

Numbers: Part 1

Edgar Valdebenito

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abstract

This note presents two elementary integrals.

keywords: number Pi, integrals

1. Two Elementary Integrals

$$\frac{\pi}{4} = \int_0^{\infty} \frac{\sqrt{2} x + \sqrt{\sqrt{4x^4 + 1} - 1}}{2\sqrt{2} x^3 + \sqrt{2} x \sqrt{4x^4 + 1} + 2x^2 \left(\sqrt{\sqrt{4x^4 + 1} - 1} + \sqrt{\sqrt{4x^4 + 1} + 1} \right)} dx \quad (1)$$

$$\frac{\pi}{4} = \int_0^{\infty} \frac{e^{-x} \coth(2x)}{1 + \sqrt{\coth(x)}} dx \quad (2)$$

Remarks:

■ (1) \implies (2), change of variable : $x \mapsto \sqrt{\frac{\sinh(x)}{2}}$.

■ $\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.1415926535 \dots$

References

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