Proof of $\sum_{n=1}^{\infty}(-1)^n = -\frac{1}{2}$

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First, ±∞ is constant at any observation point (position).
If a set of real numbers is R,then,

$$R\times(\pm\infty)=\pm\infty$$
$$R+(\pm\infty)=\pm\infty$$
$$(-1)\times(\pm\infty)\neq\mp\infty$$

On the other hand, when x (∈R)is taken on a number line, the absolute value X becomes larger toward ± ∞ as the absolute value X is expanded.
Similarly, as the size decreases, the absolute value X decreases toward 0.Furthermore, × (-1) represents the reversal of the direction of the axis.

$$\boxed{\begin{array}{l}\frac{1}{\pm\infty}=(-1)\cdot(\pm\infty)=i\\ (\pm\infty)\cdot i-1=0\end{array}}$$

$$(-1)\cdot(\pm\infty)=\frac{1}{\pm\infty}$$

$$i^2=(\pm\infty)^2\rightarrow i=\pm(\pm\infty)$$

$$\therefore i=-(\pm\infty)=(-1)(\pm\infty)=\frac{1}{\pm\infty},(\because i\neq+(\pm\infty))$$

Next,

$$\boxed{\pi=\frac{2}{\pi}+2\arctan\left(\frac{1}{\tan\left(\frac{1}{x}\right)}\right),\left(\because x\geq\frac{1}{\pi}\right)}$$

$$x=\frac{2}{x}\left(\geq\frac{1}{x}\right)$$

$$\pi=\frac{2}{\left(\frac{2}{\pi}\right)}+2\arctan\left(\frac{1}{\tan\left(\frac{\pi}{2}\right)}\right)=\pi+2\arctan\left(\frac{1}{\pm\infty}\right)$$

$$\arctan\left(\frac{1}{\pm\infty}\right)=\arctan(i)=0$$

$$\therefore \tan 0=\frac{1}{\pm\infty}=(-1)(\pm\infty)=i$$

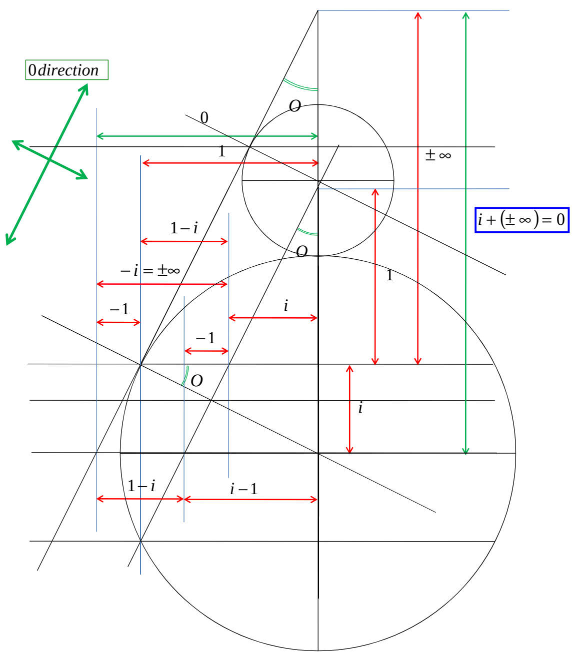

Fig.1

Second, we consider the figure below.

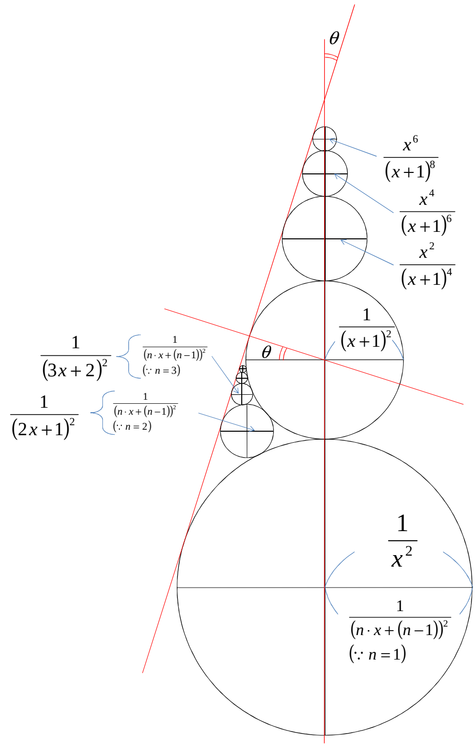

Fig.2

From the figure above, I got the following equation.

$$\theta=\arcsin\left(\frac{1}{\frac{2\cdot(x+1)^2}{2x+1}-1}\right)$$

Here, when take ±∞ to the consideration,

$$\boxed{\begin{array}{l}\tan\theta=\frac{2x+1}{2x(x+1)}=i\left(=(-1)\cdot(\pm\infty)=\frac{1}{\pm\infty}\right)\\ \therefore x=-\frac{1}{2}(1+i)\end{array}}$$

Here, when we consider the figure above,

$$\frac{x^2}{(x+1)^2}=-1$$

So, when we put x=-(1+i)/2, 1/(x^2)=2/i=-2i .
Here, from Fig.1, i+(±∞)=0.

$$\boxed{\begin{array}{l}-2i+2\cdot(-1)\cdot(-2i)+2\cdot(-2i)+2\cdot(-1)\cdot(-2i)+2\cdot(-2i)+\cdots=i+(\pm\infty)=0\\ -2i\cdot(1-2+2-2+2-2+2-\cdots)=i+(\pm\infty)=0\\ 1-2+2-2+2-2+2-\cdots=\frac{1}{2}\left(-1-\frac{\pm\infty}{i}\right)=\frac{1}{2}(-1-(-1))=0\\ 1+2((-1)+1+(-1)+1+(-1)+\cdots)=0\\ \frac{1}{2}+\sum_{n=1}^{\infty}(-1)^n=0\end{array}}$$

$$\boxed{\therefore\sum_{n=1}^{\infty}(-1)^n=-\frac{1}{2}}$$