# De Broglie interval wave and Heisenberg's uncertainty principle.

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**Abstract:** In this work a more accurate interpretation of de Broglie waves as interval waves is given, i.e., spatio-temporal waves. Also given a strict derivation of the Heisenberg's uncertainty principle from the de Broglie interval wave, that is, from wave-particle duality, is given. Proven that that the uncertainty principle expresses the fact that in the microworld our concepts (length, time, etc.) are not fundamental, and therefore they can change in a certain way. The problem of the radius of an elementary particle is also logically explained.

**Keywords:** Heisenberg's uncertainty principle, de Broglie interval wave, wave-particle duality, interval, Louis de Broglie oscillations, elementary particle.

## INTRODUCTION.

Consider the de Broglie wave**.** Suppose we have an electron that moves with speed v. In that case, with the electron we can associate a wave whose length is:

$$\lambda = h / (m^*v)$$

The foregoing applies to any elementary particle, more precisely to any microparticle. If a microparticle moves, that is, has a momentum p = m*v, then it is always possible to associate a wave of a certain length $\lambda$ = h/p with it. This ingenious assumption was made in 1923 by Louis de Broglie. De Broglie set forth his ideas in a short note, “Waves and quanta” (Ondes et quanta), which he presented at a meeting of the Paris Academy of Sciences on September 10, 1923 [1]. Louis de Broglie proceeded from the dual nature of light: by that time it had already been unequivocally established that in some cases light manifests itself as a wave, and in other cases light manifests itself as a collection of particles (photons). And therefore, he extended this duality of light to all massive particles, that is, to particles that have a rest mass. That is, de Broglie pustulated that all microparticles with a finite momentum p have wave properties, in particular, are subject to interference and diffraction.

Louis de Broglie set forth his results on waves of matter, in an expanded form, in his doctoral dissertation, “Studies on the theory of quanta”, which was defended at the Sorbonne on November 25, 1924. Further, Erwin Schrödinger, starting from the ideas of de Broglie in early 1926, developed the theory of wave mechanics. The successes of the Schrödinger theory and the experimental discovery of electron diffraction led to wide recognition of the merits of Louis de

Broglie, as evidenced by the award of the 1929 Nobel Prize in Physics with the formulation "for discovering the wave nature of the electron".

It sounds very simple now, but in fact it was one of the most daring assumptions in physics. The fact is that the nature of the de Broglie wave remained unknown. Everything is clear with light: it is an electromagnetic wave. And what is the nature of the de Broglie wave? Understanding this, Louis de Broglie called his wave “fictitious wave”, and attributed to it a velocity $c^2/v$ (this is the phase velocity of the wave). De Broglie waves always have a speed much higher than the speed of light, and this is absolutely normal, since this is the phase velocity of the wave, which can be any. Group velocity ($v$) is the velocity of the microparticle, and it is the group velocity that cannot exceed the speed of light in vacuum.

And yet, what is the nature of de Broglie waves? De Broglie waves have a nature that has no analogy among ordinary waves studied in physics. The squared module of the de Broglie wave amplitude at a given point is a measure of the probability that a microparticle is detected at that point. This interpretation of the de Broglie wave as the probability density of detection was introduced into physics by Max Born [2]. The diffraction patterns observed in the experiments brilliantly confirm this interpretation: the particles fall into certain places in the receivers, and precisely where the de Broglie wave intensity is greatest. And the particles are not found in those places where, according to the statistical interpretation, the square of the absolute value of the amplitude of the “probability wave” vanishes. Therefore, now in physics, it is believed that de Broglie waves are useful only for approximate conclusions about the scale of manifestation of the wave properties of microparticles, but do not reflect all physical reality. And therefore, de Broglie waves do not underlie the mathematical apparatus of quantum mechanics. Instead of the de Broglie waves, this role in quantum mechanics is played by the wave function ($\Psi$), and in quantum field theory by field operators. We will try to doubt this, and then strictly show that de Broglie waves are fundamental in the microworld, and it is they that most fully reflect the physical reality of the microworld. And they should be the basis of the mathematical apparatus of quantum mechanics.

**RESULTS AND DISCUSSION.**

## Interval and wave of de Broglie.

We will analyze the concepts of “interval”, “length” and “time”. Space and time form a single space-time continuum. Therefore, the fundamental concept is "interval" S, and not the concept of "length", or the concept of "time". An interval is a “distance” between two events in space-time (a generalization of the Euclidean distance between two points). The length and time in different inertial systems can vary, but the interval will always be constant (according to Einstein's STR). We write the interval for an infinitesimal displacement in space-time:

$$dS^2 = c^2 \cdot dt^2 - dx^2 - dy^2 - dz^2$$

or

$$dS^2 = c^2 \cdot dt^2 - dL^2$$

where S is the interval, L is the distance between two points, c is the speed of light, t is time. In the case of flat space-time, that is, space-time without curvature (absence of gravity), the same expression can be written for finite difference coordinates:

$$S^2 = c^2 \cdot \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

or

$$S^2 = c^2 \cdot \Delta t^2 - L^2$$

Note that in the microworld, gravity is virtually absent (due to smallness). Therefore, in the microworld, space-time can be considered flat.

Let's do some transformations.

$$S^2/L^2 = c^2 \cdot (\Delta t^2/L^2) - 1$$

$$S^2/L^2 = c^2/v^2 - 1 = (c^2 - v^2)/v^2$$

$$S^2/L^2 = (1 - v^2/c^2)/(v^2/c^2)$$

$$S^2 \cdot v^2/c^2 = L^2 \cdot (1 - v^2/c^2)$$

$$S^2 = L^2 \cdot (1 - v^2/c^2) \cdot c^2/v^2$$

From here we get a simple expression for the interval:

$$S = L \cdot (1 - v^2/c^2)^{0.5} \cdot c/v$$

This is the dependence of the interval on the length and time (for a flat, Euclidean space). Instead of time, speed is taken here. This follows from the fact that two coordinate systems in which time flows in different ways will move one relative to the other with a certain speed v. Therefore, speed as a physical concept displays the speed of time in a given frame of reference relative to another frame of reference.

In the microworld, space-time is flat, therefore, in the mathematical apparatus, it is necessary to use the fundamental concept of “interval” rather than length, etc. In fact, there is no longer a separate “length” in the microworld, but there is an interval. Therefore, in the formula of Louis de Broglie for the wavelength, you need to replace the "wavelength" with the "interval" of the wave:

$$\lambda = h/(m*v) \quad \rightarrow \quad S = h/(m*v)$$

Where S — is interval, $S = L * (1 - v^2/c^2)^{0.5} * c/v$

Then

$$L * (1 - v^2/c^2)^{0.5} * c/v = h/(m*v)$$

Now, it is easy to establish what for the microparticles, it is L. For this, we take into account the increase in the mass of the microparticles at relativistic speeds

$$m = m0 / (1 - v^2/c^2)^{0.5}$$

Then

$$L * (1 - v^2/c^2)^{0.5} * c/v = (h*(1 - v^2/c^2)^{0.5}) / (m0*v)$$

Having made the necessary reductions, we get:

$$L = h/(m0*c)$$

That is, L is equal to the Compton wavelength of the microparticle:

$$L = \lambda c.$$

Now we write the expression for the interval (for the microparticle), and for the de Broglie interval wave:

$$S = \lambda c.* (1 - v^2/c^2)^{0.5} * c/v$$

$$S = h/(m*v)$$

Interpretation of the nature of de Broglie waves as “interval waves” automatically leads to the interpretation of Max Bourne as probability density. Since in the formula

$$S = h/(m*v)$$

on the left side of the equation there is an “interval” (that is, space-time), and on the right side of the equation there is a material microparticle (its momentum, and hence energy), then depending on the characteristics of the interval (spatial, temporal), boiling “particle – antiparticle” will be different for different intervals. This is clear, since time (and therefore energy characteristics) in different intervals will be different. For a visual demonstration, we write the interval through the ratio of times.

$$S = \lambda c.* (1 - v^2/c^2)^{0.5} * c/v$$

$$\Delta t2 = \Delta t0 / (1 - v^2/c^2)^{0.5}$$

$$(1 - v^2/c^2)^{0.5} = \Delta t0/\Delta t2$$

$$S = \lambda c.* (\Delta t0/\Delta t2)* c/v$$

$$S = h/(m*v)$$

These two formulas

$$S = h/(m*v)$$

$$S = \lambda c.* (\Delta t0/\Delta t2)* c/v$$

or

$$S = \lambda c.* (1 - v^2/c^2)^{0.5} * c/v$$

transfer the “particle - antiparticle” boiling from the region of Compton wavelengths to a much larger scale, where the “particle – antiparticle” boiling also becomes possible (due to the different passage of time, the transition in the formula is shown by reference [3]. Now it becomes clear why the de Broglie wave in the Bohr orbit is 137 times longer than the Compton electron wavelength. The electron velocity in the 1st Bohr orbit is $v = 2.19*10^6$ m/s, then the de Broglie interval wavelength is 3.324 Å:

$$S = \lambda c.e.* (1 - v^2/c^2)^{0.5} * c/v$$

$$S = \lambda c.e.* 1* c/v = \lambda c.e. * 137 = 2.4263*10^{(-12)} \text{ m} * 137 = 3.324 \text{ Å}.$$

Then we assumed this: «... suppose that the wavelength of an electron on a Bohr orbit (the hydrogen atom) is the same Compton wavelength of an electron, but in another frame of reference, and as a result there is a 137-times greater Compton wavelength (due to the effects of relativity theory)...» [3, pp. 100 – 101]. And now, we have proved it strictly.

The formula of Louis de Broglie

$$S = h/(m*v)$$

which connects space-time (more precisely, the interval, de Broglie interval wave) and the momentum of the microparticle (and hence its total energy), is an analogue of A. Einstein’s well-known field equations (GR), but only for the microworld.

## Interval and Heisenberg's uncertainty principle.

The Heisenberg's uncertainty principle can be strictly deduced from wave-particle duality, more precisely from the de Broglie interval wave. To do this, we write the equation for the de Broglie interval wave.

$$S = h/(m*v)$$

$$S = ƛc. * (1 – v\text{^}2/c\text{^}2)\text{^}0.5 * c/v$$

$$S = ƛc.* (\Delta t0/\Delta t2)* c/v$$

where S — is interval, ƛc. - the Compton wavelength of the microparticle.

Assuming the de Broglie wavelength equal to the length of the interval, we proceed to a more general, and more rigorous form of the equation for the de Broglie wave. In fact, this is a general and rigorous formulation (and interpretation) of wave-particle duality, from which the Heisenberg's uncertainty principle can be derived.

Note that both wave-particle duality and the Heisenberg's uncertainty principle are a consequence of A. Einstein's STR in the microworld. An elementary particle has its own time. In fact, an elementary particle is an inertial frame of reference. Let's call it own inertial frame of reference (own IFR). In this reference frame, a certain periodic process takes place (de Broglie oscillations) [4].

$$E = m0*c\text{^}2, \qquad E = h*\gamma$$

$$E = h*\gamma = m0*c\text{^}2, \qquad \gamma = (m0*c\text{^}2)/h$$

$$\Delta t0 = 1/\gamma = h/(m0*c\text{^}2)$$

Moreover, the speed of this process is equal to the speed of light in a vacuum, and in this process the de Broglie interval wave transforms into the "ordinary" de Broglie wave (which the concept of "length" really corresponds to), more precisely, the Compton length of the microparticle.

Now we strictly derive the Heisenberg's uncertainty principle from the de Broglie interval wave. For the de Broglie interval wave, we write

$$S = h/(m*v)$$

$$S = ƛc.* (1 – v\text{^}2/c\text{^}2)\text{^}0.5 * c/v$$

$$S = ƛc.* (\Delta t0/\Delta t2)* c/v$$

When v=c we get:

$$S = h/(m*c) = \lambda c.$$

$$S = \lambda c. * (1 - v^2/c^2)^{0.5} * c/v = \lambda c. * (1 - 1)^{0.5} * c/c = 0$$

that is, the length of the interval is zero.

Another formula shows how the interval transforms (degenerates) into the usual de Broglie wave (for the observer from its own IFR), into the Compton wave:

$$S = \lambda c.* (\Delta t0/\Delta t2)* c/v = \lambda c.* (\Delta t0/\Delta t0)* c/c = \lambda c.$$

That is, in fact, the interval is also equal to zero: it is not (there is no time), but there is only “length”.

This can also be deduced from the usual record for the interval (for an observer from another IFR, IFR of device):

$$S^2 = c^2 * \Delta t^2 - L^2$$

When $S = 0$, then

$$L = c*\Delta t0= \lambda c.$$

$$\lambda c. = h/(m*c)$$

$$\Delta t0 = 1/\gamma = h/(m0*c^2)$$

Please note that in the own frame of reference of the elementary particle there is no arrow of time, but only the duration of the periodic process. This directly indicates that at the fundamental level there is no arrow of time. How the arrow of time is formed from the set of such periodic processes, see link [5].

Moreover, the dimension of space at a fundamental level should be less than 3. The dimension of space can be 2, or 1, or more likely it should tend to zero, that is, $n \to 0$ (n is the dimension of space). Why the dimension of space at a fundamental level should be less than 3 see link [6]. The confirmation of our words that the space at the fundamental level should have a dimension less than 3 is that the spin of elementary particles has (at the same time) only two projections. And is it in 3D space ??? This is a well-known fact in quantum mechanics.

We proceed to the conclusion of the Heisenberg's uncertainty principle.

Since the elementary particle actually has its own IFR, when measuring, for example, the coordinates, we get the interaction of two IFR: the IFR of the elementary particle and the IFR of the device (observer). And therefore, when measuring, we will always record some changes ($\Delta$) in physical quantities that the observer will see in the instrument's IFR (as changes in length in A. Einstein's thought experiments). Moreover, since in the elementary particle IFR the periodic process occurs at the speed of light in vacuum, then for an external observer (IFR of the device) whose IFR has a speed of $v = 0$, the values of the physical quantities will be zero. And what the external observer will record as a physical quantity (at a certain speed v) will be a deviation from zero ($\Delta$), and it is this $\Delta$ that will be perceived as the value of the physical quantity for the external observer (IFR of the device). Thus, $\Delta v = v$, $\Delta x = x$, $\Delta p = p$, and this is also true for other physical quantities (t, E, etc.).

Therefore, we get a transition from the de Broglie general formula to the Heisenberg's uncertainty principle:

$$\Delta S * \Delta p = h \quad \rightarrow \quad \Delta x * \Delta p = \hbar/2$$

We show strictly this transition. If we move from the concept of "interval" to the concept of "length", then we can write:

$$\Delta S * \Delta p = h \quad \rightarrow \quad \Delta \text{ʎ} * \Delta p = h$$

When approaching

$$\Delta S = \Delta \text{ʎ} = 2*\pi*\Delta x$$

$$\Delta \text{ʎ} = 2*\pi*\Delta x$$

which mean

$$\Delta x * \Delta p = h/(2*\pi)$$

An elementary particle can be considered as a standing wave, then the lowest energy level will be a standing wave formed from two half-waves. That is, de Broglie's real wave will be twice as large, where do we get

$$\Delta \text{ʎ} = 4*\pi*\Delta x$$

$$\Delta x = \Delta \text{ʎ}/(4*\pi)$$

And now we can write

$$\Delta x * \Delta p = h/(4*\pi)$$

i.e,

$$\Delta x * \Delta p = \hbar/2$$

Given that the formula

$$\Delta S * \Delta p = h$$

this is actually a strict form of the Heisenberg's uncertainty principle (!!!, it is Louis de Broglie's formula), which shows the relationship between the fundamental quantities (interval, momentum), then when passing to the form

$$\Delta x * \Delta p = \hbar/2$$

we will always get a certain approximation, which will be more than the value obtained from the strict formula. This gives us the opportunity to make the transition in the last formula from = to ≥ . And therefore, we can make the transition to the classical formula of the Heisenberg's uncertainty principle:

$$\Delta x * \Delta p = \hbar/2 \quad \rightarrow \quad \Delta x * \Delta p \geq \hbar/2$$

Thus, from the general formula of Louis de Broglie (considering A. Einstein's STR)

$$\Delta S = h/\Delta p$$

we got the Heisenberg's uncertainty principle in the classical form

$$\Delta x * \Delta p \geq \hbar/2$$

As we see from the above, the Heisenberg's uncertainty principle is a consequence of the particle-wave dualism of an elementary particle, which in turn is a consequence of the fact that our world is a spatio-temporal continuum. And in the space-time continuum, the fundamental value is the interval. In the transition from the interval to the length, and we do this by measuring the coordinate of the microparticle, we inevitably go from the strict formula $\Delta S = h/\Delta p$ to a certain approximation. The expression of this approximation is the Heisenberg's uncertainty principle in the classical form, that is, in the form

$$\Delta x * \Delta p \geq \hbar/2.$$

**CONCLUSION.**

Thus, we can say that the uncertainty principle expresses the fact that in the microworld our concepts (length, time, etc.) are not fundamental, and therefore they can change in a certain way (therefore, microparticles do not have a trajectory). This also expresses the Einstein's SRT, which, as it turns out, plays a fundamental role in the microworld. It is Einstein's SRT that shows how the quantities of physical quantities are formed in our macro-world, and how the arrow of time is formed in our 3D world (in the absence of the arrow of time at the fundamental (quantum) level).

Given the above, we can explain what will be the radius of an elementary particle. The radius of an elementary particle is the concept of “length”, which does not make sense at scales shorter than the Compton wavelength of a given microparticle. Recall that the periodic process of the microparticle occurs at the speed of light in vacuum. Therefore, if the radius of a microparticle will be measured by an observer with an IFR moving at a low speed (IFR of the device), then the radius of the microparticle will tend to zero $r \to 0$. Therefore, any accuracy of the instruments will always show that the radius of an elementary particle (for example, an electron, photon, quark etc.) will always be smaller. This is clear, since the radius of an elementary particle in our world really tends to zero.

That is, elementary particles are truly point objects. If the IFR of the device moves with the speed of light in vacuum, that is, the speed of time in the IFR of the device approaches the intrinsic time of an elementary particle, then such an observer will record the Compton length of the microparticle, which can be considered a kind of “radius” of the elementary particle. It inevitably follows from this that the concept of “length” on scales of shorter Compton lengths for a given particle does not make sense. Less than Compton wavelengths there is only a boiling vacuum in which the particle – antiparticle pairs are born and disappear. There is not even time (arrows of time), but there is a certain periodic process (de Broglie oscillations). The dimensions of an elementary particle really tend to zero. Therefore, elementary particles, like true point objects, by definition cannot consist of other structural elements, for example, strings. This is a rigorous conclusion that follows from the true point structure of elementary particles.

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