New recognization for the Newton's third law: the reaction force is advanced according to the mutual energy principle

Shuang-ren Zhao

August 4, 2019

Abstract

Absorber theory published in 1945 and 1949 by Wheeler and Feynman which tells us that if the sun is put in a empty space where apart from the sun is nothing, the sun cannot shine. That means only with the source, the radiation cannot be produced. The radiation is phenomena of an action-at-a-distance. The action at a distance needs at least two object: the source and the sink or the emitter and the absorber. Only with one charge even it has the acceleration, it still cannot make any radiation. However this result is not reflect at the Maxwell's theory. According to the theory of Maxwell, a single charge can produce the radiation without any help of the absorber. Hence, Maxwell theory is different with the absorber theory of Wheeler and Feynman, this author thought that Wheeler and Feynman is correct. According the absorber theory the source (emitter) sends the retarded wave. The sink (absorber) sends the advanced wave. In the electromagnetic field theory, W. J. Welch introduced the reciprocity theorem in 1960. V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula in 1963. In early of 1987 Shuang-ren Zhao (this author) introduced the mutual energy theorem in frequency domain. In the end of 1987 Adrianus T. de Hoop introduced the time-domain cross-correlation reciprocity theorem. All these theories can be seen as a same theorem in different domain: Fourier domain or in time domain. The reciprocity theorem of Welch, Rumsey and Hoop has been applied to find out the directivity diagram of receiving antenna from the corresponding transmitting antenna. The mutual energy theorem of Zhao, has been applied to define an inner product space of electromagnetic radiation fields, and hence, to applied to the sphere wave expansion and the plane wave expansion. All these theorems the transmitting antenna sends the retarded waves and the receiving antenna sends the advanced waves. The reciprocity theorems of Welch, Rumsey and Hoop talk about reciprocity theorem in which the two fields one can be real one can be virtual. The mutual energy theorem tell us the two fields the retarded wave sent out from the transmitting antenna and the advanced wave sent out from the receiving antenna are real and are physical waves with energy. After 30 years silence on this topic, finally, this author has introduced the mutual energy principle and self-energy principle which updated the Maxwell's electromagnetic field theory and Schrödinger's quantum mechanics. According to the theory of mutual energy principle, the energy of all particles are transferred through the mutual energy flows. The mutual energy flow are inner product of the retarded wave and the advanced wave. The retarded wave is the action the emitter gives to the absorber. The advanced wave is the reaction the absorber gives to the emitter. When the absorber received the retarded wave, it received a force from the emitter, that is the action from emitter to the absorber. When the emitter receive the advanced wave, it obtained the reaction from the absorber. This reaction is express as the recoil force of the particle to the emitter. Hence, action is retarded and the reaction is advanced. In this article the action is retarded and the reaction is advanced will be widen to the macroscopic object for example a stone or a piece of wood. Hence, even the waves in water, in air or wood all involve the advanced reaction. The author reviewed the Newton's third law, found that only when the reaction is advanced, the Newton's third law can be applied on arbitrary surface of the object. Hence the reaction is advanced must be correct.

Keywords: Poynting; Maxwell; Schrödinger; Self-energy; Mutual energy; Mutual energy flow; Reciprocity theorem; Radiation; Newton's third law; Action; Reaction; Advanced wave; Photon; Electron; Wave and particle duality;

1 Introduction

1.1 Action at a distance and the absorber theory

The theory about advanced wave became most interesting work for the author. The author noticed the absorber theory of Wheeler and Feynman[1, 2]. The absorber theory is based on the action-at-a-distance [10, 12, 6]. In the absorber theory, any current will send half retarded wave and half advanced wave. For a source we only notice the source sends the retarded wave, we did not notice it also sends the advanced wave. Some one will argue that if in the same time the source sends the retarded wave, it also sends the advanced wave, the source loss the energy from the retarded wave and acquire the energy from the advanced wave, and hence, it doesn't send any energy out. However we all know that the source can send the energy out. This means the absorber theory also has some thing which is not self-consistence. This is also the reason that the absorber theory has not been widely accept. But any way the absorber theory accept the advanced wave as a real wave instead of some virtual wave. The author is inspired by this a lot. The transactional interpretation of John Cramer has introduced the advance wave to the whole quantum mechanics [3, 4], that is also inspired the author. The author begin to accept the advanced wave is a real wave instead of a virtual wave. This all tell the author perhaps the mutual energy theorem is a real energy theorem. Stephenson offered a good tutorial about the advanced wave [11].

1.2 The mutual energy theorems

W.J. Welch introduced a reciprocity theorem in arbitrary time-domain [13] in 1960 (this will be referred as Welch's reciprocity theorem in this article). In 1963 V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula[9], (this will be referred as Rumsey's reciprocity theorem). In early of 1987 Shuang-ren Zhao has introduced the concept of mutual energy and the mutual energy theorem [7] (this will be referred as Zhao's mutual energy theorem). In the end of 1987 Adrianus T. de Hoop introduced the time domain cross-correlation reciprocity theorem[5], (this will be referred as Hoop's reciprocity theorem). Welch's reciprocity theorem is a special case of the Hoop's reciprocity theorem.

Assume there are two current sources $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. $\boldsymbol{J}_1$ is the current of a transmitting antenna. $\boldsymbol{J}_2$ is the current of a receiving antenna. The field of $\boldsymbol{J}_1$ is described as $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$. The field of the current $\boldsymbol{J}_2$ is $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$. Assume $\boldsymbol{J}_2$ has a some distance with $\boldsymbol{J}_1$. Hoop's reciprocity theorem can be written as,

$$-\int_{t=-\infty}^{\infty}\iiint_{V_1}\boldsymbol{J}_1(t)\cdot\boldsymbol{E}_2(t+\tau)dV=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t+\tau)dV \tag{1}$$

if $\tau=0$, we have,

$$-\int_{t=-\infty}^{\infty}\iiint_{V_1}\boldsymbol{J}_1(t)\cdot\boldsymbol{E}_2(t)dV=\int_{t=-\infty}^{\infty}\iiint_{V_2}\boldsymbol{E}_1(t)\cdot\boldsymbol{J}_2(t)dV \tag{2}$$

This is Welch's reciprocity theorem. The Fourier transform of Hoop's reciprocity theorem can be written as,

$$-\iiint_{V_1}\boldsymbol{J}_1(\omega)\cdot\boldsymbol{E}_2(\omega)^*dV=\iiint_{V_2}\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)^*dV \tag{3}$$

Where "*" is the complex conjugate operator. In this article if the variable t is applied in a formula, it is in time-domain. If ω is applied, it is in Fourier frequency domain. This is the Rumsey's reciprocity theorem and is also Zhao's mutual energy theorem. Hence this 4 theorems can be seen as one theorem in different domain: time-domain and Fourier domain.

1.3 Mutual energy principle

If there are N charges, the mutual energy principle can be widen to

$$-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt$$

$$= \sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_j + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_j) dV dt$$

$$+ \sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt \tag{4}$$

If it is two charges, the above formula becomes,

$$- \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_1 \frac{\partial}{\partial t} \boldsymbol{E}_2 + \epsilon \boldsymbol{E}_2 \frac{\partial}{\partial t} \boldsymbol{E}_1 + \mu \boldsymbol{H}_1 \frac{\partial}{\partial t} \boldsymbol{H}_2 + \mu \boldsymbol{H}_2 \frac{\partial}{\partial t} \boldsymbol{H}_1) dV dt$$

$$+ \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt \tag{5}$$

The above formulas are referred as the mutual energy principle. The mutual energy principle can be derived from Maxwell equations,

$$\begin{cases} -\frac{\partial}{\partial t}(\epsilon \boldsymbol{E}) + \nabla \times \boldsymbol{H} = \boldsymbol{J} \\ -\nabla \times \boldsymbol{E} - \frac{\partial}{\partial t}(\mu \boldsymbol{H}) = 0 \end{cases} \tag{6}$$

Maxwell equations can also be derived from the Mutual energy principle. However the mutual energy principle is not equivalent to the Maxwell equations. Because the solution of the mutual energy principle require there must be two wave solution which satisfy the Maxwell equations. The two waves one must be retarded wave, another must be advanced wave. The two wave must be synchronized. One wave satisfy Maxwell equations is not a solution of the mutual energy principle, but it is a solution of the Maxwell equations.

One wave satisfy the Maxwell equation is a probability wave. The author has introduced also the self-energy principle, that principle says that there is a time-reversal wave which can cancel the energy of the original waves. Hence, the retarded wave or the advance wave alone are canceled by the corresponding time-reversal waves. After the cancellation the waves do not carry any energy. The mutual energy flow is built from the retarded wave and the advanced wave which doesn't cancel. The energy is transferred by the mutual energy flow. In the electromagnetic field theory, the mutual energy flow is the photon or the photon's energy is transferred by the mutual energy flow. Mutual energy flow satisfied the mutual energy flow theorem which can be derived from the mutual energy principle[8].

$$- \int_{t=-\infty}^{\infty} \iiint_{V} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$$

$$= \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$$

$$= \int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt \tag{7}$$

In the above formula, $-\int_{t=-\infty}^{\infty} \iiint_V \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$ is the source $\boldsymbol{J}_1(t)$ offers energy to the system, the system include a source $\boldsymbol{J}_1(t)$ and a sink $\boldsymbol{J}_2(t)$. $\int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt$ is the received energy by the sink $\boldsymbol{J}_2(t)$. $S_{12} = (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)$ is the energy flow intensity. $Q = \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma$is the energy flow go through the surface Γ. $Eenergy = \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$ is the whole energy go through the surface Γ. Here Γ is any surface between the volume V_1 and V_2. Source $\boldsymbol{J}_1(t)$ is inside the V_1. The sink $\boldsymbol{J}_2(t)$ is inside V_2.

This theorem tell us, the source sends the retarded wave $\boldsymbol{E}_1$ to the sink, cause the current $\boldsymbol{J}_2$ on the sink. The energy received by the sink is $\int_{t=-\infty}^{\infty} \iiint_{V_2} \boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) dV dt$. This energy is equal to the sucked energy on the source by the advanced wave produced by the current $\boldsymbol{J}_2$ of the sink, which is $-\int_{t=-\infty}^{\infty} \iiint_V \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t) dV dt$. The energy is transferred in the space through the mutual energy flow $Eenergy = \int_{t=-\infty}^{\infty} \oiint_{\Gamma} (E_1(t) \times H_2(t) + E_2(t) \times H_1(t)) \cdot \hat{n} d\Gamma dt$.

This concept has been widened to whole quantum mechanics. Hence all particles are mutual energy flows.

1.4 Inner product of electromagnetic fields

Shuang-ren Zhao has define the inner product for electromagnetic fields[7]. Assume $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$, $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$ we have inner product,

$$(\xi_1, \xi_2) = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \tag{8}$$

Γ is closed surface. Shuang-ren find that this formula satisfy inner product 3 conditions[7],

(I) Conjugate symmetry:

$$(\xi_1, \xi_2) = (\xi_2, \xi_1)^* \tag{9}$$

(II) Linearity:

$$(a\xi_1' + b\xi"_1, \xi_2) = a(\xi_1', \xi_2) + b(\xi"_1, \xi_2) \tag{10}$$

(III) Positive-definiteness:

$$(\xi, \xi) > 0 \tag{11}$$

$$(\xi, \xi) = 0 \Rightarrow \xi = 0 \tag{12}$$

"⇒" means "can derive". Shuang-ren Zhao found that the mutual energy theorem can be also written as [7, 14],

$$-(J_1,\xi_2)_{V_1}=(\xi_1,\xi_2)=(\xi_1,J_2)_{V_2} \tag{13}$$

where

$$(J_1,\xi_2)_{V_1}=\iiint\limits_{V}\boldsymbol{E}_2(\omega)^*\cdot\boldsymbol{J}_1(\omega)dV \tag{14}$$

$$(\xi_1,J_2)_{V_2}=\iiint\limits_{V_2}\boldsymbol{E}_1(\omega)\cdot\boldsymbol{J}_2(\omega)^*dV \tag{15}$$

$$(\xi_1,\xi_2)=(\xi_1,\xi_2)_\Gamma=\oiint\limits_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2^*+\boldsymbol{E}_2^*\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma \tag{16}$$

It is clear that the integral at V_1 and V_2 Eq.(14,15) are also the inner product. Whether or not Eq.(16) is a inner product is not clear, but shuang-ren Zhao has discovered that is a inner product. It is found that Γ does not need to be written, since it can be proved that Γ can be taken at any surface between the 2 volumes: V_1 and V_2.

1.5 Action and reaction

We have know that there is the Newton's third law that says that the action and the reaction are equal and are in opposite direction.

2 Action and reaction

According to the section 1 we know the photon is the mutual energy flow which is consists as the retarded wave of the source and the advanced wave of the sink. The retarded wave is the action of the source to the sink. The advanced wave is the reaction of the sink to the source. It is clear the reaction is in advanced.

Action is the retarded wave, the reaction is the advanced wave means that the action and the reaction is propagated in the space. The reaction of the photon to the source is the recoil force. The action of the photon to the absorber is the light pressure of the photon to the absorbers.

If photon is the mutual energy flow, then, this concept can be widen to all particles which are all mutual energy flow. Hence, all particle is consist of the retarded wave sends from the source and the advanced wave sends from the absorber. Hence, all particle is the action from a source to the sink and the reaction from the sink to the source. And it is clear the reaction is also in advanced. That means the reaction is go through the space propagated through the advanced wave.

If one particle is the action and the reaction, what about the macroscopic object for example a stone, a piece of wood or a person, which are consist of

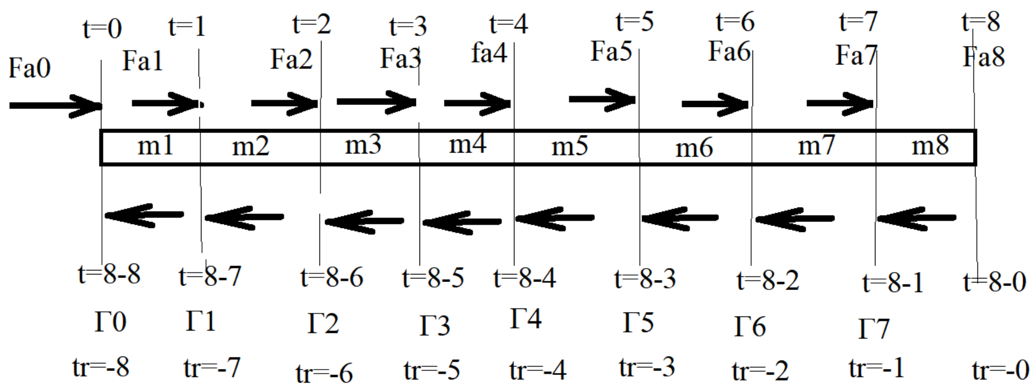

Figure 1: Action and the reaction serial.

many many particles? The author believe that principle that the reaction is in advanced is also suitable to the macroscopic object.

Now let us to prove that the reaction is in advanced through the Newton's third law only. Assume there is a piece of of wood or a wire pole.

Assume we use a hammer to knock on one end of the wire pole. Hence there are short time force act on the end of the wire pole. According the Newton's third law there is action action on the end of the wire pole; there is also a reaction act on the hammer. The two forces are equal in values and opposite in directions.

Assume the mass of wire pole is m. Assume the wire pole obtained an acceleration of $\boldsymbol{\alpha}$. Hence, the force act on the pole is:

$$\boldsymbol{F}_a = m\boldsymbol{\alpha} \tag{17}$$

The pole give the hammer a reaction force which is:

$$\boldsymbol{F}_r = -m\boldsymbol{\alpha} \tag{18}$$

This we can all understand without any problem. If we consider take any section surface on the pole, it is clear that there should be also a action and reaction on the two sides of section surface. The action should be equal and also with opposite direction. Let us consider a serial of the section surfaces. Assume the wire pole is divided as 8 parts. We can draw the action and reaction serials on the Figure 1.

We can see that the force $\boldsymbol{F}_a(t=0)$ is act at the end of the pole. $\boldsymbol{F}_a(t=1)$ is action at the surface $\Gamma = 1$. $\boldsymbol{F}_a(t=2)$ is the action act on the surface of $\Gamma = 2$. $\cdots\cdots$ $\boldsymbol{F}_a(t=7)$ is the action at on the surface $\Gamma = 7$.

In other hand the $\boldsymbol{F}_a(t=7)$ is action on m_8 the reaction of m_8 to m_7 is $\boldsymbol{F}_r(t=7-0)$.

$\boldsymbol{F}_r(t=7-1)$ is the reaction of m_8+m_7 to the m_6.

$\boldsymbol{F}_r(t=7-2)$ is the reaction of $m_8+m_7+m_6$ to the m_5.

......

$\boldsymbol{F}_r(t=7-7)$ is the reaction of $m = m_8+m_7+m_6+m_5+m_4+m_3+m_2+m_1$ to the hammer.

From this we can see it is clear action is retarded. From the hammer knock the end of the wire pole at time $t=0$ to $t=7$ the force is transmit to the last part of the pole m_8. In this time the last part of the pole m_8 offers a reaction $\boldsymbol{F}_r(t=7-0)$. This reaction transmitted to the surface Γ_0 to the hammer at time $t=7-7=0$. We can see that the actions are at

$$t = 0, 1, 2, 3, 4, 5, 6, 7 \tag{19}$$

The reaction is at

$$t = 7, 6, 5, 4, 3, 2, 1, 0 \tag{20}$$

If we assume the time t=0 is at the time m_8 offers the reaction to the m_7, the reaction is at the time,

$$t = 0, -1, -2, -3, -4, -5, -6, -7.$$

Hence, we have,

$$\boldsymbol{F}_a(t=0) = (m_1+m_2+m_3+m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=1) = (m_2+m_3+m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=2) = (m_3+m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=3) = (m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=4) = (m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=5) = (m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=6) = (m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=7) = (m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_a(t=8) = 0$$
$$\boldsymbol{F}_r(t=8-0) = -0$$
$$\boldsymbol{F}_r(t=8-1) = -(m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-2) = -(m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-3) = -(m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-4) = -(m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-5) = -(m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-6) = -(m_3+m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-7) = -(m_2+m_3+m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$
$$\boldsymbol{F}_r(t=8-8) = -(m_1+m_2+m_3+m_4+m_5+m_6+m_7+m_8)\boldsymbol{\alpha}$$

For the reaction, if the starting is chosen at $t = 8$, or we can take a new time $t_r = t - 8$, the reaction will be

$$\boldsymbol{F}_r(t_r = -0) = -0$$

$$\boldsymbol{F}_r(t_r = -1) = -(m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -2) = -(m_7 + m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -3) = -(m_6 + m_7 + m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -4) = -(m_5 + m_6 + m_7 + m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -5) = -(m_4 + m_5 + m_6 + m_7 + m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -6) = -(m_3 + m_4 + m_5 + m_6 + m_7 + m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -7) = -(m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8)\boldsymbol{\alpha}$$

$$\boldsymbol{F}_r(t_r = -8) = -(m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8)\boldsymbol{\alpha}$$

This means that the reaction is transmitted to the negative direction of the time. Hence, the reaction is advanced. This is same to the photon and particle situation.

From this we can summarized that, the action is transmitted as the retarded wave, the reaction is transmitted as the advanced wave.

If the action is not transferred as infinite speed, the reaction has to be transferred by the advanced wave, otherwise it is not possible to make the action and reaction synchronized at all surfaces.

This is our new recognization to the action and reaction force of the Newton third law.

3 Conclusion

According the mutual energy principle and the self-energy principle, for all particle include the photon, electron and so on, the energy is transferred through mutual energy flow. We can say all particles are the mutual energy flow. The mutual energy flow is consist of the retarded wave sends out from the source (emitter) and the advanced wave sends out from the sink (absorber). The retarded wave transmits the action force. The advanced wave transfers the reaction force. Since the macroscopic object are just many many particles, the reaction force should be similar to the situation of single particle. Hence the reaction should are also transferred through advanced wave.

This author checked the Newton's third law and find if the action and reaction are same in value and opposite in direction in every surface of the object, if the speed of reaction is not infinite, the reaction must be advanced. The reaction has to be transmitted through the advanced wave.

This author obtained a conclusion that all macroscopic wave for example the water wave, the sound wave, the reaction are all as advanced.

References

[1] Wheeler. J. A. and Feynman. R. P. *Rev. Mod. Phys.*, 17:157, 1945.

[2] Wheeler. J. A. and Feynman. R. P. *Rev. Mod. Phys.*, 21:425, 1949.

[3] John Cramer. The transactional interpretation of quantum mechanics. *Reviews of Modern Physics*, 58:647–688, 1986.

[4] John Cramer. An overview of the transactional interpretation. *International Journal of Theoretical Physics*, 27:227, 1988.

[5] Adrianus T. de Hoop. Time-domain reciprocity theorems for electromagnetic fields in dispersive media. *Radio Science*, 22(7):1171–1178, December 1987.

[6] A. D. Fokker. *Zeitschrift fÃŒr Physik*, 58:386, 1929.

[7] Shuang ren Zhao. The application of mutual energy theorem in expansion of radiation fields in spherical waves. *ACTA Electronica Sinica, P.R. of China*, 15(3):88–93, 1987.

[8] Shuang ren Zhao. A new interpretation of quantum physics: Mutual energy flow interpretation. *American Journal of Modern Physics and Application*, 4(3):12–23, 2017.

[9] V.H. Rumsey. A short way of solving advanced problems in electromagnetic fields and other linear systems. *IEEE Transactions on antennas and Propagation*, 11(1):73–86, January 1963.

[10] K. Schwarzschild. *Nachr. ges. Wiss. Gottingen*, pages 128,132, 1903.

[11] Lawrence M. Stephenson. The relevance of advanced potential solutions of maxwell's equations for special and general relativity. *Physics Essays*, 13(1), 2000.

[12] H. Tetrode. *Zeitschrift fÃŒr Physik*, 10:137, 1922.

[13] W. J. Welch. Reciprocity theorems for electromagnetic fields whose time dependence is arbitrary. *IRE trans. On Antennas and Propagation*, 8(1):68–73, January 1960.

[14] Shuangren Zhao. The simplification of formulas of electromagnetic fields by using mutual energy formula. *Journal of Electronics, P.R. of China*, 11(1):73–77, January 1989.