The black hole in M87 (EHT2017) may provide evidence for a Poincare Dodecahedral Space universe

by Dipl. Ing. (FH) Harry K. Hahn / Germany

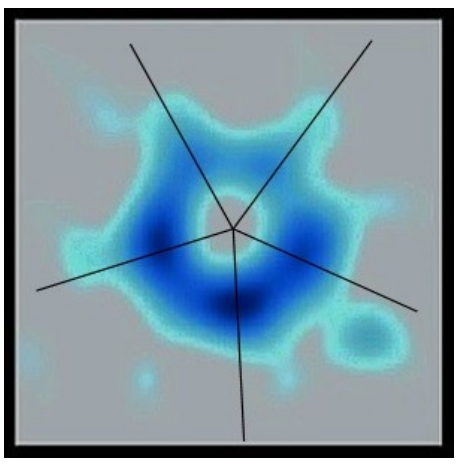
---

26. May 2019 - Update from **8. August 2020**

Abstract :

The M87 (EHT2017) black hole images indicate a dodecahedral structure of the gravitational singularity.
One feature in the final images of the black hole shadow is standing out. It's the nearly perfect pentagonal shape of the ring structure and the existence of five knots in this ring-structure ! If these five knots turn out to be real and stable, then we may look at a gravitational singularity which indicates a more complex universe than expected ! The pentagonal ring may indicate a Poincare Dodecahedral Space structure of our universe, and a dodecahedral shaped gravitational singularity (black hole) in the center of M87. Further evidence comes from the contrast- and brightness enhanced image of the black hole shadow, which indicates a lattice of 3 to 4 full pentagonal rings and probably further 4 to 5 partly visible rings. This lattice of pentagonal rings may be a projection of the assumed dodecahedral structure of the black hole onto the surrounding transparent emission region, caused by photon capture and gravitational light bending around this dodecahedral gravity source.

Contents : **Page**



1 Introduction - The EHT2017 images may provide hints for the universal physical theory

The **EHT-team** has done a great job to provide the first visible evidence of a **Black hole** with the help of the **EH-Telescope !**

One feature in your fiducial image sequence caught my eye and forced me to write this paper here ! It's the apparent existence of **5 knots** in the ring-structure and the nearly symmetrical **pentagonal-shape of the black hole shadow !**

In Paper IV you wrote that your current image reconstructions and visibility-domain analyses are not able to confirm or reject the reality of the "knots" seen in the images, and these features should therefore be interpreted with caution.

But if these knots turn out to be real and stable, then we may look at a gravitational singularity which indicates a more complex universe then expected ! The pentagonal ring structure may indicate a **Poincare Dodecahedral Space (PDS) structure** of our universe, and a **dodecahedral shaped** gravitational singularity (black hole) in the center of **M87.**
Your images may provide evidence for the **underlying universal theory of our universe** Albert Einstein was searching for.
I.V. Volovich describes the construction of a universal physical theory based on pure numbers as fundamental entities.
And I want to show the **importance of number 5** for the distribution of primes and non-primes as a base for this theory.

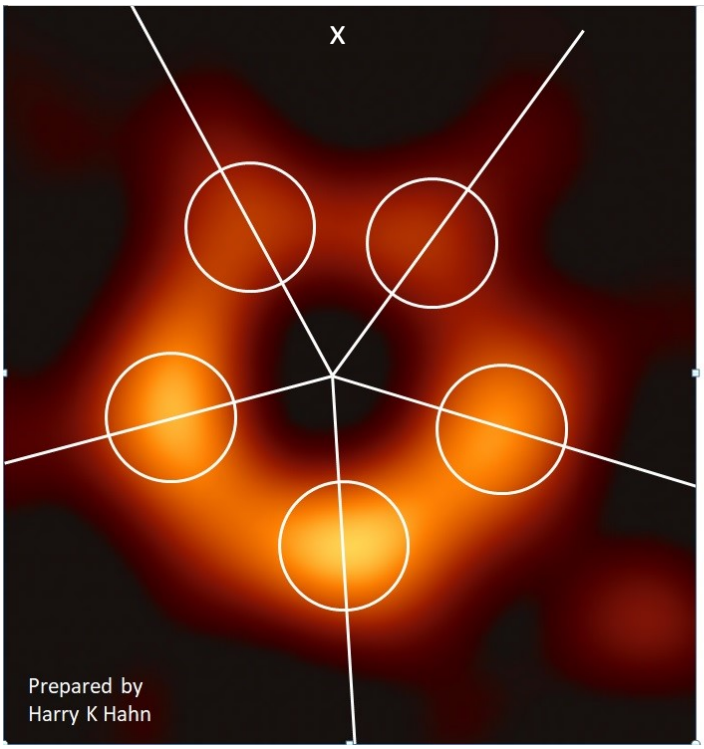
Questions to be answered :

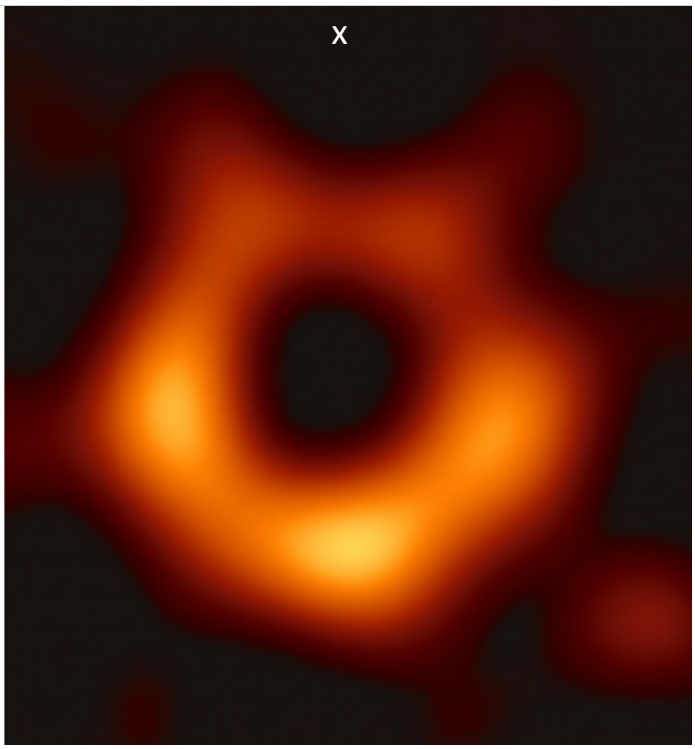
Are the five knots are real and stable in their position ?
Do the knots correspond to a slightly tilted pentagon ?
Is the visible pentagonal structure stable over time ?
Are there structures and extensions around the ring ?
Is the visible anti-clockwise flow over the knots real ?

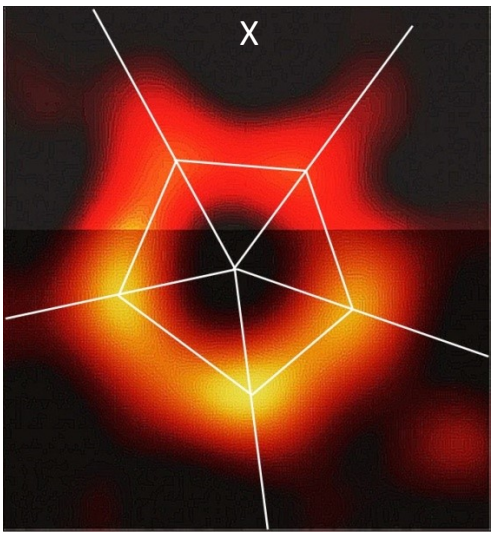
What should be done ? :

Further image processing & analyses of the sharpest images.

Computer simulations of gravitational singularities in order-4 and order-5 Poincare-Dodecahedral Spaces should be made.
How would the shadow of a comparable black hole look like in an order-4 & -5 PDS-universe ? Then compare it with M87 !







The image on the top (with and without markings) is from the fiducial image sequence of the M87 Black Hole Shadow from the 10th of April from page 18 of Paper IV)
Because the ring is brighter in the south than in the north **I have increased the brightness of the northern section** of the ring, in order to show the ring structure with more homogeneous brightness.

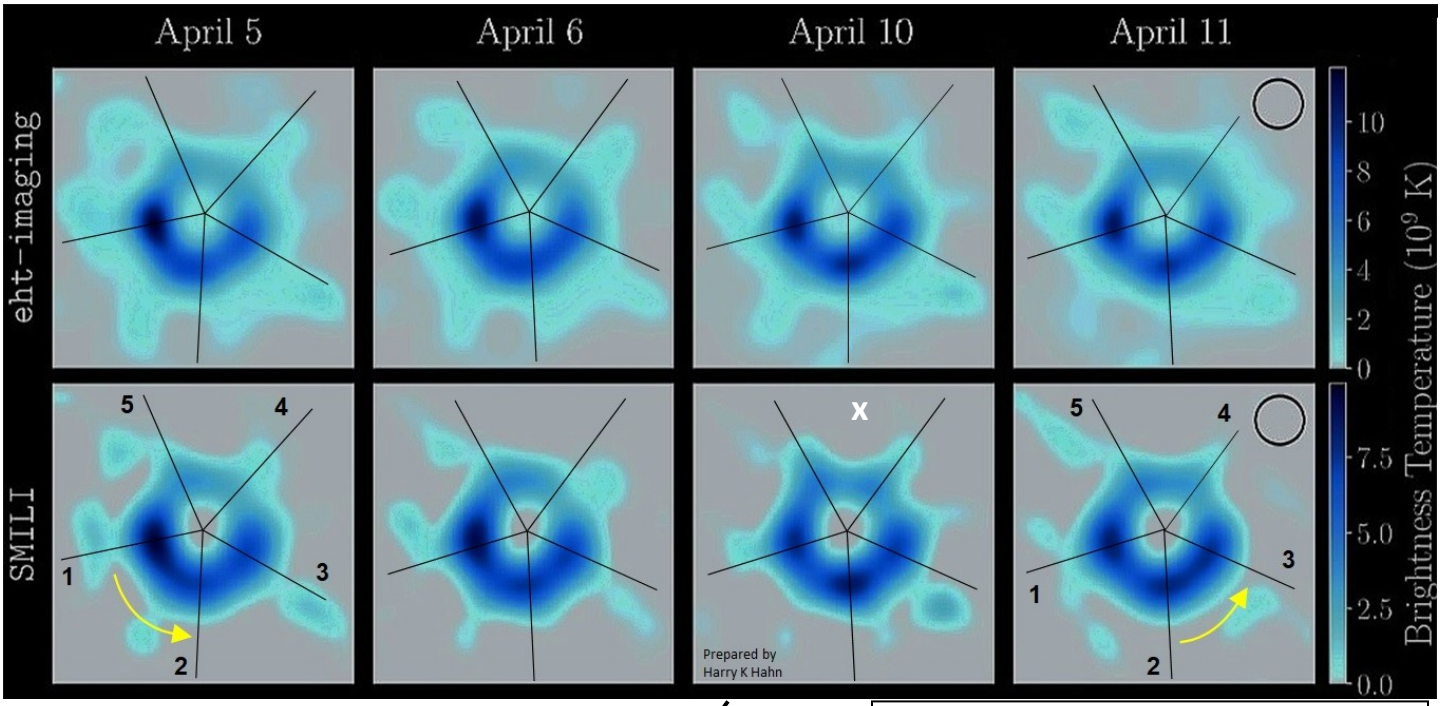
And it becomes clear that the ring has a pentagonal structure !

In Paper I you wrote that the asymmetry in brightness in the ring can be explained in terms of relativistic beaming of the emission from a plasma rotating close to the speed of light around a black hole.

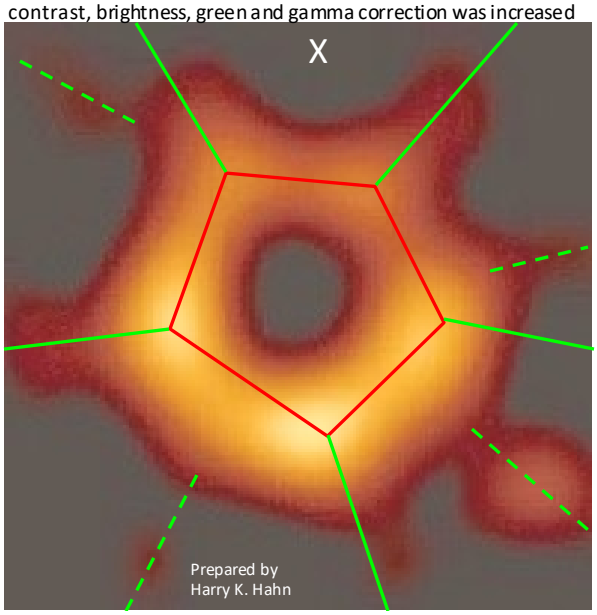
EHT2017 papers :

Paper I: The Shadow of the Supermassive Black Hole
Paper II: Array and Instrumentation
Paper III: Data processing and Calibration
Paper IV: Imaging the Central Supermassive Black Hole
Paper V: Physical Origin of the Asymmetric Ring
Paper VI: The Shadow and Mass of the Central Black Hole

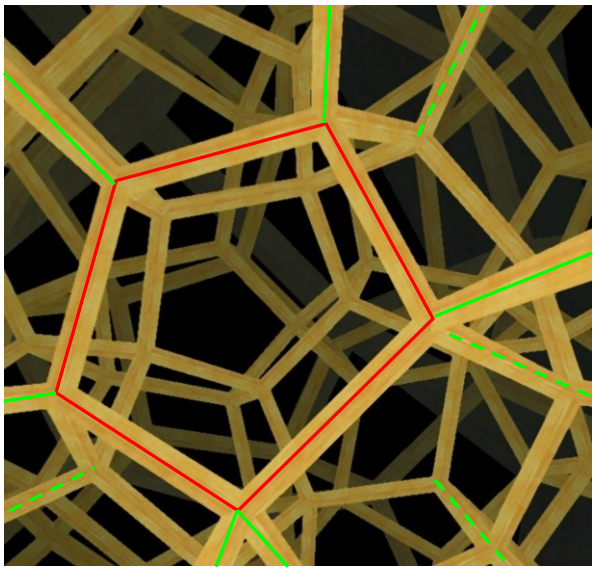
2 The M87 Black Hole Shadow seems to have a pentagonal shape and may imply a dodecahedral structure



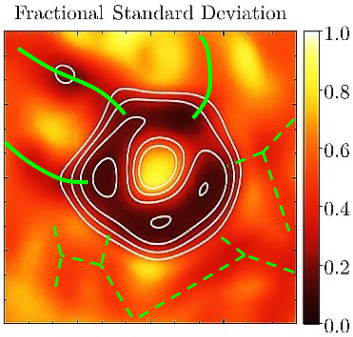
Reprocessed inverted color fiducial image sequence from Paper IV
contrast, brightness, green and gamma correction was increased



The EHT2017 images may indicate a dodecahedral lattice structure of a PDS universe around the black hole



Note : the re-processed true-color images are shown in **Appendix 1** at the end of this study



The image above is a **contrast enhanced & inverted color image of the fiducial image sequence of the M87 Black Hole Shadow** shown on page 18 of Paper IV of the EHT2017 observing campain.
Only the images processed by the **SMILI & eht-imaging algorithm** were used for this inspection.

There was no modification made on the visible structures in the images. All fiducial images calculated by the SMILI & eht-imaging algorithms **clearly indicate a pentagonal ring structure !**
The SMILI-images even show **two flow events** ! On April 5 flow is visible from knot 1 to 2, and on April 11 flow from knot 2 to 3.
This means over 24h the velocity of the flow probably must have been > 50000 km/s.

The above shown contrast-enhanced image sequence also shows lobes extending outward from the corners (knots) of the pentagonal ring structure. If these lobes are real structures, visible in the shadow of the black hole, then these lobes may indicate a Poincare Dodecahedral Space (PDS) structure around the black hole !

We may see a distorted mirror image of a dodecahedral lattice structure around the black hole as shown on the image on the left.

Several studies have proposed that the preferred model of the comoving spatial 3-hypersurface of the Universe may be a Poincare dodecahedral space (PDS) rather than a simply connected flat space

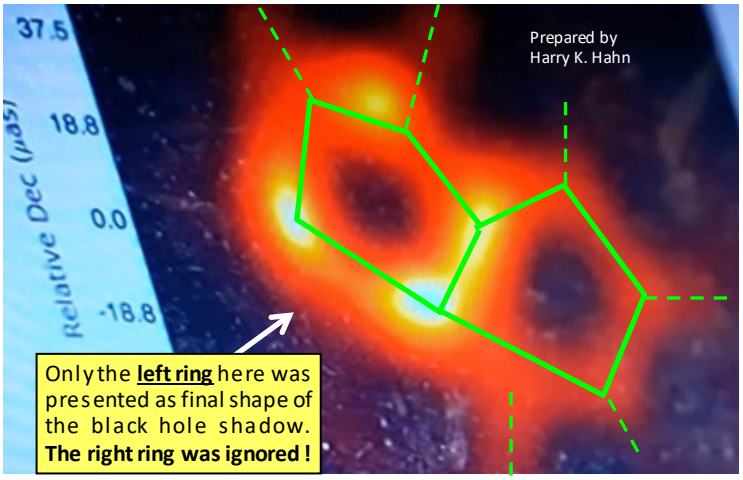
As the pentagon structure in the EHT images isn't perfect symmetric we may see the shadow of a dodecahedral shaped black hole, whose top-tile isn't pointing exactly towards the shadow area & the brightness of the dodecahedral lattice structure seems to fluctuate.

The M87 black hole images indicate a dodecahedral structure of the gravitational singularity

The following images are from the **documentation of the EHT2017-project** → movie title : **„ Black Hole Hunters“**
Weblink to → „Black Hole Hunters“ : https://www.youtube.com/watch?v=o_F3KVAPMpo (german version)
The English version (same title) is available on **www.smithsonianchannel.com** or on **www.amazon.com**

The images are from the section which shows how the algorithms calculate the first images of the M87 black hole
This is the **movie sequence from around 39:40 to 41:00 minutes.**

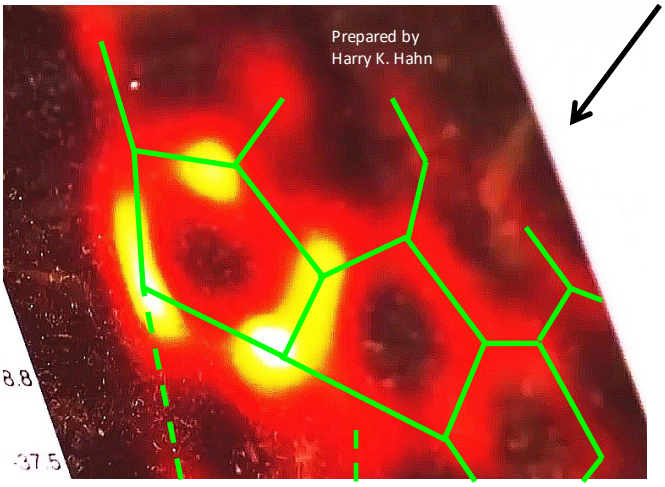
In the abstract of Paper I the scientists wrote that when surrounded by a transparent emission region, black holes are expected to reveal a dark shadow caused by gravitational light bending, and photon capture at the event horizon. When the algorithm started to calculate the first image there were distinct polygonal structures visible ! **→ see image below !**
If you have a close look at this polygonal structure then **two pentagons can be recognized** ! (→ see markings in the image)
And this **pentagon array structure** seems to **extend further into space** as the doted lines indicate !

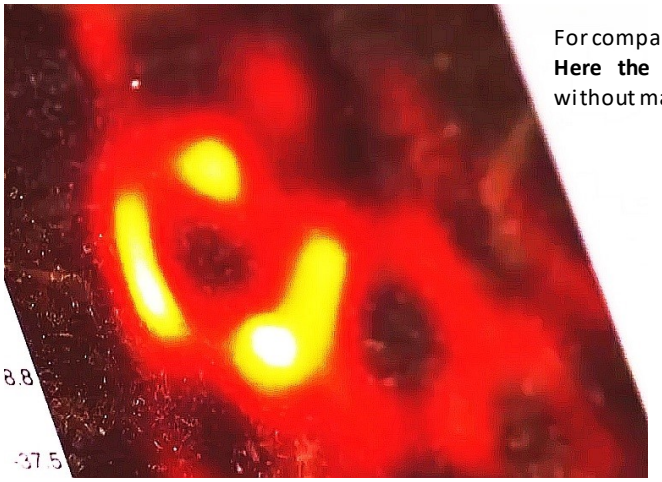


Only the **left ring** here was presented as final shape of the black hole shadow. **The right ring was ignored !**

For comparison **here the original image** of the image calculation in progress without markings :

The image below shows a brightness- and contrast-enhanced version of the above shown image :
This image shows a complex poly-pentagonal (-polygonal) structure. The shadow of the **M87 black hole** (**EHT2017**) looks like the **„flat pattern“ of a Dodecahedron !** There are 3 to 4 full pentagonal tiles and 4 to 5 half pentagonal tiles visible !





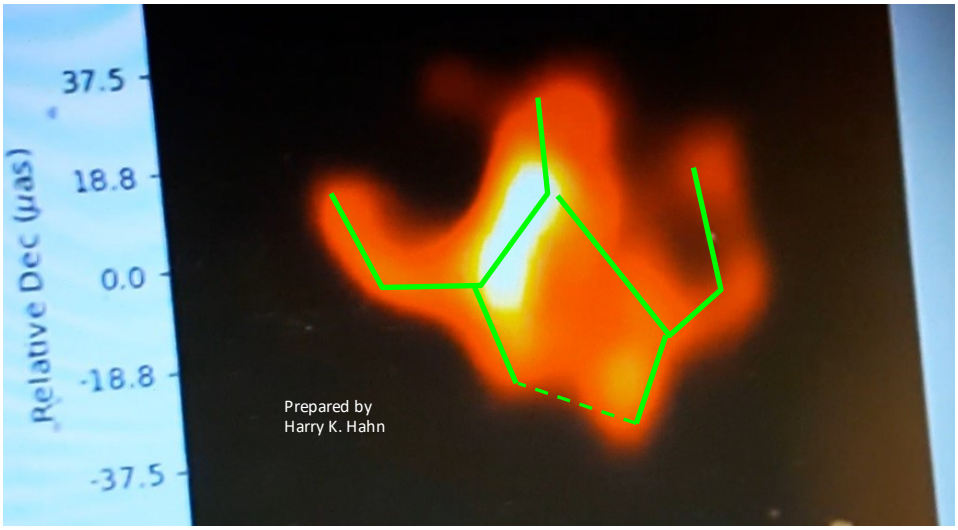
For comparison : **Here the Image** without markings

Here another image of the start of the movie sequence, which shows the calculation prozess. The first impression is that we are looking at **a pretty complex three-dimensional (3D-) structure !** ⟶

This structure also indicates that the **M87 black hole shadow** seems to be a projection of a **3D-dodecahedral structure !**

This image seems to show the dode-cahedral structure **in a slightly rotated position** compared to above shown images

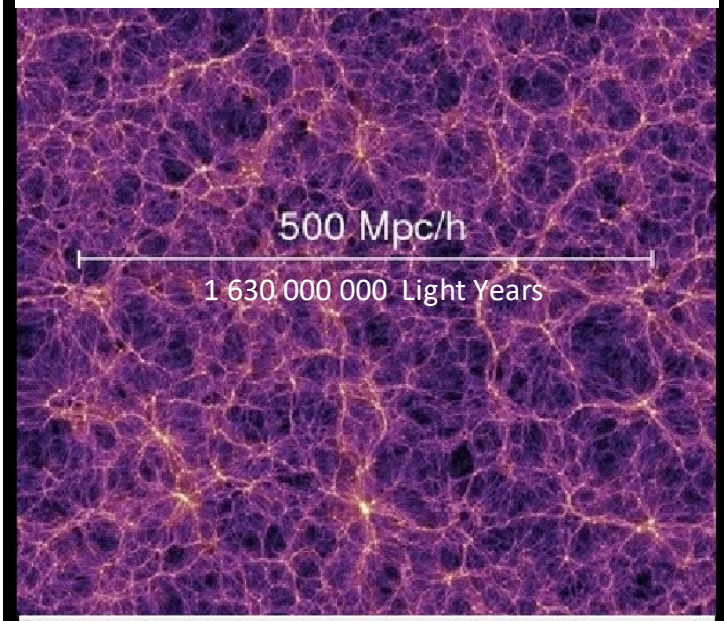
Question : Was this image taken a considerable time earlier or later than the above shown image ??



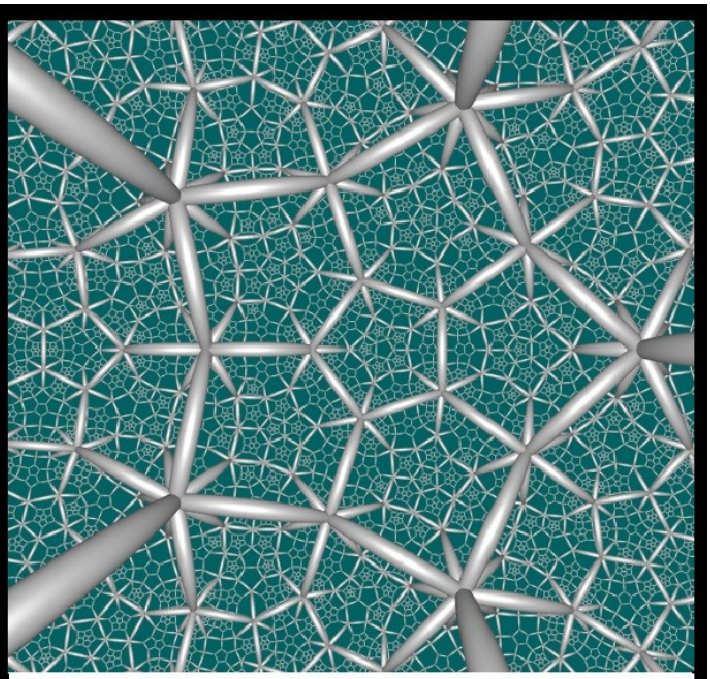
3 The large-scale distribution of matter in our universe is similar to an O-5 dodecahedral honeycomb structure

If we compare the large scale distribution of matter in our universe with the structure of a regular order-5 dodecahedral honeycomb in hyperbolic 3-space, then there are similarities visible ! The order-5 dodecahedral honeycomb has a better „visual" fit to the large-scale filament structure of our universe compared to an order-4 dodecahedral honeycomb.
In our universe matter clearly is distributed along the edges (filaments) of the visible cell structure, if we look at the universe on a large scale. Refering to the „similar-looking" order-5 dodecahedral honeycomb, that means matter (and the photons emitted by it) would be distributed and moving along the edges (filiaments) of the dodecahedral cells.
If we consider a black hole in the order-5 dodecahedral honeycomb then it probably would also have a dodecahedral shape, and matter and photons would move along the edges of the structure. However because our universe does't have a precise cell structure, **the real structure probably is caused by a superposition of all possible honeycomb structures.**

large-scale Universe Structure : The Millenium Simulation



This is how the universe looks on a large scale; a very large scale. It's not a photo but the outcome of a simulation called The Millennium Simulation. Since the outcome matches the observed distribution of galaxies in the universe very well, we can use it for visualising the biggest structures of the universe, the filaments.
The structure of the universe resembles a sponge with the matter in the form of hundreds of billions of galaxies not being uniformly distributed over space but gathering in so-called filaments. In between the filaments there are voids, which are almost empty areas in the universe with diameters of between 40 million and 400 million light years

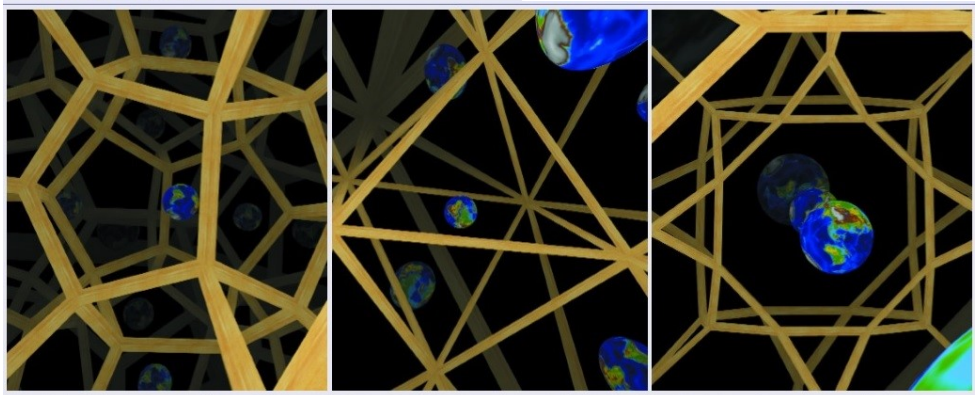


This is the regular {5,3,5} honeycomb in hyperbolic 3-space. The model is cell-centered in the Poincare Ball model, with the viewpoint then placed at the ball origin.

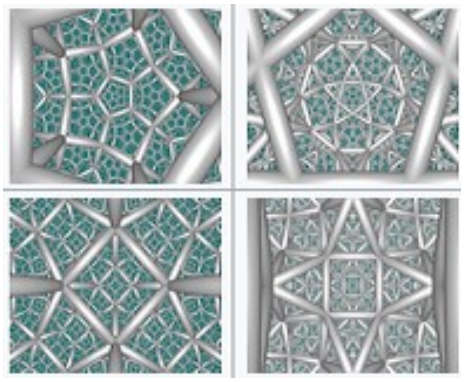
Prepared by
Harry K. Hahn

See : http://www.sun.org/images/structure-of-the-universe-1 and : https://en.wikipedia.org/wiki/Order-5_dodecahedral_honeycomb and https://en.wikipedia.org/wiki/Galaxy_filament

3 The many shapes of the universe → **See article : A cosmic hall of mirrors**



The Poincaré dodecahedral space (left) can be described as the interior of a "sphere" made from 12 slightly curved pentagons. However, there is one big difference between this shape and a football because when one goes out from a pentagonal face, one immediately comes back inside the ball from the opposite face after a 36° rotation. Such a multiply connected space can therefore generate multiple images of the same object, such as a planet or a photon. Other such well-proportioned, spherical spaces that fit the WMAP data are the tetrahedron (middle) and octahedron (right).



Other possible honeycomb structures
See :
→ **Uniform_honeycombs_in_hyperbolic_space**
e.g. Order-4_dodecahedral_honeycomb

Poincare Dodecahedral Space (PDS) Desribed as the interior of a hypersphere tiled with 12 slightly curved pentagons. When one goes out from a pentagonal face, one comes immediately back inside the PDS from the opposite face after a 36° rotation. Such a PDS is finite, although without edges or boundaries so that one can indefinitely travel within it. As a result an observer has the illusion to live in a space 120 times vaster, made of tiled dodecahedra which duplicate like in a mirror hall. As light rays crossing the faces go back from the other side, every cosmic object has multiple images

4 Analyses of the CMB indicate that the universe may have a Poincare dodecahedral space (PDS) structure

Here are the weblinks to two studies which indicate a Poincare dodecahedral space (PDS) structure of the universe :

1.) Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background (CMB)

Jean-Pierre Luminet, Jeffrey R. Weeks, Alain Riazuelo, Roland Lehoucq & Jean-Philippe Uzan

Weblink 1: http://ceadserv1.nku.edu/longa//classes/2004fall/mat115/days/luminet-nat.pdf
Weblink 2: https://luth.obspm.fr/~luminet/physworld.pdf

2.) The optimal phase of the generalised Poincare dodecahedral space hypothesis implied by the spatial cross-correlation function of the WMAP sky maps

Boudewijn F. Roukema, Zbigniew Bulin´ski, Agnieszka Szaniewska, Nicolas E. Gaudin

Weblink 1: https://arxiv.org/abs/0801.0006 Weblink to PDF: https://arxiv.org/pdf/0801.0006.pdf

ABSTRACT

Context. Small universe models predicted a cutoff in large-scale power in the cosmic microwave background (CMB). This was detected by the Wilkinson Microwave Anisotropy Probe (WMAP). Several studies have since proposed that the preferred model of the comoving spatial 3-hypersurface of the Universe may be a Poincaré dodecahedral space (PDS) rather than a simply connected, flat space. Both models assume an FLRW metric and are close to flat with about 30% matter density.

Aims. We study two predictions of the PDS model. (i) For the correct astronomical positioning of the fundamental domain, the spatial two-point cross-correlation function ξ_C of temperature fluctuations in the covering space (where the two points in any pair are on different copies of the surface of last scattering (SLS)) should have a similar order of magnitude to the auto-correlation function ξ_A on a single copy of the SLS. (ii) Consider a "generalised" PDS model for an *arbitrary* "twist" phase $\phi \in [0, 2\pi]$. The optimal orientation and identified circle radius for a generalised PDS model found by maximising ξ_C relative to ξ_A in the WMAP maps should yield one of the two twist angles $\pm 36°$.

Methods. Comparison of ξ_C to ξ_A extends the identified circles method, using a much larger number of data points. We optimise the ratio of these functions at scales $\lesssim 4.0h^{-1}$ Gpc using a Markov chain Monte Carlo (MCMC) method over orientation (l, b, θ), circle size α, and twist ϕ.

Results. Both predictions were satisfied: (i) An optimal generalised PDS solution was found for two different foreground-reduced versions of the WMAP 3-year all-sky map, both with and without the kp2 galactic contamination mask. This solution yields a strong cross-correlation between points which would be distant and only weakly correlated according to the simply connected hypothesis. The face centres are $\{(l, b)\}_{i=1,6} \approx \{(184°, 62°), (305°, 44°), (46°, 49°), (117°, 20°), (176°, -4°), (240°, 13°)\}$ (and their antipodes) to within $\approx 2°$; (ii) This solution has twist $\phi = (+39 \pm 2.5)°$, in agreement with the PDS model. The chance of this occurring in the simply connected model, assuming a uniform distribution $\phi \in [0, 2\pi]$, is about 6-9%.

Conclusions. The PDS model now satisfies several different observational constraints.

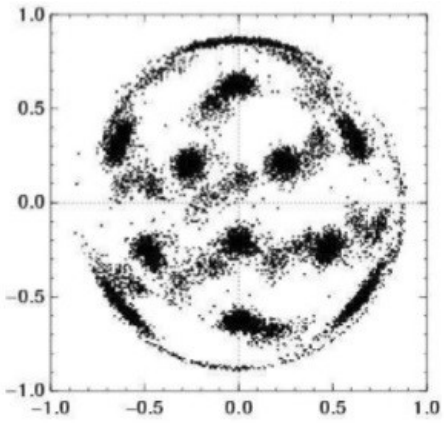


Fig. 5. Full sky map showing the optimal orientation of dodecahedral face centres based on 100,000 steps in 10 MCMC chains, using the ILC map and either the kp2 mask (upper panel) or no mask (lower panel), showing face centres for which $P > 0.5$ (see Eq. (25)). The projection is a Lambert azimuthal equal area projection (Lambert 1772) of the full sky, centred on the North Galactic Pole (NGP). The 0° meridian is the positive vertical axis and galactic longitude increases clockwise. These face centres are derived from the MCMC chains *without any constraint on the twist phase* ϕ.

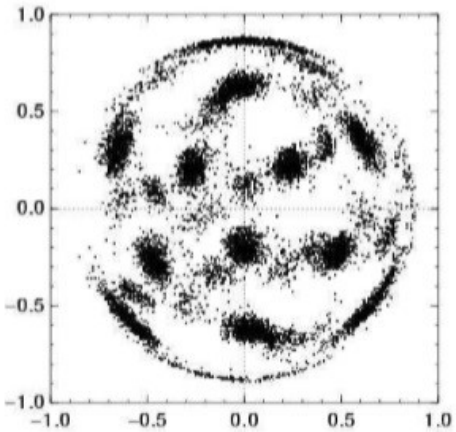


Fig. 10. Full sky map showing the optimal dodecahedral orientation for the ILC map with the kp2 mask, as for Fig. 5, centred on the North Galactic Pole, from an MCMC chain starting at the PDS orientation and circle size suggested in Roukema et al. (2004), for an initial twist of $\phi = -\pi/5$. The optimal orientation is clearly very close to what is found from arbitrary initial positions, shown in the previous figures.

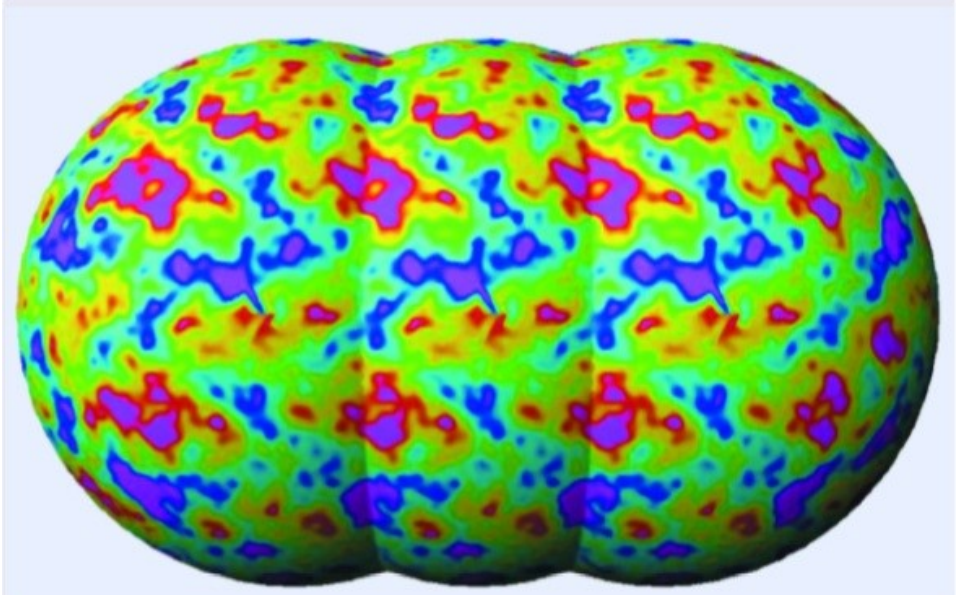
5 How the traces of a Poincare dodecahedral space unsiverse would appear on the CMB map

→ The image below is from the news article : **„A cosmic hall of mirrors“**

Weblink : **A cosmic hall of mirrors**

Extract 1 from article : If physical space is indeed smaller than the observable universe, some points on the map of the **Cosmic Microwave Background** (**CMB**) will have several copies. As first shown by Neil Cornish of Montana State University and co-workers in 1998, these ghost images would appear as pairs of so-called matched circles in the cosmic microwave background where the temperature fluctuations should be the same (figure 4). This “lensing” effect, which can be precisely calculated, is thus purely attributable to the topology of the universe. Due to its 12-sided regular shape, the **Poincare dodecahedral model** actually predicts six pairs of diametrically opposite matched circles with an angular radius of 10–50°, depending on the precise values of cosmological parameters such as the mass–energy density.

4 Simulated circle matching



The topology of the universe describes how different regions are connected and could therefore leave its imprint on the cosmic microwave background. For example, if our physical space is smaller than the observable universe (as recent data suggest it is) then the horizon sphere wraps around the universe and intersects itself. As a result, duplicated images of the cosmic microwave background (in which the colours represent temperature fluctuations) will intersect along a circle and we would observe this circle on different sides of the sky.

Extract 2 from article :

In June 2004, Boud Roukema and colleagues at the Torun Centre for Astronomy in Poland independently searched for circles in the WMAP data. By only looking for back-to-back circles within a limited range of angular sizes and neglecting all other possible matches, the computer time was reduced drastically. Remarkably, the Polish team found six pairs of matched circles distributed in a dodecahedral pattern and twisted by 36°, each with an angular size of about 11°. This implied that $\Omega = 1.010 \pm 0.001$, which is perfectly consistent with our dodecahedral model, although the result was much less publicized than the earlier negative results.

In fact, the statistical significance of the match still needs to be improved, which means that the validity of the Poincaré dodecahedron model is still open to debate. In the last few months, however, there has been much theoretical progress on well-proportioned spaces in general.

Early this year, for example, Frank Steiner and co-workers at the University of Ulm in Germany went on to prove that the fit between the power spectrum predicted by the Poincaré dodecahedron model and that observed by WMAP was even better than we had previously thought. But the German team also extended its calculations to well-proportioned tetrahedral and octahedral spherical spaces in which $\Omega > 1$ (see figure 3).

These spaces are somewhat easier to understand than a dodecahedral space, but they require higher values of the density: $\Omega > 1.015$ for octahedral spaces and $\Omega > 1.025$ for tetrahedral spaces, compared with $\Omega > 1.009$ for dodecahedral spaces. However, these values are still compatible with the WMAP data. Furthermore, Steiner and co-workers found that the signal for pairs of matched circles could have be missed by current analyses of the cosmic microwave background due to various measurement effects that damage or even destroy the temperature matching.

Studies from : Prof. Frank Steiner :

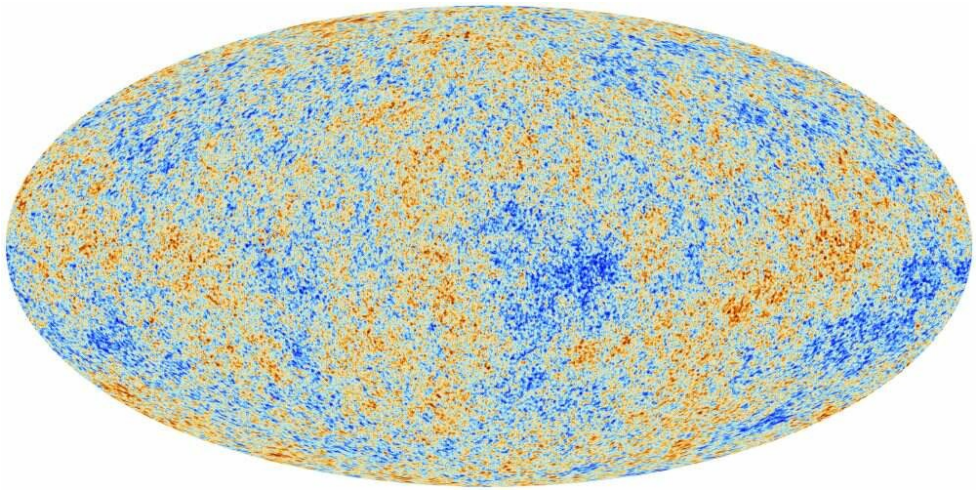
(Institute of Theoretical Physics Ulm University)

Cosmic_microwave_background_alignment_in_multi-connected_universes

CMB_Anisotropy_of_the_Poincare_Dodecahedron

other related studies :

https://www.researchgate.net/profile/Frank_Steiner5



Cosmic microwave background map (All sky map of the **CMB** created from **WMAP**-data)

Cosmic microwave background (CMB) is faint electromagnetic radiation, as remnant, from an early stage of the universe, which is filling all space

6 To Albert Einsteins's work on a Unified field theory - Some clues about the final theory he had in mind

One letter to his girlfriend Ilse Rosenthal-Schneider in the year 1945 gives some insight about his thoughts to a TOE

Essentiell he was looking for a theory where the universal constants which appear in the base equations only have units which are reduced to kg , m and s (the units for mass , length and time). Albert Einstein considered the most other physical units and constants equipped with these units as fictitious (manmade) , and said that they can be eliminated.

He went further and said that by multiplying these universal constants with factors, which are formed out of powers of c1, c2, c3 (the universal physical constants), that the new universal constants c4*, c5*, c6* are pure numbers.

Essentiell he was searching for an underlying universal mathematical theory only containing constants like Pi(π) & e

→ **Weblink to the german book which contains Einsteins original letter from 13th October 1945 in german language :**

→ http://docplayer.org/69639849-Ilse-rosenthal-schneider-begegnungen-mit-einstein-von-laue-und-planck.html
see also : - description oft he book contents in english : http://blog.alexander-unzicker.com/?p=27

The letter to Ilse Rosenthal-Schneider (dated 13.10.1945) can be found on the pages 23 bis 27 (pages of online-document 30-34) in Chapter 2 : The universal natural constants (Kapitel 2: Die universellen Naturkonstanten)

→ **my Translation of Albert Einsteins letter from Oct. 1945 - Without guarantee of a 100% correct translation ! :**

(→ my own comments are marked in pink (blue) color !)

Dear Ms Rosenthal !

I can see from your (last) letter that you haven't understood my hints regarding the universal constants of physics. Therefore I want to try now to make the subject clearer.

1.) …"There are fictitious constants and true constants. The fictitious constants can be eliminated. …the true constants are real numbers…. **I believe that these true constants must be of a -„rationell" type-, like Pi (π) and e**

z.B.: $e = 1 + 1 + 1/2! + 1/3! + \ldots$

…"The fictious constants just come from the introduction of arbitrary units. Such constants don't exist. Their existence is only based on the fact, that we haven't penetrated physics deep enough"

"In contrast to these constants of the „rationell type" there is the rest of the numbers which do not come through a transparent construction out of number 1"

„It lies in the nature of this matter, that such constants of the „rationell type" are not very different to number 1 from their order (of magnitude)

→ see for example : **Mathematical constants** - on page 24 (31)

(comment : the expression -„rationell" type- is not exactly clear. In the german language „rationell" can also mean in general : „based on logic" or „based on (logic) reason" / „well reasoned" or he precisely meant the group of irrational and transcendental constants like Pi (π) and **e** , or more in general a group of irrational numbers comprising also square roots of a selected group of small non quadratic numbers like 2, 3, 5 and 7,….

2.) „There is now a complete theory of physics, were the universal constants c1,c2,…cn appear in the base equations. The Units are reduced somehow to kg (gr), m (cm), s (sek.).……The choice of these three units is obvious very conventional. Each of the c1,c2,…cn has a dimension in these units. Now we want to chose it in such a way, that c1,c2,c3 have such dimensions, that we can't form a dimensionfree product $c^{\alpha}1\, c^{\beta}2\, c^{\gamma}3$

Then we can multiply c4, c5, etc. in such a way with factors, which are formed out of powers (potenzen) of c1, c2, c3, that these new c4*, c5*, c6* are pure numbers (without units).

„These (pure numbers) are the real universal constants of the theoretical system (of the universal mathematical physical theory) **which haven't anything to do with conventional units".** – on page 26 (33)

(comment : Albert Einstein is talking here about a universal (physical) theory (Theory of everything) which only contains true (pure) mathematical constants as mentioned in 1.) without (manmade) physical units !

3.) „My expectation is now, that these constants c4*, c5*, …etc. are of the „rationell“ type, and that their value is based on the logical base of the complete theory.“

„You can also say it so : In a reasonable theory there are no dimensionfree numbers, with a value, which can only be determined empirical.“

Of course I can't proof this. But I can't imagine a universal and reasonable theory , which contains a number, which could have been chosen just as good differently by the mood of the creator (god), whereby the world would have turned out qualitatively different in its physical legitimacy.“

„You can also say it so : A universal theory, which contains in their base equations a constant which is not of the „rationell“ type, would somehow have to be added together out of pieces, which are logically independ ent from each other. But I have confidence, that this world isn't like that, that we need such an ugly construction to capture the theory of the world.“

4.) „Of course, there is no consistent theoretical base for the complete physical science yet. And not at all a theoretical base which would meet the radical demands described above. It is not so difficult to think about possible formulations. Unfortunately these are such relativistic theories , in which it is extremely hard, to progress to proofable conclusions.

It is difficult to solve non-linear differential equations so, that there are nowhere Singularities.

But this problems haven't anything to do with the principle question.

With friendly greetings and wishes.

Your **Albert Einstein**

Some weblinks to studies about natural constants :

Looking_for_Those_Natural_Numbers_Dimensionless_Constants_and_the_Idea_of_Natural_Measurement_1
https://www.academia.edu/35881283/

- Do we live in an eigenstate of the “fundamental constants” operators?
  https://arxiv.org/pdf/1809.05355.pdf

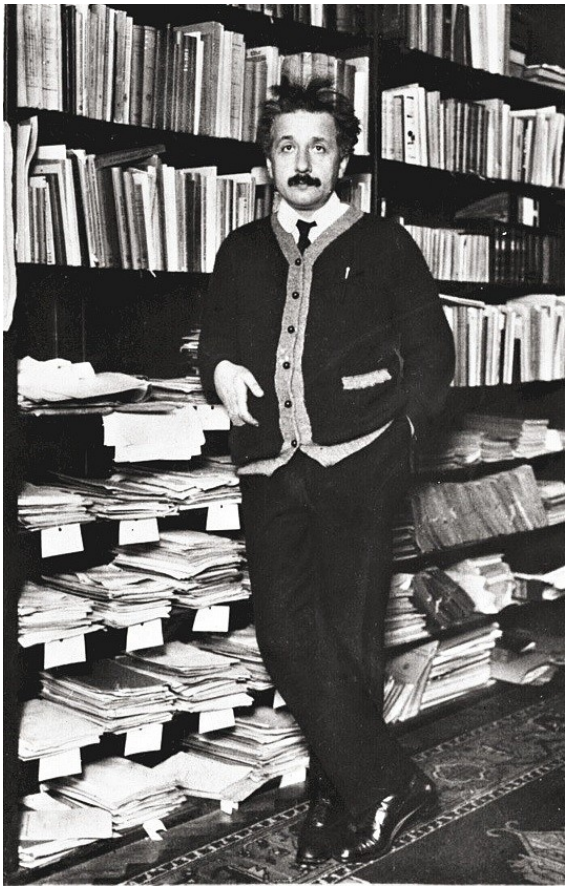
- Fine structure constant
  https://oeis.org/wiki/Fine-structure_constant

- Dimensionless Physical Constant Mysteries
  http://www.rxiv.org/pdf/1205.0050v1.pdf

Two quotes from Albert Einstein :

“If you can't explain it in a simple way, then you haven't understood it good enough“ (Wenn Du es nicht einfach erklären kannst, hast Du es nicht gut genug verstanden)

“Not everything that counts can be counted, and not everthing that can be counted counts“ (Nicht alles was zählt kann gezählt werden, und nicht alles was gezählt werden kann, zählt)



Albert Einstein in his office in Berlin
in the early 1920's

7 Number Theory as the Ultimate Physical Theory - by I. V. Volovich / Steklov Mathematical Institute

The mathematician I. V. Vololovich has written a hypothesis on the quantum fluctuation of the number field.
Our picture of space-time at the Plank-scale doesn't seem to be correct. The fundamental entities (base units) of the universe can't be particles, fields or strings. Numbers are considered to be the fundamental entities. Therefore a new quantum mechanics over an arbitrary number field must be developed. **And number theory is the base of physics.**

→ **Weblink to the study :** http://cdsweb.cern.ch/record/179558/files/198708102.pdf

Extracted text from page 1 of the study :

Weblink : I.V.Volovich_publication list

NUMBER THEORY AS THE ULTIMATE PHYSICAL THEORY

I.V. Volovich*)
CERN - Geneva

A B S T R A C T

At the Planck scale doubt is cast on the usual notion of space-time and one cannot think about elementary particles. Thus, the fundamental entities of which we consider our Universe to be composed cannot be particles, fields or strings. In this paper the numbers are considered as the fundamental entities. We discuss the construction of the corresponding physical theory.

A hypothesis on the quantum fluctuations of the number field is advanced for discussion. If these fluctuations actually take place then instead of the usual quantum mechanics over the complex number field a new quantum mechanics over an arbitrary field must be developed. Moreover, it is tempting to speculate that a principle of invariance of the fundamental physical laws under a change of the number field does hold.

The fluctuations of the number field could appear on the Planck length, in particular in the gravitational collapse or near the cosmological singularity. These fluctuations can lead to the appearance of domains with non-Archimedean p-adic or finite geometry.

We present a short review of the p-adic mathematics necessary, in this context.

Planck units
Elementary particle
Field (physics)
String (physics)
Quantum fluctuations
Quantum mechanics
Field (mathematics)
Non-Archimedean
P-adic geometry
finite geometry
Black hole

Extracted text from pages 14 and 15 of the study :

It is appropriate to recall the famous Einstein programme to reduce all physics to geometry. It is a promising programme but let us ask the question about which geometry we would like to speak of? Why should we pick Riemannian geometry? Are there the reasons in favour of Riemannian geometry, or can one also use non-Archimedean geometry? One can go farther and ask the question why geometry over the field of real numbers but not over an arbitrary field is the proper geometry for physics. We believe that the contemporary version of Einstein's programme should look like a proposal to reduce all physics to geometry over arbitrary number fields. In fact this means the reduction of the physics to number theory. One can agree with the Pythagoreans according to whom we have to understand number, in order to understand the Universe.

If these ideas are true then number theory and the corresponding branches of algebraic geometry are nothing else than the ultimate and unified physical theory.

Number theory

Of course, it is possible to generalize the above general principle and to consider some algebras instead of fields. In superanalysis[4], we exchange the field of real numbers for superalgebra with a norm. But in this paper we restrict ourselves to the case of the field.

Extracted text from pages 15 and 16 of the study :

We will discuss now an appropriate modification of quantum mechanics. In the usual quantum mechanics, the principle of superposition of probability amplitudes plays the main rôle and it can be written in the form

$$\langle a|c\rangle = \sum_b \langle a|b\rangle\langle b|c\rangle$$

where the probability has to be calculated as follows:

$$P_{ab} = |\langle a|b\rangle|^2. \qquad (1)$$

In the construction of the quantum mechanics on an arbitrary field K, one can follow two ways. The first way consists in considering complex-valued functions depending on variables belonging to K, or more generally, functions taking values in a complex Hilbert space. Here the results of representation theory on the local compact fields may be useful [see Ref. 44)] where these constructions are actually considered.

The second way one deals with wave functions taking values in the field K, i.e., $\langle a|b\rangle \in K$. Then in the case of the field K with a norm, Eq. (1) has sense if $|\ |$ means the norm in K. In our opinion it is difficult at present to tell which of these two ways is more favourable, so that it is reasonable to develop both simultaneously. Note that the first way is more closer to traditional quantum mechanics. However, there are important differences. Namely, it seems rather inappropriate to formulate dynamics in this case using the Schrödinger equation. A more appropriate way is to deal with unitary representation of the translation group.

If the above-mentioned hypothesis on the fluctuations of the number field is indeed realized then it is possible to suggest also the following hypothesis. It is common wisdom that in the Big Bang or in the final collapse, time and space do not have their usual meanings. But this is a purely negative answer to the question about the meaning of the time and space co-ordinates in such circumstances. What is a positive answer? Our proposal is as follows. The space and time co-ordinates would be, for example, p-adic. Of course this is an unusual world. For example, p-adic variables are not ordered. In this case, there is no meaning to the words "greater" or "less". But nevertheless we can write differential equations in such variables and we can try to understand processes in the Big Bang in a constructive way.

Then the strongest fluctuations take place in the Big Bang and a newly born Universe can have non-Archimedean or finite or other geometry over non-standard number fields. It may be that the corresponding exotic domain exists at present. An analogous hypothesis can also be considered in the context of the gravitational collapse. By this we mean that in the process of the collapse as a result of quantum effects, matter can collapse into a space with non-Archimedean geometry.

8 - What subjects do we have to look at to find the Universal Mathematical- and Physical Theory ?

Albert Einstein was on the right track ! There is a more profound logic (or theory) which we haven't grasped yet !

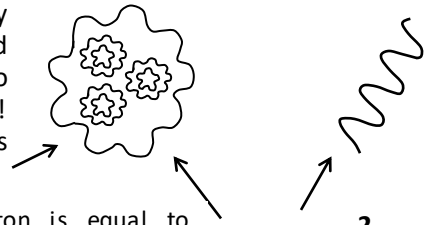
The main reason why he failed to find an universal physical theory was the fact that he tried to base it on the existing physical theories. He was coming from the physical science.

In his letter from 13.10.1945 Albert Einstein wrote that „too many arbitrary units & -constants" had been introduced in physical theories. Therefore it was an impossible and desperate task to try to eliminate all these arbitrary units and constants, in order to come to a simple and fundamental set of equations.

A better way ist to come from the mathematical side ! Especially from the branch of mathematics which deals with the foundation of mathematics : **Number theory** ! And we have to consider **Geometry** & **Algebraic Geometry** as important foundation stones for the Universal Theory. There are no units to eliminate in mathematics ! This ist the big advantage !

Regarding the existing physical theory we also have to ask the serious question if manmade units like kg (for mass) and m (for length) aren't arbitrary units too, which can all be eliminated if we only work with true base units of the creator. If we have a look to the famous equation $\mathbf{E = mc^2}$ in a more „visual way" we may see a bit clearer what is going on.

Matter is just energy where the lightspeed was diverted into an oscillating shell !! → It's Energy that's circulating on the spot !



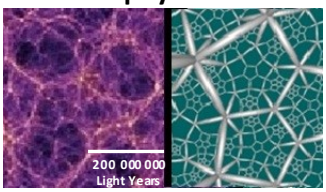
The energy of a photon is equal to **Planck's Constant h** multiplied by the photon's frequency. **This is an important fundamental law that we need to consider in the Universal Physical Theory**. But we have to use it in an abstract mathematical way **without the manmade units Kg and m attached to it**. And time should be replaced by a general variable.

I believe that we must use an „energy base unit" described by an abstract "X-dimensional mathematical-/ geometrical (wave) element" and an abstract time unit in the Universal Theory. Linear moving energy (photons) that is caught in small particle-size "oscillating shells", seems to be the fundamental cause of the strong- and weak-interaction and the cause of electromagnetism and gravitation.
The "bending" or „geometrical deformation" of the „linear moving wave-elements" (photons) into small oscillating shells may be the fundamental cause of the four known fundamental forces. The truth may be that only energy exists, either moving on open linear tracks, or oscillating in small particle-size shells, that we wrongly call „Matter" !

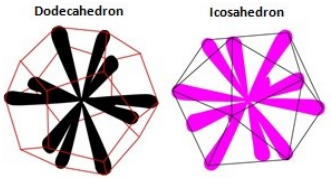
The theory of the early universe says that direct after the Big Bang only energy existed. The universe was pure energy at this stage. Then after a very short time period of 10^{-32} s energy began to condense into Protons & Neutrons.

That's why a fundamental physical theory must be able to explain how matter forms out of pure „massless energy". The biggest unsolved problem of particle physics, that quarks, the hypothetical "building stones" of matter, **never could be observed** in experiments so far, **indicates that our current understanding of matter may be fundamentally wrong !** (→ **see Weblink 1 , Weblink 2**)
My opinion is that we must explain matter & gravitation as the result of "geometrially deformed energy". The deformation of linear moving energy, into small particle-size oscillating shells is the cause of gravitation & "matter".
Energy & Time are the only fundamental physical units !

And we must consider that this energy ("matter") isn't distributed in a flat space universe but instead in a universe that has a structure similar to a spatial 3-hypersurface with a Poincare-Dodecahedral-Space (PDS)-structure → see image (→ order-5 dodecahedral honeycomb) The distribution of the **CMB**, the galaxy filiament stucture, the **M87 "black hole"**, atomic clusters & quasicrystals all indicate a frame-work based on the **Platonic Solids**



the galaxy filiament structure is similar to a **5/3/5 honeycomb in hyperbolic 3-space**



Atomic core model based on Platonic Solids

And I agree with **Igor.V.Volovich** that numbers must be fundamental entities (units) of an universal physical theory ! However numbers and number systems are manmade and therefore also can't be the fundamental units for the universal theory. I believe **Albert Einstein** was right when he was considering only important irrational mathematical constants (→ constants of the „rationell type") like **Pi** (π) and **e** as fundamental elements of the Universal Theory.

My **mathematical discovery** described in **chapter 8.1**, shows that all irrational square roots of the natural numbers and even constant **Pi** (**π**) can be expressed by only using the most irrational constant **Phi** (**φ**) and **Number 1** as base units. Therefore I even want to go further and want to claim that **only one irrational constant exists**, which is **Phi** (**φ**) ! And all other existing irrational constants are mathematical constructions out of **Phi** (**φ**) and **Number 1 !**
The base units of the Universal Theory therefore must be the most irrational constant Phi (**φ**) and **Number 1 !** Constant **PhI** (**φ**) seems to be **the fundamental constant that defines the distribution** (**scattering**) **of "particles" and waves in the universe**, and the **base unit** (number) **1** seems to represent **a base energy/wave element** (see **chapter 8.3**)
In order to arrive at an Universal Physical Theory **we should leave this base energy/wave element dimensionfree and we should consider it, togther with a dimensionfree time, in an abstract and pure mathematical / geometrical way**.

Contents of Chapter 8 :

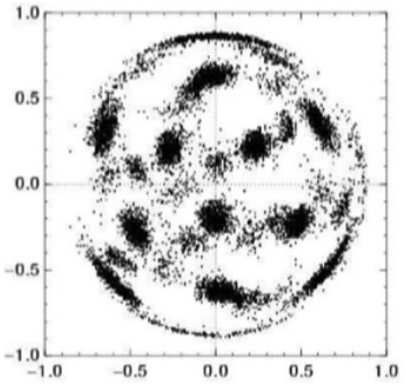
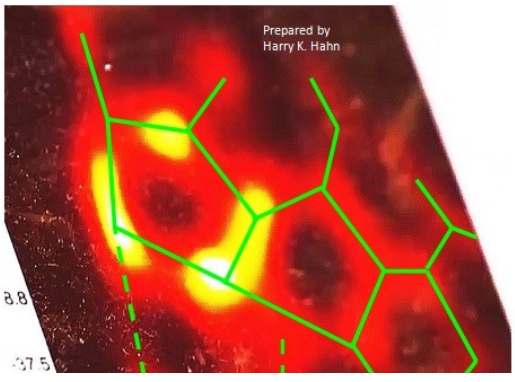
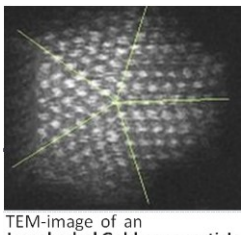


Fig. 10. Full sky map showing the optimal dodecahedral orientation for the ILC map with the kp2 mask, as for Fig. 5, centred on the North Galactic Pole



Brightness- and contrast-enhanced image of the **shadow of the M87 central black hole (EHT2017)** which **indicates a Dodecahedral structure** of the gravitational singularity



TEM-image of an
Icosahedral Gold nanoparticle

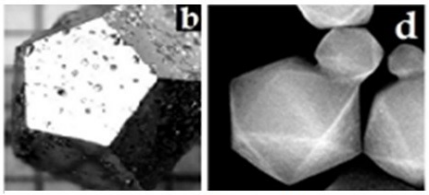


Fig 1 : **b.) Dodecahedral** Quasicrystal **Ho-Mg-Zn**
d.) Icosahedral Silica Quasicrystal (Nanocrystals)

The **Dodecahedron**, the **Icosahedron** and the other **Platonic Solids** seem to play a major role in the structure of our Universe and in the structure of matter. The cosmic microwave background (CMB) indicates that the comoving spatial 3-hypersurface of our Universe may be a Poincare Dodecahedral Space (PDS). The gravitational singularity in the center of the M87-galaxy (EHT2017) also indicates a dodecahedral structure. Atomic clusters that consist of inert atoms (e.g. Gold, Argon) with closed outer electron shells, and Quasi-Crystals often show an icosahedral- and dodecahedral structure. → This indicates that "matter" seems to be based on a "framework" which refers to the Platonic Solids ! The **Platonic Solids** can be described mathematically by only using constant **Phi** (**φ**) and **1** as fundamental units. The most irrational mathematical constant **Phi** (**φ**) appears directly in the geometry of the Dodecahedron and Icosahedron.

8.1 The important connection between Mathematics & Physics - The Mathematical Physical Triangle (MPT)

The following study regarding Special Relativity provided an **important algebraic term** for cathetus **u** of the **MPT**, which makes it possible to base Number Theory and Geometry (and Physics !) on a transparent construction out of **Number 1**, as **Albert Einstein** was hoping for ! :

➔ **„Phase spaces in Special Relativity : Towards eliminating gravitational singularities“** - by Peter Danenhower

See Weblink : https://arxiv.org/pdf/0706.2043.pdf

This study uses phase spaces in special relativity by expanding Minkowski Space to model the physical world.

The phase spaces developed in this study indicate that graviational singularities can be eliminated !

The key mathematical idea used in this study is the inclusion of a complex phase factor, such as, $e^{i\Phi}$ in the Lorentz transformation , and to use both the proper time and the proper mass as parameters. Additional a simple (invariant) parameter, the **“energy to length“ ratio**, defined by c^4/G was used for any spherical region of space-time-matter.

This study may show a way forward to combine General Relativity with Quantum Mechanics.

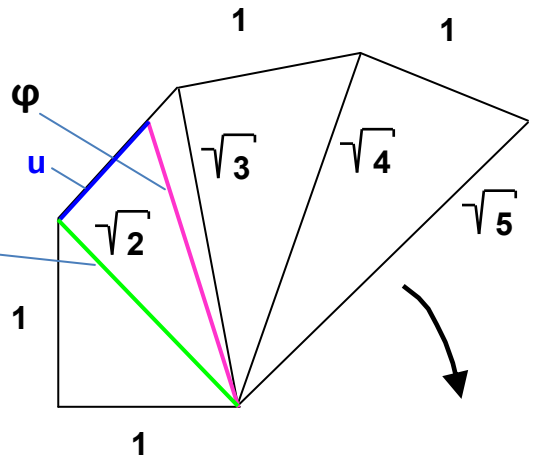
The Mathematical Physical Triangle and the start section of the Square Root Spiral, together with a more profound (number theoretical- and geometrical) analysis of the Square Root Spiral will help with this task :

The first right triangles of the Square Root Spiral, which are only defined by **constant φ and 1**, not only define the complex structure of the square root spiral, but also the Platonic Solids , and they also form the base of Number Theory, Geometry and Physics as well !

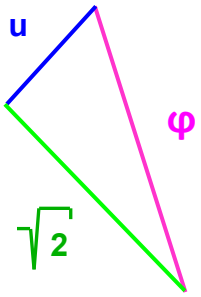
The start of the Square Root Spiral is shown with the constant φ drawn in :

$$\varphi = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+..}}}$$

$$\sqrt{2} = 1 + \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ddots}}}}.$$



The Mathematical Physical Triangle (MPT) :



With the help of the constants in this right triangle we will find an important starting point on the way to the universal theory :

From the right triangle φ , square root of 2 & u follows :

$\varphi^2 = (\sqrt{2})^2 + u^2$; application of the Pythagorean theorem

→ $u = \sqrt{\varphi^2 - 2} = 0{,}786151377.....$; we can calculate this value of **u** with the calculator

I did some research in the internet with Google, and I found a study where the constant u was expressed with an algebraic term ! With the help of this algebraic term it was possible to find interesting new properties of constant φ

→ **See next page !**

Here the abstract of the study where the algebraic term was found for constant U :

PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES

from PETER DANENHOWER → see weblink : **https://arxiv.org/pdf/0706.2043.pdf**

See also document : → space-time-matter

Abstract : This paper shows one way to construct phase spaces in special relativity by expanding Minkowski Space. These spaces appear to indicate that we can dispense with gravitational singularities. The key mathematical ideas in the present approach are to include a complex phase factor, such as, $e^{i\phi}$ in the Lorentz transformation and to use both the proper time and the proper mass as parameters. To develop the most general case, a complex parameter σ = s + im, is introduced, where s is the proper time, and m is the pro per mass, and σ and σ /|σ| are used to parameterize the position of a particle (or reference frame) in space-time-matter phase space. A new reference variable, u = m/r , is needed (in addition to velocity), and assumed to be bounded by 0 and $c^2/G = 1$, in geometrized units. Several results are derived: The equation $E = mc^2$ apparently needs to be modified to $E^2 = (s^2c^{10})/ G^2 + m^2c^4$, but a simpler (invariant) parameter is the "energy to length" ratio, which is c^4/G for any spherical region of space-time-matter. The generalized " momentum vector" becomes completely "masslike" for **u ≈ 0.7861…,** which we think indicates the existence of a maximal gravity field. Thus, gravitational singularities do not occur. Instead, as u → 1 matter is apparently simply crushed into free space. In the last section of this paper we attempt some further generalizations of the phase space ideas developed in this paper.

Extract from page 11 of the study (equation 4.9) : $$\hat{\mathbf{P}} = \frac{[(\sqrt{1-u^2}-u^2)+i(u\sqrt{1-u^2}+u)]}{\sqrt{1+u^2}}\gamma < 1, v >$$

In this form the real and imaginary part of **P** have a very interesting property, namely, if

(4.10) $u = \frac{\sqrt{2\sqrt{5}-2}}{2} \approx 0.786151377\ldots = u$, then the real part of **P** is zero, and the imaginary part takes its maximum value (= 1).

I think it makes sense to argue that when the real part of **P** = 0, **P** is entirely "mass like", which we could understand to be representative of the state of space-time-matter for which the maximal gravity field occurs. In this picture gravity is understood to be the propensity of space-time-matter to become completely mass like. The more mass-like a region of space-time-matter is, then the stronger the external gravity field. Thus, within the discussion of this paper, I think **the only reasonable interpretation of the existence of the special value of u given in equation 4.10 is that there is a maximal gravity field at this value of u.** It is important to observe that the **value of** u considered above, substantially exceeds **the value of** u **for a typical neutron star (≈ 0.1 – 0.2).**
Thus, I think the maximal gravity field concept can be used to explain all of the experimental evidence for enormous gravity fields.

→ **Now we can equate the two algebraic terms which represent the same constant ! :**

$$\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2} \quad ; \text{ we square both sides}$$

$$\rightarrow \quad 4\varphi^2 - 8 = 2\sqrt{5} - 2 \quad ; \text{ and transform}$$

$$\varphi^2 = \frac{\sqrt{5} + 3}{2} \quad ; (1) \text{ we solve for } \varphi^2$$

$$\sqrt{5} = 2\varphi^2 - 3 \quad ; (2) \text{ we solve for } \sqrt{5}$$

→ **Now we use the following right triangle :**

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \text{ Pythagorean theorem}$$

$$6 = (2\varphi^2 - 3)^2 + 1 \quad ; \text{ we replace } \sqrt{5} \text{ by } (2)$$

$$\rightarrow \quad 3 = \frac{\varphi^4 + 1}{\varphi^2} \; (3) \rightarrow \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \; (4)$$

→ square root 3 expressed by **φ** and **1** !

With the other right triangles of the square root spiral we can calculate all square roots of the natural numbers expressed only by φ and 1 : (see Appendix 2 of study !)

$$2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \; (5) \text{ and } \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \; (6)$$

$$\sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \rightarrow \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} \quad ; \quad (7)$$

$$6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \; (8) \text{ and } \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \; (9)$$

$$7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \rightarrow \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11)$$

$$8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \; (12) \quad \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \; (13)$$

$$10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \; (14) \quad \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \; (15)$$

$$11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \; (16) \quad \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \; (17)$$

$$12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \; (18) \quad \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \; (19)$$

Important for Mathematics & Physics : → The Mathematical Physical Triangle (MPT)

Albert Einstein was looking for **a universal theory based only on irrational constants like Pi (π) & e,** which are transparent constructions out of **number 1** (see **chapter 6**), and which all have values based on the logical base of the complete theory. **I.V.Volovich** said that **Albert Einstein tried to reduce all physics to algebraic geometry**. This means the reduction of physics to Number Theory (see **chapter 7**).

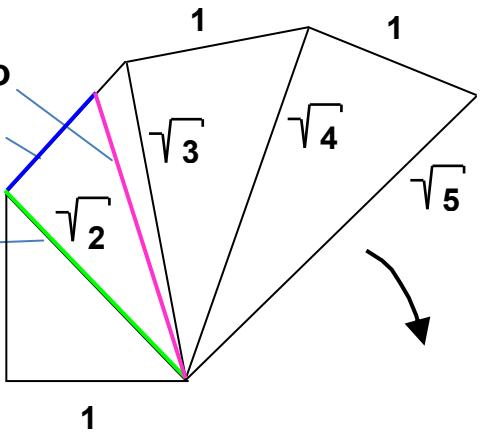
The Mathematical Physical Triangle (MPT) and the Square Root Spiral open the door to an Universal Theory !
With the algebraic term found by Peter Danenhower, all irrational square roots of natural numbers, constants like **Pi** & **e**, and Platonic Solids can be expressed with **constant φ** (Phi) and **1 ! →** all transparent constructions of 1 !

The start of the square root spiral is shown with the constant φ drawn in :

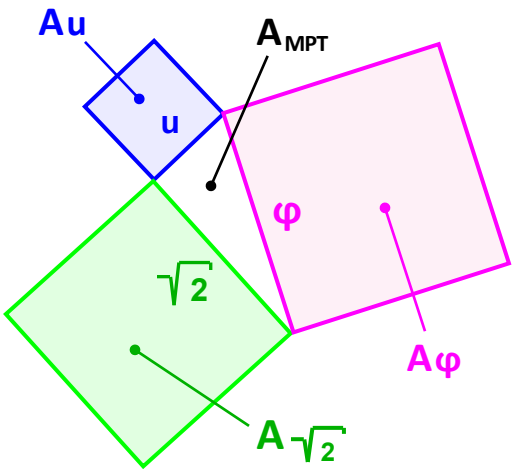
$$\varphi = \frac{1}{1+\frac{1}{1+\frac{1}{1+..}}}$$

$$\frac{1}{\sqrt{\varphi}} = u$$

$$\sqrt{2} = 1+\frac{1}{1+1+\frac{1}{1+1+\frac{1}{1+1+\frac{1}{1+1+\ddots}}}}$$



The Mathematical Physical Triangle (MPT) :



With this right triangle a simpler algebraic term for constant **u** can be calculated

$$\sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}}$$

$$\sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \quad ; \quad \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2}$$

$$A\varphi = \varphi^2$$

$$A_{\sqrt{2}} = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}$$

$$Au = \frac{\varphi^2 - 1}{\varphi^2} = \frac{\varphi}{\varphi^2}$$

Pythagorean theorem :

$$; \quad Au = A\varphi - A_{\sqrt{2}}$$

$$\rightarrow A_{MPT} = \frac{\sqrt{2} \times u}{2} = \frac{\sqrt{\varphi}}{\sqrt{\varphi^4 - \varphi^2 + 1}}$$

$$\rightarrow u = \sqrt{\frac{\varphi}{\varphi^2}} = \frac{1}{\sqrt{\varphi}} = \frac{\sqrt{2\sqrt{5} - 2}}{2} = 0{,}786151377\ldots$$

From the above shown equations (→ see last two pages) I have realized a general rule for all natural numbers > 10 :

Note : → The expression (3+n) in the rule can be replaced by products and / or sums of the equations (3) to (13)

$$\rightarrow (10+n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \text{ and } \sqrt{(10+n)} = \sqrt{\frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4}} \quad (30)$$

For $n \rightarrow \infty$

With this general formula we can express all natural numbers ≥ 10 and their square roots only with φ and 1 !

This statement is also valid for all rationals (fractions) and their square roots. This is a quite interesting discovery !!

8.2 The base of an Universal Mathematical- & Physical Theory as considered by Albert Einstein

Constant **PhI** (**φ**) seems to be **the fundamental constant that defines the scattering** (distribution) **of particles and waves in the universe**, and the **base unit** (number) **1** seems to represent **a base energy/wave element** like the **quantum of electromagnetic action** → the **Planck Constant h** that is **defined by Energy / Frequency**
The **Mathematical Physical Triangle (MPT)** and the **Square Root Spiral (SRS)** represent a starting point for the **Universal Mathematical- & Physical Theory**. To understand the meaning of the **MPT** & **SRS** for Physics we must understand the physical meaning of the most irrational constant **PhI** (**φ**) and the meaning of the **base unit 1** !

Please note that the **MPT**, the **SRS** and in all probability all existing **irrational constants** can be represented by transparent constructions out of **base unit** (**number**) **1** and the most **simple infinite continued fraction** : **Constant PhI (φ)** that is only **based on Number 1** which essentially is the set-up for a universal theory **Albert Einstein** was looking for !

$$\varphi = \frac{1}{1+\frac{1}{1+\frac{1}{1+..}}}$$

In Physics the socalled **Continued Fraction Method (MCF)** was developed for solutions of integral equations of the **Quantum Scattering Theory**, which describe particle-collisions and –scattering of particles and waves. For example the **MCF** is used for the **Fermi-Dirac Function**, which describes the macroscopic properties of a system consisting of **Fermions** (→ particles which form matter : quarks, leptons and baryons : (e.g. electrons, protons). → **see weblinks to some exemplary studies which use the MCF :**

Continued fraction representation of the Fermi Dirac function for large scale electronic structure calculations
Application of matrix valued integral continued fractions to the spectral problems on periodic graphs (see also : **Algebra of waves**)
The MCF for electron (positron)-atom scattering and **The MCF in the theory of slow electron scattering by molecules**

On the other hand there is clear indication that constant **Phi** (**φ**) also defines the distribution of matter in the astronomical-scale ! For example the **ratios of orbital periods of Solar-planetary and Exo-planetary systems show a preference for Fibonacci-Number ratios, which are defined by constant PhI** (**φ**). → **See the Study 5 in Chapter 8.5.** And if the **M87 gravitational singularity** (**EHT2017**) indeed **indicates a dodecahedral structure** then we even have a proof that **PhI** (**φ**) **defines the distribution of matter in very extreme gravitational fields.**

The Volume of the Dodecahedron and Icosahedron expressed only with constant φ and 1 !

$$V_{Dod.} = \frac{a^3}{4}\,(15 + 7\sqrt{5}) \rightarrow V_{Dod.} = \frac{\varphi^4\,(\varphi^4 - 1)}{\varphi^4 - \varphi^2 + 1}$$; for **a** = **1** (edge)

$$V_{Ico.} = \frac{5a^3}{12}\,(3 + \sqrt{5}) \rightarrow V_{Ico.} = \frac{\varphi^2\,(\varphi^4 - 1)^2}{(\varphi^4 - \varphi^2 + 1)(\varphi^4 + 1)}$$

Dodecahedral and Icosahedral Quasicrystals

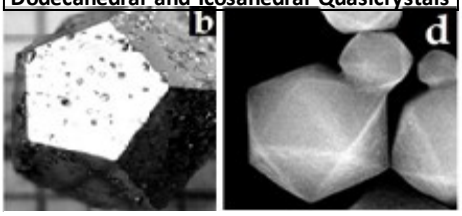
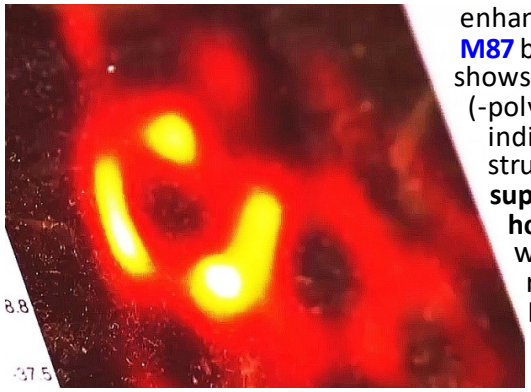


Fig 1 : b.) Dodecahedral Quasicrystal **Ho-Mg-Zn**
d.) Icosahedral Silica Quasicrystal (Nanocrystals)

→ **Three possible ways to find the Universal Theory and uncover the true geometry of our Universe :**

1.) Through advances in **Number Theory**. → By applying a **new mathematics based only on Phi and 1**
2.) By fully understanding **the Physics & Mathematics of Quasicrystals , Atom-Clusters & Atomic-cores**
3.) By **high-resolution astronomical observations of gravitational singularities**, like the one in **M87**

The M87 black hole has a complex (dodecahedral) **geometry**

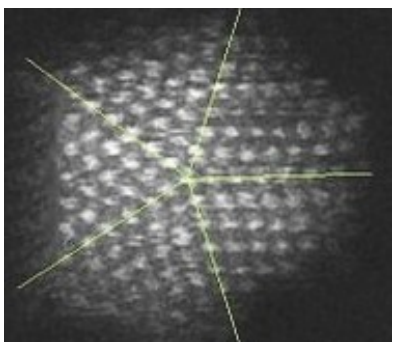


A contrast- & brightness enhanced image of the **M87** black hole shadow shows a polypentagonal (-polygonal) structure, indicating a complex structure of the **M87 super-massive black hole** (**EHT2017**), which may be the result of an O5-**Poincare-Dodecahedral Space** universe.

This image is from the section of the documentation **Black Hole Hunters** which shows how the algorithms calculate the first image of the M87 black hole shadow. **Note :** only the left (bright) ring was presented to the public ! The rest of the structure was ignored !

Icosahedral Atom-Clusters indicate an icosahedral structure of the atom's electron shell & atomic-core

Elements which are chemically inert often form **clusters** with an **icosahedral** shape.The shell structure of such **Icosahedral clusters** is defined by the **electron configuration** of the whole cluster, that is a consequence of the electron shell of the single atom, which again is **a result of the atom-core-structure** ! Image: **MacKay cluster** made of **Gold atoms**.



TEM-image of an **Icosahedral Gold nanoparticle**. A variety of nano-structures assume icosahedral form (e.g. condensing **Argon- & metal atoms**) → see : **Icosahedral Twins** (→ see also : **Superatom**)

8.3 Quasicrystals will help us to find the true geometry & dimensionality of space-time & matter

Here on Earth we can find out the true geometry and dimensionality of space-time and matter by fully understanding Quasicrystals and quasicrystalline structures from the mathematical – and physical point of view. **Quasicrystals indicate that space-time and/or matter (energy) is based on higher geometrical dimensions of at least 4 maybe even 5 or 6 !**

According to the classical Crystallographic Restriction Theorem crystals can only possess two-, three-, four- and six-fold rotational symmetries. However in 1984 **Quasicrystals with five-fold symmetry** were discovered !

Two types of Quasicrystals exist : 1.) Polygonal Quasicrystals which have one axis of 8-, 10- or 12-fold local symmetry.
2.) Icosahedral Quasicrystals. These Quasicrystals are aperiodic in all directions.

Similar to atomic clusters (→ previous page) the electron shell structure (electron configuration) of the whole quasicrystal defines ist structure → **see : Scanning tunnelling microscopy reveals a quasiperiodic order in the electronic wave functions**

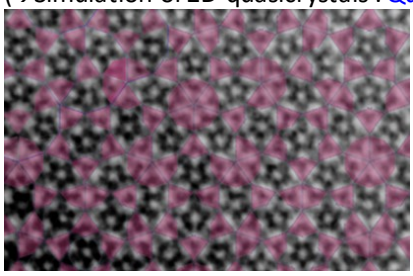
For Quasicrystals **at least 5** linearly independent vectors are necessary in order to assign integer indices to the diffraction intensities of quasicrystals. We need **5 indices for polygonal quasicrystals** and **6 indices for icosahedral quasicrystals.**

The necessary **n** vectors span a **nD-reciprocal space** in which a structure can be built that produces a diffraction pattern as observed for quasicrystals. **In higher-dimensional space we can describe a quasiperiodic structure as a periodic one.**

The mathematics which describes quasicrystals indicates that matter (energy) has higher embedded dimensions !

Nicolaas G. de Brujin showed that **aperiodic quasicrystal-like Penrose Tilings** can be viewed as **2-dimensional slices of five-dimensional hypercubic structures**. The study of Penrose Tilings is important for understanding Quasicrystals.

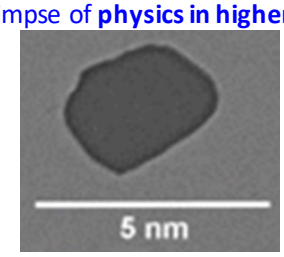
(→Simulation of 2D-quasicrystals : Quasicrystalline Bose-Einstein-Condensate provides a glimpse of **physics in higher dimensions**)



Electron Microscope image of a real Decagonal Quasicrystal (Weblink)

Interesting is the fact that **the production of Quasicrystals** in the lab **is difficult and tricky**, **requiring precise temperatures** and **strange conditions** including a **vacuum** and an **Argon atmosphere**, see Weblink 4 (→ **Argon** is an inert noble gas that forms Icosahedral clusters during condensation)

Natural (mineral) quasicrystals only seem to form under extreme high-pressure and –temperature conditions → see right image →

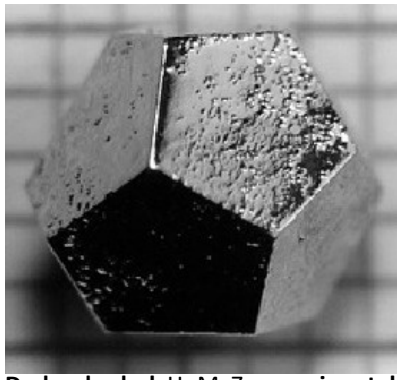


→ see :
Weblink 1
Weblink 2
Weblink 3
Weblink 4

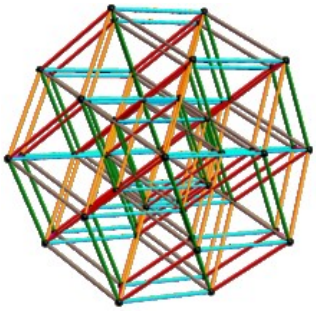
Extraterrestrial Quasicrystal
A grain of stishovite that only occurs at **ultrahigh pressures** (**≥10 Gpa**), contains a **Icosahedrite Quasicrystal** inclusion **Al63Cu24Fe13**

Three studies about the Mathematics of Quasicrystals:

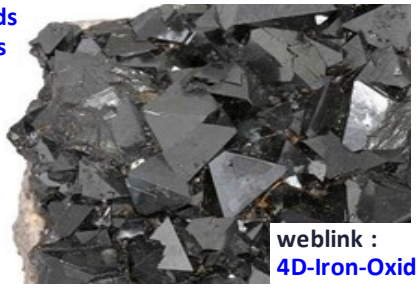
1 - The Noncommutative Geometry of Aperiodic Solids
2 - Crystallography of Quasicrystals **3 - Quasicrystals**



Dodecahedral HoMgZn **quasicrystal**
A **Quasicrystal** is a structure that is ordered but not periodic, it lacks translational symmetry.
In a simulation scientists showed that the golden ratio (Phi) governs the interaction of the atoms. **See : The World's most complex Crystal**

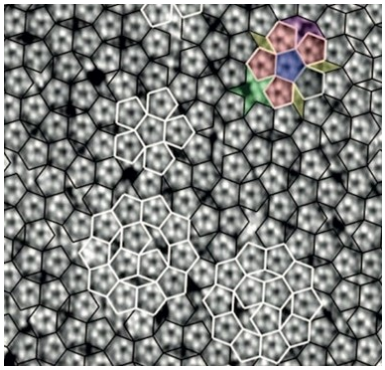


A **6-cube** projected into the rhombic triacontahedron using the golden ratio in the basis vectors. **This is used to understand the aperiodic icosahedral structure of quasicrystals**

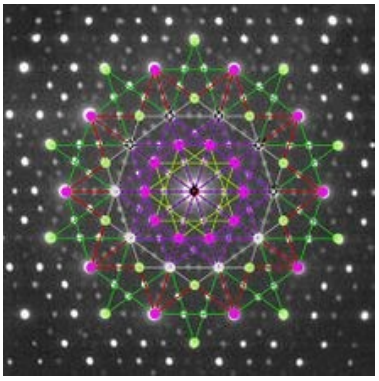


weblink : 4D-Iron-Oxid

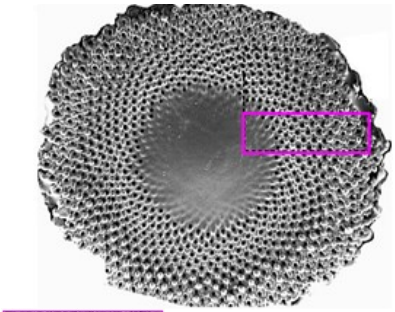
Four-Dimensional Iron-Oxid Fe_4O_5
below 150K this iron-oxid goes through an unusual phase transition related to a formation of charge-density waves—which lead to a **"four-dimensional crytal structure"**
It formed at very high temperatures and **very high pressure** hundreds of kilometres below Earth's surface.



A special **Two-dimensional quasi-crystal** made from **self-assembling organic molecules** which form pentagons, stars, boats and rhombi
→ **see : Bizarre organic Quasicrystal**



A **5-cube** as an orthographic projection into 2D using Petrie polygon basis vectors overlaid on the diffractogram from an icosahedral Ho-Mg-Zn quasicrystal



Sunflower capitulum : The **Fibonacci number spirals** (**Phyllotactic-pattern**) indicate an icosahedral quasicrystal structure, probably caused by the large icosahedral **Water-Cluster** $(H_2O)_{100}$ or $(H_2O)_{280}$ → **see my following Study**

8.4 A new calculable model of the atomic nucleus based on the Platonic Solids and on constant φ

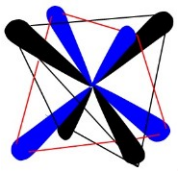
I want to introduce a new model for the **atomic nucleus** that can explain **magic numbers** in nuclear physics : (**2, 8, 20, 28, 50, 82, 126**..) easily with **a framework based on the Platonic Solids, and on constant Phi (φ).**

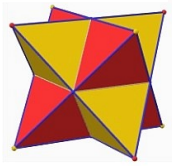
1.) In this model **2 Protons or 2 Neutrons are represented by one orbital similar to the p-orbital of Hydrogen**
2.) The **diagonals** of the Platonic Solids represent & **define the long axes of these Proton- & Neutron-Orbitals**
3.) **Each Platonic Solid framework represents a Sub-Shell of the nuclear shell model →** (more info see **here**)
Similar to the Platonic Solids these Sub-Shells can precisely be inscribed and/or superimposed to each other
4.) All Proton- & Neutron-Orbitals **can be precisely calculated** with defined **wave equations** (Schrödinger WE)
5.) **The defined nucleus orbitals influence the electron orbitals** and cause in this way the **Hyperfine-Structure**
6.) Each Sub-Shell can consist either of protons or neutrons or of a combination of both → **see examples** below

2 Protons / 2 Neutrons

one base orbital unit

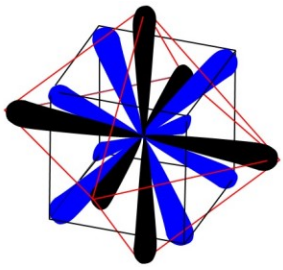
4 Protons / 4 Neutrons
(or **4 Protons** & **4 Neutrons** (for **2T**)

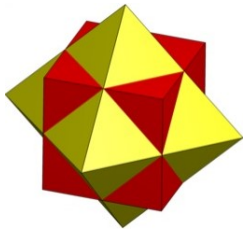




double Tetrahedron

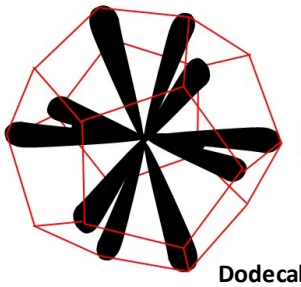
6 or **8** **Protons / Neutrons**
(or **6 Protons** & **8 Neutrons**)

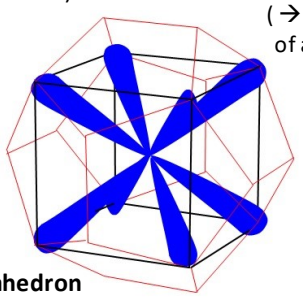




Hexahedron & Octahedron

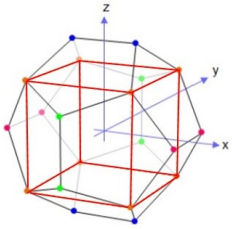
12 or **20** **Protons / Neutrons**
(or **12 Protons** & **20 Neutrons**)

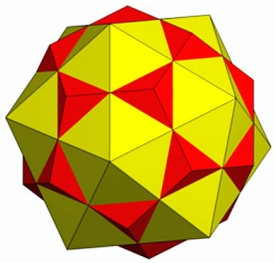




Dodecahedron

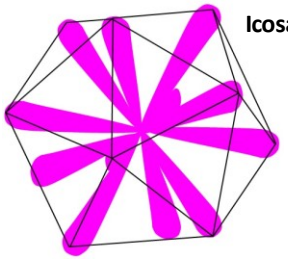
Note : **8 of 20** Dodecahedron-vertices can form a Hexahedron (→ 5 different orientations of a Hexahedron possible)



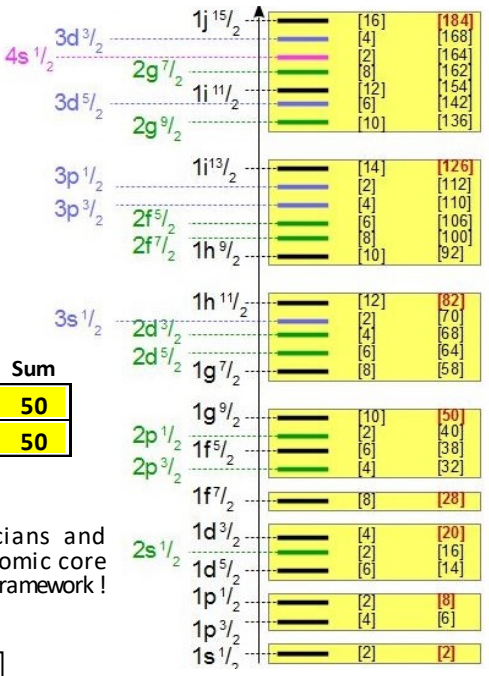


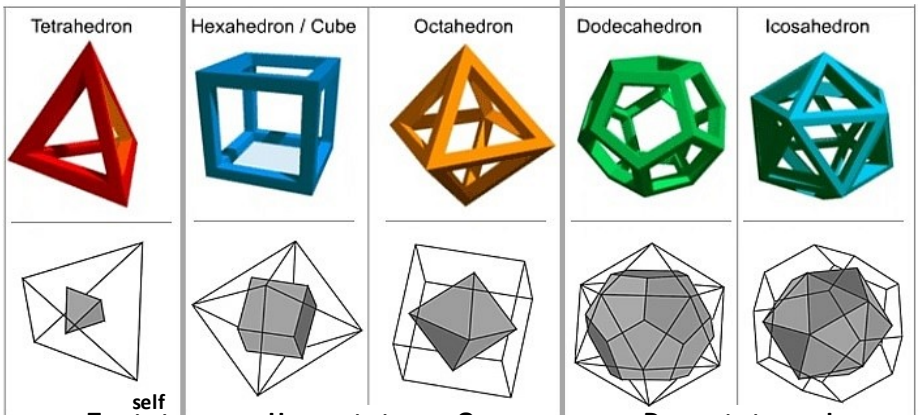
Dodecahedron & Icosahedron

Icosahedron



Nuclear Shell Model (→ Link to **Table**)





Name :	T	self dual	H	dual	O	D	dual	I	Sum
Faces	4		6		8	12		20	50
Vertices	4		8		6	20		12	50

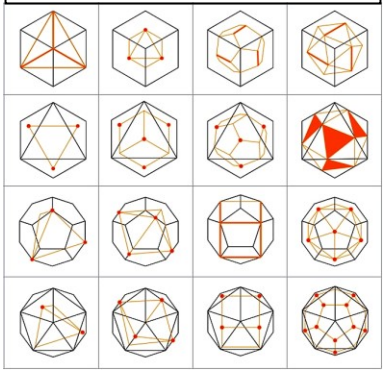
14 28 32

Magic Number System :

8 = 2 + 6 or 4 + 4
20 = 2 + 4 + (6+8) or 2 + 6 + 12
28 = 2 + 6 + 8 + 12 or 2 + 4 + 4 + 6 + 12 **or** 8 + 20
50 = 4 + (6+8) + (12+20) or 2 + 6 + 8 + 6 + 8 + 12
82 = 4 + (6+8) + (12+20) + (12 + 20) or 2 + 6 + 8 + 6 + 8 + 12 + 20 + 12
126 = **82** + 44 → **44** = 12 + 20 + 12 or **28** + (2 + 6 + 8)
184 = 126 + 58 → **58** = 6 + 8 + 12 + 20 + 12 or **44** + (6+8)
196 = **184** + 12 **or** **184** + 2 + 4 + 6
114 = **50** + 64 → **64** = 20 + 12 + 20 + 12 (→only for Neutrons)

I want to invite eager mathematicians and physicists to calculate such an atomic core model based on a Platonic-Solids-framework !

Combinations of Platonic Solids



Area & Volume of Platonic Solids if the edge length is 1 (→ Numbers replacable by φ & 1 !)

solid	A	V
cube	1	1
dodecahedron	$\frac{1}{4}\sqrt{25+10\sqrt{5}}$	$\frac{1}{4}(15+7\sqrt{5})$
icosahedron	$\frac{1}{4}\sqrt{3}$	$\frac{5}{12}(3+\sqrt{5})$
octahedron	$\frac{1}{4}\sqrt{3}$	$\frac{1}{3}\sqrt{2}$
tetrahedron	$\frac{1}{4}\sqrt{3}$	$\frac{1}{12}\sqrt{2}$

As **doubt must be cast on the theory of Quarks** (see **Link1**, **Llink2**) an orbital-model is prefered.
In the shown model **closed outer Sub-Shells have an Icosahedral- / Dodecahedral-shell-structure.**
This model can explain **dodecahedral gravitational singularities as EHT2017 which must be a result of a Dodecahedral/Icosahedral atomic core structure**
Note : All Platonic Solids are only based on **φ** & **1** !

8.5 - The prime number distribution follows a clear Logic

→ <u>see study</u> **"About the logic of the prime number distribution" :** https://arxiv.org/abs/0801.4049

I will focus on the description of this **„Wave Model" as explanation for the distribution of primes and non-primes :**

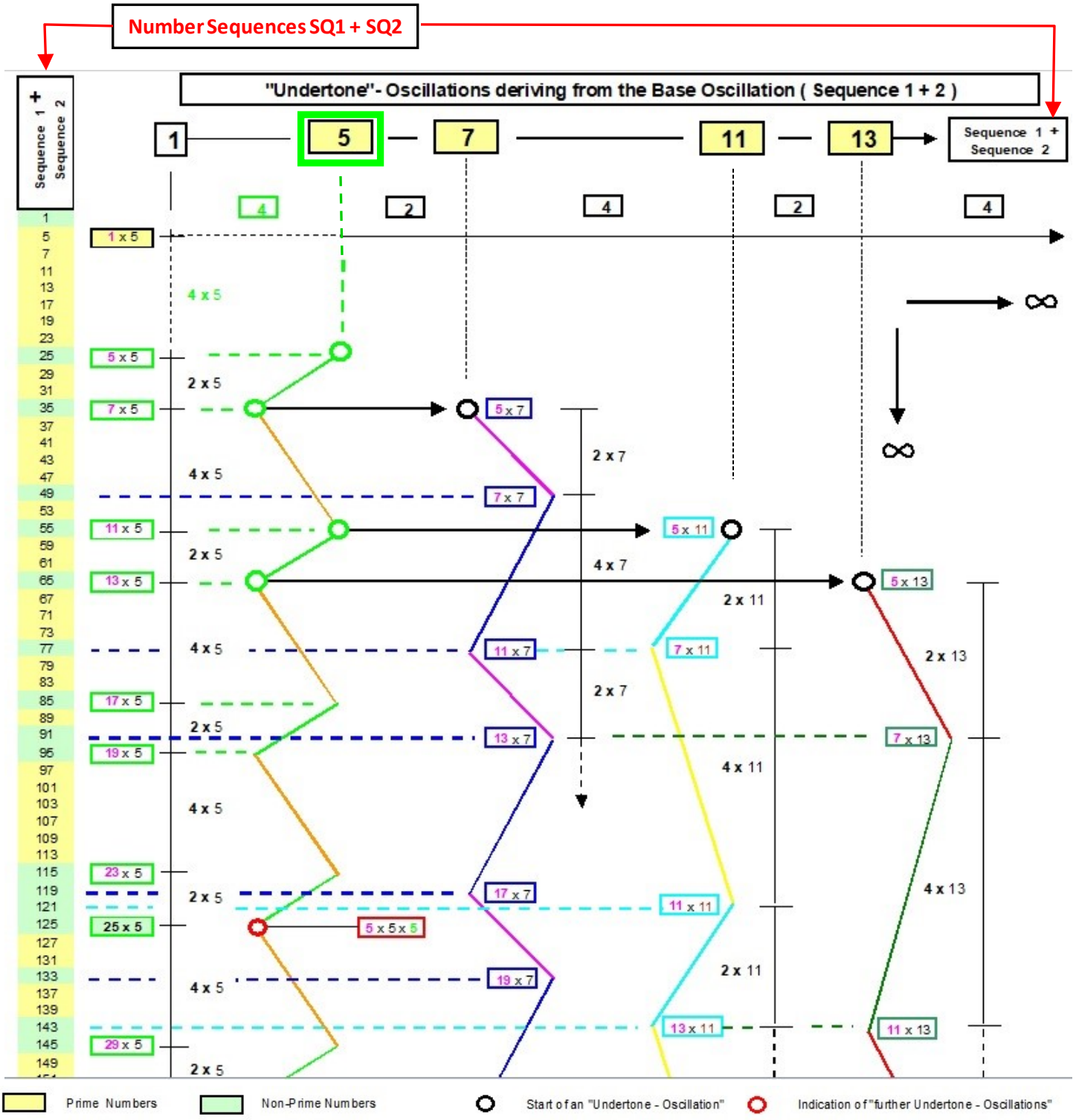
➔ **The distribution of all non-prime numbers in the number sequences SQ1 and SQ2 is defind by a simple logic :**

We consider the following two number sequences which contain all prime numbers :

Sequence 1 (SQ1) : 5, 11, 17, 23, 29, **35**, 41, 47,.... and

Sequence 2 (SQ2) : 1, 7, 13, 19, **25**, 31, 37, 43,....

These two sequences are based on the well known fact, that every prime number is either of the form 6n + 1 or 6n + 5. Or in other words, if a prime number of these two number sequences is divided by 6 the rest of -1 or +1 remains.



Here a short description of the simple rules which define the distribution of the non-prime numbers :

- **Every peak of an Undertone-Oscillation corresponds to a non-prime number in Sequence 1 & 2 (SQ1 & SQ2)**

- **On the contrary "Prime Numbers" represent places in Sequence 1 & 2 which do not correspond with any peak of an Undertone-Oscillation.**

  Prime numbers represent "spots" in the two basic Number-Sequences SQ1 & SQ2 where there is no interference caused by the Undertone Oscillations shown on the righthand side.

- **In every Undertone Oscillation "further Undertone Oscillations" occur, which again are defined by the numbers contained in Sequence 1 & 2.**

  However these "further Undertone Oscillations" are not required to explain the existence of the non prime numbers in Sequence 1 & 2, because the non prime numbers in these sequences are already explained by the undertone oscillations which directly derive from Sequence 1 & 2.

  (→ "further Undertone Oscillations" are marked by red circles on the corresponding peaks of the Undertone Oscillations. Prime factor products of the numbers which belong to these peaks are shown in red and pink boxes)

  Example : The numbers 125, 175, 275 and 325 in the Undertone Oscillation **5** (=1/5f), represent the prime factor products **5**x5x**5**, **5**x5x**7**, **5**x5x**11** and **5**x5x**13.** It is easy to see that these prime factor products form another Undertone Oscillation **5** inside the Undertone Oscillation **5** !!

The following properties are important, because they show the importance of number 5 !! :

- **On every peak of the Undertone Oscillation 5 (=1/5 f) another Undertone Oscillation starts.**
  Undertone Oscillation 5 is the cause (trigger) of all other Undertone Oscillations !
  The green circles on the first few peaks of the Undertone Oscillation **5** mark the starting points of the next 3 Undertone Oscillations **7, 11** and **13** (= 1/**7**f, 1/**11**f and 1/**13**f). More of such Undertone Oscillations will start on every peak of the Undertone Oscillation **5** ad infinitum.

  Note that Undertone Oscillations which are defined by non-prime numbers (e.g. 1/**25**f or 1/**35**f etc.) are not required to explain the non-prime numbers in Sequence 1 & 2 !!

- **If we consider Sequence 1 and 2 (SQ1 & SQ2) simultaneously then it applies that new prime factors at first only occur together with the prime factor 5 !!!**

  These are very important properties ! It shows that the number 5 oscillation is defining the distribution of all non-prime numbers in the Number Sequences 1 + 2. Mathematicians doesn't seem to know this properties !

For the distribution of the prime numbers the following simple rule applies in the „Wave Model" :

- **Every peak of an Undertone-Oscillation corresponds to a non-prime number in Sequence 1 & 2 (SQ1 & SQ2)**

  Non-Prime Numbers

- **On the contrary "Prime Numbers" represent places in Sequence 1 & 2 which do not correspond with any peak of the clearly (by simple rules) definded Undertone-Oscillation.**

  Prime Numbers

The above described simple rules (properties) clearly define the distribution of all prime numbers !

From these rules the following simple definitions for the groups of primes and non-primes can be derived :

→ **See next page :**

General description of this "Wave Model" and the prime number distribution :

Definition of **Sequence 1 & 2** (Base oscillation with frequency **f**) in mathematical terms :

SQ1 (Sequence 1) : $\mathbf{a_n = 5 + 6n}$ for example $a_0 = 5$; $a_1 = 11$; $a_2 = 17$ etc.

SQ2 (Sequence 2) : $\mathbf{b_n = 1 + 6n}$ for example $b_0 = 1$; $b_1 = 7$; $b_2 = 13$ etc.

with $n \in N = \{ 0, 1, 2, 3, 4, \ldots \}$

Description of the "**Undertone Oscillation 5**" (= 1/**5** f) :

→ undertone oscillation **5** is split into two number sequences **U-5$_1$** and **U-5$_2$** :

U-5$_1$: $\mathbf{a(5)_n = 5(5 + 6n)}$ for example $a_0 = 25$; $a_1 = 55$; $a_2 = 85$ etc.

U-5$_2$: $\mathbf{b(5)_n = 5(1 + 6n)}$ for example $b_1 = 35$; $b_2 = 65$; $b_3 = 95$ etc.

with $n \in N = \{ 0, 1, 2, 3, 4, \ldots \}$ for **U-5$_1$** and with $n \in N^* = N \setminus \{0\} = \{ 1, 2, 3, 4, \ldots \}$ for **U-5$_2$**

General description of all "**Undertone Oscillations X**" (= 1/**X** f) :

→ every undertone oscillation is split into two number sequences **U-(x)$_1$** and **U-(x)$_2$** :

U-(x)$_1$: $\mathbf{a(x)_n = x(5 + 6n)}$ with $n \in N = \{ 0, 1, 2, 3, 4, \ldots \}$

U-(x)$_2$: $\mathbf{b(x)_n = x(1 + 6n)}$ with $n \in N^* = N \setminus \{0\} = \{ 1, 2, 3, 4, \ldots \}$

and with $X \in (SQ1 \cup SQ2) \setminus \{1\} = \{ 5, 7, 11, 13, 17, 19, 23, 25 \ldots \}$ for both sequences $a(x)_n$ & $b(x)_n$

According to the above described definitions the set of prime numbers (PN) can be defined as follows :

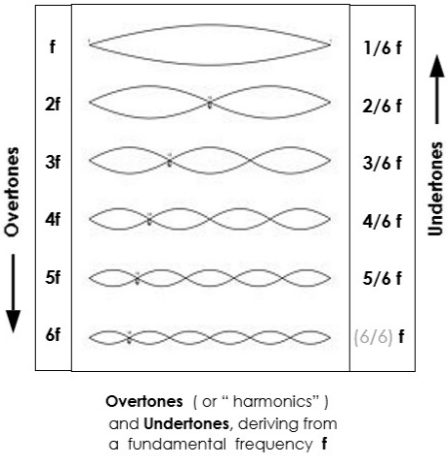
$$\mathbf{PN^* = (SQ1 \cap SQ2) \setminus (U\text{-}(x)_1 \cap U\text{-}(x)_2)}$$

$$\mathbf{PN = \{ 2, 3 \} \cap (SQ1 \cap SQ2) \setminus (U\text{-}(x)_1 \cap U\text{-}(x)_2)}$$

for **PN*** and **PN** the following definition applies :

PN = { 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...} ; **set of prime numbers**

and **PN*** = { 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...} = **PN \ { 2, 3 }**



Overtones (or " harmonics") and **Undertones**, deriving from a fundamental frequency **f**

further the following definitions applies :

$\mathbf{PN^* \subset (SQ1 \cap SQ2)}$ or $\mathbf{PN^* \subset (a_n \cap b_n)}$

$\mathbf{PN^* \not\subset (U\text{-}(x)_1 \cap U\text{-}(x)_2)}$ or $\mathbf{PN^* \not\subset (a(x)_n \cap b(x)_n)}$ with $\mathbf{x} \in (SQ1 \cup SQ2) \setminus \{1\}$

$\mathbf{NPN^* = (U\text{-}(x)_1 \cap U\text{-}(x)_2)}$ or $\mathbf{NPN^* = (a(x)_n \cap b(x)_n)}$; **NPN*** = non-prime-numbers divisible by 2 or 3

8.6 - There are three fundamental Number Oscillations :

The "Wave Model" for the logic of the prime number distribution described on the previous pages is **based on all natural numbers not divisible by 2 or 3.**
But there is the rest of the natural numbers which are divisible by 2 or (and) 3, which also have to be taken in consideration !

These numbers represent two more fundamental oscillations which exist parallel to the "base oscillation SQ1 + SQ2" described in Table 2. And the same physical principle of the creation of "Undertones" would of course also apply to **these two additional fundamental oscillations**, whose highest mode frequencies **could be named f2 and f3.**

Accordingly the highest mode frequency of **the fundamental oscillation SQ1 + SQ2 would then be named f1**.

The image on the righthand side (FIG. 7) shall give an idea of the coexistence of the described „Three fundamental Number Oscillations".

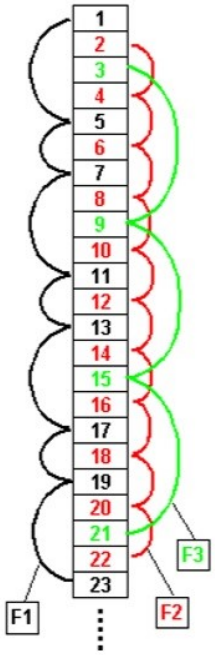


FIG. 7 : There are three fundamental oscillations

8.7 The Platonic Solids Icosahedron & Dodecahedron seem to play an important role in the Universal Theory

The Platonic Solids **Icosahedron** & **Dodecahedron** are closely related to **constant Phi (φ)**. The Icosahedron plays an important role in the structure of extremely stable atomic- and molecular- clusters. And together with the Dodecahedron the Icosahedron can **describe quasi-crystalline structures with five-fold symmetry**.

The Dodecahedron seems to play an important role in the large scale structure of the universe, especially in the formation of very strong gravitational singularities (supermassive black holes) like the one in **M87**.

The **Icosahedron** & **Dodecahedron** form Quasicrystals that can only be described in „higher-dimensional" space Therefore these Platonic Solids must play a crucial role in unifying Quantum Mechanics with General Relativity

To the Icosahedron :

An **Icosahedron** is one of the five **Platonic Solids**

It has 30 edges and 12 vertices, and 20 faces which are equilateral triangles. It has the Schläfli symbol {3,5}

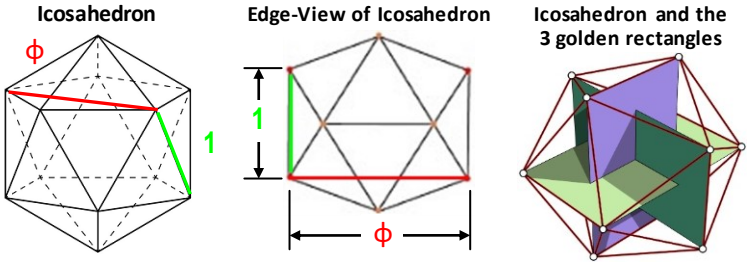


Fig 1 : shows how the **Golden Ratio** (→ **the ratio of constant Phi (ϕ) and 1**) is built into the Geometry of the **Icosahedron**

Formulas for the Icosahedron in reference to the sphere :

If the edge length of the Icosahedron is ***a***
the radius of a circumscribed sphere **r_u** is : $r_u = \frac{a}{4} \cdot \sqrt{10 + 2 \cdot \sqrt{5}}$

and the radius of an inscribed sphere **r_i** is : $r_i = \frac{a}{12} \cdot \sqrt{3} \cdot (3 + \sqrt{5})$

while the mid-radius **r_k** (mid of edges) is : $r_k = \frac{a}{4} \cdot (1 + \sqrt{5})$

Dodecahedral & Icosahedral Quasicrystals :

In the higher-dimensional space we can describe a quasi-periodic structure as a periodic one. The actual quasiperiodic structure in the 3D-physical space can then be obtained by appropriate projection/section techniques. Thus it is enough to define a single unit cell of the nD-structure. The contents of that nD-unit cell consists of "hyperatoms" (occupation domains, ..) in analogy to the atoms in a normal unit cell. This enables us to describe the whole quasicrystal structure with a finite set of parameters. If we described it in 3D-space only, we needed thousands of atoms to obtain a representative volume segment of the whole structure as well as all parameters that go with it (eg. thousands of positions). → see also : **weblink**

→ **Image Source**

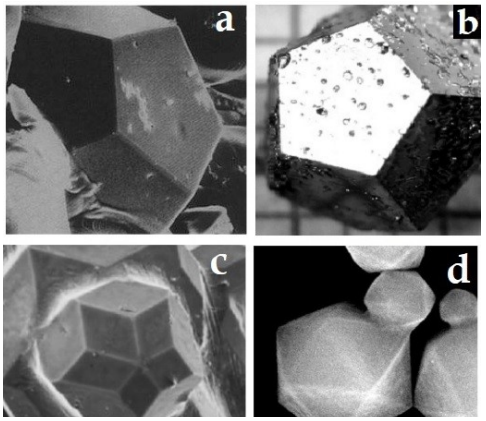


Fig 2 : **a.)** Dodecahedral Quasicrystal **Al65Cu20Fe15**
b.) Dodecahedral Quasicrystal **Ho-Mg-Zn**
c.) Quasicrystal **Al-Cu-Fe** with Triacontahedron shape
d.) Icosahedral **Silica** Quasicrystal (Nanocrystals)

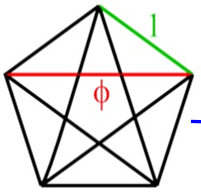
To the Dodecahedron :

A **regular** **Dodecahedron** or **pentagonal dodecahedron** is a dodecahedron that is regular, which is composed of twelve regular pentagonal faces (**Pentagons**), three meeting at each vertex.

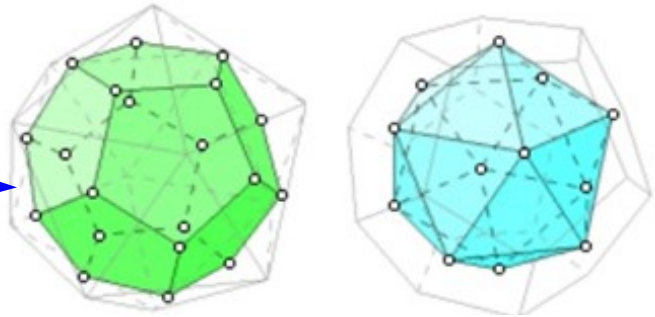
It is one of the five **Platonic Solids.** And it is obviously a very important one ! , as the M87 gravitational singularity indicates !

It has 12 faces, 20 vertices, 30 edges, and 160 diagonals (60 face diagonals, 100 space diagonals). It is represented by the Schläfli symbol {5,3}.

Pentagon



Dodecahedron



Phi (**ϕ**) and **1** are built into the Geometry of the Pentagon and the Dodecahedron

The **Dodecahedron** is the **dual** of the **Icosahedron** Connecting the centers of adjacent faces of the dodecahedron results in an icosahedron, and connecting the centers of the icosahedron faces results in a dodecahedron.

Formulas for the Dodecahedron :

If the edge length of a regular dodecahedron is ***a*** , the radius of a circumscribed sphere r_u (one that touches the regular dodecahedron at all vertices) is :

and the radius of an inscribed sphere r_i (tangent to each of the regular dodecahedron's faces) is :

while the midradius r_m, which touches the middle of each edge, is :

Weblink to more formulas of the dodecahedron **see :** polyhedra_dodecahedron

$$r_u = a\frac{\sqrt{3}}{4}\left(1+\sqrt{5}\right) \rightarrow r_u = a\frac{\sqrt{3}}{2}\phi$$

$$r_i = a\frac{1}{2}\sqrt{\frac{5}{2}+\frac{11}{10}\sqrt{5}} \rightarrow r_i = a\frac{\phi^2}{2\sqrt{3-\phi}}$$

$$r_m = a\frac{1}{4}\left(3+\sqrt{5}\right) \rightarrow r_m = a\frac{\phi^2}{2}$$

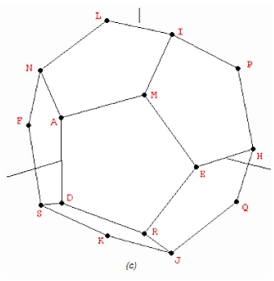
These quantities can also be expressed as :

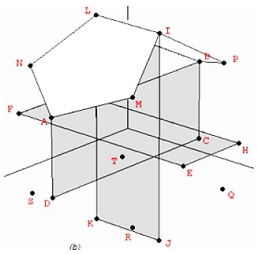
where ϕ is the golden ratio.

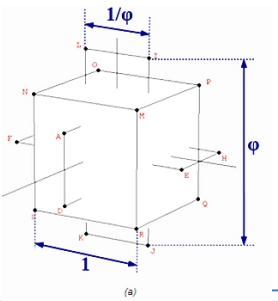
Please have a look at the following websides
→ The golden ratio Phi (φ) in Platonic Solids : → Phi_sacred_Solids

The Dodecahedron in cartesian coordinates :

The vertices of the dodecahedron obtained from the **cube** and **three orthogonal Golden Rectangles** with the side relationship **1 / φ²** (= 2/φ : 2φ)







A	φ,	0,	1/φ
B	-φ,	0,	1/φ
C	-φ,	0,	-1/φ
D	φ,	0,	-1/φ
E	1/φ,	φ,	0
F	1/φ,	-φ,	0
G	-1/φ,	-φ,	0
H	-1/φ,	φ,	0
I	0,	1/φ,	φ
J	0,	1/φ,	-φ
K	0,	-1/φ,	-φ
L	0,	-1/φ,	φ

M	1,	1,	1
N	1,	-1,	1
O	-1,	-1,	1
P	-1,	1,	1
Q	-1,	1,	-1
R	1,	1,	-1
S	1,	-1,	-1
T	-1,	-1,	-1

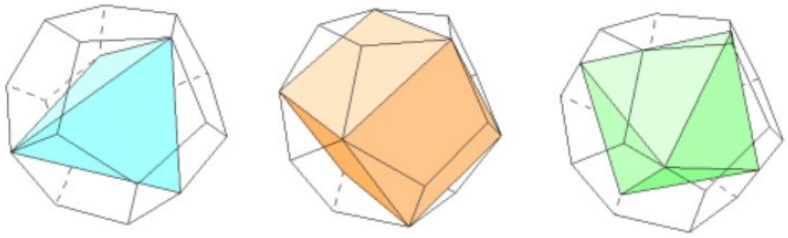
A_1	0,	φ²,	φ,
B_1	0,	φ²,	-φ,
C_1	0,	-φ²,	-φ,
D_1	0,	-φ²,	φ,
E_1	φ²,	φ	0
F_1	φ²,	-φ	0
G_1	-φ²,	-φ	0
H_1	-φ²,	φ	0

I_1	φ,	0,	φ²
J_1	-φ,	0,	φ²
K_1	-φ,	0,	-φ²
L_1	φ,	0,	-φ²

Table 4: The cartesian coordinates of the small stellated dodecahedron. The vertices of the original dodecahedron (Table 3), though visible, are not vertices of the stellation.

The Dodecahedron has geometric relations to the other four Platonic Solids (see also image above → dual of the Icosahedron) :

By connecting selected vertices of the dodecahedron, it is possible to form a **Tetrahedron** or a **Cube.** By connecting midpoints of certain edges, it is possible to form an **Octahedron :**



The small stellated Dodecahedron contains three powers of φ

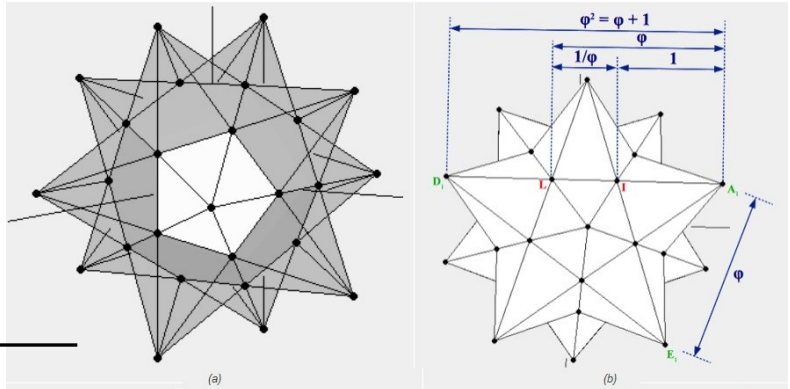


Figure 6: The small stellated dodecahedron contains three powers of the Golden Ratio. This can be clearly appreciated in its pentagram faces. Its outer vertices define an icosahedron whose edge length (A_1E_1) is in a proportion 1:φ² with the edge of the original dodecahedron (IL).

8.8. The mathematical constant φ (Φ) appears everywhere in Nature

The asymptotic ratio of successive Fibonacci numbers leads to the Golden Ratio constant Phi φ (or Φ)

The Fibonacci Sequence describes morphological patterns in a wide range of living organisms. It is one of the most remarkable organizing principles mathematically describing natural and manmade phenomena

$$\varphi = x = \frac{1+\sqrt{(5)}}{2} = 1.618034...$$

The constant φ is the positive solution of the following quadratic equation :

$$x + 1 = x^2$$

Constant φ can be written in terms of itself :

$$\frac{1.618 + 1}{1.618} = 1.618 \quad \& \quad \frac{1.618}{1} = 1.618$$

We can write the Golden Ratio in terms of itself! Which is totally awesome.

The **Fibonacci numbers** defined by φ :

1/1 = 1
2/1 = 2
3/2 = 1.5
5/3 = 1.667
8/5 = 1.6
13/8 = 1.625
21/13 = 1.615
34/21 = 1.619
55/34 = 1.618 → φ

The following most simple periodic continued fraction describes φ :

$$\varphi = \frac{\varphi + 1}{\varphi} \longrightarrow \varphi = 1 + \frac{1}{\varphi} \longrightarrow \varphi = 1 + \frac{1}{1 + \frac{1}{\varphi}} \ldots$$

Rewritten with ϕ in place of 1.618.

Substitute φ = 1 + 1/φ for φ in the denominator.

periodic continued fraction of φ

→ **Please read my study about the possible cause of Phyllotaxis :**

Microscope Images indicate that Water Clusters are the cause of Phyllotaxis (**alternative : Weblink 2**)

Phyllotactic spirals form a distinctive class of patterns in nature. The basic arrangement of leaves and seeds is an opposite alternate (spiral) pattern. The current theory says that plant hormones like Auxin are responsible for the pattern. But because phyllotactic patterns defined by the Fibonacci sequence also appear outside biology there must be a more profound physical cause that is responsible for these Fibonacci number patterns. **The following extracts from 6 selected studies indicate that there must be a fundamental physical cause responsible for the appearance of constant Phi in nature !**

Study 1 : http://surface.iphy.ac.cn/sf03/articles/2006-2007/2007APL-chirality.pdf

→ This study shows how phyllotactic patterns appear outside biology on spherical or nearly spherical surfaces :

Stressed Fibonacci spiral patterns of definite chirality

Chaorong Li
Zhejiang Sci-Tech University, Hangzhou 310018, China and Institute of Physics, P.O. Box 603, Beijing 100080, China

Ailing Ji and Zexian Cao[a)]
Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, P.O. Box 603, Beijing 100080, China

(Received 21 August 2006; accepted 22 February 2007; published online 18 April 2007)

Fibonacci spirals are ubiquitous in nature, but the spontaneous assembly of such patterns has rarely been realized in laboratory. By manipulating the stress on Ag core/SiO_2 shell microstructures, the authors obtained a series of Fibonacci spirals (3×5 to 13×21) of definite chirality as a least elastic energy configuration. The Fibonacci spirals occur uniquely on conical supports-spherical receptacles result in triangular tessellations, and slanted receptacles introduce irregularities. These results demonstrate an effective path for the mass fabrication of patterned structures on curved surfaces; they may also provide a complementary mechanism for the formation of phyllotactic patterns.
 [DOI: 10.1063/1.2728578]

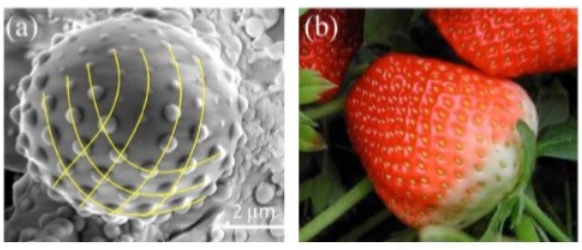


FIG. 4. Parastichous spirals on frustrating surfaces. (a) Stressed pattern on a slanted Ag core/SiO_2 shell microstructure, and (b) the "X pattern" of achenes in a strawberry.

In summary, we demonstrated that the Fibonacci spiral patterns of definite chirality can be reproduced through stress manipulation on the Ag core/SiO_2 shell microstructures. These results will be very helpful for the design and fabrication of patterned structures on curved surfaces that can find useful applications in photonics and foldable electronics. Furthermore, these results obtained in a purely inorganic material system hint at the role of stress in influencing the plant patterns. We speculate that the prerequisite for the occurrence of Fibonacci spiral patterns as stressed buckling modes be the availability of a conical support. The robust adherence of the stressed patterns to the geometry of the supports sheds some light on the mechanical rationale underlying the formation of particular plant patterns. Of course, a comprehensive model for the formation of plant patterns should incorporate as well the biochemical and genetic processes that alter growth at deeper levels.

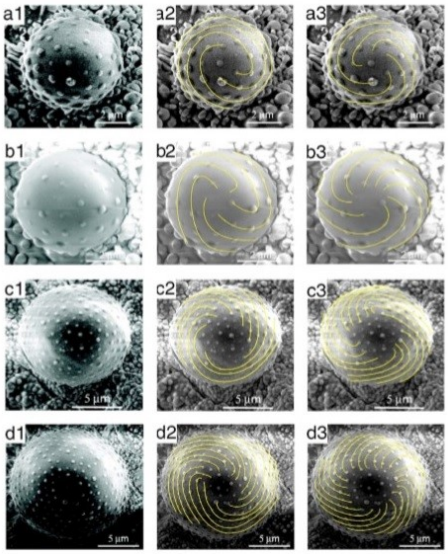


FIG. 1.
Fibonacci spiral patterns in the sinister form grown **on Ag-core/SiO2 shell microstructures: (a) 3x5, (b) 5x8, (c) 8x13 and (d) 13x21.**
Each individual pattern is presented in triad, one original and two with plotted ounterclockwise and clockwise spirals to guide the eyes.

Study 2 : https://hal.archives-ouvertes.fr/jpa-00212565/document

→ Crystalline phases with the cubic symmetry close to the icosahedral one cause **Fibonacci Crystals in Al-Mn/-Si alloys** :

Cubic approximants in quasicrystal structures

V. E. Dmitrienko

Institute of Crystallography, 117333, Moscow, U.S.S.R.

(*Received 7 mai 1990, accepted in final form 7 August 1990*)

Abstract. — The regular deviations from the exact icosahedral symmetry, usually observed at the diffraction patterns of quasicrystal alloys, are analyzed. It is shown that shifting, splitting and asymmetric broadening of reflections can be attributed to crystalline phases with the cubic symmetry very close to the icosahedral one (such pseudo-icosahedral cubic approximants may be called the Fibonacci crystals). The Fibonacci crystal is labelled as $\langle F_{n+1}/F_n \rangle$, if in this crystal the most intense vertex reflections have the Miller indices $\{0, F_n, F_{n+1}\}$ where F_i are the Fibonacci numbers (F_i = 1, 1, 2, 3, 5, 8, 13, 21, 34...). The deviations of x-ray and electron reflections from their icosahedral positions are calculated. The comparison with available experimental data shows that at least four different Fibonacci crystals have been observed in Al-Mn and Al-Mn-Si alloys : ⟨2/1⟩ (MnSi structure), ⟨5/3⟩ (α-Al-Mn-Si), ⟨13/8⟩, and ⟨34/21⟩ with the lattice constants 4.6 Å, 12.6 Å, 33.1 Å, 86.6 Å respectively. It is interesting to note that there are no experimental evidences for the intermediate approximants ⟨3/2⟩, ⟨8/5⟩ and ⟨21/13⟩. The possible space groups of the Fibonacci crystals and their relationships with quasicrystallographic space groups are discussed.

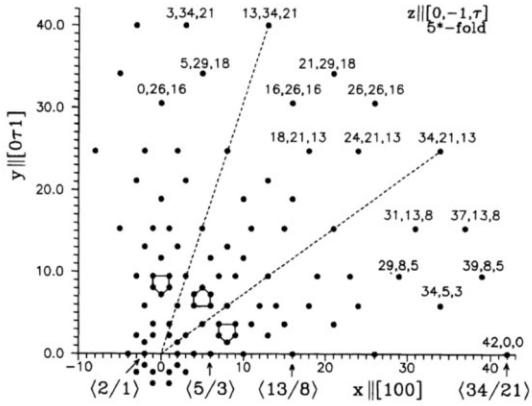
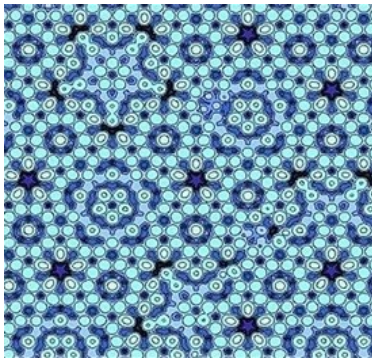


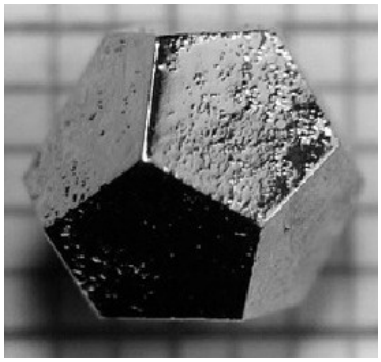
Fig. 3. — Universal pseudo-fivefold electron-diffraction pattern of the Fibonacci crystals (incident beam falls along − z direction). All x-coordinates are integer : $x_{tot} = h$. Only about one fourth of the pattern is shown ; the complete picture can be obtained by the mirror reflections : $x \to -x$ and (or) $y \to -y$. The scale of the pattern are determined by the positions of the most intense edge reflections ($Q \simeq 3$ Å^{-1}) ; for the different Fibonacci crystals those positions are marked by arrows. The Miller indices are shown for outer reflections only ; the indices of inner reflections can be obtained from the outer ones after multiplication by factor τ^{-i} (i = 1, 2, 3, ...) and round-up to the closest integer. The innermost ten reflections are of the type {200} and {211}. The approximate τ-inflation is clearly seen in the figure (pentagons with strongest distortions are marked with solid lines). Pseudo-twofold axes are shown by dashed lines.

Quasi-Crystals :

A quasiperiodic crystal, or **Quasicrystal** , is a structure that is ordered but not periodic. A quasicrystalline pattern can continuously fill all available space, but it lacks translational symmetry. While crystals, according to the classical crystallographic restriction theorem, can possess only two, three, four, and six-fold rotational symmetries, the Bragg diffraction pattern of quasicrystals shows sharp peaks with other symmetry orders, for instance **five-fold.**



Atomic model of a Al-Pd-Mn Quasicrystal



Ho-Mg-Zn Quasicrystal with a dodecahedral shape

Aperiodic tilings were discovered by mathematicians in the early 1960s, and, some twenty years later, they were found to apply to the study of natural quasicrystals. The discovery of these aperiodic forms in nature has produced a paradigm shift in the fields of crystallography. Roughly, an ordering is non-periodic if it lacks translational symmetry, which means that a shifted copy will never match exactly with its original. The more precise mathematical definition is that there is never translational symmetry in more than $n - 1$ linearly independent directions, where n is the dimension of the space filled, e.g., the three-dimensional tiling displayed in a quasicrystal may have translational symmetry in two directions. Symmetrical diffraction patterns result from the existence of an indefinitely large number of elements with a regular spacing, a property loosely described as long-range order. Experimentally, the aperiodicity is revealed in the unusual symmetry of the diffraction pattern, that is, symmetry of orders other than two, three, four, or six.

Study 3 : [252736598_Fibonacci_order_in_the_period-doubling_cascade_to_chaos]

→ The **Fibonacci number sequence (φ)** appears **in the Feigenbaum scaling of the period-doublin cascade** to chaos.

Fibonacci order in the period-doubling cascade to chaos

G. Linage [a], Fernando Montoya [a], A. Sarmiento [b], K. Showalter [c], P. Parmananda [a,*]

Abstract

In this contribution, we describe how the Fibonacci sequence appears within the Feigenbaum scaling of the period-doubling cascade to chaos. An important consequence of this discovery is that the ratio of successive Fibonacci numbers converges to the golden mean in every period-doubling sequence and therefore the convergence to ϕ, the most irrational number, occurs in concert with the onset of deterministic chaos.

Two of the most remarkable organizing principles mathematically describing natural and man-made phenomena are the Fibonacci number sequence and the Feigenbaum scaling of the period-doubling cascade to chaos. The Fibonacci sequence describes morphological patterns in a wide variety of living organisms [1], and the asymptotic ratio of successive Fibonacci numbers yields the golden mean. The Feigenbaum scaling [2] for the period-doubling cascade to chaos has been observed in a wide range of dynamical systems, from turbulence to cell biology to chemical oscillators [3,4]. Here we describe how the Feigenbaum scaling and the Fibonacci sequence are intimately intertwined.

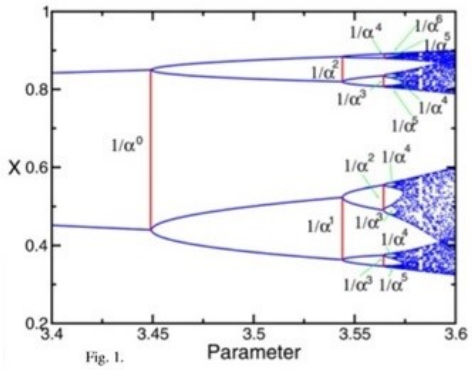


Fig. 1.

Table 1 reveals how the Fibonacci series develops in the period-doubling cascade. We see that after the first bifurcation gives rise to the period-2 oscillation, there is one segment of width $1/\alpha^0$. An examination of the successive bifurcations shows how the Fibonacci sequence expands with each bifurcation. We also see that there is a pattern of branch widths within each period. Every bifurcation contains a $1/\alpha^n$ branch, which is the first component of a binomial expansion of branch widths according to $(1/\alpha + 1/\alpha^2)^n$ as well as the remaining components of the expansion. The exponent n corresponds to the bifurcation, with $n = 0$ for period-2, $n = 1$ for period-4, and so on.

We emphasize that the Fibonacci sequence will be found in all dynamical systems exhibiting the period-doubling route to chaos, as it is directly linked to the Feigenbaum scaling constant α. However, since the scaling of α holds only in the asymptotic limit, the reported relation also holds only for asymptotic values. It follows that the ratio of successive Fibonacci numbers converges to the golden mean [7], ϕ, in every period-doubling cascade. Thus, the convergence to ϕ, the "most irrational number" [8], occurs in concert with the onset of deterministic chaos.

A typical period-doubling bifurcation diagram for the logistic map is shown in Fig. 1. We normalize the width of the period-2 branch to unity ($1/\alpha^0$), allowing the widths of the branches corresponding to higher periods to be written as inverse powers of α. With this normalization, we notice that the number of branches corresponding to the various powers of $1/\alpha$ follows the sequence:

$1/\alpha^0$, $1/\alpha^1$, $2/\alpha^2$, $3/\alpha^3$, $5/\alpha^4$, $8/\alpha^5$, $13/\alpha^6$, $21/\alpha^7$, $34/\alpha^8$, $55/\alpha^9$, ...,

where the coefficients 1, 1, 2, 3, 5, 8, ... correspond to the number of branches with widths $1/\alpha^0$, $1/\alpha^1$, $1/\alpha^2$, $1/\alpha^3$, $1/\alpha^4$, $1/\alpha^5$, ..., respectively. These coefficients are the beginning of the Fibonacci sequence,

Table 1
Distribution of powers of $1/\alpha$ in successive period-doubling bifurcations

	P2 $n=0$	P4 $n=1$	P8 $n=2$	P16 $n=3$	P32 $n=4$	P64 $n=5$		F.N.
$1/\alpha^0$	1							1
$1/\alpha^1$		1						1
$1/\alpha^2$		1	1					2
$1/\alpha^3$			2	1				3
$1/\alpha^4$			1	3	1			5
$1/\alpha^5$				3	4	1		8
$1/\alpha^6$				1	6	5	...	13
$1/\alpha^7$					4	10	...	21
$1/\alpha^8$					1	10	...	34

Study 4 : http://www.itp.uni-bremen.de/prichter/download/DoppelpendelIWF.pdf ; **Movie : https://av.tib.eu/media/14902**

→ The pendulum (P1) orbit with the winding number W= g^2=0.3820 (g= φ -1) can resist longest to the chaos

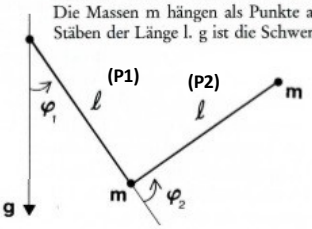
The Planar Double Pendulum — Das ebene Doppelpendel

Verfasser der Publikation: PETER H. RICHTER und HANS-JOACHIM SCHOLZ

Summary of the Film:

The Planar Double Pendulum. Computer experiments made it possible to describe the complex dynamics of this classical example in mechanics. To begin with, the various types of motion of the double pendulum are presented. With the help of the method of Poincaré sections, a qualitative survey of the complex dynamics follows, with special emphasis on irrational winding numbers (golden ratio).

Abb. 1b. Eine mathematische Idealisierung. Die Massen m hängen als Punkte an starren Stäben der Länge l. g ist die Schwerkraft



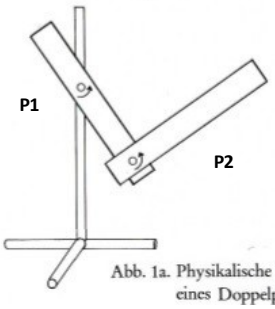


Abb. 1a. Physikalische Realisierung eines Doppelpendels

„Dreihundert Jahre nach Newton sollten wir eigentlich wissen, was seine Gleichungen uns über das qualitative Verhalten konservativer Systeme mit zwei Freiheitsgraden lehren" – so MICHAEL V. BERRY in einem vielbeachteten Übersichtsartikel ([2]), der das aktuelle Interesse von Physikern und Mathematikern an der Klassischen Mechanik reflektiert. Tatsache aber ist, daß wir erst jetzt zu sehen beginnen, was alles in diesen scheinbar so einfachen Gleichungen steckt. Denn erst seit kurzem haben wir Zugang zu ihren Lösungen. Es bedurfte der Entwicklung moderner Computer, den Reichtum an Komplexität aufzudecken, der schon den einfachsten Systemen innewohnt. Gegenwärtig erleben wir eine Phase, in der ausgiebiges mathematisches Experimentieren zunächst die Phänomene zutage fördert, während die theoretische Analyse dann immer wiederkehrende Szenarien zu verstehen sucht.

Figure 5 shows Poincare-Sections for the Double Pendulum at different values of the total energy of the pendulum. Different colors mean different orbits.

X-axis = angle φ1 of Pend. 1 (P1)

y-axis = angular momentum P1

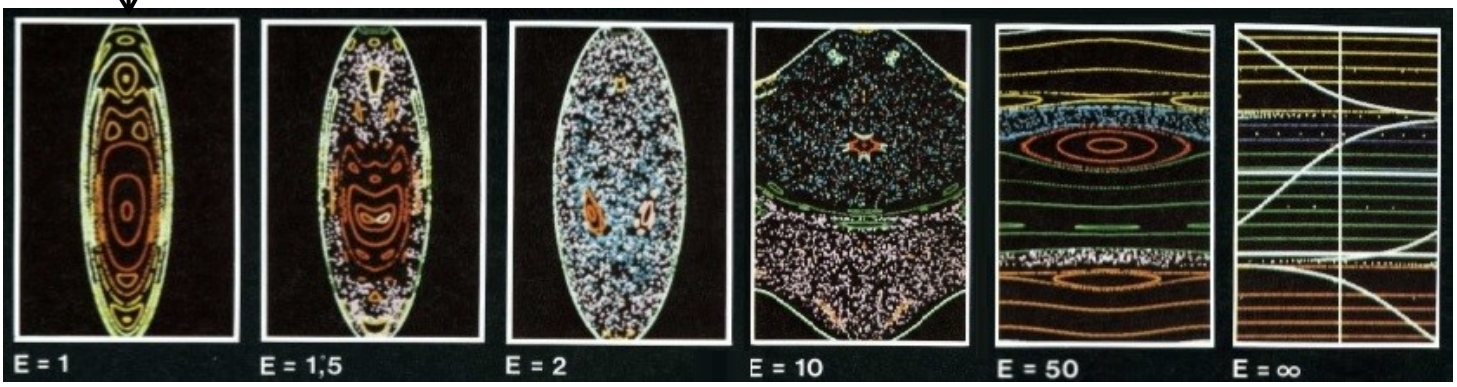


Abb. 5. Poincaré-Schnitte für das ebene Doppelpendel bei verschiedenen Werten der Gesamtenergie E (in Einheiten von mgl). Auf der Abszisse ist der Winkel φ_1 des inneren Pendels, auf der Ordinate der entsprechende Drehimpuls L_1 aufgetragen. Die Bewegung wird immer dann aufgezeigt, wenn $\varphi_2 = 0$ und $\dot{\varphi}_2 > 0$ sind. Jedes Bild wurde aus etwa 10 bis 20 Anfangsbedingungen und 300 Folgepunkten generiert. Mit Hilfe der Farbe können verschiedene Orbits unterschieden werden. Das letzte Bild zeigt den integrierbaren Fall E = ∞ oder g = 0, wo der Drehimpuls konstant bleib

ARNOLD ([1]) und MOSER ([9]) haben unabhängig voneinander mit mathematischer Strenge gezeigt, daß invariante Linien mit Windungszahlen W auch unter Störungen noch existieren, wenn diese hinreichend klein sind und wenn W eine sog. diophantische Bedingung erfüllt: Für jede rationale Approximation p/q muß eine Abschätzung der Art

$$\left| W - \frac{p}{q} \right| > \frac{c}{q^\tau} \qquad (4)$$

W = winding number of pendulum L1 per winding of pendulum L2 ; (p, q → two numbers not simply divisible)

möglich sein, mit festem c und τ. Es läßt sich zeigen, daß die meisten irrationalen Zahlen eine solche Bedingung erfüllen. Sie läßt sich dazu verwenden, den Grad der Irrationalität einer Zahl W zu definieren. Denn der nach (4) geforderte Abstand zu den rationalen Zahlen wird größer, wenn τ kleiner und c größer werden. Wir nennen eine Zahl W_1 irrationaler als eine Zahl W_2, wenn das zu W_1 gehörige minimale τ kleiner und das maximale c größer sind als die entsprechenden Werte für W_2.

In diesem Sinne nun erweist sich das Verhältnis des Goldenen Schnitts als die irrationalste Zahl. Erinnern wir uns an die Definition: der Wert g teilt die Strecke 1 im goldenen Verhältnis, wenn der kleinere Teil 1 – g sich zu g verhält wie g zu 1.

Also (1 – g) : g = g : 1. Daraus folgt sofort

$$g^2 + g - 1 = 0,\ g = \frac{\sqrt{5}-1}{2} = 0.618034 \qquad (5)$$

Gelegentlich werden auch die Zahlen

$$G = 1/g = 1 + g = 1.618034$$

oder $g^2 = 1 - g = 0.381966$

als Goldener Schnitt bezeichnet. Ihnen allen ist gemeinsam, daß sie den größtmöglichen Abstand von den rationalen Zahlen haben, denn wenn W = g, G oder g^2 und p, q beliebig, so gilt

$$\left| W - \frac{p}{q} \right| \geq \frac{g^2}{q^2} \qquad (6)$$

Keine Zahl erlaubt eine noch schärfere Abschätzung. Für eine genauere Diskussion verweisen wir auf Bücher über Zahlentheorie, etwa SCHRÖDER ([11]). Wir begnügen uns hier mit dem Hinweis auf die Kettenbruchentwicklung der Zahl g, die sich aus (5) unmittelbar ergibt:

$$g = \frac{1}{1+g} \longrightarrow g = \frac{1}{1+\frac{1}{1+\frac{1}{1+..}}} \qquad (7)$$

Brechen wir die Folge der Brüche nach dem n-ten Schritt ab, so erhalten wir die n-te Kettenbruchapproximation g_n für g. Es gilt

$$\{g_1, g_2, \ldots, g_n \ldots\} = \left\{1, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{3}{5}, \tfrac{5}{8}, \ldots \frac{F_n}{F_{n+1}}, \ldots\right\} \qquad (8)$$

wobei die F_n die bekannten Fibonacci-Zahlen sind: $F_1 = F_2 = 1$, $F_{n+1} = F_n + F_{n-1}$. Für G ist die n-te Kettenbruchapproximation $G_n = F_n/F_{n-1}$, für g^2 finden wir $(g^2)_n = F_n/F_{n+2}$. Die Bedeutung dieser Kettenbruchapproximationen liegt darin, daß sie für gegebene (oder kleinere) Nenner jeweils die beste rationale Annäherung an die irrationale Zahl W darstellen. Die langsame Konvergenz der Folgen im Sinne von (6) rührt daher, daß die Entwicklung (7) nur Einsen enhält, also die kleinstmöglichen ganzen Zahlen.

Was aber hat das mit der Dynamik des Doppelpendels zu tun? Abb. 7 gibt darauf die Antwort. Wenn wir, vom integrablen Grenzfall E = ∞ ausgehend, die Energie erniedrigen und dabei anhand der Poincaré-Schnitte die Ausbreitung des Chaos verfolgen, dann sehen wir zunächst noch viele invariante Linien. Die einzelnen Chaosbänder sind relativ schmal, die Bewegung hat vorwiegend regelmäßigen Charakter. Allmählich aber verschwinden mehr und mehr dieser Linien; Chaosbänder verschmelzen zu größeren, und schließlich – etwa bei E = 10 – gibt es nur noch eine letzte solche „K A M - L i n i e" (so genannt nach den Mathematikern KOLMOGOROFF, ARNOLD und MOSER). Diese letzte Form regelmäßiger Bewegung, ehe ein Chaos die ganze Energieschale überschwemmt, hat als Windungszahl das goldene Verhältnis:

$$W = g^2 = 0.381\,966 \qquad (9)$$

Keine Zahl erlaubt eine noch schärfere Abschätzung. Für eine genauere Diskussion verweisen wir auf Bücher über Zahlentheorie, etwa SCHRÖDER ([11]). Wir begnügen uns hier mit dem Hinweis auf die Kettenbruchentwicklung der Zahl g, die sich aus (5) unmittelbar ergibt:

$$g = \frac{1}{1+g} \longrightarrow g = \frac{1}{1+\frac{1}{}}$$

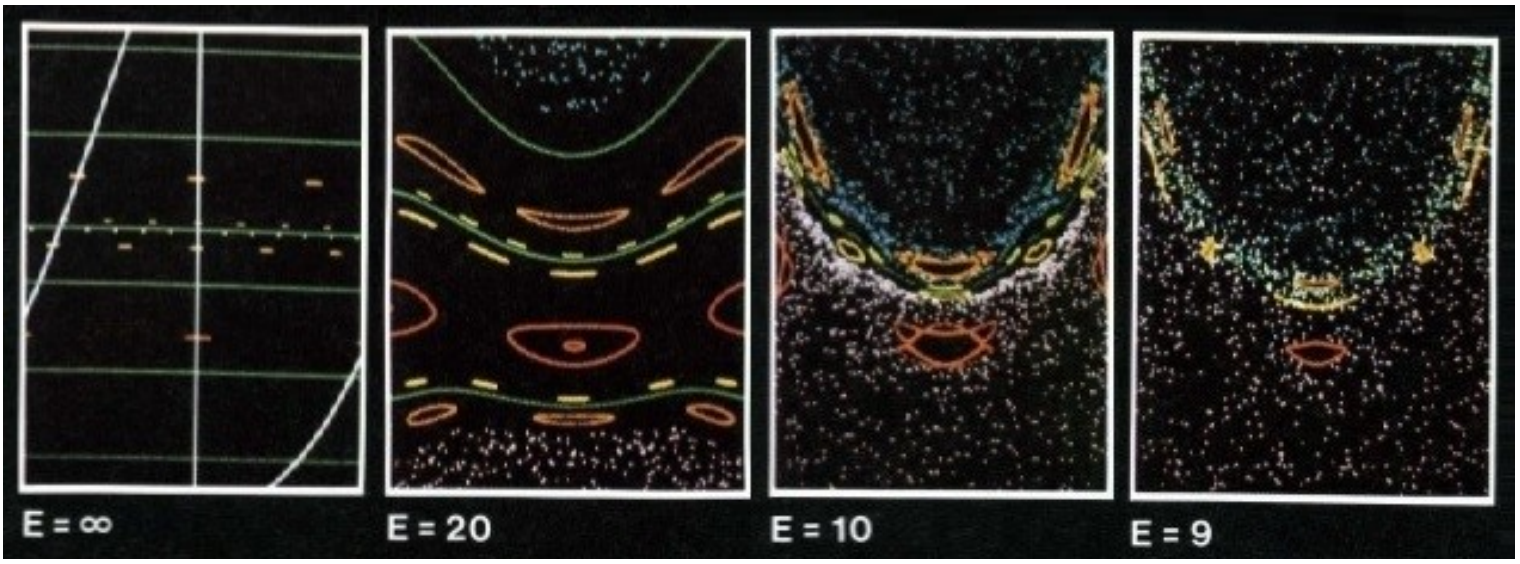


Figure 7 shows details from Images of **Figure 5**

Abb. 7. Diese Serie zeigt jeweils Ausschnitte der entsprechenden Bilder von Abbildung 5. Sie demonstriert das Schicksal einiger periodischer (W = 1/2, 1/3, 2/5, 3/8) und einiger quasiperiodischer Orbits unter Störung. Der Orbit mit der goldenen Windungszahl W = g^2 = 0,3820 hält dem Einbruch des Chaos am längsten stand (Bild c). Bei E = 9 ist er auch zerfallen

Figure 7 (Abb.7) shows Poincare-Sections for the double pendulum at different values of the energy motion equation. It shows the fate of some periodic (W=1/2, 1/3, 2/5, 3/8) and of some quasiperiodic orbits under disturbance.

If W = g, G(φ) or g^2 (golden ratio numbers, see above), and p, q (any numbers), then the equation (6) is valid.

→ The golden ratio numbers g, G or g^2 (most irrational numbers) have the biggest possible distance to the rational numbers.

→ **Note :** **The orbit with the winding number W= g^2=0.3820 (g = φ -1) can resist longest to the chaos !**

→ This behaviour indicates that **φ** somehow must be connected to gravitation ! (H.K. Hahn)

Study 5 : → **Ratios of orbital periods clearly show a preference for Fibonacci-Number ratios → Another indication for a link between the constant φ and gravitation !**

→ weblink : https://arxiv.org/pdf/1803.02828

Orbital Period Ratios and Fibonacci Numbers in Solar Planetary and Satellite Systems and in Exoplanetary Systems

Technology and Engineering Center for Space Utilization,
Chinese Academy of Sciences, Beijing, China
Vladimir.Pletser@csu.ac.cn

Abstract

It is shown that orbital period ratios of successive secondaries in the Solar planetary and giant satellite systems and in exoplanetary systems are preferentially closer to irreducible fractions formed with Fibonacci numbers between 1 and 8 than to other fractions, in a ratio of approximately 60% vs 40%. Furthermore, if sets of minor planets are chosen with gradually smaller inclinations and eccentricities, the proximity to Fibonacci fractions of their period ratios with Jupiter or Mars' period tends to increase. Finally, a simple model explains why the resonance of the form $\frac{P_1}{P_2} = \frac{p}{p+q}$, with P_1 and P_2 orbital periods of successive secondaries and p and q small integers, are stronger and more commonly observed.

1 Introduction

The discovery of the Trappist-1 system of seven planets (Gillon et al., 2017; Luger et al., 2017) with five out of six orbital period ratios being close to ratios of Fibonacci integers (Pletser and Basano, 2017) has prompted a search among other planetary and satellite systems of the Solar System and of exo-planetary systems to assess whether Fibonacci numbers intervene more often in integer fractions close to ratios of orbital periods. It is found that ratios of Fibonacci numbers outnumber significantly ratios formed with other integers, when limited to the most significant ratios of small integers between 1 and 8.

6. Conclusions

It was shown that orbital period ratios of successive secondaries in the Solar planetary and giant satellite systems and in exoplanetary systems are preferentially and significantly closer to irreducible fractions formed with the second to the sixth Fibonacci numbers (between 1 and 8) than to other fractions, in a ratio of approximately 60% vs 40%, although there are less irreducible fractions formed with Fibonacci integers between 1 and 8 than other fractions.

Furthermore, if sets of minor planets are chosen with gradually smaller inclinations and eccentricities, one observes that the proximity to Fibonacci fractions of their period ratios with Jupiter or Mars' period tends to increase for more "regular" sets with minor planets on less eccentric and less inclined orbits. Therefore, orbital period ratios closer to Fibonacci fractions could indicate a greater regularity in the system.

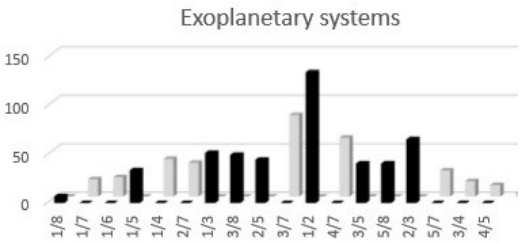


Figure 5: Histogram of the numbers of ratios of orbital periods of all adjacent planets in all exoplanetary systems, close to irreducible fractions of two Fibonacci numbers (black) and other fractions (grey).

It is seen that the highest peak is observed for 1/2 with 135 cases, followed by 3/7 with 84 cases and 2/3 with 66 cases, and in total, there are 473 out of 791 ratios (59.80%) close to Fibonacci fractions, disregarding 115 cases of ratios smaller than 0.1181.

Ratios of orbital periods in exoplanetary systems :

Fibonacci Number ratios :

1/2 (135 cases), 2/3 (66 cases), 3/5 (~35 cases), 3/8 (~40 cases), 5/8 (~35 cases), 3/8 (~35 cases),

Lucas Number ratios :

1/3 (~45 cases), 3/7 (84 cases) , 4/7 (~60 cases)

Study 6 : - Extracts from a study produced by Dr. Iliya Iv. Vakarelov, University of Forestry, Bulgaria (1982-1994)

Changes in phyllotactic pattern structure (Fibonacci Sequences) in Pinus mugo due to changes in altitude

from the book „Symmetry in Plants“ by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. – Chapter 9 , pages 213 – 229) , **weblinks :** **Weblink 1** (Google Books), **Weblink 2**

Fig : *Pinus mugo*

Research Site and methods :

Pinus Mugo grows in high mountainous parts at altitudes up to 2500m forming vast communities. The vertical profile of the research sites for *Pinus mugo* was situated along the northern slopes of the eastern part of the Rila mountain, and test specimens were collected from the following altitudes : 1900, 2200 and 2500 m. Test specimens were also collected from the city of Sofia (at 550 m) where *Pinus mugo* is grown as decorative plant.

The research was carried out over a period of 12 years (except of altitude 550m here research was carried out only around 6 years). The initation of leaf primordia in the bud (meristem) occurs at the end of the growing period. The apical meristem of *Pinus mugo* starts this process around the beginning of mid of August and ends in autumn when the air temperature goes below a certain point.

The interesting results of the study :

(3) With the increase of altitude from 1900m to 2500m the phyllotactic pattern structure of “Pinus mugo“ twigs changes considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main sequence decreases from 88 % to 38 %.

At the upper boundary of Pinus mugo natural distribution – at about 2500m, the variation of phyllotactic twig pattern structure (entropy) becomes cyclic, with six year duration of the cycles.

(5) The changes in temperature during the period of phyllotactic pattern formation of Pinus mugo twigs determine about 48 % of the changes in pattern structure, the latter lagging behind with one or two years.

It is obvious that when the altitude increases, the number of phyllotactic patterns (Fibonacci-Sequences) of the vegetative organs of *Pinus mugo* also increases above a given altitude. → see Table below !

Sequence No.	FIBONACCI–Sequences present in given altitude	Altitute in (m) 550 Frequency	550 Relative Frequency	1900 Frequency	1900 Relative Frequency	2200 Frequency	2200	2200 Relative Frequency	2500 Frequency	2500	2500 Relative Frequency	Total Frequency	Total Relative Frequency
F1	⟨1,2,3,5,8,13,…⟩	231	0.902	431	0.885	619	F1	0.812	246	F1	0.381	1527	0.710
F3	2⟨1,2,3,5,8,13,…⟩	16	0.063	34	0.070	35	F3	0.046	111	F3	0.172	196	0.092
F2	⟨1,3,4,7,11,18,…⟩	3	0.012	22	0.045	49	F2	0.064	86	F2	0.133	160	0.074
F4	3⟨1,2,3,5,13,…⟩	6	0.023	–	–	29	F4	0.038	98	F4	0.152	133	0.062
F8	⟨2,5,7,12,19,31,…⟩	–	–	–	–	10		0.013	50		0.077	60	0.028
F11	⟨3,7,10,17,27,44,…⟩	–	–	–	–	5		0.007	18		0.028	23	0.011
F6	⟨1,4,5,9,14,23,…⟩	–	–	–	–	1		0.001	8		0.012	9	0.004
F9	2⟨1,3,4,7,11,18,…⟩	–	–	–	–	4		0.005	7		0.011	11	0.005
(?) F6	⟨1,7,8,15,23,38,…⟩	–	–	–	–	2		0.003	7		0.011	9	0.004
F5	4⟨1,2,3,5,8,13,…⟩	–	–	–	–	8		0.011	9		0.013	17	0.008
(?) F13	⟨1,6,7,13,20,33,…⟩	–	–	–	–	–		–	3		0.005	3	0.001
F10	⟨2,7,9,16,25,41,…⟩	–	–	–	–	–		–	3		0.005	3	0.001

Note : The number of Fibonacci-Sequences is **increasing** with altitude !

Table 1 : Data on the frequency and relative frequency of the different phyllotactic patterns for ***Pinus mugo*** twigs at different altitudes. Specimen formed during the period 1982-1994 have been tested for all sites except for the one at 550 m where the period covers the years 1989 – 1993.

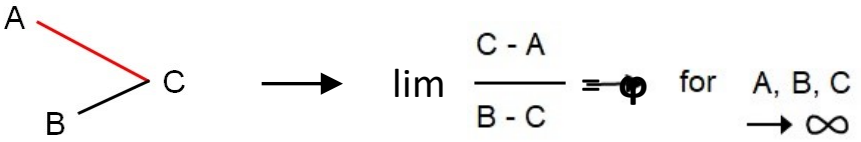
8.9 From the Fibonacci-Sequences shown by *Pinus mugo* at 2500m an infinite Fibonacci-Table was developed

There are clear spatial interdependencies noticable between the different Fibonacci-Sequences, which are connected by the golden ratio **φ** . There is a complex network visible between the numbers of all Sequences. This table of Fibonacci-Number Sequences can be extended towards infinity and all natural numbers are contained in the lower half only once!

For 3 numbers A, B and C in the below shown arrangement, which belong to the same 3 (or 2) different Fibonacci-Sequences, the following rule is true :

The ratio of the difference (C-A) indicated by a "red line", to the difference (B-C) indiated by a "black line" is approaching the golden ratio **φ** for the further progressing Fibonacci-Number Sequences towards infinity (downwards in the table).

„**Main Bow-Structures**“ are also linked by the „golden ratio“ **φ** !



FIBONACCI – Number Sequences No. 1 to 14 (F1 - F14) → see extended Table F1-F33 in the Appendix 4 !

Row No.	F1 Fibonacci-Base-Sequence	F2 Lucas-Sequence	F3 Fibonacci-Sequence (x 2)	F4 Fibonacci-Sequence (x 3)	F5 Fibonacci-Sequence (x 4)	F6	F7	F8	F9 Lucas-Sequence (x 2)	F10	F11	F12	F13 Lucas-Sequence (x 3)	F14
1	1	1				1	1							
2	2		2					2	2	2				
3	3	3		3							3	3	3	
4		4	4		4	4								4
5	5					5	5	5						
6			6	6			6		6					
7	7							7		7	7			
8	8				8				8			8		
9			9			9				9			9	9
10	10										10			
11	11						11					11		
12					12		12						12	
13	13													13
14						14								
15	15													
16	16									16				
17							17				17			
18	18													
19								19				19		
20	20													
21	21												21	
22									22					22
23	23													
24	24													
25										25				
26	26													
27											27			
28	28													
29	29													
30												30		
31	31													
32	32													
33													33	
34	34													
35														35
36	36													
37	37													
38														
39	39													
40														
41	41													
42	42													
43														
44	44													
45	45													
46														
47	47													
48														
49	49													
50	50													
51														
52	52													
53														
54	54													
55	55													
56														
57	57													
58	58													
59														
60	60													

3 5 7 8 11 13 21 34 55

Lucas-Sequence (bottom main bows)

Note : Below this line all natural numbers are contained in the Fibonacci Sequences just once !

Fibonacci-Sequence (top main bows)

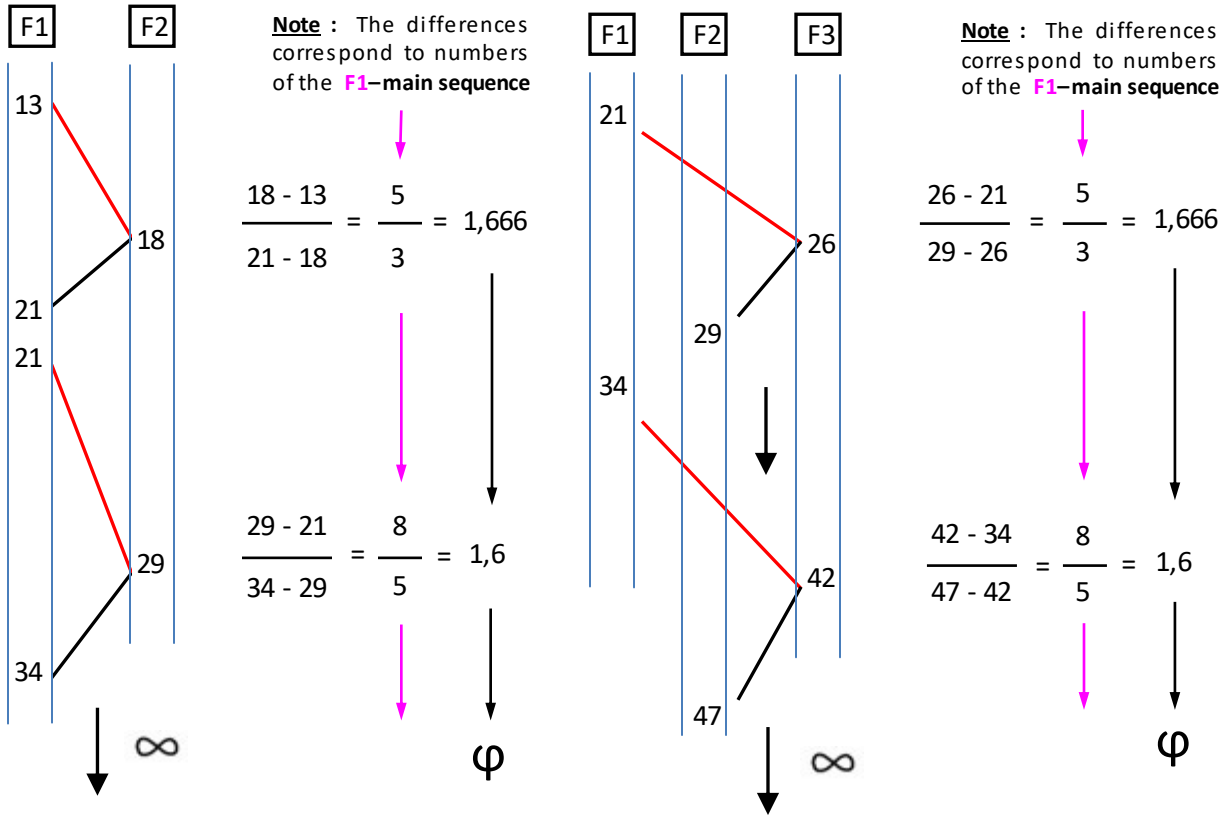
11 13 ∞

→ Weblink to this Study : → An infinite Fibonacci Number Sequence Table

➔ **The following two examples explain the rule which was described in general on the previous page :**

The examples show how the quotient of the differences between the numbers of designated Fibonacci-Sequences (indicated by red- and black-lines in the table), is approaching the golden ratio for the number sequences progressing towards infinity.

For the examples we look at the Fibonacci Sequences **F1, F2** and **F3** (→ F2 is the Lucas-Sequence, F3 = F1 x 2)



➔ **Interesting properties of the Fibonacci-F1 Sequence (and other Fibonacci-Sequences) :**

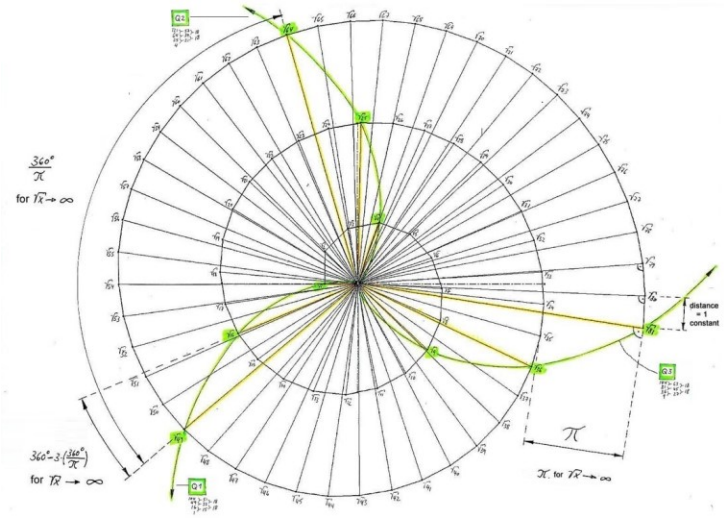
- The numbers of the **Fibonacci F1** – Number Sequence seem to contain all prime numbers as prime factors !
- This is not the case for all other Fibonacci-Sequences where certain prime factors are missing ! (**see Appendix 3**)
- And all prime factors appear periodic in defined "number-distances" in the sequence (see left side of table)
- This is the case for all Fibonacci-Sequences ! (→ These mentioned properties must be analysed in more detail !)

Table 2 : Periodicity of the prime factors of the **Fibonacci F1** - Number Sequence :

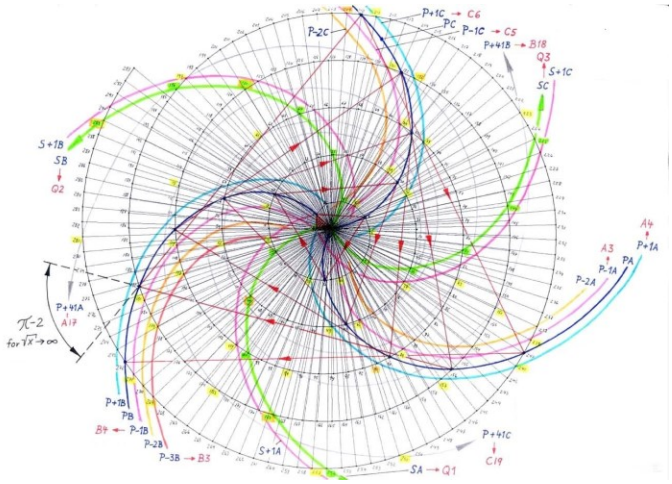
some prime factors shown in table form												in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F''	Nr.
															1	1			1
															1	1			2
															2	2	1		3
															3	3	1		4
															5	5	2	1	5
												2^3		2x2x2	8	8	3	1	6
															4	13	5	2	7
									7		3			3x7	3	21	8	3	8
						17						2		2x17	7	34	13	5	9
								11		5				5x11	10	55	21	8	10
															17	89	34	13	11
											3^2	2^4	2x2x2x	3x3	9	144	55	21	12
															8	233	89	34	13
			29				13							13x29	17	377	144	55	14
										5		2		2x5x61	7	610	233	89	15
									7		3		3x7x	47	24	987	377	144	16
															22	1597	610	233	17
					19	17						2^3	2x17x	2x2x19	19	2584	987	377	18
	37													37x113	14	4181	1597	610	19
41								11		5	3		5x11x	3x41	24	6765	2584	987	20

➔ **See some selected Fibonacci-Sequences in more detail in Appendix 3**

8.10 The Square Root Spiral represents a two-dimensional projection of the Universal Theory (Number Theory)



Study : The Ordered Distribution of Natural Numbers on the Square Root Spiral
http://front.math.ucdavis.edu/0712.2184 **PDF :** http://arxiv.org/pdf/0712.2184



The Distribution of Prime Numbers on the Square Root Spiral
http://front.math.ucdavis.edu/0801.1441
PDF : http://arxiv.org/pdf/0801.1441

The complex **square root spiral develops out of a simple right triangle** with the **cathetus lengths of 1** and the **hypotenuse lenght square root of 2**, by the application of the Pythagorean theorem. And it is obvious to see in the square root spiral that **constant Pi (π), which defines the distance between the winds of the square root spiral, is developing out of this base triangle and the continued application of the Phytagoran Theorem**. It is not the opposite way, that the square root of 2 is developing out of π ! (By the way a perfect circle only exists in theory not in reality !)

Pi (π) is developing out of square root of 2 ! Therefore **square root of 2** must be considered as a more fundamental constant than **π** ! Note that in the square root spiral **all other irrational numbers develop out of this base triangle !**

The importance (simplicity) of **square root of 2** is also indicated by a comparison of the Continued fraction of **square root of 2** and **π** : see also weblink to : „Periodic Continued Fraction“ , from : mathworld.wolfram. com

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}}.$$

The simple continued fraction for π does not exhibit any obvious pattern
But mathematicians have discovered several generalized continued fractions that do, such as :

$$\pi = \cfrac{4}{1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \ddots}}}}}} = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cfrac{9^2}{6 + \ddots}}}}}$$

and by **Viete's formula from 1593 :**

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \cdots$$

It is also possible to derive from Viète's formula a related formula for π that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \lim_{k \to \infty} 2^k \underbrace{\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2}}}}}}}_{k \text{ square roots}}$$

However the most important mathematical constant is Phi (**φ**)
This is already indicated by the continued fraction of **φ** :

$$\varphi = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$$

Note : **φ** Is the most irrational number and therefore logically the most important constant !

8.11 Constant Pi (π) can also be expressed by only using constant φ and 1 !

Again to **Viete's formula from 1593 :**

$$\pi = \frac{2}{\sqrt{2}} \frac{2}{\sqrt{2+\sqrt{2}}} \frac{2}{\sqrt{2+\sqrt{2+\sqrt{2}}}} \cdots$$

→ It is also possible to derive from Viète's formula a related formula for π that still involves nested square roots of two, but uses only one multiplication :

$$\pi = \lim_{k\to\infty} 2^k \underbrace{\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2}}}}}}}_{k \text{ square roots}}$$

If we replace the number 2 in the above shown formulas by the found equation (5) where number 2 can be expressed by constant φ and 1, then we can express constant Pi (π) also by only using the constant φ and 1 !

Replace Number 2 in the above shown formulas with this term.

→ $2 = \frac{\varphi^4 + 1}{\varphi^2} - 1$ → $2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}$ (5) and $\sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}}$ (6)

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 ! Numbers don't exist ! Only φ & 1 exist !

Constant Pi (π) can now be expressed in this way, by only using constant φ and 1 :

$$\pi = \lim_{k\to\infty} \left[\frac{\varphi^4-\varphi^2+1}{\varphi^2}\right]^k \underbrace{\sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} - \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} + \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2} + \cdots + \sqrt{\frac{\varphi^4-\varphi^2+1}{\varphi^2}}}}}}_{k \text{ square roots}}$$

It becomes clear that the irrationality of Pi (π) is also only based on the constant φ and 1, in the same way as the irrationality of all irrational square roots, is only based on constant φ & 1 !

Numbers don't seem to exist ! Natural Numbers, their square roots and irrational transcendental constants like Pi (π) can be expressed by only using constant φ and 1 !!

This is an interesting discovery because it allows to describe most (maybe all) geometrical objects only with φ & 1 !

The result of this discovery may lead to a new base of number theory. Not numbers like 1, 2, 3,….. and constants like Pi (π) etc. are the base of number theory ! Only the constant φ and the base unit 1 (which shouldn't be considered as a number) form the base of mathematics and geometry. This will certainly also have an impact on physics !

And constant φ and the base unit 1 must be considered as fundamental „Matter (energy) structure constants" of the real physical world ! With constant φ and 1 all geometrical objects including the Platonic Solids can be expressed !

There probably isn't something like a base unit if we consider a „wave model" as the base of physics and if we see the universe as one oscillating unit. In the universe everyting is connected with everything. see : Quantum Entanglement

8.12 Referring to my discovery regarding constant φ (Phi) , I have defined these 12 Conjectures :

Conjectures : (→ you can call them **Harry K. Hahn's conjectures**)

1.) All Natural Numbers and their square roots can be expressed (calculated) by only using the mathematical constant Phi (golden mean = 1.618..) and number 1. This statement is also valid for all rationals (fractions) and their square roots

2.) All existing irrational numbers seem to be constructions out of Phi and 1.
For example the irrational transcendental constant Pi (3.1415926....) can also be expressed by only using Phi and 1 !

3.) Phi and 1 are the base units of Mathematics ! Numbers and number-systems don't exist ! They are manmade and therefore can be eliminated. In principle Mathematical Science can be carried out by only using Phi and 1, as base units.

4.) All geometrical objects, including the Platonic Solids can also be described by only using constant Phi and 1.
Because all natural numbers, their square roots, rationals (fractions) and probably all irrational and all transcendental numbers too, can be expressed by only using Phi and 1.

5.) Point 4.) leads me to the conclusion that in the physical world the geometries of all possible crystal-lattice-structures are fundamentally based on Phi and 1. The more fundamental the lattice the simpler it can be expressed by Phi and 1.

6.) Point 4.) 5.) & 7.) lead me to the conclusion that on the molecular- and atomic-level, as well as on the macroscopic (cosmic) level the distribution and structure of matter (=energy), is fundamentally based on constant Phi and 1.
→ **Constant Phi (φ) therefore must be a fundamental physical "Energy (Matter) Structure Constant"**
Because at the beginning of the universe (BigBang) matter formed out of pure energy, the property to form matter with defined structure must be a property of energy, from the logical point of view ! Without energy no matter could have formed, and without energy & matter no space would exist ! → Energy must store the structural information !
Together with Point 7.) this indicates that the curvature of spacetime at the molecular level (crystals) and at the atomic level and on the macroscopic level is defined only by this "Energy Structure Constant" Phi and the base unit 1 which represents a base energy/wave element → This idea will help to unify General Relativity with Quantum Physics !
If the gravitational singularity in M87 really has a dodecahedral structure, then there is strong indication that gravitation, like matter, is defined by the same constant duo : Phi and 1 in Quantum Mechanics and at the cosmic level !

7.) The structure of the M87 black hole (→ **EHT2017**) indicates a dodecahedral structure. The distribution of matter in gravitational singularities therefore seems to be defined essentially by constant Phi and base unit 1 ! The largescale distribution of matter in the universe seems to be predominantly based on an order-5 Poincare-Dodecahedral-Space.
(See this study **: "EHT2017 may provide evidence for a Poincare Dodecahedral Space Universe"**)

8.) The natural numbers can be assigned to a defined infinite set of Fibonacci-Number Sequences.

9.) This infinite set of Fibonacci-Number Sequences , and the numbers contained in these sequences, are connected to each other by a complex precisely defined spatial network based on constant Phi. For the progressing Fibonacci-Sequences towards infinity, the connections between the numbers approach constant Phi.
→ see my study : **"Creation of an infinite Fibonacci Number Sequence Table"**

10.) Constant Phi (golden mean = 1.618..) must be a fundamental constant of the final equation(s) of the universal mathematical and physical theory. (→ It may be the only irrational constant that appears in the(se) equation(s))

11.) The number-5-oscillation (→ the numbers divisible by 5) in the two number sequences 6n+5 (Sequence 1) and 6n+1 (Sequence 2), with n=(0,1,2,3,...) , defines the distribution of the prime numbers and non-prime-numbers. The number-5-oscillation defines the starting point and the wave length of defined non-prime-number-oscillations in these Sequences 1+2 (SQ1 & SQ2). (Note : the combination of the two sequences SQ1 & SQ2 is considered here)
→ weblink to my study : https://arxiv.org/abs/0801.4049 (or alternatively here : http://vixra.org/abs/1907.0355)
→ For a quick overview please see the **Chapter 8.2** of this study (**"EHT2017 may provide evidence..."**)

12.) The importance of the number-5-oscillation for the distribution of primes and non-primes is a further indication for the conjecture that the largescale structure of the universe seems to be predominantly (mainly) based on an order-5 Poincare-Dodecahedral-Space structure. → The space structure of the universe seems to be based essentially on the **5.Platonic Solid : the Dodecahedron** (→ consisting of 12 regular pentagonal faces, three faces meeting at each vertex)

The time will show if my Conjectures are correct !

References : → **Regarding the M87 black hole (EHT2017) and a possible Poincare Dodecahedral Space universe**

Webside of the Event Horizon Telescope Organization
https://eventhorizontelescope.org/

The Event Horizon Telescope
https://en.wikipedia.org/wiki/Event_Horizon_Telescope

Scientific papers to the M87-black hole observation (EHT2017)-Project :

Paper I: The Shadow of the Supermassive Black Hole
Paper II: Array and Instrumentation
Paper III: Data processing and Calibration
Paper IV: Imaging the Central Supermassive Black Hole
Paper V: Physical Origin of the Asymmetric Ring
Paper VI: The Shadow and Mass of the Central Black Hole

Movie about the EHT-Project (in german or english language)
→ **Image Calculation process_live** in action in the movie **at around 39:40 to 41:00 minutes in the movie.**
See weblink at YouTube.com :
→ **Title „Black Hole Hunters" :** https://www.youtube.com/watch?v=o_F3KVAPMpo (german version)
The **English version** (same title) is available on www.smithsonianchannel.com (for free in USA & UK)
or alternatively on www.amazon.com → Just type-in the title in the search-box : „Black Hole Hunters"

Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background (CMB)
Jean-Pierre Luminet, Jeffrey R. Weeks, Alain Riazuelo, Roland Lehoucq & Jean-Philippe Uzan
Weblink 1 : http://ceadserv1.nku.edu/longa//classes/2004fall/mat115/days/luminet-nat.pdf
Weblink 2 : https://luth.obspm.fr/~luminet/physworld.pdf

The optimal phase of the generalised Poincare dodecahedral space hypothesis implied by the spatial cross-correlation function of the WMAP sky maps
Boudewijn F. Roukema, Zbigniew Bulin´ski, Agnieszka Szaniewska, Nicolas E. Gaudin
Weblink 1 : https://arxiv.org/abs/0801.0006 Weblink to PDF : https://arxiv.org/pdf/0801.0006.pdf

Studies to a Dodecahedral Space Universe and multi-connected universes - from Prof. Frank Steiner :
Cosmic_microwave_background_alignment_in_multi-connected_universes
CMB_Anisotropy_of_the_Poincare_Dodecahedron
other related studies from Prof. Frank Steiner : https://www.researchgate.net/profile/Frank_Steiner5

The large-scale structure of our universe : http://www.sun.org/images/structure-of-the-universe-1
The Millenium Simulation : https://wwwmpa.mpa-garching.mpg.de/galform/millennium/
see also : https://en.wikipedia.org/wiki/Galaxy_filament

Order-5 Dodecahedral Honeycomb Structure in hyperbolic space
https://en.wikipedia.org/wiki/Order-5_dodecahedral_honeycomb

A cosmic hall of mirrors - Jean-Pierre Luminet - https://arxiv.org/ftp/physics/papers/0509/0509171.pdf
News article : https://www.slideshare.net/Convergent_Technology/the-dodecahedron-universe
https://physicsworld.com/a/a-cosmic-hall-of-mirrors/

Cosmic microwave background map : https://en.wikipedia.org/wiki/Cosmic_microwave_background

Wilkinson Microwave Anisotropy Probe : https://en.wikipedia.org/wiki/Wilkinson_Microwave_Anisotropy_Probe

References : → Regarding the **Universal Physical Theory**

PHASE SPACES IN SPECIAL RELATIVITY : TOWARDS ELIMINATING GRAVITATIONAL SINGULARITIES
from PETER DANENHOWER → see weblink : https://arxiv.org/pdf/0706.2043.pdf

---

Unified Field Theory : https://en.wikipedia.org/wiki/Unified_field_theory

Number Theory as the Ultimate Physical Theory
by **I. V. Volovich** / Steklov Mathematical Institute - **Study** : http://cdsweb.cern.ch/record/179558/files/198708102.pdf

Space-Time-Matter – by Gerald E. Marsh : **Study :** https://arxiv.org/ftp/arxiv/papers/1304/1304.7766.pdf

Letters of Albert Einstein, including his letter to natural constants from 13th October 1945 in german language :
http://docplayer.org/69639849-Ilse-rosenthal-schneider-begegnungen-mit-einstein-von-laue-und-planck.html
see also : - description of the book contents in english : http://blog.alexander-unzicker.com/?p=27

Looking for those Natural Numbers Dimensionless Constants & the Idea of Natural Measurement
https://www.academia.edu/35881283/

Spectral-Temporal Correlations in the X-ray Emission of Cygnus X-1 (→ Study about the first confirmed black hole)
by Moritz Boeck - Uni Erlangen-Nuremberg → http://www.sternwarte.uni-erlangen.de/docs/theses/2008-10_Boeck.pdf

Continued fraction representation of the Fermi Dirac function for large scale electronic structure calculations
by Taisuke Ozaki – RICS, Ibariki, Japan - **Study :** http://www.openmx-square.org/tech_notes/CF_Fermi.pdf

Crystallography of Quasicrystals - by Walter Steurer & Sofia Deloudi - **Study :** http://www.xray.cz/kryst/kvazi.pdf

The Noncommutative Geometry of Aperiodic Solids – by Jean Bellissard – GIT-Mathematical Department (USA)
http://people.math.gatech.edu/~jeanbel/Publi/ncg02.pdf

Quasicrystals – by Uwe Grimm & Peter Kramer - **Study :** https://arxiv.org/pdf/1906.10392.pdf

Creating a new frontier through synergy of Quasicrystals and strongly correlated Electron Systems
https://www.tfc.tohoku.ac.jp/program/2156.html

Mysterious four dimensional Iron Oxide - https://phys.org/news/2016-04-mysterious-four-dimensional-iron-oxide.html

What goes on in a Proton ? Quark Math still conflicts with experiments - News article from the **QuantaMagazine**
https://www.quantamagazine.org/what-goes-on-in-a-proton-quark-math-still-conflicts-with-experiments-20200506/

Quark-gluon plasma paradox - by D. Miskowjec – Gesellschaft für Schwerionenforschung, Darmstadt, Germany
https://web-docs.gsi.de/~misko/noqgp.pdf

Tightly Circumscribed Regular Polygons – by Richard J. Mathar - https://arxiv.org/abs/1301.6293

About the logic of the prime number distribution - by Harry K. Hahn : https://arxiv.org/abs/0801.4049

The golden ratio Phi (φ) in Platonic Solids : http://www.sacred-geometry.es/?q=en/content/phi-sacred-solids

Extracts from the following studies (presented in Chapter 8.8) :

Stressed Fibonacci spiral patterns of definite chirality - by Chaorong Li ; Ailing Ji & Zexian Cao
http://surface.iphy.ac.cn/sf03/articles/2006-2007/2007APL-chirality.pdf

Cubic approximants in quasicrystal structures – by V. E. Dimitrienko
https://hal.archives-ouvertes.fr/jpa-00212565/document

Fibonacci order in the period-doubling cascade to chaos - by G.Linage, Fernando Montoya, A. Samiento…
https://www.researchgate.net/publication/252736598_Fibonacci_order_in_the_period-doubling_cascade_to_chaos

The Planar Double Pendulum - by Peter H. Richter & Hans-Joachim-Scholz
http://www.itp.uni-bremen.de/prichter/download/DoppelpendelIWF.pdf ; → **Movie :** https://av.tib.eu/media/14902

Orbital Period Ratios and Fibonacci Numbers in Solar Planetary and Satellite Systems and in Exoplanetary Systems – by Vladimir Pletser - weblink : https://arxiv.org/pdf/1803.02828

Changes in phyllotactic pattern structure (Fibonacci Sequences) in Pinus mugo due to changes in altitude
Longterm botanical research study by **Dr. Iliya Iv. Vakarelov**, University of Forestry, Bulgaria (1982-1994)

From the book **„Symmetry in Plants"** by Roger V. Jean and Denis Barabe, Universities of Quebec and Montreal, Canada (Part I. – Chapter 9 , pages 213 – 229) – 1998 by World Scientific Publishing , **ISBN : 981-02-2621-7**

Further interesting studies which are connected to the presented discoveries :

Microscope Images indicate that Water Clusters are the cause of Phyllotaxis - by Harry K. Hahn
https://vixra.org/abs/2005.0118
alternative : https://archive.org/details/microscope-images-indicate-that-water-clusters-are-the-cause-of-phyllotaxis

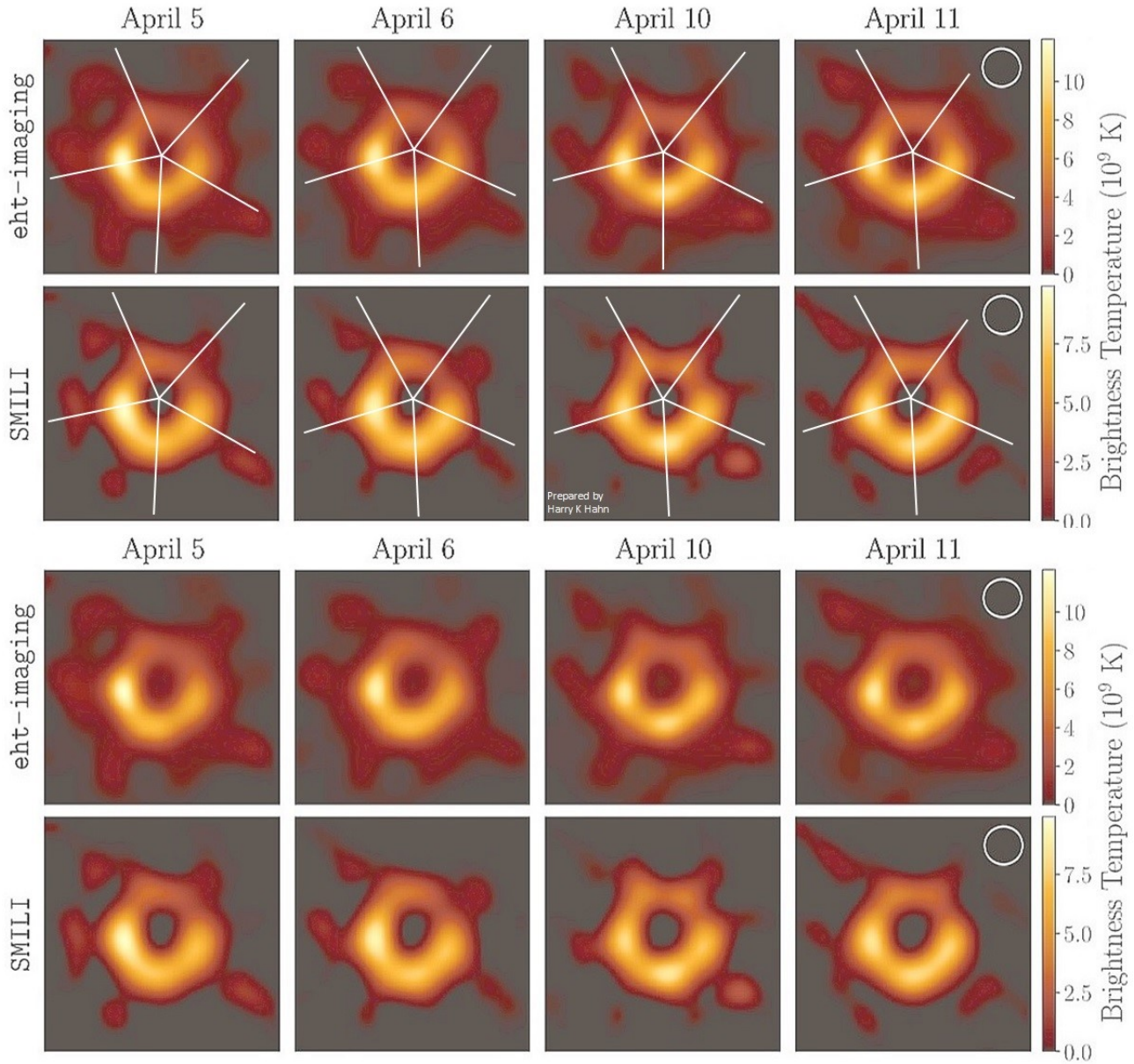
Creation of an infinite Fibonacci Number Sequence Table - by Harry K. Hahn
https://vixra.org/abs/2008.0027
alternative : https://archive.org/details/AnInfiniteFibonacciNumberSequenceTable

The Ordered Distribution of Natural Numbers on the Square Root Spiral - by Harry K. Hahn
http://front.math.ucdavis.edu/0712.2184 **PDF :** http://arxiv.org/pdf/0712.2184

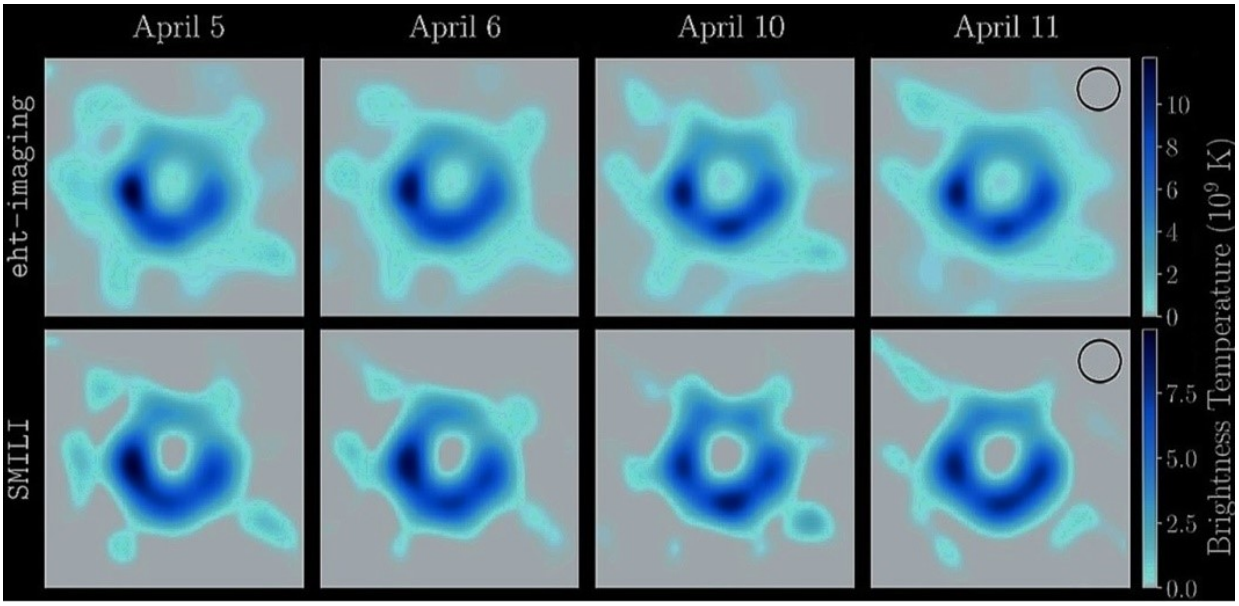
The Distribution of Prime Numbers on the Square Root Spiral – by Harry K. Hahn
http://front.math.ucdavis.edu/0801.1441 **PDF :** http://arxiv.org/pdf/0801.1441

<u>Appendix 1</u> : Here are the re-processed images of the EHT2017 fiducial image sequence :

The image below was in steps contrast enhanced, brightness was slightly increased, a gamma correction was carried out and the color green was slightly increased



The image below is just the inverted color image of the above shown image :



Appendix 2 : → **Here the calculations from Chapter 8**

With the algebraic term of constant u we can calculate all square roots of all natural numbers expressed only by constant φ and 1 :

$$\sqrt{\varphi^2 - 2} = \frac{\sqrt{2\sqrt{5} - 2}}{2} \quad ; \quad \text{we equate the two algebraic terms which represent the same constant !}$$

$$\rightarrow \quad 4\varphi^2 - 8 = 2\sqrt{5} - 2 \quad ; \quad \text{we square both sides and transform}$$

$$\varphi^2 = \frac{\sqrt{5} + 3}{2} \quad ; \quad (1) \quad \text{we solve for } \varphi^2$$

$$\sqrt{5} = 2\varphi^2 - 3 \quad ; \quad (2) \quad \text{we solve for } \sqrt{5}$$

Now we go back to the square root spiral and use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem}$$

$$6 = (2\varphi^2 - 3)^2 + 1 \quad ; \quad \text{we replace } \sqrt{5} \text{ by equation (2) and transform}$$

$$\rightarrow \quad 3 = \frac{\varphi^4 + 1}{\varphi^2} \quad (3) \quad \rightarrow \quad \sqrt{3} = \sqrt{\frac{\varphi^4 + 1}{\varphi^2}} \quad (4) \quad ; \text{ square root 3 expressed by } \varphi \text{ and 1}$$

Now we use the following right triangle :

$$(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem \& inserting equation (3)}$$

$$\rightarrow \quad 2 = \frac{\varphi^4 + 1}{\varphi^2} - 1 \quad \rightarrow \quad 2 = \frac{\varphi^4 - \varphi^2 + 1}{\varphi^2} \quad (5) \text{ and } \sqrt{2} = \sqrt{\frac{\varphi^4 - \varphi^2 + 1}{\varphi^2}} \quad (6)$$

Now we insert equation (3) in equation (2) : ; square root 2 expressed by φ and 1

$$\rightarrow \quad \sqrt{5} = 2\varphi^2 - \frac{\varphi^4 + 1}{\varphi^2} \quad \rightarrow \quad \sqrt{5} = \frac{\varphi^4 - 1}{\varphi^2} \quad ; \quad (7) \quad ; \quad \rightarrow \quad 5 = \left(\varphi^2 - \frac{1}{\varphi^2}\right)^2$$

square root 5 expressed by φ and 1

Now we use the following right triangle :

$$(\sqrt{6})^2 = (\sqrt{5})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem \& inserting equation (7)}$$

$$\rightarrow \quad 6 = \left(\frac{\varphi^4 - 1}{\varphi^2}\right)^2 + 1 \quad \rightarrow \quad 6 = \frac{\varphi^8 - \varphi^4 + 1}{\varphi^4} \quad (8) \text{ and } \sqrt{6} = \sqrt{\frac{\varphi^8 - \varphi^4 + 1}{\varphi^4}} \quad (9)$$

We can now continue and use the following right triangles of the square root spiral :

$$(\sqrt{7})^2 = (\sqrt{6})^2 + 1^2 \quad ; \quad \text{application of the Pythagorean theorem \& inserting equation (8)}$$

$$\rightarrow \quad 7 = \frac{\varphi^8 + 1}{\varphi^4} \quad (10) \quad \rightarrow \quad \sqrt{7} = \sqrt{\frac{\varphi^8 + 1}{\varphi^4}} \quad (11) \quad \rightarrow \quad 7 = \varphi^4 + \frac{1}{\varphi^4}$$

In the same way we can now calculate all square roots of all natural numbers with the next right triangles :

$$\rightarrow \quad 8 = \frac{\varphi^8 + \varphi^4 + 1}{\varphi^4} \quad (12) \text{ and } \sqrt{8} = \sqrt{\frac{\varphi^8 + \varphi^4 + 1}{\varphi^4}} \quad (13)$$

$$\rightarrow \quad 10 = \frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4} \quad (14) \text{ and } \sqrt{10} = \sqrt{\frac{\varphi^8 + 3\varphi^4 + 1}{\varphi^4}} \quad (15)$$

$$\rightarrow \quad 11 = \frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4} \quad (16) \text{ and } \sqrt{11} = \sqrt{\frac{\varphi^8 + 4\varphi^4 + 1}{\varphi^4}} \quad (17)$$

$$\rightarrow \quad 12 = \frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4} \quad (18) \text{ and } \sqrt{12} = \sqrt{\frac{\varphi^8 + 5\varphi^4 + 1}{\varphi^4}} \quad (19)$$

From the above shown formulas (equations 3 to 19) we can read a general rule for all natural numbers > 10 :

Note : → The expression (3+n) in the rule can be replaced by products or sums of the equations (3) to (13)

$$\rightarrow \quad (10 + n) = \frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4} \quad (20) \text{ and } \sqrt{(10 + n)} = \sqrt{\frac{\varphi^8 + (3+n)\varphi^4 + 1}{\varphi^4}} \quad (30)$$

For $n \rightarrow \infty$

With these formulas we can express all natural numbers and their square roots only with φ and 1 ! This is a very interesting discovery, because it allows to describe probably most (if not all) geometrical objects only with φ and 1 !

If we transfom the equations (3) to (19) into the standard-form for polynomials then we get the following equations :

$$0 = \varphi^4 - 3\varphi^2 + 1 \quad (40) \qquad \text{or} \qquad 0 = \varphi^8 - 7\varphi^4 + 1 \quad (50)$$

Appendix 3 : Additional information to Study 6 described in the main document (carried out by myself)

➔ **To the periodicity of the prime factors of the Fibonacci-Numbers from selected Sequences**

Note: The numbers of the Fibonacci F1 – Number Sequence seem to contain all prime numbers as prime factors !
and all prime factors appear periodic in defined "number-distances" in the sequence (see left side of table)

Table 2 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F1** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F1			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		F	F'	F''	Nr.
															1	1			1
															1	1			2
															2	2	1		3
															3	3	1		4
															5	5	2	1	5
												2^3		2x2x2	8	8	3	1	6
															4	13	5	2	7
									7		3			3x7	3	21	8	3	8
						17						2		2x17	7	34	13	5	9
								11		5				5x11	10	55	21	8	10
															17	89	34	13	11
											3^2	2^4	2x2x2	2x3x3	9	144	55	21	12
															8	233	89	34	13
			29				13							13x29	17	377	144	55	14
										5		2		2x5x61	7	610	233	89	15
									7		3		3x7x	47	24	987	377	144	16
															22	1597	610	233	17
					19	17						2^3	2x17x	2x2x19	19	2584	987	377	18
	37													37x113	14	4181	1597	610	19
41								11		5	3		5x11x	3x41	24	6765	2584	987	20
							13					2		2x13x421	20	10946	4181	1597	21
														89x199	17	17711	6765	2584	22
															28	28657	10946	4181	23
				23					7		3^2	2^5	2x2x2x2x3x3x	2x7x23	27	46368	17711	6765	24
										5^2				5x5x3001	19	75025	28657	10946	25
														233x521	19	121393	46368	17711	26
						17						2		2x17x53x109	29	196418	75025	28657	27
			29				13				3		13x29x	3x281	21	317811	121393	46368	28
															23	514229	196418	75025	29
		31						11		5		2^3	2x5x61x	2x2x11x31	17	832040	317811	121393	30
														557x2417	31	1346269	514229	196418	31
									7		3		3x7x47x	2207	30	2178309	832040	317811	32
												2		2x89x19801	34	3524578	1346269	514229	33
														1597x3571	37	5702887	2178309	832040	34
							13			5				5x13x141961	35	9227465	3524578	1346269	35
					19	17					3^3	2^4	2x2x2x17x19x	2x3x3x3x107	27	14930352	5702887	2178309	36
														73x149x2221	35	24157817	9227465	3524578	37
	37												37x113x	9349	44	39088169	14930352	5702887	38
												2		2x233x135721	43	63245986	24157817	9227465	39
41								11	7	5	3		3x5x11x41x	7x2161	24	102334155	39088169	14930352	40
														2789x59369	31	165580141	63245986	24157817	41
			29				13					2^3	2x13x421x	2x2x29x211	46	267914296	102334155	39088169	42
															41	433494437	165580141	63245986	43
											3		89x199x	3x43x307	33	701408733	267914296	102334155	44
						17				5		2		2x5x17x61x109441	29	1134903170	433494437	165580141	45
														139x461x28657	35	1836311903	701408733	267914296	46
															37	2971215073	1134903170	433494437	47
				23					7		3^2	2^6	2x2x2x2x2x3x3x7x23x	2x47x1103	54	4807526976	1836311903	701408733	48

Note : all prime numbers are marked in yellow and all numbers not divisible by 2, 3 or 5 are marked in orange

Note: **In the numbers of the Fibonacci F2 – Number Sequence some prime numbers, as prime factors, are missing!**

Table 3 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F2 (Lucas)** - Number Sequence :

some prime factors shown in table form													in prime factors factorized Fibonacci-Numbers		sum of digits	Fibonacci-Sequence F2 (Lucas-Sequence)			
41	37	31	29	23	19	17	13	11	7	5	3	2	repeating products	new products		L	L'	L"	No.
	Prime factor is missing					Prime factor is missing	Prime factor is missing			Prime factor is missing					1	1			1
															3	3			2
												2^2		2x2	4	4	1		3
															7	7	3		4
															4	11	4	1	5
											3^2	2		2x3x3	9	18	7	3	6
															11	29	11	4	7
															11	47	18	7	8
					19							2^2	2x2x	19	13	76	29	11	9
41											3			3x41	6	123	47	18	10
															19	199	76	29	11
				23					7			2		2x7x23	7	322	123	47	12
															8	521	199	76	13
											3			3x281	15	843	322	123	14
		31						11				2^2		2x2x11x31	14	1364	521	199	15
															11	2207	843	322	16
															16	3571	1364	521	17
											3^3	2	2x3x3x	3x107	27	5778	2207	843	18
															25	9349	3571	1364	19
									7					7x2161	16	15127	5778	2207	20
			29									2^2		2x2x29x211	23	24476	9349	3571	21
											3			3x43x307	21	39603	15127	5778	22
														139x461	26	64079	24476	9349	23
												2		2x47x1103	20	103682	39603	15127	24
								11						11x101x151	28	167761	64079	24476	25
											3			3x90481	21	271443	103682	39603	26
					19							2^2	2x2x19x	5779	22	439204	167761	64079	27
									7					7x7x14503	25	710647	271443	103682	28
														59x19489	29	1149851	439204	167761	29
41											3^2	2	3x41x	2x3x2521	36	1860498	710647	271443	30
															20	3010349	1149851	439204	31
														1087x4481	38	4870847	1860498	710647	32
												2^2		2x2x199x9901	40	7881196	3010349	1149851	33
											3			3x67x63443	24	12752043	4870847	1860498	34
			29					11						11x29x71x911	28	20633239	7881196	3010349	35
				23					7			2	2x7x23x	103681	34	33385282	12752043	4870847	36
															26	54018521	20633239	7881196	37
											3			3x29134601	33	87403803	33385282	12752043	38
												2^2		2x2x79x521x859	23	141422324	54018521	20633239	39
														47x1601x3041	38	228826127	87403803	33385282	40
															34	370248451	141422324	54018521	41
											3^2	2	3x281x	2x3x83x1427	54	599074578	228826127	87403803	42
														6709x144481	43	969323029	370248451	141422324	43
									7					7x263x881x967	52	1568397607	599074578	228826127	44
		31			19			11				2^2	2x2x11x31x	19x181x541	41	2537720636	969323029	370248451	45
											3			3x4969x275449	30	4106118243	1568397607	599074578	46
															62	6643838879	2537720636	969323029	47
												2		2x769x2207x3167	47	10749957122	4106118243	1568397607	48
			29											29x599786069	46	17393796001	6643838879	2537720636	49
41											3			3x41x401x570601	39	28143753123	10749957122	4106118243	50

Note : **all prime numbers are marked in yellow** **and all numbers not divisible by 2, 3 or 5 are marked in orange**

Note: **In the numbers of the Fibonacci F6 – Number Sequence some prime numbers, as prime factors, are missing!**

Table 4 : Periodicity of some of the prime factors of the numbers of the **Fibonacci F6** - Number Sequence :

Periodicity of the prime factors **2 - 41** shown in table form													in prime factors factorized Fibonacci-(F6)-Numbers	sum of digits	**Fibonacci-F6 Sequence**			
41	37	31	29	23	19	17	13	11	7	5	3	2			F6	F6'	F6''	Nr.
???			prime factor is missing			prime factor is missing	prime factor is missing	prime factor is missing							1			1
												2^2	2x2		4			2
															5	1		3
											3^2		3x3		9	4		4
									7			2	2x7		14	5	1	5
															23	9	4	6
															37	14	5	7
										5	3	2^2	2x2x3x5		60	23	9	8
															97	37	14	9
															157	60	23	10
												2	2x127		254	97	37	11
											3		3x137		411	157	60	12
					19				7	5			5x7x19		665	254	97	13
												2^2	2x2x269		1076	411	157	14
															1741	665	254	15
											3^2		3x3x313		2817	1076	411	16
												2	2x43x53		4558	1741	665	17
										5^3			5x5x5x59		7375	2817	1076	18
															11933	4558	1741	19
											3	2^2	2x2x3x1609		19308	7375	2817	20
									7				7x4463		31241	11933	4558	21
															50549	19308	7375	22
										5		2	2x5x8179		81790	31241	11933	23
		31									3		3x31x1423		132339	50549	19308	24
															214129	81790	31241	25
	37											2^2	2x2x37x2341		346468	132339	50549	26
															560597	214129	81790	27
										5	3^2		3x3x5x6719		907065	346468	132339	28
									7			2	2x7x79x1327		1467662	560597	214129	29
				23									23x223x463		2374727	907065	346468	30
					19								19x202231		3842389	1467662	560597	31
											3	2^2	2x2x3x379x1367		6217116	2374727	907065	32
										5			5x227x8863		10059505	3842389	1467662	33
															16276621	6217116	2374727	34
												2	2x641x20543		26336126	10059505	3842389	35
											3		3x1637x8677		42612747	16276621	6217116	36
									7				7x181x54419		68948873	26336126	10059505	37
										5		2^2	2x2x5x5578081		111561620	42612747	16276621	38
															180510493	68948873	26336126	39
											3^2		3x3x32452457		292072113	111561620	42612747	40
												2	2x1109x213067		472582606	180510493	68948873	41
													67x2083x5479		764654719	292072113	111561620	42
										5^2			5x5x49489493		1237237325	472582606	180510493	43
											3	2^2	2x2x3x53x3147629		2001892044	764654719	292072113	44
	37								7^2				7x7x37x1786613		3239129369	1237237325	472582606	45
													71x3613x20431		5241021413	2001892044	764654719	46
												2	2x167x3607x7039		8480150782	3239129369	1237237325	47
										5	3		3x5x914744813		13721172195	5241021413	2001892044	48
					19								19x83x14078201		22201322977	8480150782	3239129369	49
												2^2	2x2x337x2664083		35922495172	13721172195	5241021413	50
													129631x448379		58123818149	22201322977	8480150782	51
											3^2		3x3x2671x3912239		94046313321	35922495172	13721172195	52
									7	5		2	2x5x7x2173859021		152170131470	58123818149	22201322977	53
		31		23									23x31x345324607		246216444791	94046313321	35922495172	54
															398386576261	152170131470	58123818149	55

Note : **all prime numbers are marked in yellow** **and all numbers not divisible by 2, 3 or 5 are marked in orange**

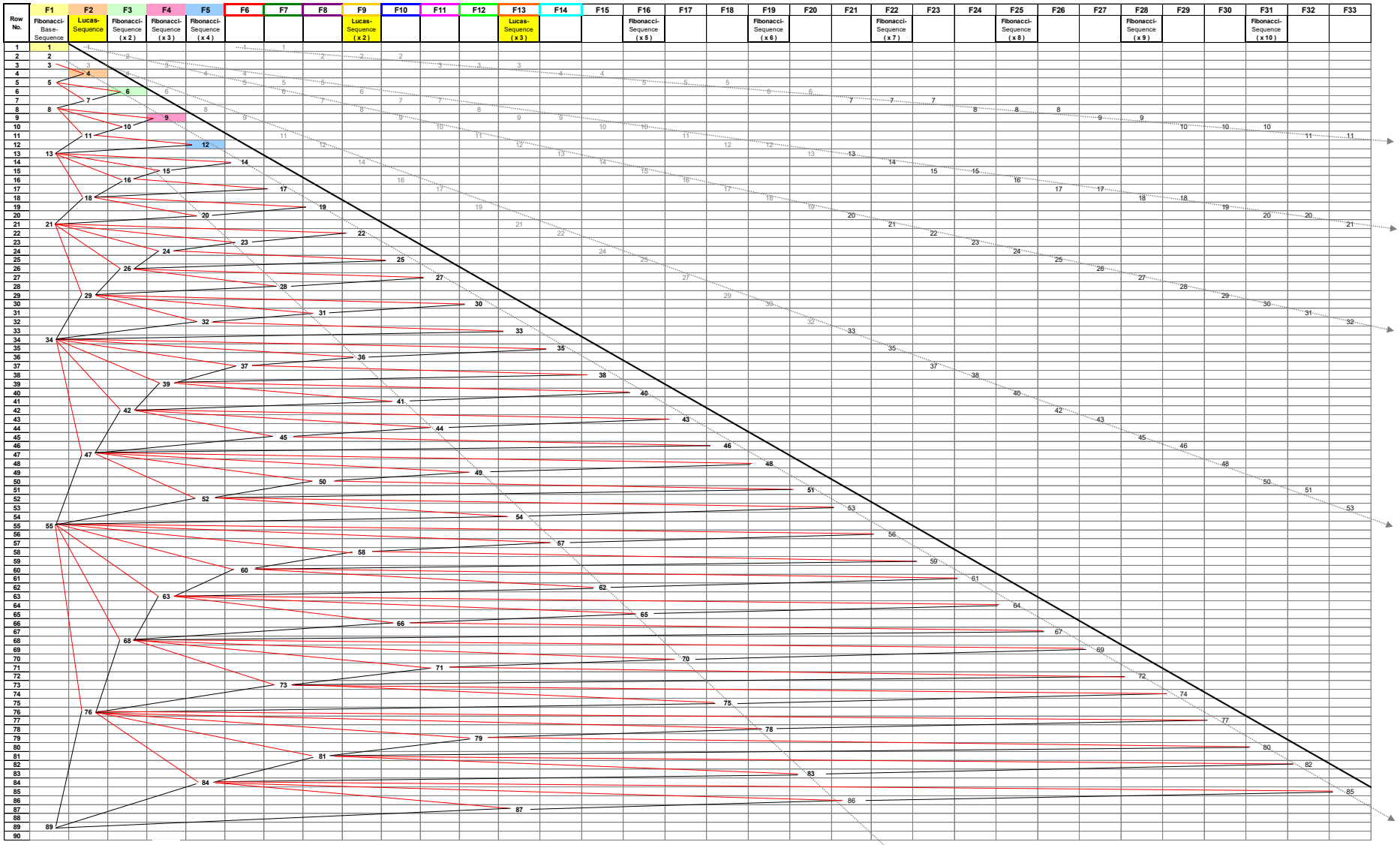
Appendix 4 : Infinite Fibonacci Number Sequence Table : Sequences No. 1 to 33 shown (F1 – F33) : → Weblink to the explanatory Study - by Dipl. Ing.(FH) Harry K. Hahn

Abstract :

A Fibonacci-Number-Sequences-Table was developed, which contains infinite Fibonacci-Sequences. This was achieved with the help of research results from an extensive botanical study. This study examined the phyllotactic patterns (Fibonacci-Sequences) which appear in the tree-species "Pinus mugo" at different altitudes (from 550m up to 2500m) With the increase of altitude above around 2000m the phyllotactic patterns change considerably, the number of patterns (different Fibonacci Sequences) grows from 3 to 12, and the relative frequency of the main Fibonacci Sequence decreases from 88 % to 38 % The appearance of more Fibonacci-Sequences in the plant clearly is linked to environmental (physical) factors changing with altitude. Especially changes in temperature- / radiation- conditions seem to be the main cause which defines which Fibonnacci-Patterns appear in which frequency.

The developed (natural) Fibonacci-Sequence-Table shows interesting spatial dependencies between numbers of different Fibonacci-Sequences, which are connected to each other, by the golden ratio (constant Phi). In botany Phyllotaxis describes the arrangement of leaves on spiral paths on a plant's stem. Phyllotactic spirals form a distinctive class of patterns in nature. But the true cause of these phyllotactic spirals , which appear everywhere in nature, still isn't found yet ! → Please read my own **hypothesis** : → **Microscope Images indicate that Water Clusters are the cause of Phyllotaxis** (Weblink 2)

FIBONACCI - Number Sequences No. 1 - 33 (F1 - F33)



Meaning of the line colors :

For 3 numbers A, B and C in the shown arrangement the following is true :

The ratio of the difference (C-A) indicated by a "red line" to the difference (B-C) indiated by a "black line" is approaching the golden ratio φ for the further progressing number sequences (which contain these numbers) towards infinity (->downwards).

$$\frac{C - A}{B - C} \rightarrow \varphi \quad \text{for} \quad A, B, C \rightarrow \infty$$