—
—

Bell's correlation formula and anomalous spin

Han GEURDES
Address
Geurdes Datascience, KvK 64522202, C vd Lijnstraat 164, 2593 NN, Den Haag Netherlands
E-mail:*han.geurdes@gmail.com*

Koji NAGATA
Address
Department of Physics, Korea advanced Institute of Science and Technology, Daejeon, 34141, Korea

Abstract In this paper it is demonstrated that a hidden spin component may exist that in a local hidden variables, but quantum, manner invalidates the non-locality analysis with e.g. inequalities such as CHSH.

Keywords Inconsistency, Bell's theorem.

1 Introduction

It is perhaps not so well known that Einstein already had doubts about quantum mechanics from its very early beginning [1] and [2]. Einstein was not very satisfied with the paper containing the EPR paradox [4]. This paper did contain the criticism of Einstein on the completeness of quantum theory. However, according to Einstein in a letter to Schrödinger: die Hauptsage is sozusagen durch Gelehrsamkeit verschüttet. This is most likely not some sort of artisitic need for simplicity but reflects that he and his co-authors were not able to express Einstein's real point of view. There is a difference between the view in the EPR paradox and Einstein's criticism of quantum mechanics [1]. In the paper, together with Rosen and Podolsky, Einstein argued that the quantum description must be supplemented with extra variables to explain the entanglement phenomenon. Einstein's own views were more directed to inseparability than to the existence of extra hidden parameters [1].

In 1964, John Bell wrote an important paper [3] on the possibility of local hidden variables [4] causing the spin-spin entanglement correlation $E(a, b)$ between two particles. Bell's paper opened the possibility of an experiment to see if Einstein local hidden variables could be present in nature.

In a recently submitted paper we demonstrated an inconsistency in the starting formula of Bell [3]. This paper was a continuation of / response to [12]. The aim of our present short paper is to come with the possibility of hidden quantum numbers that always disallow a consistent Bell analysis. Let us, please, note here that Einstein never intended to *exclusively* have classical variables explaining the inseparability or local hidden variables in entanglement.

...

Bell, based his hidden variable description on particle pairs with entangled spin, originally formulated by Bohm [5]. Bell used hidden variables λ that are elements of a universal set Λ and are distributed with a density $\rho(\lambda) \geq 0$. Suppose, $E(a,b)$ is the correlation between measurements with distant A and B that have unit-length, i.e. $||a|| = ||b|| = 1$, real 3 dim parameter vectors a and b.

Then with the use of the λ we can write the classical probability correlation between two simultaneously measured spins of the particles. This is what we will call Bell's formula.

$$E(a,b) = \int_{\lambda \in \Lambda} \rho(\lambda) A(a,\lambda) B(b,\lambda) d\lambda \tag{1.1}$$

The spin measurement functions are, $A(a,\lambda) \in \{-1,1\}$ and $B(b,\lambda) \in \{-1,1\}$. The probability density is normalized, $\int \rho(\lambda) d\lambda = 1$.

2 What about the sgn in Bell's formula for spin measurement?

This section contains a study on the obvious use of a sign distribution representing a (part of a) measurement function, for either A or B in (1.1). We refer for definition of sign to [14] and [15].

2.1 A sub-model that can be incorporated in any hidden variable theory

Suppose we look at a a probability density function in a single real variable $x \in \mathbb{R}$.

$$\rho(x) = \begin{cases} -x, & x \in [-1,0] \\ +x, & x \in [0,1] \\ 0, & otherwise \end{cases} \tag{2.1}$$

It can be easily verified that $\rho(x) \geq 0$ for all $x \in [-1,1]$. Moreover, it is also easy to establish that

$$\int_{-1}^{+1} \rho(x) dx = -\int_{-1}^{0} x dx + \int_{0}^{+1} x dx = -\frac{1}{2}(0-(-1)^2) + \frac{1}{2}(1^2 - 0) = 1 \tag{2.2}$$

Hence, ρ is a real possibility for (part of a) probablity desity in (1.1). It is subsequently noted that $\rho(x) = |x|$ for all $-1 \leq x \leq 1$. Let us look at a part of a more complete model. We have e.g.

$$E = \int_{-1}^{1} |x| \mathrm{sgn}(x) dx \tag{2.3}$$

The E defined previously can be the result of any model where, $|x|\rho(\lambda)$, with, $-1 \leq x \leq 1$, replaces the density $\rho(\lambda)$ and $A(a,\lambda)\mathrm{sgn}(x)$ replaces the $A(a,\lambda)$. Then, E will occur in the evaluation of the $E(a,b)$ from (1.1).

2.2 sign algebra

Our object of study will be $|x|\mathrm{sgn}(x)$. We have $\mathrm{sgn}(x) = 1$ when $x \geq 0$ and $\mathrm{sgn}(x) = -1$ when $x < 0$.

2.2.1 Fractional exponents & possible hidden quantum numbers

Let us look at the integral in (2.3). The integrand $|x|\mathrm{sgn}(x)$ can be written as

$$|x|\mathrm{sgn}(x) = x \times \mathrm{sgn}(x) \times \mathrm{sgn}(x) \tag{2.4}$$

Here $\times$ is used for emphasis on multiplication. In our previous paper we wrote

$$\mathrm{sgn}(x) \times \mathrm{sgn}(x) = \{\mathrm{sgn}(x)\}^1 \times \{\mathrm{sgn}(x)\}^1 = \{\mathrm{sgn(x)}\}^{1/2} \times \{\mathrm{sgn(x)}\}^{3/2} \tag{2.5}$$

which is allowed because obviously $1 + 1 = 2 = \frac{1}{2} + \frac{3}{2}$. The anomaly was obtained from $\{\mathrm{sgn(x)}\}^{3/2}$ with

$$\left[\{\mathrm{sgn}(x)\}^3\right]^{1/2} \not\equiv \left[\{\mathrm{sgn}(x)\}^{1/2}\right]^3 \tag{2.6}$$

despite $3 \times (1/2) = (1/2) \times 3$.

2.2.2 Anomalous spin introduction

It is interesting to find out if other forms of breakdown of $1 + 1 = 2$ gives a similar sort of anomaly. Let us for example look at $1 + 1 = 2 = \frac{1}{4} + \frac{1}{4} + \frac{3}{4} + \frac{3}{4}$. Therefore

$$\mathrm{sgn}(x) \times \mathrm{sgn}(x) = \{\mathrm{sgn}(x)\}^{1/4} \times \{\mathrm{sgn}(x)\}^{1/4} \times \{\mathrm{sgn}(x)\}^{3/4} \times \{\mathrm{sgn}(x)\}^{3/4} \tag{2.7}$$

Let us repeat the principles upon which the (2.6) is based. The principles are itemized below and refer to the treatment of e.g.

$$\{\mathrm{sgn(x)}\}^{3/2} = \{\mathrm{sgn(x)}\}^{-1/2} \tag{2.8}$$

reflected in the exponentials above.

- *Principle 1:* In the evaluation of $\{\mathrm{sgn}(x)\}^{3/2}$ of (2.8), the evaluation is based on first the power 3 then the power 1/2 and this concurs with, on the right hand of (2.8), first the power -1 then the power 1/2.

- *Principle 2:* In the evaluation of $\{\mathrm{sgn}(x)\}^{3/2}$ of (2.8), the evaluation is based on first the power 1/2 then the power 3 and this concurs with, on the right hand of (2.8), first the power 1/2 then the power -1.

Obviously the examples in the principles above are just there to present the differences in the mathematical operations. Let us look at $\{\mathrm{sgn}(x)\}^{3/4}$. Let us take $x < 0$. Then, given $i = (-1)^{1/2}$

$$\{\mathrm{sgn}(x)\}^{3/4} = \{\{\mathrm{sgn}(x)\}^3\}^{1/4} = (-1)^{1/4} = \{(-1)^{1/2}\}^{1/2} = i^{1/2} \tag{2.9}$$

This is an evaluation according to principle 1. According to principle 2 we see

$$\{\mathrm{sgn}(x)\}^{3/4} = \{\{\mathrm{sgn}(x)\}^{1/4}\}^3 = i^{1/2} \times i^{1/2} \times i^{1/2} = i \times i^{1/2} \tag{2.10}$$

with, again, $(-1)^{1/4} = i^{1/2}$. Hence, as in (2.6) we see again

$$\left[\{\mathrm{sgn}(x)\}^3\right]^{1/4} \not\equiv \left[\{\mathrm{sgn}(x)\}^{1/4}\right]^3 \tag{2.11}$$

despite $3 \times (1/4) = (1/4) \times 3$. If we evaluate according to principle 1 and equation (2.9) then it follows from (2.7) and $x < 0$ that

$$\mathrm{sgn}(x) \times \mathrm{sgn}(x) = i^{1/2} \times i^{1/2} \times i^{1/2} \times i^{1/2} = i \times i = -1 \tag{2.12}$$

Hence the part of the E integral in (2.3) over the negative axis, according to principle 1, looking at (2.12), equals

$$E_1^{neg} = \int_{-1}^{0} x \times \mathrm{sgn}(x) \times \mathrm{sgn}(x) dx = -\int_{-1}^{0} x dx = \frac{1}{2} \tag{2.13}$$

If we, on the other hand, evaluate according to principle 2 and equation (2.10) then it follows from (2.7) and $x < 0$ that

$$\mathrm{sgn}(x) \times \mathrm{sgn}(x) = i^{1/2} \times i^{1/2} \times i \times i^{1/2} \times i \times i^{1/2} = i \times i = (-1) \times (-1) \tag{2.14}$$

Hence the part of the E integral in (2.3) over the negative axis, according to principle 2, looking at (2.14), equals

$$E_2^{neg} = \int_{-1}^{0} x \times \mathrm{sgn}(x) \times \mathrm{sgn}(x) dx = (-1) \times (-1) \int_{-1}^{0} x dx = -\frac{1}{2} \tag{2.15}$$

Therefore the same anomaly arises in the E evaluation.

2.2.3 Anomalous spin

We now ask ourselves the question if this anomalous spin-like functions hold a general structure. Let us in the very first plce not forget that one can read Einstein's criticism on quantum mechanics [1] without being forced to *by necessity* to have classical variables. Bell was perhaps leaning too much on that side without acknowledging that his "simple" approach contains many riddles itself.

Of course the anomaly pointed at previously may look weird. But then again, when quantum mechanics was founded in the previous century, many weird phenomena and riddles in both mathematics and physics experiment were observed. Who are we to pretend that the mathematics we commonly use is the *only* valid way to describe nature. Is there no reason to believe in concrete mathematical incompleteness [7]. What about incompletenss in Bell's claim iteself [6]. Are there no other ways, e.g. tropical algebra [8], possible?

Returning to the physics we might insist that the weirdness is an expression of hidden quantum numbers in the theorem. The authors believe that the success of quantum mechanics is in fact the unclocking of weird aspects of the language mathematics. This here presented anomaly is just another example of it.

This also implies, looking at the experimental side, e.g. [9], that there are Einsteinian hidden -but at the same time quantum- structures behind the classical Bell scene. Again we stress, Einstein never said that incompleteness (in the physical sense) must be supplemented with *classical* local hidden variables. To our minds and looking at [1] it was about inseparability and the implication for causality and locality. The here found local hidden -but quantum- structures are not there to allow one party to be right and to ridicule all the others. On the contrary. The hidden states behave precisely as could have been expected from quantum anomalies. The bold claim here, based on the mathematics thus far, is that one never can have anything but those anomalies when working with correlations such as Bell's in a spin-spin entanglement experiment [10].

The general form of the anomaly spin breakdown is given below. We gently remind the reader that the validity of the anomaly is based on simple arithmetics like e.g. $3 \times (1/2) = (1/2) \times 3$ or $3 \times (1/4) = (1/4) \times 3$ and the in equivalences in (2.6) and (2.11).

The general anomalous spin function can be written down in parts. Firstly let us define

$$\psi_{\alpha_j}^{(n)}(x) = \{\mathrm{sgn}(x)\}^{\alpha_j} \tag{2.16}$$

Here the $n = 2^m$ and $m \in \{0, 1, 2, 3 \dots\}$ stands for the $2n$ number of terms the $\mathrm{sgn}(x) \times \mathrm{sgn}(x)$

can be broken down into. For instance with $n = 1(m = 0)$ we have (2.5). For $n = 2(m = 1)$ we have (2.7). Secondly, the α_j, for $j \in \{1, 2, 3, \dots, 2n\}$, are defined by

$$\alpha_j = \frac{2k_j + 1}{2n} \tag{2.17}$$

Here, $k_j = \{0, 1, \dots, n-1, n, \dots\}$, and $n = 2^m$, together with

$$\sum_{j=1}^{2n} \alpha_j = 2. \tag{2.18}$$

We can write down an expression for $\mathrm{sgn}(x) \times \mathrm{sgn}(x)$ and know that (2.16) allows anomaly in the breakdown

$$\mathrm{sgn}(x) \times \mathrm{sgn}(x) = \prod_{j=1}^{2n} \psi_{\alpha_j}^{(n)}(x) \tag{2.19}$$

It is likely that some anomalous spin structure has been lost when selecting $n = 2^m$ and $m \in \{0, 1, 2, 3 \dots\}$. But in this manner we know that (2.19) always contains anomalous spin functions. Unclarity with e.g. $3/6 = 1/2$ is avoided in this way. The left hand $\{\mathrm{sgn}(x)\}^{3/6}$ allows anomaly but the right hand $\{\mathrm{sgn}(x)\}^{1/2}$ does not.

3 Conclusion and discussion

In the first place it is noted that the anomalous breaking down of $\mathrm{sgn}(x) \times \mathrm{sgn}(x)$ is unavoidable. The claim is true because the functions $\psi_{\alpha_j}^{(2^m)}(x)$ with $\alpha_j = (2k_j + 1)2^{-m-1}$ and $j \in \{1, 2, \dots, 2^m\}$ with $k_j \in \mathbb{N} \cup \{0\}$ and $\alpha_1 + \alpha_2 + \cdots + \alpha_{2\times 2^m} = 2$, and $m \in \{0, 1, 2, \dots\}$, contain forms with

$$\{\{\mathrm{sgn}(x)\}^{(2k_j+1)}\}^{2^{-m-1}} \not\equiv \{\{\mathrm{sgn}(x)\}^{2^{-m-1}}\}^{(2k_j+1)}$$

and we can always write

$$\mathrm{sgn}(x) \times \mathrm{sgn}(x) = \prod_{j=1}^{2^{m+1}} \psi_{\alpha_j}^{(2^m)}(x)$$

such as was demonstrated in the paper. We emphasize that $\alpha_1 + \alpha_2 + \cdots + \alpha_{2\times 2^m} = 2$ is the reason behind the rewriting. Therefore the integral that can always be attached to any Bell formula,

$$E = \int_{-1}^{1} x \times \mathrm{sgn}(x) \times \mathrm{sgn}(x)\ dx$$

is ambiguous.

Secondly we claimed that this is in line with Einstein's conception of "there is inseparability in quantum mechanics and therefore quantum mechanics is an incomplete theory". This is surely not about a statistical explanation of quantum phenomena although this is definitely not ruled out either. The authors believe that we must show forgiveness to a searching mind to find where possible lacunes in such a difficult theory reside. There is no room for belittling or ridicule in this domain of study. People who use the social tactics of belittling a searching mind for what might be the cause of quantum weirdness, have surely to hide their blatant (and anybody elese's) ignorance from sight.

The anomalous spin structure presented here shows that Bell's hidden variable approach and experimental methodology is invalid. One can always have forms of $\psi_{\alpha_j}^{(2^m)}(x)$ spoiling a consistent claim to be derived from Bell's formula. We believe that the anomaly found is directly in line with all the other anomalies and weirdness to be found in quantum mechanics. We can posdtulate that those $\psi_{\alpha_j}^{(2^m)}(x)$ are somehow present at spin measurement.

Nevertheless quantum mechanics is a construction of the human mind and it has been shown many times -and in what can be found in any decent textbook of quantum mechanics- that our mental conception of the quantum reality is warped. Our present analysis underscores this. We note that it is rather remarkable that people believe at the same time in a quantum weirdness like e.g. nonlocality and think that mathematically and experimentally one can act as though the reality behind the scences is possibly ruled by classical statistics. If there is weirness classical theory cannot be tested at all. We would also like to note that perhaps there is no world behind the scences in an ontological sense [11, Chap Introduction: a la recherche de l'etre].

Although what we say here is theory, there is a whole lot more in quantum physics that is taken for granted but is in fact theory. E.g. nobody has ever witnessed a wave function collapsing. The interpretation of the experiments of Aspect for instance are but interpretation. Nobody ever was able to go beyond Bell statistics. This statistics is flawed and that flaw can be a reflection of what is happening in quantum reality. This is what we claim.

We cannot but agree with professor Howard when he claims that valuable points of view are ignored when Einstein his approach to quantum mevhanics is dismissed as a forgiveable but wrong point of view on modern physics. We add that it is unforgiveable to close the door on Einstein because one is in favour of another point of view. On the contrary we believe that Einstein was far ahead in his way of looking at quantum reality; perhaps without him even completely realizing it. Concrete mathematical incompleteness and quantum mechanics are intertwined.

Finally note, the experimentally demonstrated difference between classical and quantum mechanics does not at all need Bell methodology. This state of affairs can be found in the recent literature, [12] and [13].

References

[1] D. Howard, "Einstein on locality and separability", Stud. Hist. Phil. Sci. **16**, 171-201, (1985).

[2] D. Howard, "Nicht sein kann was nicht sein darf, or the pre-history of EPR, 1909-1935: Einstein's early worries about the quantum mechanics of composite systems", Sixty-two Years of Uncertainty, Ed. A. Miller, 1-51, Plenum (1990).

[3] J.S. Bell, "On the Einstein Podolsky Rosen paradox," Physics, 1, 195, (1964).

[4] A. Einstein, B., Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," Phys. Rev. 47, 777, (1935).

[5] D. Bohm, "Quantum Theory," pp 611-634, Prentice-Hall, Englewood Cliffs, 1951.

[6] H. Geurdes, K. Nagata, T. Nakamura & A. Farouk, A note on the possibility of incomplete theory, arxiv 1704.00005, (2017).

[7] H. Friedman, https://m.youtube.com/watch?v=CygnQSFCA80, uploaded U Gent 2013.

[8] S.M. Avdoshin, V.V Belov, V.P. Maslov and A.M. Chebotarev, "Design of computational media: Mathematical aspects", p 9-142, in: Mathematical aspects of computer engineering, ed V.P. maslov & K.A. Volosov, MIR Publishers, Moscow, 1988.

[9] A. Aspect, P. Grangier & G. Roger, "Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedanken-experiment: A New Violation of Bell's Inequalities", Phys. Rev. Lett. **49**, 91-94, (1982).

[10] A. Aspect,J.Dalibard & G. Roger, "Experimental test of Bell's inequality using time-varying analyzers", Phys. Rev. Lett. **49**, 1804-1811, (1982).

[11] J.P. Sartre, L'être et le néant, Editions, Galimard, (1943)

[12] B. Nordén, Entangled photons from single atoms and molecules, Chemical Phys. 507, 28-33, (2018).

[13] B. Nordén, Quantum entanglement: facts and fiction - how wrong was Einstein after all ?Quart.Rev.Biophys. 49, 1-13, (2016).

[14] M.J. Lighthill, *Einfürung in die Theorie der Fourieranalysis und der verallgemeinerten Funktionen: chap 2, Die theorie der verallgemeinerte Funktionen*, Hochschultaschenbücher (1966).

[15] F. Frasat, *Introduction to Generalized Functions with applications in Aerodynamics and Aeroaucoustics*, NASA, 1996.