frac{267^{5/12}}{4 \times 2^{11/12} \left(\int_0^1 \sqrt{1-t^2} dt \right)^{5/6}}$$

[Open code](#)

$$\sqrt[5]{\frac{12}{-3 \times 89} \pi^2} = \frac{267^{5/12}}{4 \sqrt[12]{2} \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{5/6}}$$

Now, we have that:

4 RGE for the 4-Fermi theory

Using $\mu \frac{d\alpha}{d\mu} = \beta(\alpha)$, the solution to this differential equation is

$$G_R(\mu) = G_R(\mu_0) \exp \left[\int_{\alpha(\mu_0)}^{\alpha(\mu)} \frac{\gamma_G(\alpha)}{\beta(\alpha)} d\alpha \right] \quad (50)$$

In particular, with $\beta(\alpha) = -\frac{\alpha^2}{2\pi} \beta_0 = \frac{2\alpha^2}{3\pi}$ at leading order we find

$$G_R(\mu) = G_R(\mu_0) \exp \left[-\frac{9}{4} \int_{\alpha(\mu_0)}^{\alpha(\mu)} \frac{d\alpha}{\alpha} \right] = G_R(\mu_0) \left(\frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{-\frac{9}{4}} \quad (51)$$

Now, we are assuming that we know the value for G at the scale $\mu_0 = m_W$ where the W boson (or its equivalent in our toy model) is integrated out and we would like to know the value of G at the scale relevant for muon decay, $\mu = m_\mu$. Using Eq. (30), we find $\alpha(m_\mu) = 0.00736$ and $\alpha(m_W) = 0.00743$ so that

$$G_R(m_\mu) = 1.024 \times G(m_W) \quad (52)$$

We obtain, from (51), for the values 0.00775 and 0.00782, that are the values of the rest mass of Omega meson 782.65 and charged Rho meson 775.4:

$$(0.00775/0.00782)^{-9/4}$$

Input:

$$\left(\frac{0.00775}{0.00782}\right)^{-9/4}$$
 Open code

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Result:
 More digits
 1.02044...

1.02044... that is very near to the value in TeV of rest mass of Phi meson 1019.445.

Now:

$$\left(\left(\left(10^3 \left(\frac{0.00775}{0.00782}\right)^{-9/4}\right)\right)\right)^{1/14}$$

Input:

$$\sqrt[14]{10^3 \left(\frac{0.00775}{0.00782}\right)^{-9/4}}$$
 Open code

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Result:
 More digits
 1.64026...

$$1.64026... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

And:

$$2\sqrt{\left(\left(\left(6 \cdot \left(\left(10^3 \left(\frac{0.00775}{0.00782}\right)^{-9/4}\right)\right)\right)^{1/14}\right)\right)\right)}$$

Input:

$$2\sqrt{6 \sqrt[14]{10^3 \left(\frac{0.00775}{0.00782}\right)^{-9/4}}}$$
 Open code

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Result:
 More digits
 6.274257...

$$6.274257... \approx 2\pi$$

Now, we have that:

Since we are just interested in counterterm we take $k \gg q_1, q_2$. Then this reduces to

$$i\mathcal{M}_{1\text{-loop}}^\mu = \frac{i}{q_1 - m} \gamma^\mu \frac{i}{q_2 - m} \left[-ie_R^2 \mu^{4-d} \frac{(2-d)^2}{d} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^4} + \delta_J \right] \quad (58)$$

$$= i\mathcal{M}_{\text{tree}}^\mu \left\{ \frac{e_R^2}{16\pi^2} \left[\frac{2}{\varepsilon} \right] + \delta_J \right\} \quad (59)$$

Thus $\delta_J = \frac{e_R^2}{16\pi^2} \left[-\frac{2}{\varepsilon} \right]$, which also happens to equal δ_2 and δ_1 . Thus $Z_2 = Z_J$ at 1-loop.

From:

$$\delta_J = \frac{e_R^2}{16\pi^2} \left[-\frac{2}{\varepsilon} \right]$$

For $e_R^2 = 89$ and $\varepsilon = 1/24$ we obtain:

$$i\left(\left(\left(\left(89/(16\pi^2)\right)\right)\right)\left(-2/(1/24)\right)\right)$$

Input:

$$i\left(\frac{89}{16\pi^2}\left(-\frac{2}{24}\right)\right)$$

[Open code](#)

- i is the imaginary unit

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Result:

- Approximate form
- Step-by-step solution

$$-\frac{267i}{\pi^2}$$

Decimal approximation:

- More digits

$$-27.0527560325041849755158166769972795874637928374898285417... i$$

[Open code](#)

Property:

$$-\frac{267i}{\pi^2} \text{ is a transcendental number}$$

[Open code](#)

Polar coordinates:

- Exact form
- $r \approx 27.0528$ (radius), $\theta = -90^\circ$ (angle)

[Open code](#)

$$27.0528$$

$$\left(\left(\left(\left(\left(i\left(\left(\left(\left(89/(16\pi^2)\right)\right)\right)\left(-2/(1/24)\right)\right)\right)\right)\right)\right)\right)^2$$

Input:

$$\left(i \left(\frac{89}{16 \pi^2} \left(-\frac{2}{24} \right) \right) \right)^2$$

[Open code](#)

- i is the imaginary unit

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Approximate form
- Step-by-step solution

$$-\frac{71\,289}{\pi^4}$$

Decimal approximation:

- More digits

-731.851608954191571299512766329624586676762166344018110915...

[Open code](#)

Property:

$-\frac{71\,289}{\pi^4}$ is a transcendental number

$$10^3 - \left(\left(-i \left(\frac{89}{16 \pi^2} \left(-\frac{2}{24} \right) \right) \right)^2 \right)$$

Input:

$$10^3 - \left(i \left(\frac{89}{16 \pi^2} \left(-\frac{2}{24} \right) \right) \right)^2$$

[Open code](#)

- i is the imaginary unit

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Result:

$$1000 + \frac{71\,289}{\pi^4}$$

Decimal approximation:

- More digits

1731.851608954191571299512766329624586676762166344018110915...

[Open code](#)

1731.8516...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Property:

$1000 + \frac{71\,289}{\pi^4}$ is a transcendental number

Series representations:

- More

$$10^3 - \left(-\frac{i 89 (-2)}{16 \pi^2 24} \right)^2 = 1000 + \frac{71\,289}{256 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4}$$

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$$10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}} \right)^2 = 1000 + \frac{71\,289}{\left(\sum_{k=0}^{\infty} \frac{4 (-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4}$$

[Open code](#)

$$10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}} \right)^2 = 1000 + \frac{71\,289}{\left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4}$$

Integral representations:

• [More](#)

$$10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}} \right)^2 = 1000 + \frac{71\,289}{256 \left(\int_0^1 \sqrt{1-t^2} dt \right)^4}$$

[Open code](#)

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$$10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}} \right)^2 = 1000 + \frac{71\,289}{16 \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^4}$$

[Open code](#)

$$10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}} \right)^2 = 1000 + \frac{71\,289}{16 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^4}$$

Now, we have that:

Then, since δ_λ starts at order λ_R we have $\mu \frac{d}{d\mu} \lambda_R = -\varepsilon \lambda_R + \mathcal{O}(\lambda_R^2)$. Although not necessary for the running of m_R , it's not hard to calculate δ_λ at 1-loop. We can extract it from the radiative correction to the 4-point function. With zero external momenta, the loop gives

$$(-i\lambda_R)^2 \frac{3}{2} \mu^{2(4-d)} \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2} \frac{i}{k^2} = \mu^{2(4-d)} \frac{3\lambda_R^2}{16\pi^2} \frac{i}{\varepsilon} \quad (93)$$

So that $\delta_\lambda = \frac{3\lambda_R}{16\pi^2} \frac{1}{\varepsilon}$ and then the β -function to order λ_R^2 is

$$\beta(\lambda_R) \equiv \mu \frac{d}{d\mu} \lambda_R(\mu) = -\varepsilon \lambda_R - \frac{3\lambda_R^2}{16\pi^2} \frac{1}{\varepsilon} (-\varepsilon) = -\varepsilon \lambda_R + \frac{3\lambda_R^2}{16\pi^2} \quad (94)$$

We will use this result below.

From (94) for $\lambda_R^2 = 89$ and $\varepsilon = 1/24$ we obtain:

$$-1/24 * (\text{sqrt}(89)) + (3 * 89 / (16 \text{Pi}^2))$$

Input:

$$-\frac{\sqrt{89}}{24} + 3 \times \frac{89}{16 \pi^2}$$

[Open code](#)

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Result:

$$\frac{267}{16 \pi^2} - \frac{\sqrt{89}}{24}$$

Decimal approximation:

More digits

1.297714704862486402164711026578053277675502775950817562166...

[Open code](#)

Property:

$-\frac{\sqrt{89}}{24} + \frac{267}{16 \pi^2}$ is a transcendental number

[Open code](#)

Series representations:

More

$$\frac{1}{24} \sqrt{89} (-1) + \frac{3 \times 89}{16 \pi^2} = \frac{267}{16 \pi^2} - \frac{1}{24} \sqrt{88} \sum_{k=0}^{\infty} 88^{-k} \binom{\frac{1}{2}}{k}$$

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$$\frac{1}{24} \sqrt{89} (-1) + \frac{3 \times 89}{16 \pi^2} = \frac{267}{16 \pi^2} - \frac{1}{24} \sqrt{88} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{88}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$\frac{1}{24} \sqrt{89} (-1) + \frac{3 \times 89}{16 \pi^2} = \frac{267}{16 \pi^2} - \frac{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 88^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{48 \sqrt{\pi}}$$

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- $\Gamma(x)$ is the gamma function
- $\operatorname{Res}_{z=z_0} f$ is a complex residue
- [More information](#)

We have also:

$$(27*4) * (((-1/24*(\text{sqrt}(89)))+(3*89/(16\text{Pi}^2))))$$

Input:

$$(27 \times 4) \left(-\frac{\sqrt{89}}{24} + 3 \times \frac{89}{16 \pi^2} \right)$$

[Open code](#)

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Result:

$$108 \left(\frac{267}{16 \pi^2} - \frac{\sqrt{89}}{24} \right)$$

Decimal approximation:

More digits

140.1531881251485314337887908704297539889542998026882967140...

[Open code](#)

140.153188... result very near to the rest mass of Pion meson 139.57

Property:

$$108 \left(-\frac{\sqrt{89}}{24} + \frac{267}{16 \pi^2} \right) \text{ is a transcendental number}$$

Series representations:

More

$$\left(-\frac{\sqrt{89}}{24} + \frac{3 \times 89}{16 \pi^2} \right) 27 \times 4 = \frac{7209}{4 \pi^2} - \frac{9}{2} \sqrt{88} \sum_{k=0}^{\infty} 88^{-k} \binom{\frac{1}{2}}{k}$$

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$$\left(-\frac{\sqrt{89}}{24} + \frac{3 \times 89}{16 \pi^2}\right)^2 \times 4 = \frac{7209}{4 \pi^2} - \frac{9}{2} \sqrt{88} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{88}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$\left(-\frac{\sqrt{89}}{24} + \frac{3 \times 89}{16 \pi^2}\right)^2 \times 4 = \frac{7209}{4 \pi^2} - \frac{9 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 88^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{4 \sqrt{\pi}}$$

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- $\Gamma(x)$ is the gamma function
- $\operatorname{Res}_{z=0} f$ is a complex residue
- [More information](#)

$$\left(\left(-\frac{1}{24} \times \sqrt{89}\right) + \left(\frac{3 \times 89}{16 \pi^2}\right)\right)^2 - \left(\frac{1}{10} \times 0.3464719\right)$$

Where 0.3464719 is a Ramanujan mock theta function

Input interpretation:

$$\left(-\frac{\sqrt{89}}{24} + 3 \times \frac{89}{16 \pi^2}\right)^2 - \frac{1}{10} \times 0.3464719$$

[Open code](#)

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Result:

More digits

1.64941627...

$$1.64941627... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$\left(\frac{1}{24} \sqrt{89} (-1) + \frac{3 \times 89}{16 \pi^2}\right)^2 - \frac{0.346472}{10} = -0.0346472 + \left(\frac{267}{16 \pi^2} - \frac{1}{24} \sqrt{88} \sum_{k=0}^{\infty} 88^{-k} \binom{\frac{1}{2}}{k}\right)^2$$

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$$\left(\frac{1}{24} \sqrt{89} (-1) + \frac{3 \times 89}{16 \pi^2} \right)^2 - \frac{0.346472}{10} =$$

$$-0.0346472 + \left(\frac{267}{16 \pi^2} - \frac{1}{24} \sqrt{88} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{88}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2$$

Open code

$$\left(\frac{1}{24} \sqrt{89} (-1) + \frac{3 \times 89}{16 \pi^2} \right)^2 - \frac{0.346472}{10} =$$

$$-0.0346472 + \left(\frac{267}{16 \pi^2} - \frac{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 88^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{48 \sqrt{\pi}} \right)^2$$

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- $\Gamma(x)$ is the gamma function
- $\text{Res}_{z=z_0} f$ is a complex residue

Now, we have that:

by Polchinski. Polchinski's idea was first to cut off the path integral more smoothly by writing

$$Z[J] = \int \mathcal{D}\phi e^{iS + \phi J} = \int \mathcal{D}\phi \exp \left\{ i \int d^4x \left(-\frac{1}{2} \phi(\square + m^2) e^{\frac{\square}{\Lambda^2}} \phi + \frac{g_3}{3!} \phi^3 + \frac{g_4}{4!} \phi^4 + \dots + \phi J \right) \right\} \quad (108)$$

The e^{\square/Λ^2} factor makes the propagator go like $e^{-p^2/\Lambda^2} \rightarrow 0$ at high energy. You can get away with this only in a scalar theory in Euclidean space, but we will not let such technical details prevent us from making very general conclusions. It's easiest to proceed in momentum space, where $\phi(x)^2 \rightarrow \phi(p)\phi(-p)$. Then

$$Z[J] = \int \mathcal{D}\phi e^{iS + \phi J} = \int \mathcal{D}\phi \exp \left\{ i \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{2} \phi(p)(p^2 - m^2) e^{-\frac{p^2}{\Lambda^2}} \phi(-p) + \mathcal{L}_{\text{int}}(\phi) + \phi J \right) \right\}$$

which is an exact solution to Eqs (114) and (115). In these solutions, $\lambda_4(\Lambda_0)$ and $\lambda_6(\Lambda_0)$ are free parameters to be set by boundary conditions.

What we would like to know is the sensitivity of λ_6 at some low scale Λ_L to its initial condition at some high scale Λ_H for fixed, renormalized, value of $\lambda_4(\Lambda_L)$. For simplicity, let us take $\lambda_6(\Lambda_H) = 0$ (any other boundary value would do just as well, but the solution is messier). Then, Eqs. (118) and (119) can be combined into

$$\lambda_6(\Lambda) = \frac{2c \left[\left(\frac{\Lambda}{\Lambda_H} \right)^\Delta - 1 \right]}{(2+d-a+\Delta) - (2+d-a-\Delta) \left(\frac{\Lambda}{\Lambda_H} \right)^\Delta} \lambda_4(\Lambda) \quad (120)$$

Setting $\Lambda = \Lambda_L \ll \Lambda_H$ and assuming $a, b, c, d \ll 2$, so that $\Delta \approx 2$, we find

$$\lambda_6(\Lambda_L) = -\frac{c}{2} \left(1 - \frac{\Lambda_L^2}{\Lambda_H^2} \right) \lambda_4^L(\Lambda_L) \quad (121)$$

In particular, the limit $\Lambda_H \rightarrow \infty$ exists. Back in terms of g_4 and g_6 we have fixed $g_4(\Lambda_L)$ and set $g_6(\Lambda_H) = 0$. Thus as $\Lambda_H \rightarrow \infty$ we have $g_6(\Lambda_L) = -\frac{c}{2} \frac{1}{\Lambda_L^2} g_4(\Lambda_L)$. That is, the boundary condition at large Λ_H is totally irrelevant to the value of g_6 at the low scale. That is why operators with

From (121), for $c = 1$, $\Lambda_L = 443,716125$, $\Lambda_H = 41,569219$ and $\lambda_4^L = 8$, we obtain:

$$-1/2(1-(443.716125^2/41.569219^2)*8*443.716125)$$

We note that 443.716125^2 and 41.569219^2 are equal to 196884 and 1728

Input interpretation:

$$-\frac{1}{2} \left(1 + \frac{443.716125^2}{41.569219^2} \times 8 \times (-443.716125) \right)$$

[Open code](#)

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Result:

More digits

202223.1272558000835103079432785462628692481389730875869753...
202223.1272558...

From this expression, we obtain, by the ln, the following result:

$$\ln\left(\left(-1/2(1-(443.716125^2/41.569219^2)*8*443.716125)\right)\right)$$

Input interpretation:

$$\log\left(-\frac{1}{2} \left(1 + \frac{443.716125^2}{41.569219^2} \times 8 \times (-443.716125) \right)\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

More digits

12.2171270...

12.2171... result that is very near to the value of black hole entropy 12.1904

Series representations:

More

$$\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)(-1) = \log(202\,222.) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-12.2171k}}{k}$$

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$$\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)(-1) = 2i\pi \left\lfloor \frac{\arg(202\,223. - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (202\,223. - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)(-1) = \left\lfloor \frac{\arg(202\,223. - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(202\,223. - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (202\,223. - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

- $\arg(z)$ is the complex argument
 - $\lfloor x \rfloor$ is the floor function
 - i is the imaginary unit
 - [More information](#)

Integral representations:

$$\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)(-1) = \int_1^{202\,223.} \frac{1}{t} dt$$

[Open code](#)

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$$\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)(-1) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-12.2171s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

[Open code](#)

- $\Gamma(x)$ is the gamma function

$$\left(\left(\left(\left(\ln\left(\left(-\frac{1}{2}\left(1-\frac{443.716125^2}{41.569219^2}\right)*8*443.716125\right)\right)\right)\right)\right)\right)^{1/5}$$

Input interpretation:

$$\sqrt[5]{\log\left(-\frac{1}{2}\left(1+\frac{443.716125^2}{41.569219^2}\times 8\times(-443.716125)\right)\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

More digits

1.64965762...

$$1.64965762... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

And:

$$2\sqrt{\left(\left(\left(\left(\left(6*\left(\left(\ln\left(\left(-\frac{1}{2}\left(1-\frac{443.716125^2}{41.569219^2}\right)*8*443.716125\right)\right)\right)\right)\right)\right)\right)^{1/5}\right)\right)\right)}$$

Input interpretation:

$$2\sqrt{6\sqrt[5]{\log\left(-\frac{1}{2}\left(1+\frac{443.716125^2}{41.569219^2}\times 8\times(-443.716125)\right)\right)}}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

More digits

6.292200162...

$$6.2922... \approx 2\pi$$

Series representations:

More

$$2\sqrt{6\sqrt[5]{\log\left(\frac{1}{2}\left(1-\frac{443.716^2\times 8\times 443.716}{41.5692^2}\right)\right)}_{(-1)}} = 2\sqrt{-1+6\sqrt[5]{\log(202\,223.)}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+6\sqrt[5]{\log(202\,223.)}\right)^{-k}}$$

[Open code](#)

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$$2 \sqrt{6 \sqrt[5]{\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)} (-1)} =$$

$$2 \sqrt{6 \sqrt[5]{\log(202\,222.) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-12.2171k}}{k}}}$$

[Open code](#)

$$2 \sqrt{6 \sqrt[5]{\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)} (-1)} =$$

$$2 \sqrt{-1 + 6 \sqrt[5]{\log(202\,223.)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + 6 \sqrt[5]{\log(202\,223.)}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

[Open code](#)

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- [More information](#)

Integral representations:

$$2 \sqrt{6 \sqrt[5]{\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)\right)} (-1)} = 2 \sqrt{6 \sqrt[5]{\int_1^{202\,223.} \frac{1}{t} dt}}}$$

[Open code](#)

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$$2 \sqrt{6 \sqrt[5]{\log\left(\frac{1}{2}\left(1 - \frac{443.716^2 \times 8 \times 443.716}{41.5692^2}\right)(-1)\right)}} =$$

$$2 \sqrt{3 \times 2^{4/5} \sqrt[5]{\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-12.2171s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}} \quad \text{for } -1 < \gamma < 0$$

[Open code](#)

- $\Gamma(x)$ is the gamma function
- i is the imaginary unit
- [More information](#)

$$1.0061571663 \left(\left(\left(\left(10^3 - \left(\left(-i \left(\frac{89}{16\pi^2} \left(-\frac{2}{24} \right) \right) \right) \right) \right) \right) \right) \right)^2 \right) \right)^{1/15}$$

Where 1.0061571663 is a Ramanujan mock theta function

Input interpretation:

$$1.0061571663 \sqrt[15]{10^3 - \left(-i \left(\frac{89}{16\pi^2} \left(-\frac{2}{24} \right) \right) \right)^2}$$

[Open code](#)

- i is the imaginary unit

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Result:

More digits

1.6541181865...

1.654118... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = \left(G_{505}/G_{101/5} \right)^3 = 1164,2696 \text{ i.e. } 1,65578...$$

Series representations:

More

$$1.00615716630000 \sqrt[15]{10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}} \right)^2} =$$

$$0.695210773690925 \sqrt[15]{256\,000 - \frac{71\,289 i^2}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4}}$$

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$$1.00615716630000 \sqrt[15]{10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}}\right)^2} =$$

$$1.00615716630000 \sqrt[15]{1000 - \frac{71\,289 i^2}{\left(-2 + 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^4}}$$

Open code

$$1.00615716630000 \sqrt[15]{10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}}\right)^2} =$$

$$1.00615716630000 \sqrt[15]{1000 - \frac{71\,289 i^2}{\left(x + 2 \sum_{k=1}^{\infty} \frac{\sin(kx)}{k}\right)^4}} \text{ for } (x \in \mathbb{R} \text{ and } x > 0)$$

Open code

- $\binom{n}{m}$ is the binomial coefficient

- \mathbb{R} is the set of real numbers

Integral representations:

More

$$1.00615716630000 \sqrt[15]{10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}}\right)^2} =$$

$$0.836355966104201 \sqrt[15]{16\,000 - \frac{71\,289 i^2}{\left(\int_0^{\infty} \frac{1}{1+t^2} dt\right)^4}}$$

Open code

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$$1.00615716630000 \sqrt[15]{10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}}\right)^2} =$$

$$0.695210773690925 \sqrt[15]{256\,000 - \frac{71\,289 i^2}{\left(\int_0^1 \sqrt{1-t^2} dt\right)^4}}$$

Open code

$$1.00615716630000 \sqrt[15]{10^3 - \left(-\frac{i 89 (-2)}{\frac{16\pi^2}{24}}\right)^2} =$$

$$0.836355966104201 \sqrt[15]{16\,000 - \frac{71\,289 i^2}{\left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^4}}$$

[Open code](#)

Now, we have that:

Returning to our toy RGEs, suppose we set $\lambda_4(\Lambda_H) = 0$. Then we would have found

$$\lambda_4(\Lambda) = \frac{2b \left[1 - \left(\frac{\Lambda}{\Lambda_H}\right)^\Delta\right]}{2 + d - a - \Delta - (2 + d - a + \Delta) \left(\frac{\Lambda}{\Lambda_H}\right)^\Delta} \lambda_6(\Lambda) \quad (122)$$

Expanding this for $a, b, c, d \ll 2$ gives

$$\lambda_4(\Lambda_L) = \frac{b}{2} \left(1 - \frac{\Lambda_H^2}{\Lambda_L^2}\right) \lambda_6(\Lambda_L) \quad (123)$$

From (123), for $b = 1$, $\Lambda_L = 443,716,125$, $\Lambda_H = 41,569,219$ and $\lambda_6 = 8$, as for eq. (121), we obtain:

$$1/2 * (1 - (41.569219^2 / 443.716125^2)) * 8 * 443.716125$$

Input interpretation:

$$\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125$$

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Result:

More digits

1759.286973100160955385608309817453670406952192688512277979...

[Open code](#)

1759.2869...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

$$\left(\left(\left(\left(\frac{1}{2} * (1 - (41.569219^2 / 443.716125^2)) * 8 * 443.716125\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2} \right) \times 8 \times 443.716125}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.645719364...

$$1.645719... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$\text{sqrt}(\text{(((((((6 * (((1/2 * (1 - (41.569219^2/443.716125^2)) * 8 * 443.716125))))^1/15))))))$$

Input interpretation:

$$\sqrt[6]{\sqrt[15]{\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2} \right) \times 8 \times 443.716125}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3.142342467...

3.14234...

$$2\text{sqrt}(\text{(((((((6 * (((1/2 * (1 - (41.569219^2/443.716125^2)) * 8 * 443.716125))))^1/15))))))$$

Input interpretation:

$$2 \sqrt[6]{\sqrt[15]{\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2} \right) \times 8 \times 443.716125}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

6.284684935...

6.284684... $\approx 2\pi$

We have also that:

$$\text{((((1/2 * (1 - (41.569219^2/443.716125^2)) * 8 * 443.716125)) - (9^3 - 1^3 + 13))$$

Input interpretation:

$$\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2} \right) \times 8 \times 443.716125 - (9^3 - 1^3 + 13)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1018.286973100160955385608309817453670406952192688512277979...

[Open code](#)

1018.2869... very near to the rest mass of Phi meson 1019.445

And:

$$\left(\left(\left(\frac{1}{2} \cdot \left(1 - \frac{41.569219^2}{443.716125^2}\right)\right) \cdot 8 \cdot 443.716125\right)\right) + 27 \cdot 4$$

Input interpretation:

$$\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125 + 27 \times 4$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1867.286973100160955385608309817453670406952192688512277979...

1867.2869... result very near to the rest mass of D meson 1869.62

Furthermore:

$$\left(\left(\left(\frac{1}{2} \cdot \left(1 - \frac{41.569219^2}{443.716125^2}\right)\right) \cdot 8 \cdot 443.716125\right)\right) \frac{1}{4\pi}$$

Input interpretation:

$$\left(\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125\right) \times \frac{1}{4\pi}$$

[Open code](#)

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Result:

More digits

139.999609...

139.9996... result very near to the rest mass of Pion meson 139.57

Series representations:

More

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right) (8 \times 443.716)}{(4\pi)^2} = \frac{109.955}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

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$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right) (8 \times 443.716)}{(4\pi)^2} = \frac{219.911}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

[Open code](#)

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} = \frac{439.822}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

Open code

$\binom{n}{m}$ is the binomial coefficient

Integral representations:

More

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} = \frac{219.911}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

Open code

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$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} = \frac{109.955}{\int_0^1 \sqrt{1-t^2} dt}$$

Open code

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} = \frac{219.911}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

$$\left(\left(\left(\left(\frac{1}{2} \cdot \left(1 - \frac{41.569219^2}{443.716125^2}\right)\right) \cdot 8 \cdot 443.716125\right)\right)\right) \frac{1}{4\pi} - 5$$

Input interpretation:

$$\left(\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125\right) \times \frac{1}{4\pi} - 5$$

Open code

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Result:

More digits

134.999609...

134.9996... result practically equal to the rest mass of Pion meson 134.9766

Series representations:

More

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} - 5 = -5 + \frac{109.955}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} - 5 = -5 + \frac{219.911}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

Open code

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} - 5 = -5 + \frac{439.822}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

Open code

• $\binom{n}{m}$ is the binomial coefficient

Integral representations:

• More

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} - 5 = -5 + \frac{219.911}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} - 5 = -5 + \frac{109.955}{\int_0^1 \sqrt{1-t^2} dt}$$

Open code

$$\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2} - 5 = -5 + \frac{219.911}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

From the result 139.9996..., that is very near to the Pion rest mass, we obtain:

((((((((((1/2*(1-(41.569219^2/443.716125^2))*8*443.716125))))))1/(4Pi))))))^1/10

Input interpretation:

$$\sqrt[10]{\left(\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125\right) \times \frac{1}{4\pi}}$$

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Result:

More digits

1.639127295...

1.639127...

$$1.0061571663 \left(\left(\left(\left(\left(\left(\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2} \right) \right) \times 8 \times 443.716125 \right) \right) \right) \right) \right) \frac{1}{4\pi} \right)^{1/10}$$

Where 1.0061571663 is a Ramanujan mock theta function

Input interpretation:

$$1.0061571663 \sqrt[10]{\left(\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125\right) \times \frac{1}{4\pi}}$$

[Open code](#)

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Result:

More digits

1.649219674...

$$1.649219... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$1.00615716630000 \sqrt[10]{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right) (8 \times 443.716)}{(4\pi)^2}} = 1.60986 \sqrt[10]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}$$

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$$1.00615716630000 \sqrt[10]{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right) (8 \times 443.716)}{(4\pi)^2}} = 1.7254 \sqrt[10]{\frac{1}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}}$$

[Open code](#)

$$1.00615716630000 \sqrt[10]{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = 1.84924 \sqrt[10]{\frac{1}{x + 2 \sum_{k=1}^{\infty} \frac{\sin(kx)}{k}}}$$

for $(x \in \mathbb{R}$ and $x > 0)$

[Open code](#)

- $\binom{n}{m}$ is the binomial coefficient
- \mathbb{R} is the set of real numbers
- [More information](#)

Integral representations:

More

$$1.00615716630000 \sqrt[10]{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = 1.7254 \sqrt[10]{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

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$$1.00615716630000 \sqrt[10]{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = 1.60986 \sqrt[10]{\int_0^1 \sqrt{1-t^2} dt}$$

[Open code](#)

$$1.00615716630000 \sqrt[10]{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = 1.7254 \sqrt[10]{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

$$\left(\left(\left(\left(\left(\left(\left(\left(\frac{1}{2} \cdot \left(1 - \frac{41.569219^2}{443.716125^2}\right) \cdot 8 \cdot 443.716125\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{\frac{1}{85\pi/26}}$$

Input interpretation:

$$85 \times \frac{\pi}{26} \sqrt[10]{\left(\frac{1}{2} \left(1 - \frac{41.569219^2}{443.716125^2}\right) \times 8 \times 443.716125\right) \times \frac{1}{4\pi}}$$

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Result:

More digits

1.617925229...

1.617925229... $\approx \phi = 1.61803398 \dots$

Series representations:

More

$$\frac{85\pi}{26} \sqrt{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = 439.822^{26 / \left(85 \times \sum_{k=1}^{\infty} 4^{-k} (-1+3^k) \zeta(1+k)\right)}$$

$$\left(\frac{1}{\sum_{k=1}^{\infty} 4^{-k} (-1+3^k) \zeta(1+k)}\right)^{26 / \left(85 \times \sum_{k=1}^{\infty} 4^{-k} (-1+3^k) \zeta(1+k)\right)}$$

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$$\frac{85\pi}{26} \sqrt{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} =$$

$$439.822^{26 / \left(85 \sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}\right)} \left(\frac{1}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}\right)^{26 / \left(85 \sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}\right)}$$

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$$\frac{85\pi}{26} \sqrt{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} =$$

$$e^{0.824846 / \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)} \left(\frac{1}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}\right)^{13 / \left(85 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)\right)}$$

[Open code](#)

- $\zeta(s)$ is the Riemann zeta function
- $\binom{n}{m}$ is the binomial coefficient

[More information](#)

Integral representations:

More

$$\frac{85\pi}{26} \sqrt{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = e^{0.824846 / \left(\int_0^\infty \frac{1}{1+t^2} dt\right)} \left(\frac{1}{\int_0^\infty \frac{1}{1+t^2} dt}\right)^{13} / \left(85 \int_0^\infty \frac{1}{1+t^2} dt\right)$$

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$$\frac{85\pi}{26} \sqrt{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = e^{0.824846 / \left(\int_0^\infty \frac{\sin(t)}{t} dt\right)} \left(\frac{1}{\int_0^\infty \frac{\sin(t)}{t} dt}\right)^{13} / \left(85 \int_0^\infty \frac{\sin(t)}{t} dt\right)$$

[Open code](#)

$$\frac{85\pi}{26} \sqrt{\frac{\left(1 - \frac{41.5692^2}{443.716^2}\right)(8 \times 443.716)}{(4\pi)^2}} = e^{0.824846 / \left(\int_0^\infty \frac{\sin^2(t)}{t^2} dt\right)} \left(\frac{1}{\int_0^\infty \frac{\sin^2(t)}{t^2} dt}\right)^{13} / \left(85 \int_0^\infty \frac{\sin^2(t)}{t^2} dt\right)$$

[Open code](#)

• [More information](#)

From: String Theory “**Superstring Theory and beyond**” Vol. II– J. Polchinski

discovered by imposing closure directly. It has a representation theory parallel to that of the Virasoro algebra, and in particular has a series of unitary degenerate representations of central charge

$$c = 2 - \frac{24}{(k+3)(k+4)}. \quad (15.5.18)$$

The $(k_1, k_2, n) = (k, 1, 3)$ cosets produce these representations. As it happens, the first nontrivial case is $k = 1$, $c = \frac{4}{5}$, which as we have seen also has a parafermionic algebra. The number of extended chiral algebras is

picture indicates that this is the next minimal model down. We can compute the central charge from the c -theorem. Taking from the literature the value $c_{111} = 4/3^{1/2}$ for the large- m minimal model yields

$$c' = c - \frac{12}{m^3}. \quad (15.9.20)$$

15.8 Renormalization group flows

A perturbation with $h_i > 1$ is thus termed *irrelevant*, because its effect drops away at long distance and we return to the conformal theory. A perturbation with $h_i < 1$ is termed *relevant*. It grows more important at low energies, and we move further from the original conformal theory. A perturbation with $h_i = 1$ is termed *marginal*.

the C function. With $T_{\bar{z}z} = -\pi\beta^i\phi_i$, the result (15.8.13) for the C function becomes to leading order

$$\dot{C} = -12\pi^2\beta^i\beta^jG_{ij}, \quad (15.8.29)$$

where

$$G_{ij} = z^2\bar{z}^2 \langle \phi_i(z, \bar{z})\phi_j(0, 0) \rangle \quad (15.8.30)$$

is evaluated at $\lambda^i = 0$. Observe that

$$\beta^i = \frac{\partial}{\partial \lambda_i} U(\lambda), \quad (15.8.31a)$$

$$U(\lambda) = (h_i - 1)\lambda^i\lambda_i + \frac{2\pi}{3}c_{ijk}\lambda^i\lambda^j\lambda^k, \quad (15.8.31b)$$

indices being lowered with G_{ij} . Using this and $\beta^i = -2\dot{\lambda}^i$ gives

$$\dot{C} = 24\pi^2\beta_j\dot{\lambda}^j = 24\pi^2\dot{U}. \quad (15.8.32)$$

This integrates to

$$C = c + 24\pi^2U \quad (15.8.33)$$

with c being the central charge at the conformal point $\lambda^i = 0$.

Now let us apply this to the case of a single slightly relevant operator,

$$\dot{\lambda} = (1 - h)\lambda - \pi c_{111}\lambda^2, \quad (15.8.34)$$

normalized so that $G_{11} = 1$. If λ starts out positive it grows, but not indefinitely: the negative second order term cuts off the growth. At long

Now let us apply this to the case of a single slightly relevant operator,

$$\dot{\lambda} = (1 - h)\lambda - \pi c_{111}\lambda^2, \quad (15.8.34)$$

normalized so that $G_{11} = 1$. If λ starts out positive it grows, but not indefinitely: the negative second order term cuts off the growth. At long distance we arrive at a *new* conformal theory, with coupling

$$\lambda' = \frac{1 - h}{\pi c_{111}}. \quad (15.8.35)$$

From the string spacetime point of view, we can interpret $U(\lambda)$ as a potential energy for the light field corresponding to the world-sheet coupling λ , and the two conformal theories correspond to the two stationary points of the cubic potential. Note that $\lambda = 0$ is a local maximum: relevant operators on the world-sheet correspond to tachyons in spacetime. The central charge of the new fixed point is

$$c' = c - 8 \frac{(1 - h)^3}{c_{111}^2}. \quad (15.8.36)$$

Remembering that if λ starts out positive it grows, $h < 1$ is termed relevant, taking from the literature the value $c_{111} = 4/3^{1/2}$ and, from (15.5.18), if $k = 1$, $c = 4/5$, we have, for $\lambda = 1.617861\dots$, $h = 0.6181$, $c = 4/5$ and $c_{111} = \sqrt{4/3}$:

$$U(\lambda) = (h_i - 1)\lambda^i \lambda_i + \frac{2\pi}{3} c_{ijk} \lambda^i \lambda^j \lambda^k$$

$$C = c + 24\pi^2 U$$

$$c' = c - 8 \frac{(1 - h)^3}{c_{111}^2}.$$

$$(0.6181-1)*1.617861^2+(2\pi/3)*(4/5)*(1.617861^3)$$

Input interpretation:

$$(0.6181 - 1) \times 1.617861^2 + \left(2 \times \frac{\pi}{3}\right) \times \frac{4}{5} \times 1.617861^3$$

[Open code](#)

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Result:

- More digits
6.09571...
6.09571...

Series representations:
More

$$(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3} = -0.999613 + 9.03405 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3} = -5.51664 + 4.51702 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

Open code

$$(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3} = -0.999613 + 2.25851 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

Open code

• $\binom{n}{m}$ is the binomial coefficient

Integral representations:
More

$$(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3} = -0.999613 + 4.51702 \int_0^{\infty} \frac{1}{1+t^2} dt$$

Open code

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$$(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3} = -0.999613 + 9.03405 \int_0^1 \sqrt{1-t^2} dt$$

Open code

$$(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3} = -0.999613 + 4.51702 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$-1.0064637980 * -1 + 1 / (((((0.6181 - 1) * 1.617861^2 + (2\pi/3) * (4/5) * (1.617861^3))))))^{1/4}$$

Where -1.0064637980 is given by:

Input interpretation:

$$\frac{1}{2} \times \frac{1.897512108}{6} + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right)$$

[Open code](#)

where 1.897512108 is a Ramanujan mock theta function

Input interpretation:

$$-1.0064637980 \times (-1) + \frac{1}{\sqrt[4]{(0.6181 - 1) \times 1.617861^2 + \left(2 \times \frac{\pi}{3}\right) \times \frac{4}{5} \times 1.617861^3}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.642884...

$$1.642884... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$\begin{aligned}
 & -(-1) 1.00646379800000 + \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3}}} = \\
 & 1.00646379800000 + \frac{1}{\sqrt[4]{-0.999613 + 9.03405 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}
 \end{aligned}$$

[Open code](#)

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$$\begin{aligned}
 & -(-1) 1.00646379800000 + \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2\pi}{5 \times 3}}} = \\
 & 1.00646379800000 + \frac{1}{\sqrt[4]{-5.51664 + 4.51702 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}}
 \end{aligned}$$

[Open code](#)

$$-(-1) 1.00646379800000 + \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2 \pi}{5 \times 3}}} =$$

$$1.00646379800000 + \frac{1}{\sqrt[4]{-0.999613 + 2.25851 x + 4.51702 \sum_{k=1}^{\infty} \frac{\sin(k x)}{k}}}$$

for $(x \in \mathbb{R}$ and $x > 0)$

[Open code](#)

- $\binom{n}{m}$ is the binomial coefficient
- \mathbb{R} is the set of real numbers
- [More information](#)

Integral representations:

More

$$-(-1) 1.00646379800000 + \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2 \pi}{5 \times 3}}} =$$

$$1.00646379800000 + \frac{1}{\sqrt[4]{-0.999613 + 4.51702 \int_0^{\infty} \frac{1}{1+t^2} dt}}$$

[Open code](#)

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$$-(-1) 1.00646379800000 + \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2 \pi}{5 \times 3}}} =$$

$$1.00646379800000 + \frac{1}{\sqrt[4]{-0.999613 + 9.03405 \int_0^1 \sqrt{1-t^2} dt}}$$

[Open code](#)

$$-(-1) 1.00646379800000 + \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(4 \times 1.61786^3) 2 \pi}{5 \times 3}}} =$$

$$1.00646379800000 + \frac{1}{\sqrt[4]{-0.999613 + 4.51702 \int_0^{\infty} \frac{\sin(t)}{t} dt}}$$

$$2\sqrt{\left(-6 - 1.0064637980 \times 1 - \frac{1}{\sqrt[4]{(0.6181 - 1) \times 1.617861^2 + (2\pi/3) \times (4/5) \times (1.617861^3)}}}\right)^{1/4}}$$

Input interpretation:

$$2 \sqrt{-6 \left(-1.0064637980 \times 1 - \frac{1}{\sqrt[4]{(0.6181 - 1) \times 1.617861^2 + \left(2 \times \frac{\pi}{3}\right) \times \frac{4}{5} \times 1.617861^3}}}\right)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

6.2792686...

6.2792686... $\approx 2\pi$

Series representations:

More

$$2 \sqrt{-6 \left(-1.00646379800000 - \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(2\pi)4 \times 1.61786^3}{3 \times 5}}}\right)} =$$

$$2 \sqrt{5.03878278800000 + \frac{6}{\sqrt[4]{-0.999613 + 2.25851 \pi}}}$$

$$\sum_{k=0}^{\infty} \left(5.03878278800000 + \frac{6}{\sqrt[4]{-0.999613 + 2.25851 \pi}} \right)^{-k} \binom{\frac{1}{2}}{k}$$

[Open code](#)

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$$2 \sqrt{-6 \left(-1.00646379800000 - \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(2\pi)4 \times 1.61786^3}{3 \times 5}}}\right)} =$$

$$2 \sqrt{5.03878278800000 + \frac{6}{\sqrt[4]{-0.999613 + 2.25851 \pi}}}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(5.03878278800000 + \frac{6}{\sqrt[4]{-0.999613 + 2.25851 \pi}} \right)^{-k}}{k!} \left(-\frac{1}{2} \right)_k$$

[Open code](#)

$$2 \sqrt{-6 \left(-1.00646379800000 - \frac{1}{\sqrt[4]{(0.6181 - 1) 1.61786^2 + \frac{(2\pi)^4 \times 1.61786^3}{3 \times 5}}} \right)} =$$

$$2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(6.03878278800000 + \frac{6}{\sqrt[4]{-0.999613 + 2.25851\pi}} - z_0\right)^k}{k!} z_0^{-k}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$4/5 + (24\pi^2 * 6.0957103853482426118171915159953128944456292874611917)$$

Input interpretation:

$$\frac{4}{5} + 24\pi^2 \times 6.0957103853482426118171915159953128944456292874611917$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1444.6940011279790110413552765601421661343847731572377...

1444.694...

Series representations:

More

$$\frac{4}{5} + 24\pi^2 \cdot 6.09571038534824261181719151599531289444562928746119170000 =$$

$$\frac{4}{5} + 2340.75278797372516293780154214220015146712164638509761280$$

$$\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{4}{5} + 24\pi^2 \cdot 6.09571038534824261181719151599531289444562928746119170000 =$$

$$\frac{4}{5} + 146.297049248357822683612596383887509466695102899068600800$$

$$\left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}} \right)^2$$

[Open code](#)

$$\frac{4}{5} + 24\pi^2 \cdot 6.09571038534824261181719151599531289444562928746119170000 =$$

$$\frac{4}{5} + 146.297049248357822683612596383887509466695102899068600800$$

$$\left(x + 2 \sum_{k=1}^{\infty} \frac{\sin(kx)}{k}\right)^2 \text{ for } (x \in \mathbb{R} \text{ and } x > 0)$$

Open code

- $\binom{n}{m}$ is the binomial coefficient
- \mathbb{R} is the set of real numbers
- [More information](#)

Integral representations:

More

$$\frac{4}{5} + 24\pi^2 \cdot 6.09571038534824261181719151599531289444562928746119170000 =$$

$$\frac{4}{5} + 585.188196993431290734450385535550037866780411596274403200$$

$$\left(\int_0^{\infty} \frac{1}{1+t^2} dt\right)^2$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{4}{5} + 24\pi^2 \cdot 6.09571038534824261181719151599531289444562928746119170000 =$$

$$\frac{4}{5} + 2340.75278797372516293780154214220015146712164638509761280$$

$$\left(\int_0^1 \sqrt{1-t^2} dt\right)^2$$

Open code

$$\frac{4}{5} + 24\pi^2 \cdot 6.09571038534824261181719151599531289444562928746119170000 =$$

$$\frac{4}{5} + 585.188196993431290734450385535550037866780411596274403200$$

$$\left(\int_0^{\infty} \frac{\sin(t)}{t} dt\right)^2$$

$$1/48 + [((((((((4/5 + (24\pi^2 * 6.095710385348242)))))))))]^{1/15}$$

Input interpretation:

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 24\pi^2 \times 6.095710385348242}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.6450790293492541...

$$1.645079029... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Series representations:

More

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 24\pi^2 \cdot 6.0957103853482420000} =$$

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 2340.7527879737249280 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 24\pi^2 \cdot 6.0957103853482420000} =$$

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 585.18819699343123200 \left(-1.00000000000000000000 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^2}$$

[Open code](#)

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 24\pi^2 \cdot 6.0957103853482420000} =$$

$$\frac{1}{48} + 15\sqrt{\frac{4}{5} + 146.29704924835780800 \left(x + 2 \sum_{k=1}^{\infty} \frac{\sin(kx)}{k} \right)^2} \text{ for } (x \in \mathbb{R} \text{ and } x > 0)$$

[Open code](#)

- $\binom{n}{m}$ is the binomial coefficient
- \mathbb{R} is the set of real numbers
- [More information](#)

Integral representations:

More

$$\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 24 \pi^2 \cdot 6.0957103853482420000} =$$
$$\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 585.18819699343123200 \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^2}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 24 \pi^2 \cdot 6.0957103853482420000} =$$
$$\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 2340.7527879737249280 \left(\int_0^1 \sqrt{1-t^2} dt \right)^2}$$

Open code

$$\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 24 \pi^2 \cdot 6.0957103853482420000} =$$
$$\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 585.18819699343123200 \left(\int_0^\infty \frac{\sin(t)}{t} dt \right)^2}$$

$$2\text{sqrt}(\text{(((((((6 ((((((1/48+((((((((((4/5 + (24Pi^2*6.095710385348242))))))))))^1/15))))))))))))))$$

Input interpretation:

$$2 \sqrt[15]{6 \left(\frac{1}{48} + 15 \sqrt[15]{\frac{4}{5} + 24 \pi^2 \times 6.095710385348242} \right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

$$6.2834621590634330\dots$$
$$6.283462\dots \approx 2\pi$$

Series representations:

More

$$2\sqrt{6\left(\frac{1}{48} + \sqrt[15]{\frac{4}{5} + 24\pi^2 6.0957103853482420000}\right)} =$$

$$2\sqrt{-\frac{7}{8} + 6\sqrt[15]{\frac{4}{5} + 146.29704924835780800\pi^2}}$$

$$\sum_{k=0}^{\infty} \left(-\frac{7}{8} + 6\sqrt[15]{\frac{4}{5} + 146.29704924835780800\pi^2}\right)^{-k} \binom{\frac{1}{2}}{k}$$

[Open code](#)

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$$2\sqrt{6\left(\frac{1}{48} + \sqrt[15]{\frac{4}{5} + 24\pi^2 6.0957103853482420000}\right)} =$$

$$2\sqrt{-\frac{7}{8} + 6\sqrt[15]{\frac{4}{5} + 146.29704924835780800\pi^2}}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{7}{8} + 6\sqrt[15]{\frac{4}{5} + 146.29704924835780800\pi^2}\right)^{-k}}{k!} \left(-\frac{1}{2}\right)_k$$

[Open code](#)

$$2\sqrt{6\left(\frac{1}{48} + \sqrt[15]{\frac{4}{5} + 24\pi^2 6.0957103853482420000}\right)} =$$

$$2\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{8} + 6\sqrt[15]{\frac{4}{5} + 146.29704924835780800\pi^2} - z_0\right)^k}{k!} z_0^{-k}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\left(\left(\left(\frac{4}{5} - 8\left(\left(\left(1 - 0.6181\right)^3\right) / \left(\frac{4}{3}\right)\right)\right)\right)\right)$$

Input:

$$\frac{4}{5} - 8 \times \frac{(1 - 0.6181)^3}{\frac{4}{3}}$$

[Open code](#)

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Result:

0.465804786446

[Open code](#)

0.4658047...

$$\text{sqrt}(\left(\left(\left(\left(\left(\left(1.081849^3 / \left(\left(\frac{4}{5} - 8\left(\left(\left(1 - 0.6181\right)^3\right) / \left(\frac{4}{3}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)$$

where 1.081849 is a Ramanujan mock theta function

Input interpretation:

$$\sqrt{\frac{1.081849^3}{\frac{4}{5} - 8 \times \frac{(1-0.6181)^3}{\frac{4}{3}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.64872...

$$1.64872... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$2\sqrt{6 * \sqrt{(((((((1.081849^3 / (((4/5 - 8((((1-0.6181)^3) / ((4/3))))))))))))))}}$$

Input interpretation:

$$2 \sqrt{6 \sqrt{\frac{1.081849^3}{\frac{4}{5} - 8 \times \frac{(1-0.6181)^3}{\frac{4}{3}}}}}$$

[Open code](#)

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Result:

More digits

6.29042...

$$6.29042... \approx 2\pi$$

From the product of the three results obtained 6.09571... 1444.694... 0.4658047..., we obtain:

$$27 * \sqrt{6.09571 * 1444.694 * 0.4658047}$$

Input interpretation:

$$27 \sqrt{6.09571 \times 1444.694 \times 0.4658047}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1729.28...

1729.28

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross-

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$(((27*\text{sqrt}(6.09571*1444.694*0.4658047))))^{1/15}$$

Input interpretation:

$$\sqrt[15]{27 \sqrt{6.09571 \times 1444.694 \times 0.4658047}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.643833...

$$1.643833... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$2\text{sqrt}[6*(((27*\text{sqrt}(6.09571*1444.694*0.4658047))))^{1/15}]$$

Input interpretation:

$$2 \sqrt{6 \sqrt[15]{27 \sqrt{6.09571 \times 1444.694 \times 0.4658047}}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

6.2810822...

$$6.2810822... \approx 2\pi$$

$$8+1/\text{Pi}(6.09571*1444.694*0.4658047)$$

Input interpretation:

$$8 + \frac{1}{\pi} (6.09571 \times 1444.694 \times 0.4658047)$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1313.73...

1313.73... result very near to the rest mass of Xi baryon 1314.86

Series representations:

More

$$8 + \frac{6.09571 \times 1444.69 \times 0.465805}{\pi} = 8 + \frac{1025.52}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$8 + \frac{6.09571 \times 1444.69 \times 0.465805}{\pi} = 8 + \frac{2051.04}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

Open code

$$8 + \frac{6.09571 \times 1444.69 \times 0.465805}{\pi} = 8 + \frac{4102.08}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

Open code

• $\binom{n}{m}$ is the binomial coefficient

Integral representations:

More

$$8 + \frac{6.09571 \times 1444.69 \times 0.465805}{\pi} = 8 + \frac{2051.04}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$8 + \frac{6.09571 \times 1444.69 \times 0.465805}{\pi} = 8 + \frac{1025.52}{\int_0^1 \sqrt{1-t^2} dt}$$

Open code

$$8 + \frac{6.09571 \times 1444.69 \times 0.465805}{\pi} = 8 + \frac{2051.04}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

$$21 + 1/e(6.09571 * 1444.694 * 0.4658047)$$

Input interpretation:

$$21 + \frac{1}{e} (6.09571 \times 1444.694 \times 0.4658047)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1530.07...

1530.07... result very near to the rest mass of Xi baryon 1531.80

Series representations:

More

$$21 + \frac{6.09571 \times 1444.69 \times 0.465805}{e} = 21 + 4102.08 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$21 + \frac{6.09571 \times 1444.69 \times 0.465805}{e} = 21 + \frac{4102.08}{\sum_{k=0}^{\infty} \frac{1}{k!}}$$

[Open code](#)

$$21 + \frac{6.09571 \times 1444.69 \times 0.465805}{e} = 21 + \frac{8204.16}{\sum_{k=0}^{\infty} \frac{1+k}{k!}}$$

[Open code](#)

• $n!$ is the factorial function

$$(6.09571 \times 1444.694 \times 0.4658047)^{1/17}$$

Input interpretation:

$$\sqrt[17]{6.09571 \times 1444.694 \times 0.4658047}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.631284...

1.631284...

We have that:

In quantum field theory, the behavior of matrix elements under a rigid scale transformation is governed by a differential equation, the *renormalization group equation*. Let us derive such an equation. Consider a general quantum field theory in d -dimensional spacetime; spacetime here corresponds to the string world-sheet, which is the case $d = 2$. The scale transformation of a general expectation value is

$$\begin{aligned} \epsilon^{-1} \delta_s \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle &= -\frac{1}{2\pi} \int d^2\sigma \left\langle T_a^a(\sigma) \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle \\ &\quad - \sum_n \Delta_{i_n}^j \left\langle \mathcal{A}_j(\sigma_n) \prod_{m \neq n} \mathcal{A}_{i_m}(\sigma_m) \right\rangle, \end{aligned} \quad (15.8.5)$$

where \mathcal{A}_i is a complete set of local operators. The second term is from the action of the scale transformation on the operators,

$$\epsilon^{-1} \delta_s \mathcal{A}_i(\sigma) = -\Delta_i^j \mathcal{A}_j(\sigma). \quad (15.8.6)$$

The integrated trace of the energy-momentum tensor can be expanded in terms of the complete set,

$$\int d^d\sigma T_a^a = -2\pi \sum_i' \int d^d\sigma \beta^i(g) \mathcal{A}_i. \quad (15.8.7)$$

The prime on the sum indicates that it runs only over operators with dimension less than or equal to d , because this is the dimension of the energy-momentum tensor. We can similarly write a general renormalizable action as a sum over all such terms

$$S = \sum_i' g^i \int d^d\sigma \mathcal{A}_i(\sigma). \quad (15.8.8)$$

Here g^i is a general notation that includes the interactions as well as the masses and the kinetic term normalizations. The expansions (15.8.7) and (15.8.8) can be used to rewrite the scale transformation (15.8.5) as the *renormalization group equation*,

$$\begin{aligned} \epsilon^{-1} \delta_s \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle &= -\sum_i' \beta^i(g) \frac{\partial}{\partial g^i} \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle \\ &\quad - \sum_n \Delta_{i_n}^j \left\langle \mathcal{A}_j(\sigma_n) \prod_{m \neq n} \mathcal{A}_{i_m}(\sigma_m) \right\rangle. \end{aligned} \quad (15.8.9)$$

There may also be contact terms between T_a^a and the other operators, and terms from the g_i -derivative acting on the local operators. These are dependent on definitions (the choice of renormalization scheme) and can all be absorbed into the definition of Δ_i^j . Eq. (15.8.9) states that a scale

Remembering that if λ starts out positive it grows, $h < 1$ is termed relevant, taking from the literature the value $c_{111} = 4/3^{1/2}$ and, from (15.5.18), if $k = 1$, $c = 4/5$, we have, for $\lambda = 1.617861\dots$, $h = 0.6181$, $c = 4/5$ and $c_{111} = \sqrt{4/3}$

$$\epsilon^{-1} \delta_s \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle = -\frac{1}{2\pi} \int d^2\sigma \left\langle T_a^a(\sigma) \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle - \sum_n \Delta_{i_n}^j \left\langle \mathcal{A}_j(\sigma_n) \prod_{m \neq n} \mathcal{A}_{i_m}(\sigma_m) \right\rangle, \quad (15.8.5)$$

renormalization group equation,

$$\epsilon^{-1} \delta_s \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle = -\sum_i' \beta^i(g) \frac{\partial}{\partial g^i} \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle - \sum_n \Delta_{i_n}^j \left\langle \mathcal{A}_j(\sigma_n) \prod_{m \neq n} \mathcal{A}_{i_m}(\sigma_m) \right\rangle. \quad (15.8.9)$$

$$\beta^i = -2\lambda^i$$

$$\mathcal{A}_n = \left\langle \mathcal{O}_{1,2}(z_1, \bar{z}_1) \prod_{i=2}^n \mathcal{O}_i(z_i, \bar{z}_i) \right\rangle_{S_2}. \quad (15.3.4)$$

$$\mathcal{O}_{1,1} : h = 0, \quad \mathcal{O}_{1,2} : h = \frac{1}{16}, \quad \mathcal{O}_{1,3} : h = \frac{1}{2}. \quad (15.9.7)$$

A different generalization of the Ising model is the \mathbf{Z}_k Ising model (the clock model). Here the spins take k values $\sigma_i = \exp(2\pi i n/k)$ for $n = 0, 1, \dots, k-1$, and there is a \mathbf{Z}_k symmetry $\sigma_i \rightarrow \exp(2\pi i/k) \sigma_i$. The energy is

$$H = - \sum_{\text{links}} \text{Re}(\sigma_i \sigma_i^*). \quad (15.9.12)$$

We obtain:

$$A_i = 1/16 * 2 \text{ product } (i/16)*2i, i=2..5$$

Input interpretation:

$$\frac{1}{16} \times 2 \prod_{i=2}^5 \frac{i}{16} \times 2i$$

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Result:

More digits

$$\frac{225}{512} \approx 0.439453$$

0.439453

$$1 + \sqrt{\left(\frac{1}{16} * 2 \text{ product } (i/16)*2i, i=2..5\right)}$$

Input interpretation:

$$1 + \sqrt{\frac{1}{16} \times 2 \prod_{i=2}^5 \frac{i}{16} \times 2i}$$

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Result:

More digits

$$1 + \frac{15}{16\sqrt{2}} \approx 1.66291$$

1.66291 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696 \text{ i.e. } 1,65578\dots$$

Alternate forms:

$$\frac{15\sqrt{2}}{32} + 1$$

$$\frac{1}{32} (32 + 15\sqrt{2})$$

[Open code](#)

$$\frac{15 + 16\sqrt{2}}{16\sqrt{2}}$$

$$A_j = \left(\left(\frac{1}{2} * 2 \text{ product } (i/2)*2i, i=2..5\right)\right)$$

Input interpretation:

$$\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2i$$

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Result:

14400

14400

We have that:

$$\left(\left(\frac{1}{2} * 2 \text{ product } (i/2)*2i, i=2..5\right)\right)^{1/19}$$

Input interpretation:

$$\sqrt[19]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2 i}$$

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Result:

More digits

$$2^{6/19} \times 15^{2/19} \approx 1.65524$$

1.65524 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578\dots$$

And:

$$(((1/2 * 2 \text{ product } (i/2)*2i, i=2..5)))^{1/20}$$

Input interpretation:

$$\sqrt[20]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2 i}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

$$2^{3/10} \sqrt[10]{15} \approx 1.61405$$

Alternate form:

$\text{root of } x^{10} - 120 \text{ near } x = 1.61405$

[Open code](#)

1.61405

We note that:

$$1/2(((((((1/2 * 2 \text{ product } (i/2)*2i, i=2..5)))^{1/19} + (((1/2 * 2 \text{ product } (i/2)*2i, i=2..5)))^{1/20}))))))$$

Input interpretation:

$$\frac{1}{2} \left(\sqrt[19]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2 i} + \sqrt[20]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2 i} \right)$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

$$\frac{1}{2} \left(2^{3/10} \sqrt[10]{15} + 2^{6/19} \times 15^{2/19} \right) \approx 1.63465$$

Alternate forms:

$$\frac{\sqrt[10]{15}}{2^{7/10}} + \frac{15^{2/19}}{2^{13/19}}$$

[Open code](#)

$$\frac{\sqrt[10]{15} \left(1 + 2^{3/190} \sqrt[190]{15} \right)}{2^{7/10}}$$

[Open code](#)

1.63465

From this result, we obtain also:

$$2\sqrt{6 * 1/2(((((((1/2 * 2 \text{ product } (i/2)*2i, i=2..5))))^1/19 + (((1/2 * 2 \text{ product } (i/2)*2i, i=2..5))))^1/20))))]}$$

Input interpretation:

$$2 \sqrt{6 \times \frac{1}{2} \left(\sqrt[19]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2i} + \sqrt[20]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2i} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

$$2 \sqrt{3 \left(2^{3/10} \sqrt[10]{15} + 2^{6/19} \times 15^{2/19} \right)} \approx 6.26351$$

Alternate form:

$$2 \times 2^{3/20} \times 3^{11/20} \sqrt[20]{5} \sqrt{1 + 2^{3/190} \sqrt[190]{15}}$$

[Open code](#)

$$6.26351 \approx 2\pi$$

While, from the two results 1.66291 and 1.65524, we obtain:

$$2\sqrt{6 * 1/2(((((((1/2 * 2 \text{ product } (i/2)*2i, i=2..5))))^1/19 + (((1+\sqrt{((1/16 * 2 \text{ product } (i/16)*2i, i=2..5))})^1/19 + (((1/16 * 2 \text{ product } (i/16)*2i, i=2..5))))^1/20))))]}$$

Input interpretation:

$$2 \sqrt{6 \times \frac{1}{2} \left(\sqrt[19]{\frac{1}{2} \times 2 \prod_{i=2}^5 \frac{i}{2} \times 2i} + \left(1 + \sqrt{\frac{1}{16} \times 2 \prod_{i=2}^5 \frac{i}{16} \times 2i} \right) \right)}$$

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Result:

More digits

$$2 \sqrt{3 \left(1 + \frac{15}{16\sqrt{2}} + 2^{6/19} \times 15^{2/19} \right)} \approx 6.31014$$

Alternate forms:

More

$$2 \sqrt{3 + \frac{45}{16\sqrt{2}} + 3 \times 2^{6/19} \times 15^{2/19}}$$

[Open code](#)

$$\frac{1}{2} \sqrt{\frac{3}{2} \left(32 + 15\sqrt{2} + 32 \times 2^{6/19} \times 15^{2/19} \right)}$$

[Open code](#)

$$\frac{\sqrt{3(15 + 16\sqrt{2} + 16 \times 2^{31/38} \times 15^{2/19})}}{2\sqrt[4]{2}}$$

$$6.3104 \approx 2\pi$$

Now, we have:

derivative of $(-2 \times 1.617861)g$

Derivative:

Step-by-step solution

$$\frac{d}{dg}((-2 \times 1.617861)g) = -3.23572$$

[Open code](#)

$$-3.23572$$

Thence, from the eq. (15.8.9)

renormalization group equation,

$$\epsilon^{-1} \delta_s \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle = -\sum_i' \beta^i(g) \frac{\partial}{\partial g^i} \left\langle \prod_m \mathcal{A}_{i_m}(\sigma_m) \right\rangle - \sum_n \Delta_{i_n}^j \left\langle \mathcal{A}_j(\sigma_n) \prod_{m \neq n} \mathcal{A}_{i_m}(\sigma_m) \right\rangle. \quad (15.8.9)$$

We obtain:

$$-(-2 \times 1.617861)x * (-3.23572) * 0.439453 - 27 * (0.439453 * 14400)$$

Input interpretation:

$$-(-2 \times 1.617861)(x \times (-3.23572) \times 0.439453) - 27(0.439453 \times 14400)$$

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Result:

$$-4.60102x - 170859.$$

$$-4.60102x - 170589$$

$$\text{For } x = (1729 * (23.6954 + 11.8477 - 0.53)^{(1/3)}) = 5656.38...$$

where 23.6954 and 11.8477 are black hole entropies and $A = 0.53$ concerning the Polchinski equation (see Tables)

we obtain:

$$-170859. - 4.60102(1729 * (23.6954 + 11.8477 - 0.53)^{(1/3)})$$

Input interpretation:

$$-170859. + \left(1729 \sqrt[3]{23.6954 + 11.8477 - 0.53}\right) \times (-4.60102)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

$$-1.96884... \times 10^5$$

Input interpretation:

$$-1.96884 \times 10^5$$

[Open code](#)

Result:

-196884

-196884

Indeed:

$$-170859. - 4.60102 (5656.38)$$

Input interpretation:

$$-170859. + 5656.38 \times (-4.60102)$$

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Result:

-196884.1175076

-196884.1175...

Further:

$$-(((-170859. - 4.60102 (5656.38))))$$

Input interpretation:

$$-(-170859. + 5656.38 \times (-4.60102))$$

[Open code](#)

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Result:

196884.1175076

[Open code](#)

196884.1175 result practically equal to the value of following q -expansion

$$\begin{aligned} Z_{24}(\tau) &= j(\tau) - 744 \\ &= q^{-1} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots \end{aligned}$$

And:

$$\ln -(((-170859. - 4.60102 (5656.38))))$$

Input interpretation:

$$\log(-(-170859. + 5656.38 \times (-4.60102)))$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

More digits

12.19037...

12.19037 result that correspond to the value of the black hole entropy 12.1904

We note that:

$$-(((-170859. - 4.60102 (5619.4+34+3))))$$

Input interpretation:

$$-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

196884.209528

[Open code](#)

196884.209528

and we observe that 5619.4 is practically equal to the rest mass of bottom Lambda baryon 5619.4

In conclusion, we obtain:

$$[-(((-170859. - 4.60102 (5619.4+34+3))))]^{1/25}$$

Input interpretation:

$$\sqrt[25]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.628428...

1.628428...

And:

$$[-(((-170859. - 4.60102 (5619.4+34+3))))]^{1/24}$$

Input interpretation:

$$\sqrt[24]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))}$$

[Open code](#)

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Result:

More digits

1.661851...

1.661581... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$1/2((((([-((-170859. - 4.60102 (5619.4+34+3)))]^{1/24} + [-((-170859. - 4.60102 (5619.4+34+3)))]^{1/25}))))))$$

Input interpretation:

$$\frac{1}{2} \left(\sqrt[24]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))} + \sqrt[25]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))} \right)$$

[Open code](#)

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Result:

More digits

1.645139...

$$1.645139... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$\sqrt{(((((((6 * 1/2((((([-((-170859. - 4.60102 (5619.4+34+3)))]^{1/24} + [-((-170859. - 4.60102 (5619.4+34+3)))]^{1/25}))))))))))))))$$

Input interpretation:

$$\sqrt{\left(6 \times \frac{1}{2} \left(\sqrt[24]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))} + \sqrt[25]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))} \right) \right)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3.141789...

3.141789...

$$2\sqrt{\left(\left(\left(\left(\left(\left(\left(6 * \frac{1}{2}\left(\left(\left(\left(\left(-\left(\left(-170859. - 4.60102 (5619.4+34+3)\right)\right)\right)^{1/24} + \left[-\left(\left(-170859. - 4.60102 (5619.4+34+3)\right)\right)^{1/25}\right)\right)\right)\right)\right)\right)\right)\right)\right)$$

Input interpretation:

$$2 \sqrt{\left(6 \times \frac{1}{2} \left(\sqrt[24]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))} + \sqrt[25]{-(-170859. + (5619.4 + 34 + 3) \times (-4.60102))} \right) \right)}$$

[Open code](#)

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Result:

More digits

6.283577...

6.283577... $\approx 2\pi$

TABLES

From:

Polchinski equation, reparameterization invariance and the derivative expansion

Jordi Comellas

Departament d'Estructura i Constituents de la Matèria - Facultat de Física, Universitat de Barcelona-Diagonal, 647, 08028 Barcelona, Spain-comellas@sophia.ecm.ub.es

A	B	$Z(0)$	$f'(0)$	η
0.55	0.39	1.31	-0.230 17	0.041 3
0.53	0.40	1.31	-0.228 99	0.042 0
0.52	0.41	1.32	-0.228 07	0.042 6
0.49	0.42	1.33	-0.227 11	0.043 3

Table 2: Comparison of different FPs on the "hollow" (see Section 6). Recall that $f'(0) = -0.288 60$ within the LPA.

From:

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Three-dimensional AdS gravity and extremal CFTs at $c = 8m$

Spyros D. Avramis, Alex Kehagias and Constantina Mattheopoulou - Department of Engineering Sciences, University of Patras, 26110 Patras, Greece – Physics Department, National Technical University of Athens, 15780 Zografou Campus, Athens, Greece

m	L_0	d	S	S_{BH}
3	1	196883	12.1904	12.5664
	2	21296876	16.8741	17.7715
	3	842609326	20.5520	21.7656
4	2/3	139503	11.8458	11.8477
	5/3	69193488	18.0524	18.7328
	8/3	6928824200	22.6589	23.6954
5	1/3	20619	9.9340	9.3664
	4/3	86645620	18.2773	18.7328
	7/3	24157197490	23.9078	24.7812

m	L_0	d	S	S_{BH}
6	1	42987519	17.5764	17.7715
	2	40448921875	24.4233	25.1327
	3	8463511703277	29.7668	30.7812
7	2/3	7402775	15.8174	15.6730
	5/3	33934039437	24.2477	24.7812
	8/3	16953652012291	30.4615	31.3460
8	1/3	278511	12.5372	11.8477
	4/3	13996384631	23.3621	23.6954
	7/3	19400406113385	30.5963	31.3460

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of m and L_0 .

From:

String Theory “An introduction to the bosonic strings” Vol. I– J. Polchinski

We have that:

$$\begin{aligned}
 S_{cl}(q_i, q_f) &= \frac{1}{2} \int_0^U du \left[(\partial_u q_{cl})^2 + \omega^2 q_{cl}^2 \right] \\
 &= \omega \frac{(q_i^2 + q_f^2) \cosh \omega U - 2q_i q_f}{2 \sinh \omega U} \quad (\text{A1.57.a})
 \end{aligned}$$

$$\langle q_f, U | q_i, 0 \rangle = \left(\frac{\omega}{2\pi \sinh \omega U} \right)^{1/2} \exp(-S_{cl}) \quad (\text{A.1.66})$$

Remembering that if λ starts out positive it grows, $h < 1$ is termed relevant, taking from the literature the value $c_{111} = 4/3^{1/2}$ and, from (15.5.18), if $k = 1$, $c = 4/5$, we have: $\lambda = 1.617861\dots$, $h = 0.6181$, $c = 4/5$ and $c_{111} = \sqrt{4/3}$. Thence for:

$q_i = 4/5$, $q_f = 4/3^{1/2}$, $U = 6.09571$ and $\omega = 0.8975$, we obtain:

$$0.8975 * (((((16/25 + 4/3) \cosh(0.8975 * 6.09571) - 2((4/5 * \sqrt{4/3})))))) / ((2 \sinh(0.8975 * 6.09571))))$$

Input interpretation:

$$0.8975 \times \frac{\left(\frac{16}{25} + \frac{4}{3}\right) \cosh(0.8975 \times 6.09571) - 2 \left(\frac{4}{5} \sqrt{\frac{4}{3}}\right)}{2 \sinh(0.8975 \times 6.09571)}$$

[Open code](#)

- $\cosh(x)$ is the hyperbolic cosine function
- $\sinh(x)$ is the hyperbolic sine function

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Result:

- More digits

0.878588...

0.878588...

Series representations:

- More

$$\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} = \frac{0.442767 \left(I_0(5.4709) + 2 \sum_{k=1}^{\infty} I_{2k}(5.4709) - 0.810811 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)}{\sum_{k=0}^{\infty} I_{1+2k}(5.4709)}$$

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$$\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} =$$

$$\frac{1}{\sum_{k=0}^{\infty} I_{1+2k}(5.4709)} 0.442767$$

$$\left(\sum_{k=0}^{\infty} \frac{e^{3.39889k}}{(2k)!} - 0.810811 \exp \left(i\pi \left\lfloor \frac{\arg\left(\frac{4}{3} - x\right)}{2\pi} \right\rfloor \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{4}{3} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

[Open code](#)

$$\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} = \frac{1}{\sum_{k=0}^{\infty} I_{1+2k}(5.4709)}$$

$$0.442767 \left(i \sum_{k=0}^{\infty} \frac{\left(5.4709 - \frac{i\pi}{2} \right)^{1+2k}}{(1+2k)!} - 0.810811 \exp \left(i\pi \left\lfloor \frac{\arg\left(\frac{4}{3} - x\right)}{2\pi} \right\rfloor \right) \right)$$

$$\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{4}{3} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

[Open code](#)

Integral representations:

More

$$\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} =$$

$$\frac{0.885533 \left(0.182785 + \int_0^1 \sinh(5.4709 t) dt - 0.148204 \sqrt{\frac{4}{3}} \right)}{\int_0^1 \cosh(5.4709 t) dt}$$

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$$\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} =$$

$$\frac{0.161862 \left(\int_{\frac{i\pi}{2}}^{5.4709} \sinh(t) dt - 0.810811 \sqrt{\frac{4}{3}} \right)}{\int_0^1 \cosh(5.4709 t) dt}$$

[Open code](#)

$$\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} =$$

$$\frac{0.13124 \left(i \pi \sqrt{\frac{4}{3}} - 0.616667 \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{7.48269/s+s}}{\sqrt{s}} ds \right)}{i \pi \int_0^1 \cosh(5.4709 t) dt} \quad \text{for } \gamma > 0$$

Open code

$$1 + \left(\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - 2 \left(\frac{4}{5} \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} \right)^\pi$$

Input interpretation:

$$1 + \left(0.8975 \times \frac{\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - 2 \left(\frac{4}{5} \sqrt{\frac{4}{3}} \right)}{2 \sinh(0.8975 \times 6.09571)} \right)^\pi$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function
- $\sinh(x)$ is the hyperbolic sine function

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Result:

More digits

1.66588...

1.66588... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = \left(G_{505}/G_{101/5} \right)^3 = 1164,2696 \text{ i.e. } 1,65578...$$

Series representations:

More

$$1 + \left(\frac{0.8975 \left(\left(\frac{16}{25} + \frac{4}{3} \right) \cosh(0.8975 \times 6.09571) - \frac{2}{5} \left(4 \sqrt{\frac{4}{3}} \right) \right)}{2 \sinh(0.8975 \times 6.09571)} \right)^\pi =$$

$$1 + \left(\frac{0.442767 I_0(5.4709) + 0.885533 \sum_{k=1}^{\infty} I_{2k}(5.4709) - 0.359 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{3}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{\sum_{k=0}^{\infty} I_{1+2k}(5.4709)} \right)^\pi$$

Open code

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- $\cosh(x)$ is the hyperbolic cosine function
- $\sinh(x)$ is the hyperbolic sine function

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.63569...

1.63569...

We note that the mean between the two values obtained, is: 1,650785

For $S_{cl} = 0.878588$, from:

$$\langle q_f, U | q_i, 0 \rangle = \left(\frac{\omega}{2\pi \sinh \omega U} \right)^{1/2} \exp(-S_{cl})$$

we obtain:

$$\left(\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)} \right)^{1/2} \exp(-0.878588)$$

Input interpretation:

$$\sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588)$$

[Open code](#)

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Result:

More digits

0.0144009...

0.0144009...

Series representations:

$$\sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) = 0.473682 \exp(-0.878588) \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} I_{1+2k}(5.4709)}}$$

[Open code](#)

- $\sinh(x)$ is the hyperbolic sine function

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$$\sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) = 0.669888 \exp(-0.878588) \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} \frac{5.4709^{1+2k}}{(1+2k)!}}}$$

Open code

$$\sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) = 0.669888 \exp(-0.878588) \sqrt{\frac{1}{i\pi \sum_{k=0}^{\infty} \frac{(5.4709 - \frac{i\pi}{2})^{2k}}{(2k)!}}}$$

Open code

- $I_n(z)$ is the modified Bessel function of the first kind
- $n!$ is the factorial function

Integral representations:

$$\sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) = 0.2864 \exp(-0.878588) \sqrt{\frac{1}{\pi \int_0^1 \cosh(5.4709 t) dt}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) = 0.5728 \exp(-0.878588) \sqrt{\frac{i}{\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{7.48269/s+s}}{s^{3/2}} ds}} \text{ for } \gamma > 0$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function

We have that:

$$12 \cdot 10^4 \cdot \left(\left(\left(\frac{0.8975}{(2\pi \cdot \sinh(0.8975 \cdot 6.09571))} \right) \right) \right)^{1/2} \cdot \exp(-0.878588)$$

$$(89 + 2 \times 13) \left(\sqrt{\frac{0.8975}{2 \pi \sinh(0.8975 \times 6.09571)} \exp(-0.878588)} \right)$$

Open code

- $\sinh(x)$ is the hyperbolic sine function

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Result:

More digits

1.65610...

1.65610... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

Series representations:

$$(89 + 2 \times 13) \sqrt{\frac{0.8975}{2 \pi \sinh(0.8975 \times 6.09571)} \exp(-0.878588)} =$$

$$54.4735 \exp(-0.878588) \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} I_{1+2k}(5.4709)}}$$

Open code

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$$(89 + 2 \times 13) \sqrt{\frac{0.8975}{2 \pi \sinh(0.8975 \times 6.09571)} \exp(-0.878588)} =$$

$$77.0371 \exp(-0.878588) \sqrt{\frac{1}{\pi \sum_{k=0}^{\infty} \frac{5.4709^{1+2k}}{(1+2k)!}}}$$

Open code

$$(89 + 2 \times 13) \sqrt{\frac{0.8975}{2 \pi \sinh(0.8975 \times 6.09571)} \exp(-0.878588)} =$$

$$77.0371 \exp(-0.878588) \sqrt{\frac{1}{i \pi \sum_{k=0}^{\infty} \frac{(5.4709 - \frac{i\pi}{2})^{2k}}{(2k)!}}}$$

Open code

- $I_n(z)$ is the modified Bessel function of the first kind
- $n!$ is the factorial function
- [More information](#)

Integral representations:

$$(89 + 2 \times 13) \sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) =$$

$$32.936 \exp(-0.878588) \sqrt{\frac{1}{\pi \int_0^1 \cosh(5.4709 t) dt}}$$

[Open code](#)

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$$(89 + 2 \times 13) \sqrt{\frac{0.8975}{2\pi \sinh(0.8975 \times 6.09571)}} \exp(-0.878588) =$$

$$65.872 \exp(-0.878588) \sqrt{\frac{i}{\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{7.48269/s+s}}{s^{3/2}} ds}} \quad \text{for } \gamma > 0$$

[Open code](#)

Appendix A

From:

<https://plus.maths.org/content/ramanujan>

Jf

$$(i) \frac{1 + 53x + 9x^2}{1 - 82x - 82x^2 + x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$
$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$$

$$(ii) \frac{2 - 26x - 12x^2}{1 - 82x - 82x^2 + x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$$
$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$$

$$(iii) \frac{2 + 8x - 10x^2}{1 - 82x - 82x^2 + x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$$
$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } \alpha_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

https://plus.maths.org/content/sites/plus.maths.org/files/news/2015/ramanujan/page_large.jpg

Ramanujan's manuscript. The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$. Image courtesy [Trinity College library](#).

We note the two sum of two cubes:

$$9^3 + 10^3 = 12^3 + 1^3 = 1729 \quad \text{and} \quad 6^3 + 8^3 = 9^3 - 1^3 = 728$$

$$729 + 1000 = 1728 + 1 = 1729 \quad 216 + 512 = 729 - 1 = 728$$

This two numbers 728 and 1729 are very important for the development of mathematical connections with the String Theory and the Particle Physics

From:

<https://eylemmath.weebly.com/algebra/category/ramanujan>

$$\sqrt[6]{\sqrt[3]{6860} - 19} = ?$$

$$x = \sqrt[6]{\sqrt[3]{6860} - 19} = \sqrt[6]{7 \cdot \sqrt[3]{20} - 19}$$

$$x^6 = 7 \cdot \sqrt[3]{20} - 19 = -19 + 5 \cdot \sqrt[3]{20} + 2 \cdot \sqrt[3]{20} = -19 + \sqrt[3]{2500} + 2 \cdot \sqrt[3]{20}$$

$$= 1 - 20 + \sqrt[3]{2500} + \sqrt[3]{400} - 2 \cdot \sqrt[3]{50} + 2 \cdot \sqrt[3]{20}$$

$$= 1 + \sqrt[3]{2500} + \sqrt[3]{400} + 2 \cdot (\sqrt[3]{20} - 10 - \sqrt[3]{50})$$

$$x^6 = (1 - \sqrt[3]{50} + \sqrt[3]{20})^2 \Rightarrow x^3 = 1 - \sqrt[3]{50} + \sqrt[3]{20}$$

$$x^3 = 1 - \sqrt[3]{50} + \sqrt[3]{20} = \frac{5}{3} - \frac{2}{3} + \frac{3 \cdot \sqrt[3]{20}}{3} - \frac{3 \cdot \sqrt[3]{50}}{3}$$

$$= \frac{5}{3} - \frac{2}{3} + 3 \cdot \sqrt[3]{\frac{20}{27}} - 3 \cdot \sqrt[3]{\frac{50}{27}} = \frac{5}{3} - 3 \cdot \sqrt[3]{\frac{25}{9} \cdot \frac{2}{3}} + 3 \cdot \sqrt[3]{\frac{5}{3} \cdot \frac{4}{9}} - \frac{2}{3}$$

$$= \frac{5}{3} - 3 \cdot \sqrt[3]{\left(\frac{5}{3}\right)^2 \cdot \frac{2}{3}} + 3 \cdot \sqrt[3]{\frac{5}{3} \cdot \left(\frac{2}{3}\right)^2} - \frac{2}{3}$$

$$x^3 = \left[\sqrt[3]{\left(\frac{5}{3}\right)} - \sqrt[3]{\left(\frac{2}{3}\right)} \right]^3 \Rightarrow x = \sqrt[3]{\frac{5}{3}} - \sqrt[3]{\frac{2}{3}}$$

$(((((6860)^{1/3})-19)))^{1/6}$

Input:

$$\sqrt[6]{\sqrt[3]{6860} - 19}$$

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Result:

$$\sqrt[6]{7 \times 2^{2/3} \sqrt[3]{5} - 19}$$

Decimal approximation:

- More digits
0.312050636760388732954831113932305104912886422925268300387...

Alternate forms:

- Step-by-step solution
 $\frac{1}{3} \times 3^{2/3} (\sqrt[3]{5} - \sqrt[3]{2})$

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$$\sqrt[3]{1 + 2^{2/3} \sqrt[3]{5} - \sqrt[3]{2} 5^{2/3}}$$

[Open code](#)

$$1/2 \ 1/(((((((6860)^{1/3})-19)))^{1/6}))))$$

Input:

$$\frac{1}{2} \times \frac{1}{\sqrt[6]{\sqrt[3]{6860} - 19}}$$

[Open code](#)

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Exact result:

$$\frac{1}{2 \sqrt[6]{7 \times 2^{2/3} \sqrt[3]{5} - 19}}$$

Decimal approximation:

- More digits
1.602304052928211469453715934738684182814550588031745937844...

[Open code](#)

1.602304...

This result is a golden number very near to the electric charge of positron

$$-144 - 10^3 \ln(((((((6860)^{1/3})-19)))^{1/6}))))$$

Input:

$$-144 - 10^3 \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$-144 - \frac{500}{3} \log \left(7 \times 2^{2/3} \sqrt[3]{5} - 19 \right)$$

Decimal approximation:

- More digits
1020.589807032499874529513601205580851745919749291574508515...

[Open code](#)

1020.589... result very near to the rest mass of Phi meson 1019.445

Property:

$$-144 - \frac{500}{3} \log\left(-19 + 7 \times 2^{2/3} \sqrt[3]{5}\right) \text{ is a transcendental number}$$

Series representations:

- More

$$-144 - 10^3 \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = -144 + \frac{500}{3} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-20 + 7 \times 2^{2/3} \sqrt[3]{5}\right)^k}{k}$$

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$$-144 - 10^3 \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = -144 - \frac{1000}{3} i \pi \left[\frac{\arg\left(-19 + 7 \times 2^{2/3} \sqrt[3]{5} - x\right)}{2 \pi} \right] -$$

$$\frac{500 \log(x)}{3} + \frac{500}{3} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-19 + 7 \times 2^{2/3} \sqrt[3]{5} - x\right)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$-144 - 10^3 \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = -144 - \frac{500}{3} \left[\frac{\arg\left(-19 + 7 \times 2^{2/3} \sqrt[3]{5} - z_0\right)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) -$$

$$\frac{500 \log(z_0)}{3} - \frac{500}{3} \left[\frac{\arg\left(-19 + 7 \times 2^{2/3} \sqrt[3]{5} - z_0\right)}{2 \pi} \right] \log(z_0) +$$

$$\frac{500}{3} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-19 + 7 \times 2^{2/3} \sqrt[3]{5} - z_0\right)^k z_0^{-k}}{k}$$

[Open code](#)

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function

Integral representation:

$$-144 - 10^3 \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = -144 - \frac{500}{3} \int_1^{-19+7 \times 2^{2/3} \sqrt[3]{5}} \frac{1}{t} dt$$

$$\left(\left(\left(\left(\left(10^3 \ln\left(\left(\left(\left(6860\right)^{1/3}-19\right)\right)^{1/6}\right)\right)\right)\right)\right)^{1/14}$$

Input:

$$\sqrt[14]{10^3 \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Exact result:

$$\sqrt[7]{2} 5^{3/14} \sqrt[14]{\frac{1}{3} \log\left(7 \times 2^{2/3} \sqrt[3]{5} - 19\right)}$$

Decimal approximation:

More digits

1.6143022811587665721531689867273266232830617180190313114... +
 0.36845396136762156301595049474067144077050511720648002329... *i*

[Open code](#)

Property:

$$\sqrt[7]{2} 5^{3/14} \sqrt[14]{\frac{1}{3} \log\left(-19 + 7 \times 2^{2/3} \sqrt[3]{5}\right)}$$
 is a transcendental number

[Open code](#)

Polar coordinates:

Exact form

$r \approx 1.65582$ (radius), $\theta \approx 12.8571^\circ$ (angle)

1.65582 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696 \text{ i.e. } 1,65578\dots$$

Series representations:

More

$$\sqrt[14]{10^3 \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right)} = \frac{\sqrt[7]{2} 5^{3/14} \sqrt[14]{-\sum_{k=1}^{\infty} \frac{(-1)^k \left(-20 + 7 \times 2^{2/3} \sqrt[3]{5}\right)^k}{k}}}{\sqrt[14]{3}}$$

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$$\sqrt[14]{10^3 \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right)} = \frac{\sqrt[7]{2} 5^{3/14} \sqrt[14]{2 i \pi \left\lfloor \frac{\arg(-19+7 \times 2^{2/3} \sqrt[3]{5} - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (-19+7 \times 2^{2/3} \sqrt[3]{5} - x)^k x^{-k}}{k}}{\sqrt[14]{3}}$$

for $x < 0$

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$$\sqrt[14]{10^3 \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right)} = \frac{1}{\sqrt[14]{3}} \sqrt[7]{2} 5^{3/14} \left(\log(z_0) + \left\lfloor \frac{\arg(-19 + 7 \times 2^{2/3} \sqrt[3]{5} - z_0)}{2 \pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (-19 + 7 \times 2^{2/3} \sqrt[3]{5} - z_0)^k z_0^{-k}}{k} \right)^{(1/14)}$$

[Open code](#)

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function

Integral representation:

$$\sqrt[14]{10^3 \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right)} = \frac{\sqrt[7]{2} 5^{3/14} \sqrt[14]{\int_1^{-19+7 \times 2^{2/3} \sqrt[3]{5}} \frac{1}{t} dt}}{\sqrt[14]{3}}$$

$$1/2(0.0660+0.05393)+\ln((((((6860)^{1/3}-19))^{1/6}))))$$

Input:

$$\frac{1}{2} (0.066 + 0.05393) + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

• More digits

-1.104625...

-1.104625... result very near to the value of cosmological constant with minus sign:

$$\Lambda = 1.1056 \times 10^{-52} \text{ m}^{-2},$$

Series representations:

More

$$\frac{1}{2} (0.066 + 0.05393) + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right) =$$

$$0.059965 - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} \right)^k}{k}$$

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$$\frac{1}{2} (0.066 + 0.05393) + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right) =$$

$$0.059965 + 2i\pi \left[\frac{\arg \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - x \right)}{2\pi} \right] +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - x \right)^k}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\frac{1}{2} (0.066 + 0.05393) + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right) =$$

$$0.059965 + \left[\frac{\arg \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - z_0 \right)}{2\pi} \right] \log \left(\frac{1}{z_0} \right) +$$

$$\log(z_0) + \left[\frac{\arg \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - z_0 \right)}{2\pi} \right] \log(z_0) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - z_0 \right)^k}{k} z_0^{-k}$$

Integral representation:

$$\frac{1}{2} (0.066 + 0.05393) + \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = 0.059965 + \int_1^{\sqrt[6]{-19+7 \times 2^{2/3} \sqrt[3]{5}}} \frac{1}{t} dt$$

We have also that:

$$\frac{1}{2}(1.897512108/6)+\ln(((((((6860)^{1/3})-19)))^{1/6}))))$$

where 1.897512108 is a Ramanujan mock theta function

Input interpretation:

$$\frac{1}{2} \times \frac{1.897512108}{6} + \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

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Result:

More digits

-1.0064637980...

-1.0064637980... We note that this result is very near to the Ramanujan mock theta function -1.0061571663... and to the equation of state of dark energy $w = -1.006 \pm 0.045$.

Series representations:

More

$$\frac{1.89751}{6 \times 2} + \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = 0.158126 - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}}\right)^k}{k}$$

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$$\frac{1.89751}{6 \times 2} + \log\left(\sqrt[6]{\sqrt[3]{6860} - 19}\right) = 0.158126 + 2i\pi \left[\frac{\arg\left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - x\right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - x\right)^k}{k} \text{ for } x < 0$$

[Open code](#)

$$\frac{1.89751}{6 \times 2} + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right) =$$

$$0.158126 + \left[\frac{\arg \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - z_0 \right)}{2 \pi} \right] \log \left(\frac{1}{z_0} \right) +$$

$$\log(z_0) + \left[\frac{\arg \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - z_0 \right)}{2 \pi} \right] \log(z_0) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}} - z_0 \right)^k}{k} z_0^{-k}$$

Integral representation:

$$\frac{1.89751}{6 \times 2} + \log \left(\sqrt[6]{\sqrt[3]{6860} - 19} \right) = 0.158126 + \int_1^{\sqrt[6]{-19 + 7 \times 2^{2/3} \sqrt[3]{5}}} \frac{1}{t} dt$$

Open code

From:

<https://www.quora.com/Why-is-Ramanujan-considered-one-of-the-great-mathematicians>

This is one approximation formula of Pi mentioned in Ramanujan's letters:

$$\pi \approx 768 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}$$

$$\approx 3.141590463236763.$$

$$((729+34+5)(\text{sqrt}(2-\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+1))$$

Input:

$$(729 + 34 + 5) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}$$

[Open code](#)

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Exact result:

$$768 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 3}}}}}}}}}$$

Decimal approximation:

More digits

3.141590463228050095738458505930951723554282308675797050065...

3.1415904632...

$$((729+34+8+3+1)(\text{sqrt}(2-\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+1))$$

Input:

$$(729 + 34 + 8 + 3 + 1) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}}}$$

[Open code](#)

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Exact result:

$$775 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}}}$$

Decimal approximation:

More digits

3.170224751304347427340241330854801543951261444301748325261...

[Open code](#)

3.17022475130434742734024133085480154395126144430174825261...

Where 775 is practically equal to the rest mass of charged Rho meson 775.4

Indeed, we have that:

$$3.1702247513043474 / (((((\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}})))))$$

Input interpretation:

3.1702247513043474

$$\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}}$$

[Open code](#)

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Result:

More digits

774.9999999999999999...

775

$$1/6((((((((((((((((((((((729+34+5)(\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}})))))))))))))))))^2$$

Input:

$$\frac{1}{6} \left((729 + 34 + 5) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}}}} \right)^2$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Exact result:

$$98\,304 \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \right)$$

Decimal approximation:

More digits

1.644931773107572396794727202545688972145105950077786906831...

[Open code](#)

$$1.64493177... = \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Alternate form:

$$196\,608 - 98\,304 \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \right)$$

Or:

$$(295188 - 196884) \left(\left(\left(\left(\left(\left(\left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \right) + \sqrt{3} \right) \right) \right) \right) \right) \right) \right)$$

Input:

$$(295\,188 - 196\,884) \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \right)$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Exact result:

$$\frac{1}{6} \times (-1.0061571663) \times (-1) \left((729 + 34 + 5) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}}}} \right)^2$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.6550598916...

1.6550598... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

And:

$$1/(-1.0061571663^e) * -1/6((((((((([((((((((((729+34+5)(\text{sqrt}(2-\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+1)))))))))))))^2$$

Input interpretation:

$$\frac{1}{6} \left(-\frac{1}{1.0061571663^e} \right) \times (-1) \left((729 + 34 + 5) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}} \right)^2$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.617712817...

1.617712... very near to the value of golden ratio

$$1/(-1.0061571663^{((21+5)/5)}) * -1/6((((((((([((((((((((729+34+8+3)(\text{sqrt}(2-\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+\text{sqrt}(2+1)))))))))))))^2$$

Input interpretation:

$$\frac{1}{6} \left(-\frac{1}{1.0061571663^{(21+5)/5}} \right) \times (-1)$$

$$\left((729 + 34 + 8 + 3) \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}} \right)^2$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.618248011...

1.618248011... result very near to the value of golden ratio

Mock ϑ -functions (of 7th order)

$$(i) \quad 1 + \frac{q}{1 - q^2} + \frac{q^4}{(1 - q^3)(1 - q^4)} + \frac{q^9}{(1 - q^4)(1 - q^5)(1 - q^6)} + \dots$$

$$(ii) \quad \frac{q}{1 - q} + \frac{q^4}{(1 - q^2)(1 - q^3)} + \frac{q^9}{(1 - q^3)(1 - q^4)(1 - q^5)} + \dots$$

$$(iii) \quad \frac{1}{1 - q} + \frac{q^2}{(1 - q^2)(1 - q^3)} + \frac{q^6}{(1 - q^3)(1 - q^4)(1 - q^5)} + \dots$$

Thence $q = \text{coefficient} * -e^{-t}$; for $t = 0.5$, $q = (-e^{-0.5}) * -21.79216$ for each q .

For example: $q^5 = ((-e^{-0.5}) * -21.79216)^5$ and so on.

We have that $q = 13,217613181363261588521776074453$

From the (ii), we have:

-1.081849047367565973116419938674252971482398018961922 +
0.0761251367814440464022202749466671971676215118725857
-0.000433255719961759072744149660169833646052283127278

Input interpretation:

-1.081849047367565973116419938674252971482398018961922 +
0.0761251367814440464022202749466671971676215118725857 -
0.000433255719961759072744149660169833646052283127278

[Open code](#)

Result:

-1.0061571663060836857869438133877556079608287902166143

The result is -1.0061571663...

-1.0061571663060836857869438133877556079608287902166143

From:

Astronomy & Astrophysics manuscript no. planck`parameters`2015 c ESO 2016
June 20, 2016

Planck 2015 results. XIII. Cosmological parameters

ABSTRACT

This paper presents cosmological results based on full-mission *Planck* observations of temperature and polarization anisotropies of the cosmic microwave background (CMB) radiation. Our results are in very good agreement with the 2013 analysis of the *Planck* nominal-mission temperature data, but with increased precision. The temperature and polarization power spectra are consistent with the standard spatially-flat 6-parameter Λ CDM cosmology with a power-law spectrum of adiabatic scalar perturbations (denoted “base Λ CDM” in this paper). From the *Planck* temperature data combined with *Planck* lensing, for this cosmology we find a Hubble constant, $H_0 = (67.8 \pm 0.9) \text{ km s}^{-1} \text{ Mpc}^{-1}$, a matter density parameter $\Omega_m = 0.308 \pm 0.012$, and a tilted scalar spectral index with $n_s = 0.968 \pm 0.006$, consistent with the 2013 analysis. Note that in this abstract we quote 68 % confidence limits on measured parameters and 95 % upper limits on other parameters. We present the first results of polarization measurements with the Low Frequency Instrument at large angular scales. Combined with the *Planck* temperature and lensing data, these measurements give a reionization optical depth of $\tau = 0.066 \pm 0.016$, corresponding to a reionization redshift of $z_{\text{re}} = 8.8^{+1.7}_{-1.4}$. These results are consistent with those from WMAP polarization measurements cleaned for dust emission using 353-GHz polarization maps from the High Frequency Instrument. We find no evidence for any departure from base Λ CDM in the neutrino sector of the theory; for example, combining *Planck* observations with other astrophysical data we find $N_{\text{eff}} = 3.15 \pm 0.23$ for the effective number of relativistic degrees of freedom, consistent with the value $N_{\text{eff}} = 3.046$ of the Standard Model of particle physics. The sum of neutrino masses is constrained to $\sum m_\nu < 0.23 \text{ eV}$. The spatial curvature of our Universe is found to be very close to zero, with $|\Omega_K| < 0.005$. Adding a tensor component as a single-parameter extension to base Λ CDM we find an upper limit on the tensor-to-scalar ratio of $r_{0.002} < 0.11$, consistent with the *Planck* 2013 results and consistent with the B -mode polarization constraints from a joint analysis of BICEP2, *Keck Array*, and *Planck* (BKP) data. Adding the BKP B -mode data to our analysis leads to a tighter constraint of $r_{0.002} < 0.09$ and disfavors inflationary models with a $V(\phi) \propto \phi^2$ potential. The addition of *Planck* polarization data leads to strong constraints on deviations from a purely adiabatic spectrum of fluctuations. We find no evidence for any contribution from isocurvature perturbations or from cosmic defects. Combining *Planck* data with other astrophysical data, including Type Ia supernovae, the equation of state of dark energy is constrained to $w = -1.006 \pm 0.045$, consistent with the expected value for a cosmological constant. The standard big bang nucleosynthesis predictions for the helium and deuterium abundances for the best-fit *Planck* base Λ CDM cosmology are in excellent agreement with observations. We also analyse constraints on annihilating dark matter and on possible deviations from the standard recombination history. In neither case do we find no evidence for new physics. The *Planck* results for base Λ CDM are in good agreement with baryon acoustic oscillation data and with the JLA sample of Type Ia supernovae. However, as in the 2013 analysis, the amplitude of the fluctuation spectrum is found to be higher than inferred from some analyses of rich cluster counts and weak gravitational lensing. We show that these tensions cannot easily be resolved with simple modifications of the base Λ CDM cosmology. Apart from these tensions, the base Λ CDM cosmology provides an excellent description of the *Planck* CMB observations and many other astrophysical data sets.

Note that, regarding: “.....the equation of state of dark energy is constrained to $w = -1.006 \pm 0.045$, consistent with the expected value for a cosmological constant.” This value is practically equal to the result of (ii) Mock \mathfrak{J} -function of 7th order $-1.0061571663\dots$, value, which for the time in which it was calculated, coincides (and perhaps is even more precise) with the data detected by *Planck* observations 2015

thence:

$$w = -1.006 \pm 0.045 \quad \text{Mock } \mathfrak{J}\text{-function of 7th order} = -1.0061571663$$

We have also that:

$$5,608437 - 1,962364 - 0,59578 = 3.050293, \text{ value very near to the relativistic degrees of freedom } N_{\text{eff}} = 3.046$$

$$27,208033129 \times 2 + 8,0442562 + 4,58732381 + 0.5447171 = 67,592363368, \text{ value very near to the Hubble constant } H_0 = (67.8 \pm 0.9)$$

$$1.61052934 * 1/5 = 0,322105868, \text{ value very near to the matter density parameter } \Omega_m = 0.308 \pm 0.012,$$

or:

$$1.22734321 * 1/4 = 0,30683\dots, \text{ value very near to } \Omega_m = 0.308 \pm 0.012.$$

We note that all the values of the numbers utilized for the computations, are Ramanujan Mock theta functions!

From:

Three-dimensional AdS gravity and extremal CFTs at $c = 8m$

Spyros D. Avramis, Alex Kehagiasb and Constantina Mattheopoulou

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m	L_0	d	S	S_{BH}	m	L_0	d	S	S_{BH}
3	1	196883	12.1904	12.5664	6	1	42987519	17.5764	17.7715
	2	21296876	16.8741	17.7715		2	40448921875	24.4233	25.1327
	3	842609326	20.5520	21.7656		3	8463511703277	29.7668	30.7812
4	2/3	139503	11.8458	11.8477	7	2/3	7402775	15.8174	15.6730
	5/3	69193488	18.0524	18.7328		5/3	33934039437	24.2477	24.7812
	8/3	6928824200	22.6589	23.6954		8/3	16953652012291	30.4615	31.3460
5	1/3	20619	9.9340	9.3664	8	1/3	278511	12.5372	11.8477
	4/3	86645620	18.2773	18.7328		4/3	13996384631	23.3621	23.6954
	7/3	24157197490	23.9078	24.7812		7/3	19400406113385	30.5963	31.3460

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of m and L_0 .

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