Causality Between Events with Space-Like Separation

G. L. Harnagel *

April 21, 2020

Abstract

There was a young lady named Bright
Whose speed was much faster than light.
She left one day, in a relative way,
And returned the previous night.
– attributed to A. H. R. Buller, 1923

Since the first part of the twentieth century, it has been maintained that faster-than-light motion could produce time travel into the past with its accompanying causality-violating paradoxes. Part of the problem is that the Lorentz transformation (LT) presumes that time is isotropic, as does the Minkowski diagram based upon it, whereas entropy and the arrow of time govern in the real world. This paper demonstrates that time travel into the past and causality violation occur only when speeds "greater than" infinity are involved, and this absurdity is refuted by studying relativistic dynamics in certain scenarios that purportedly lead to causality violation and allowing it to instruct us in limiting the LT in certain other scenarios. Thus there is no justification for the block universe concept and the implication that the past is "back there somewhere" and can be accessed from the present, thus preventing causality paradoxes.

1 Introduction

G. Feinberg coined the name "tachyon"[1] for a particle that always travels faster than light, satisfies the principle of relativity and is Lorentz-invariant. The limiting value is c, but, as Feinberg points out, a limit has two sides. Measurements of tritium decay[2] offer some evidence that $m^2 = -0.6\ eV^2$ for neutrinos, indicating that they may have imaginary mass and may be tachyons. More recent measurements[3] support this with $m^2 = -1.1\ ev^2$. Substantial error bars in both measurements, however, provide only weak affirmation for tachyonic neutrinos; however, it's interesting that the more recent, presumably improved, result is tilted toward rather than away from a tachyonic interpretation. Other possibilities also exist for getting from point A to point B faster than light can do it, such as the Alcubierre warp metric,[4] the Natario metric,[5] and others.[6]. This paper demonstrates that superluminal communication, presumed to be consistent with a hypothetical tachyonic technology, does *not* necessarily present the bizarre absurdities of going backward in time and bringing multiple objects into existence which mainstream thought purports to occur with superluminal motion.

In 1907 A. Einstein considered it to be "sufficiently proven" that any velocity greater than that of light is an impossibility[7] by analysis of the

*Retired Physicist

[1] G.Feinberg, "Possibility of Faster-Than-Light Particles," Physical Review, 159, (5): 1089-1105 (1967)

[2] C. Kraus *et al*, "Final Results from phase II of the Mainz Neutrino Mass Search in Tritium Decay," Euro. Phys. J. C, 40:4 (2005), Pp 447-468

[3] M. Aker *et al.*,"Improved Upper Limit on the Neutrino Mass from a Direct Kinematic Method by KATRIN," Phys. Rev. Lett. 123, 221802 (2019)

[4] M. Alcubierre, "The warp drive: hyper-fast travel within general relativity," https://arxiv.org/pdf/gr-qc/0009013.pdf

[5] F. Loup, "An Extended Version of the Natario Warp Drive Equation ...," viXra:1712.0348

[6] F. Loup, "The Analysis of Chris Van Den Broeck Applied to the Natario Warp Drive Spacetime ...," viXra:1702.0110

[7] A. Einstein, "Uber das Relativitatsprinzip ...," Jahrb.

Lorentz transformation equation for time. Given an inertial frame moving at velocity v with respect to a "stationary" frame, the time differential in the moving frame over a distance Δx in the stationary frame is

$$\Delta t' = \gamma(\Delta t - \frac{v\Delta x}{c^2}) \tag{1}$$

where Δt refers to the time differential in the "stationary" frame. He concluded that for Δt less than $v\Delta x/c^2$, $\Delta t'$ would be negative, implying that any such speedy object would arrive at its destination before it departed from its origination point, according to a moving observer. Similarly, R. C. Tolman pointed out in 1917 that velocities greater than the speed of light presented the possibility that effect could precede cause.[8]

The assertion that causality can be violated by faster-than-light travel is also mainstream thought in this century. N. D. Mermin[9] wrote, "In the [moving] frame the object is in two different places at the same time! This is such a bizarre situation that *one's* suspicion is strengthened that the difficulty we have already encountered in producing an object moving faster than light must be a reflection of the impossibility of such motion." This is another aspect of a causality violation, but perhaps the "impossibility" is not in the movement of such an object but, rather, in insisting that the LT in its temporally isotropic form is superior to the reality of our world, which includes dynamics, entropy and the "arrow of time."

Figure 1 is a Minkowski diagram depicting the conventional view that infinitely-fast communication results in causality violation.[10]. The vertical axis is the time axis in the "stationary" frame (labeled t), and the axes in the moving frame are labeled x' and t'. What is considered "stationary" and what is considered "moving"

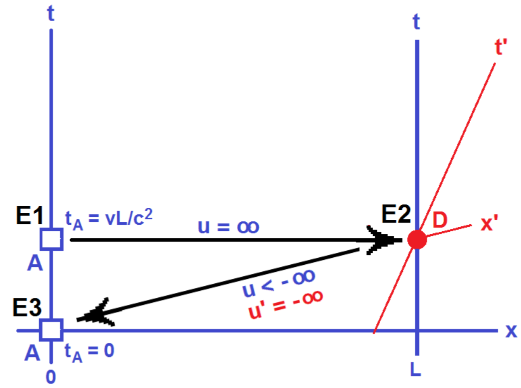

Figure 1: Typical Minkowski Diagram Showing Purported Causality Violation. A and D are assumed to have some technology that allows superluminal communication.

are, of course, arbitrary. A and D are observers that have the hypothetical capability of sending signals to each other instantaneously. The word "observer" means a conscious entity or a device that can indicate position and local time and relay that data to an observer. Observer D is moving at some positive velocity, v, with respect to A, where v is less than c. According to the Lorentz transform, the axes of the moving frame, x' and t', are tilted with respect to the stationary frame, the t' axis of the moving frame being defined by $t = x/v$ and the x' axis being defined by $t = vx/c^2$, where t and x are coordinates of the stationary frame. D is at x = L, and its time is $t_D' = 0$.

According to this view, A originates a signal at Event E1, at time $t = vL/c^2$, and transfers it instantaneously to D (horizontal black arrow) at time $t_D' = 0$, at Event E2, then D transfers the signal instantaneously back to A at time $t_A = 0$, at Event E3. The downward-sloping, leftward-going black arrow follows the x' axis, indicating that the signal is infinitely-fast in the moving frame ($\Delta t' = 0$). Thus A receives the signal at Event E3 before he sends it at Event E1. This means that A at $t = vL/c^2$ could not have originated the signal in the first place since he can be influenced by the signal received by his earlier self, hence, a causality violation.

Radioakt. Elektron. 4, 411 (1907)

[8] R. C. Tolman, The Theory of Relativity of Motion, (Berkeley, California, 1917), p. 54

[9] N. D. Mermin, *It's* About Time, (2005), pp. 53-54.

[10] *e.g.,* P. A. Tipler and R. A. Llewellyn, Modern Physics, (2008), p. 55.

The concern about causality violation also occurs in general relativity involving closed timelike curves,[11] which have the possibility of creating causality violation. The Novikov self consistency principle,[12] however, proclaims that the past cannot be changed. Friedman *et al.* demonstrated that causal solutions always existed in the case of closed timelike curves, and this paper likewise demonstrates that causal solutions also exist, even for infinite speeds in flat spacetime.

The downward-sloping arrow in Figure 1 is the consequence of allowing the signal propagation time, Δt, to be less than zero. For a velocity, defined by $u = \Delta x/\Delta t$, having a negative value for Δx, as it does for the signal propagating from E2 to E3, if Δt is also negative, u has a positive value, although it clearly is moving in the negative direction. Similarly, the signal propagating from E1 to E2, although it takes zero time in the stationary frame, takes negative time in the moving frame. Allowing this sort of thing to occur creates a very peculiar and perplexing condition. For the positive-propagating signal, $\Delta x > 0$, consider the logical sequence of the following:

(1) $\Delta t > 0$ means that the velocity u is less than infinite

(2) $\Delta t = 0$ means the velocity is infinite

(3) $\Delta t < 0$ means the velocity is *greater* than infinity

Since infinity is greater than any number, Point (3) is logically impossible, yet *that* is what Figure 1 attempts to rationalize. It is conceivable that the minus sign in the denominator of velocity equations should be interpreted as setting a limit on speeds observed in relatively-moving inertial frames. When $\Delta t = v\Delta x/c^2$, $\Delta t' = 0$, thus the velocity, $u' = \Delta x'/\Delta t'$, of an object so described will be infinity in one frame but c^2/v in a different frame, where v is the velocity difference between the two frames. This is a rational thing to do because, in the case of Figure 1, a signal with negative propagation time is moving *faster* than infinity! Disallowing this absurdity prevents the bizarre incongruities of going backward in time and bringing multiple objects into existence which are purported to occur with superluminal movement. In fact, the "young lady named Bright whose speed was much faster than light" would have to travel *faster* than infinity (in the frame where her destination was stationary) in order to return "the previous night." This paper explores reasons why this is not possible.

2 Tachyon Dynamics

The Lorentz transorm and the Minkowski diagram are kinetimatic representations of reality, concerned with geometrically possible motion, but does not include dynamics, which considers the effects of energy and forces. We have posited the hypothetical existence of tachyons which obey the relativistic energy equation,[13]

$$E^2 = p^2c^2 + m^2c^4 \tag{2}$$

where m is imaginary for tachyons and $p = \gamma mu$, and u is the velocity of the tachyon. Rewriting Equation (2) with m replaced with im,

$$E^2 = \frac{m^2u^2c^2}{u^2/c^2 - 1} - m^2c^4 \tag{3}$$

shows that E, the energy of a tachyon, approaches zero as the tachyon velocity, u, approaches infinity. As a practical matter, any signal transmission requires at least *some* expenditure of energy, hence it is not physically possible to send a tachyon signal at infinite speed. Infinite speed represents a barrier which cannot be breached, even by a tachyon.

Figure 2 presents the two situations that can occur with tachyon communication. The signal

[11] J. Friedman, *et al.*, "Cauchy problem in spacetimes with closed timelike curves," Phys. Rev. D 42, 1915 (1990)

[12] see wikipedia: Novikov self-consistency principle

[13] http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/releng.html

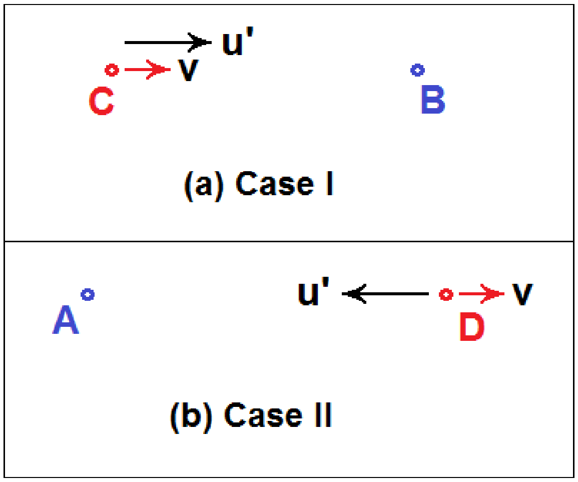

Figure 2: The two cases of superluminal communication.

moves in the same direction as the source (Figure 2a), referred to as Case I, or it moves in the opposite direction (Figure 2b), which is called Case II. In Figure 2a, Observer C is moving toward stationary observer B at velocity, v, and C sends an (almost) infinitely-fast signal directly to B. The signal has almost no energy relative to C, but it is observed by B as having significantly more energy since the energy of C's motion is added (relativistically) to the signal's energy. Consequently, the signal travels *slower* according to Equation (3). When a frame moving at velocity v with respect to a stationary frame sends out a signal or object at velocity u' (with respect to the moving frame), the velocity of said signal or object with respect to the stationary frame, according to the kinematics of the Lorentz transformation, is[14]

$$u = \lim_{u' \to \infty} \frac{u' + v}{(1 + \frac{u'v}{c^2})} = \frac{c^2}{v} \tag{4}$$

This equation, valid for v and u' having the same sign, shows that when the signal velocity relative to the moving frame, u', is (nearly) infinite, the velocity relative to the stationary frame, u, is c^2/v. This kinematic result is in agreement with energy considerations.

[14] J. D. Jackson, Classical Electrodynamics, (1965), p.361

In Figure 2b, Observer D moves away from Observer A and sends an infinitely-fast signal back to A. The signal has almost no energy relative to D, but its energy relative to A must be *subtracted* from its energy relative to D. Unfortunately, it cannot have negative energy so A cannot detect the signal from D. D must give the signal more energy so it will have positive energy when it reaches A, which means that the signal velocity relative to D is slower. The maximum velocity can be determined from the relativistic velocity composition equation for the signal moving in the opposite direction from Equation (4):

$$u = \frac{-u' + v}{1 - u'v/c^2} \tag{5}$$

where u' is positive for propagation in the negative x' direction for illustration purposes. Equation (5) shows that for leftward-going u', it is limited to c^2/v, at which point u becomes infinite. This is exactly the limit needed to successfully send a signal to A.

Thus in Figure 1, if A sends an infinitely-fast signal, which has (nearly) zero energy relative to D, it will have even less energy relative to D since D's motion is subtracted from the signal, and D will not be able to detect it. Similarly, when D sends the infinitely-fast signal back to A, A will not be able to receive it. Hence the scenario presented in Figure 1 cannot happen and does *not* represent a causality violation. The correct Minkowski diagram with allowable signal transmission is shown in Figure 3. Neither signal goes backward in time despite the fact that both signals are superluminal.

3 Instantaneous Motion, the Ontology of Time and Causality

Dynamic considerations appear to have quashed causality violation concerns; however, Figure 4 is a variation on Figure 1 that seems to avoid dynamics. In this scenario, stationary Observer B originates a signal at time $t_B = vL/c^2$ and transfers it to moving Observer D at time $t_D{}' = 0$ by

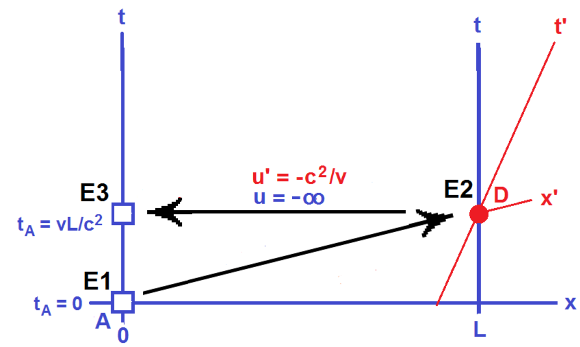

Figure 3: Correctedl Minkowski Diagram Limited by Dynamic Considerations.

ordinary subluminal transmission. This takes no appreciable time since D is adjacent to B when this occurs. D then sends the signal superluminally to moving Observer C, which is traveling at the same speed as D, so there is no dynamical limitation on speed. The signal arrives at C at $t_C{}' = 0$, and C passes it subluminally to an adjacent A, which received it at $t_A = 0$. A then sends the message instantaneously back to B at $t_B = 0$, who receives it before it was originated, thus presenting a causality violation. The signal traveling from E1 to E2 still propagates faster than infinity in the stationary frame, but no one at rest in that frame receives it directly.

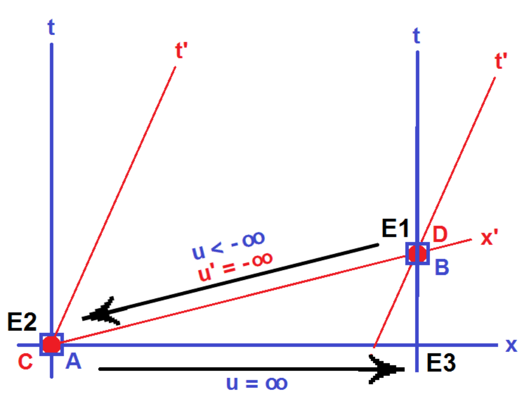

Figure 4: Minkowski Diagram without Dynamic Limitations.

One problem with Figure 4 assumes that the past actually, physically exists and is accessible from the present. This is a philosophical speculation called the "block universe" concept.[15] All experimental evidence says that only the present exists: no one has sent a signal or other object into the past. All claims to the contrary are in the domain of science fiction fantasies. In fact, all evidence points to the past only existing in memory of one kind or another (rocks, tree rings, neurons, tablets, silicon, etc.). This agrees with the philosophy of the ontology of time called presentism[16] and is adamantly opposed to the block universe concept.

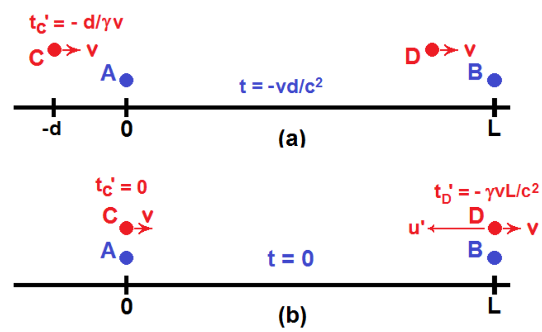

Figure 5: Laboratory Diagram with Initial Signal Event Occurring at t = 0 in the Stationary Frame.

Consider the viewpoint of an experimental physicist who has designed a laboratory experiment to mimic Figure 4. This physicist would set up the experiment as shown in Figure 5. Clocks would be positioned at C and D at the back and front ends, respectively, of a projectile. The time on the stationary clocks are shown in blue and the times on the moving clocks, as calculated from the Lorentz transform, are shown in red. If D sent a superluminal signal to C, it could not be greater than $u' = (0 - L)/(0 + vL/c^2) = -c^2/v$ in the moving frame, in agreement with the dynamical argument made previously.

Another problem uncovered by the laboratory experiment presumes the time at A and B in Figure 4 need not be synchronized. This violates the principle of events and observers

[15] see: "Growing Block Universe" at wikipedia.org

[16] $https://plato.stanford.edu/entries/presentism/$

in a given inertial reference frame: "The clocks are all identical and we, of course, want them all to read the 'same time' as one another at any instant; i.e., they must be *synchronized*."[17] Thus the time at A and the time at B are *always* the same in any practical experiment. Since the time at B is $t = vL/c^2$ in Figure 4, the time at A must be $t = vL/c^2$ also. Furthermore, the time at C must also advance according to the Lorentz transform, so neither the time at A nor the time at C is zero when B initiates the signal and transfers it to D, thus D cannot send a signal back to x = 0, t = 0 in the stationary frame. Furthermore, if it were valid for clocks at A and B to be unsynchronized, then it would be valid for clocks at C and D also to be unsynchronized, yet they *are* synchronized in Figure 4 since they are shown to lie along the x' axis. Allowing one set to be unsynchronized and the other set to be synchronized is a glaring inconsistency. In fact, just allowing A to be unsynchronized from B is all that is necessary to pretend to "send a signal into the past." A moving frame is not even necessary! But, of course, any self-respecting experimental physicist would *always* sysnchronize the clock at A with the clock at B and would check to make sure they remained sysnchronized.

Disallowing the denominator of Equation (1) to be negative agrees with dynamical limitations, laboratory scenarios and the theory of presentism, as well as entropy and the arrow of time. Thus having speeds greater than infinity in any frame and sending signals into the past are prevented. Contrary to logic and rational experimental physics, these artifices are inherent in conventional scenarios in one degree or another and invalidate the conclusion therein proposed that superluminal motion violates causality.

Figure 6 summarizes the possibilities and limitations described by limiting the denominator of the Lorentz transformation equation for time to positive values, covering both Case I and II conditions. The right side depicts the situation of a signal launched from a source moving at velocity v and moving at $u' = \infty$ relative to that source (horizontal red line) is observed in the stationary frame as moving at $u = c^2/v$ (blue curved line). When the source is moving in the opposite direction, as depicted on the left side of Figure 6 (equivalent to the scenario presented in Figures 1 and 4), the speed in the stationary frame may be infinite (horizontal blue line) but the speed in the moving frame is limited to $u' = c^2/v$ (curved red line).

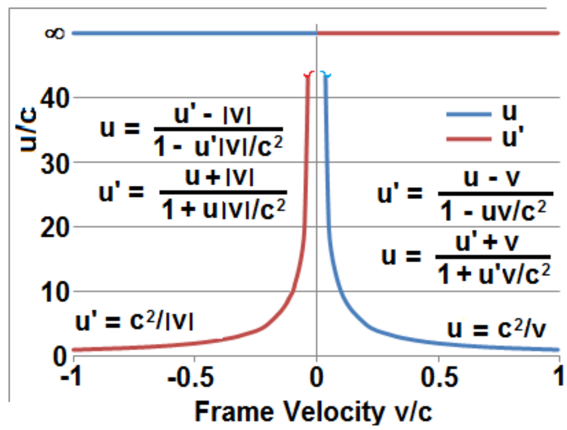

Figure 6: Limitations of u and u' As Functions of Moving Frame (v).

4 Conclusion

Since relativistic velocity composition is a valid consequence of the Lorentz transformation equations, Equation (4) clearly speaks to the fact that if a speed, u', in the moving frame grows without limit, the speed in the stationary frame is observed to be c^2/v. However, simply rearranging Equation (4) so that u' is the dependent variable

$$u' = \frac{u - v}{(1 - uv/c^2)} \tag{6}$$

seems to allow the possibility of causality violation, but kinematic analysis alone gives an incomplete and false picture. Besides the disturbing implications of speeds greater than infinity and unsynchronized clocks within an inertial

[17] P. A. Tipler and R. A. Llewellyn, Modern Physics, (2008), p. 13.

frame, kinematics fails to consider the dynamics of superluminal particles. Clearly, limiting u to c^2/v satisfies tachyon energy considerations and retains the context of Equation (4), which is also consistent with the theory of presentism and demonstrates that superluminal signals cannot result in causality violations.

The conventional view of superluminal motion claims that clocks of observers at rest with respect to each other are not synchronized in order to refute said superluminal motion. This is a conspicuous error in that any rational experiment requires that the clocks at rest in any given frame *must* be synchronized. If the clocks of A and B in Figure 4 are allowed to be unsynchronized, then why should the clocks at C and D require synchronization, as they are in Figure 4? If either set of clocks are not synchronized, any conclusion based upon time are completely invalid. Thus it is impossible to perform a valid experiment as presented in Figure 4 because of the synchronization problem as well because of tachyon dynamics. The ontology of time also weighs in against the block universe speculation which has absolutely no evidence supporting it.

Consequently, causality violation as a disproof of faster-than-light speeds is a canard. Physical phenomena prevent time in the denominator of the relativistic velocity composition equation from being less than zero. Among other things, this is because, contrary to the symmetry of time in the Lorentz transform, time in the real world is not symmetrical. Rather, it is anisotropic, since it is restricited by dynamical considerations as well as by entropy and the arrow of time. This is confirmed by the requirement that clocks of the stationary observers *must* be synchronized (a condition which is violated in the conventional views of Figures 1 and 4). This results in the unusual situation that infinite speeds are possible only under certain conditions, analogously to the way that the Novikov self-consistency principle prevents[17] closed timelike curves from violating causality in general relativity. Of course, there is no solid experimental evidence at present for faster-than-light physical phenomena, but if and when it becomes reality, we need not worry that our past histories can be altered or erased.

5 Acknowledgments

The author thanks R.T. Longo for helpful discussions and members of the Google sci.physics.relativity discussion group for ardently presenting the mainstream view.