Testing the number of space-time dimensions by the 5.9 years repeated Millikan's oil drop experiments

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(June 1, 2020)

Abstract. The basic aim of the 4+D-dimensional Kaluza–Klein theory is the unification of gravity and electromagnetism. A feature of unification theories is the relation between electromagnetic coupling, e^2, gravitational coupling, G_N, and the radius of the fifth dimension, R_c. The radius of the fifth dimension, R_c, is fixed by the elementary electric charge. From the known value of the elementary charge, we find that R_c is of the order of the Planck length. At energies well below the Planck energy, massive Kaluza-Klein states are extreme black holes. This can describe stable elementary particles (M.J.Duf, 1994). Based on this interpretation, we show that, if the observed harmonic pattern of the laboratory-measured values of G_N is due to environmental or theoretical errors, these errors must also affect the elementary electric charge, e. We calculate the fundamental electric charge (e) values predicted by a 4- and a 4+D-dimensional space-time model. We find that, in the case of 4+D dimensions, the fundamental electric charge (e) value oscillates with the 5.9-year LOD oscillation cycle. In the case of 4-space-time dimensions, the fundamental electric charge, e, is constant. Furthermore, we propose an automated Millikan oil drop experiment over 5.9 years, to discriminate between 4 and 4+D space-time dimensions. The comparison between the drop distribution predicted by the 4+D-dimensional space-time model and the Stanford Linear Accelerator results is briefly discussed.

PACS numbers: 04.60.-m, 14.80.-j, 04.80.-y
Key-words: Extra Dimensions, Quantum gravity, Experimental studies of gravity, Atomic and Molecular Physics

1. Introduction

Newton's gravitational constant, *G*, has been measured about a dozen times over the last 40 years. Recently, John D. Anderson and coauthors [1] found that the measured *G* values oscillate over time like a sine wave with a period of 5.9 years. They propose that this oscillation of measured G values does not register variation of G itself, but rather the effect of unknown factors on the measurements [46]. C.S.Unnikrishnan [47] provides a possible explanation to the 5.9-year period of G values oscillation by the gravitational link between the

Earth and Jupiter: there is a 5.9 year periodicity in the length of the Earth's and an 11.86 year periodicity in Jupiter's around the Sun.

Klein (2015[2]) suggests that the observed discrepancies between G values determined in different experiments may be associated with a differential interpretation of Modified Newtonian Dynamics (MOND) theory applied to the galaxy rotation curves. Recent quantitative analysis (Lorenzo Iorio, 2015[3]) rules out the possibility that the harmonic pattern observed in laboratory-measured values of G_N is due to some long-range modification of the currently accepted laws of gravitational interaction. This analytical approach may guide future investigations of the systematic uncertainties that plague measurements of G_N.

Based on Symbolic Gauge Theory (SGT), a formalism applied to General Relativity (GR) by R. Mignani, E. Pessa and G. Resconi [4,5] and further developed by I. Licata and G. Resconi, M.E. Rodrigues and E.Koorambas [8,9,10], the E. Koormabas and G. Resconi recently proposed a Non-Conservative Theory of Gravity (NCTG) which can explain the observed variations of G at a 5.9-year scale [9].

The strength of the gravitational force depends on the scale at which the latter is measured by Cavendish-type experiments where two masses (one of which is a test mass) are precisely known, or by (equivalent in principle) gravitational scattering experiments [11]. At laboratory scales, the strength of gravity is characterized by the reduced Planck mass $M_{\mathrm{pl}} = 2.435 \times 10^{18}$ GeV, which determines Newton's constant $G_N = M_{pl}^{-2}$. Conventionally, the Planck scale M_{Pl} is interpreted as the fundamental scale at which quantum gravitational effects become important in nature. Like all other interactions in nature, nevertheless, the effective strength of gravity is affected by quantum corrections. This effect depends on the characteristic energy of the process probing gravitational interactions (see [12,13] for reviews of an effective theory of gravity). Potential problems of running gravitational couplings by focusing only on physically observable quantities (e.g. amplitudes, cross sections) are discussed in [14,15]. New approaches to the physics of particles with masses greater than 1TeV could offer insights to the problem of the variation of measured G_N values. In such models there is no hierarchy problem [16], whereas quantum gravity can be assessed through experiments at TeV energy levels. That this can be the case in extra-dimensional models is already established [17,18]. Is such modification of gravity also possible in four dimensions [19,39]. Current data from the Large Hadron Collider (LHC) experiments at the European Laboratory for Particle Physics (CERN) do not confirm that gravity becomes stronger around 1 TeV [40-44].

Recently, E.Koorambas suggested that if the observed harmonic pattern of the laboratory-measured values of G is due to some environmental or theoretical errors, these errors must also affect the true value of momentum k transferred by the graviton in scattering experiments at the LHC [45]. Furthermore, environmental or theoretical errors could shift the scale of Quantum gravity at 100TeV. Quantum gravity can be investigated by a 100 TeV Proton-Proton Collider as long as environmental or theoretical errors are present. This proposition may explain the current null results for black hole production at the LHC [45].

Although our world appears to consist of 3+1 dimensions (three dimensions of space; and time), it is possible that other dimensions exist, and that these appear at higher energy scales. From the point of view of physics, the concept of extra dimensions received great attention after Kaluza's proposition, in 1921[48], to unify electromagnetism with gravity by identifying the extra components of the metric tensor with the usual gauge fields.

No experimental or observational signs of extra dimensions have been reported. Many theoretical techniques for detecting Kaluza–Klein resonances by means of their mass coupling with the top quark have been proposed. However, until the LHC reaches full operational power, observation of such resonances is unlikely. An analysis of results from the LHC in December 2010 severely constrains theories with large extra dimensions. [49]

The observation of a Higgs-like boson at the LHC establishes a new empirical test that can be applied to the search for Kaluza–Klein resonances and supersymmetric particles. The loop Feynman diagrams for the Higgs interactions allow any particle with electric charge and mass to run in such a loop. Standard Model particles other than the top quark and W boson do not make large contributions to the cross-section observed in the H → γγ decay. However, if there are new particles beyond those predicted by the Standard Model, they could potentially change the ratio of the predicted Standard Model H → γγ cross-section to the experimentally observed cross-section. A measurement of any change to the H → γγ cross-section predicted by the Standard Model is, therefore, crucial in probing the physics beyond the Standard Model.

A recent paper (July 2018 [50]), bodes well for this theory: in this, the authors dispute that gravity is leaking into higher dimensions as in brane theory. However, the paper does demonstrate that electromagnetism and gravity share the same number of dimensions. This lends support to the Kaluza–Klein theory, regardless of whether the number of dimensions is 3+1 or in fact 4+D. The number D of compact extra dimensions is subject to further debate.

2. The compact extra dimensions' hypothesis

The initial theory has five-dimensional general coordinate invariance. However, it is assumed that one of the spatial dimensions compactifies, so as to have the geometry of a circle S^1 of very small radius [48,51,52]. Then, there is a residual four-dimensional general coordinate invariance, and an Abelian gauge invariance associated with transformations of the coordinate of the compact manifold, S^1[48,51,52]. Put another way, the original five-dimensional general coordinate invariance is breaks spontaneously in the ground state. In this way, we arrive at an ordinary theory of gravity in four dimensions and a theory of an Abelian gauge field A_μ . The parameters of the two theories are connected because both theories derive from the same initial five-dimensional Einstein gravity theory [48,51,52].

We adopt the coordinates x^m, m = 1, 2,5 with

$$x^m = (x^\mu, x^5) \quad , \tag{1}$$

where

$$\bar{x}^\mu = x^\mu, \mu = 0,1,2,3 \tag{2}$$

being coordinates for ordinary four-dimensional space-time, and

$$\bar{x}^5 = \theta \tag{3}$$

being an angle to parametrize the compact dimension with the geometry of a circle S^1.

The line element is given by

$$d\bar{s}^2 = \bar{g}_{mn} dx^m dx^n \quad , \tag{4}$$

where m,n=1,2,..,5,and $\bar{g}_{mn}$ is the five-dimensional metric.

The five-dimensional Einstein equations yields the following results:

a) For $\bar{g}_{\mu\nu} = g_{\mu\nu}$, the four-dimensional Einstein equations for gravity;

b) For $\bar{g}_{\mu 5} = A_{\mu}$, the Maxwell equations for electromagnetism;

c) For $\bar{g}_{55} = \phi(x)$, the Klein-Gordon equation.

A feature of these theories is the relation between electromagnetic coupling, e^2, gravitational coupling, G_N and p^5, the momentum of the particle in the fifth dimension:

$$e^2 = 16\pi G_N (p^5)^2 . \tag{5}$$

Following [82], by applying the old Bohr–Sommerfeld quantization rule to the periodic motion, $2\pi r(p_5)_0 = 2\pi n\hbar$, we deduce that, $(p^5)_0 = (n / R_c)\hbar$. This implies that

$$q_n^2 = n^2 e^2 = n^2 \hbar^2 \frac{16\pi G_N}{R_c^2} . \tag{6}$$

The radius of the fifth dimension R_c is thus fixed by the elementary electric charge. From the known value of the elementary charge, we find that R_c is of the order of the Planck length:

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}, \qquad R_c = \frac{2}{\sqrt{\alpha}} \sqrt{\frac{2\pi G_N \hbar}{8\pi c}} = 3.7\times 10^{-32} cm \text{ [82]}. \tag{7}$$

Many years ago [53,54], it was pointed out that the field equations of N = 1 supergravity in d = 11 dimensions admit vacuum solutions corresponding to AdS x S^7, and that, since S^7 admits 8-Killing spinors, and since its isometry group is SO(8), this gives rise (via a Kaluza—Klein mechanism) to an effective d = 4 theory with N = 8 supersymmetry and local SO(8) invariance. There is now a considerable literature on S^7 compactification of d = 11 supergravity [53-59]. An up to date account, paying particular attention to the Brout—Englert--Higgs—Kibble spontaneous symmetry breaking interpretation of the different S^7 solutions, [57].

In the case of S^7 compactification of d = 11 supergravity, $R_c = m^{-1}$ is just the S^7 radius. However, for more complicated geometries one must be more precise about the meaning of R_c. Weinberg [58] has shown how this is done for an arbitrary geometry with Killing vectors in terms of appropriate root-mean-square circumferences. The precise constants of proportionality in (5) depend crucially on the field content of the higher-dimensional theory. Although at the classical level the "size" of S^7 is undetermined, Candelas and Weinberg [59] have pointed out that in a certain class of theories admitting a compactification due to one-loop radiative corrections one may calculate R, and hence, in a realistic theory, the fine structure constant $\alpha = e^2 / 4\pi$.

3. The sinusoidal variations of Newton's coupling constant

Measurements of the gravitational constant (G) are notoriously difficult due to the gravitational force being by far the weakest of the four known forces. Recent advances, making use of electronically controlled torsion strip balances at the *Bureau International des Poids et Mesures* (BIPM) in the last 15 years, have improved the accuracy of G measurements (see [20] for details on experimental methods). These recent measurements have also revealed a peculiar type of oscillatory variation, seemingly following a 5.9 years cycle akin to the so called Length-of-Day (LOD) [1].

Although we recognize that the correlation between G measurements and the 5.9 year LOD cycle could be fortuitous, we think that this is unlikely, given the striking match between these two (Fig. 1).

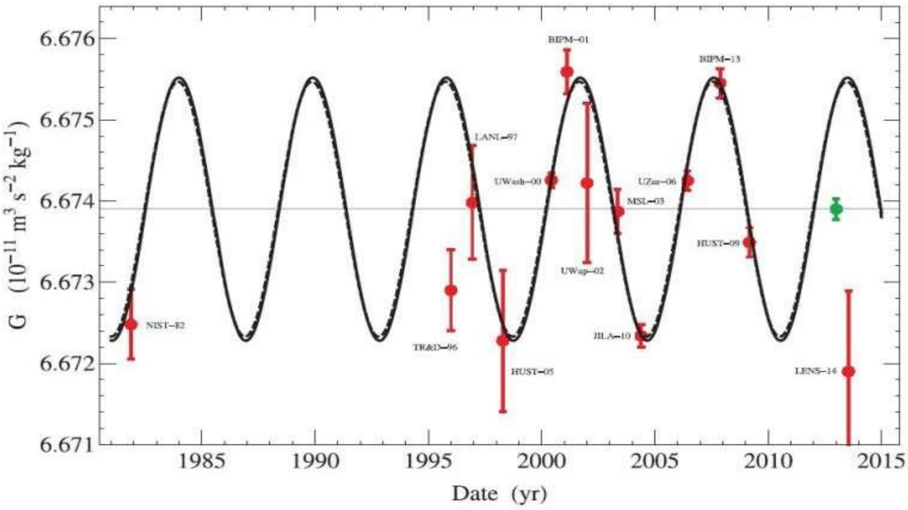


Fig. 1: Comparison of the CODATA set of G measurements) with a fitted sine wave (solid curve) and the 5.9 year oscillation in LOD daily measurements (dashed curve), scaled in amplitude to match the fitted G sine wave. Acronyms for the measurements follow the CODATA convention. Also included are a relatively recent BIPM result from Quinn *et al.* [21] and measurement LENS-14 from the MAGIA collaboration [22] that uses a new technique of laser-cooled atoms and quantum interferometry, rather than the macroscopic masses of all the other experiments. The green filled circle represents the weighted mean of the included measurements, along with its one-sigma error bar, determined by minimizing the L1 norm for all 13 points and taking into account the periodic variation.

The observed correlation cannot be due to centrifugal force acting on the experimental apparatus, since changes in LOD are too small by a factor of about 10^5 to explain changes in G. This is because the Earth's angular velocity ω_E is by definition

$$\omega_E = \omega_0 \left(1 - LOD\right), \tag{8}$$

where ω_0 is an adopted sidereal frequency equal to 72921151.467064 prad s^{-1} and the LOD is in ms d^{-1} (www.iers.org). The total centrifugal acceleration is given by:

$$a_c = r_s \omega_0^2 \left[1 - 2A\sin\left(\frac{2\pi}{P}(t - t_0)\right)\right], \tag{9}$$

where A is the amplitude of the 5.9 year sinusoidal LOD variation (= 0.000150/86400), and r_s is the distance of the apparatus from the Earth's spin axis. The maximum percentage variation of the LOD term is 3.47×10^{-9} of the steady-state acceleration, while dG/G is 2.4×10^{-4}. Even the full effect of the acceleration with no experimental compensation changes G by only 10^{-5} of the amplitude shown in Fig. 1.
Following Anderson et al. 2015a [1], the shift from the true value of renormalized gravitational constant is given by:

$$\begin{aligned} G(t)_{ren}^{shift} &= G_{ren} + \delta G(t)_{ren}^{Error} = G_{ren} + B_G \sin(a_G t + \varphi) \\ &= G_{ren} + 2G_{ren} A_G \sin(a_G t + \varphi), \end{aligned} \tag{10}$$

where

$$B_G = 2G_{ren}A_G \ , \tag{11}$$

and

$$A_G = 10^{-4}, \varphi = \ 80.9\ deg, a_G = 2\pi / P_G, P_G = \ 5.899\ yr. \text{ (Anderson et al. 2015a) [1].} \tag{12}$$

Here, the variation term due to environmental or theoretical errors $\delta G(t)_{ren}^{Error}$ in equation (10) is given by

$$\begin{aligned} &\delta G(0,t)_{ren}^{error} = G_{ren} f(t)^{error}, \\ &f(t)^{error} = 2A_G \sin(a_G t + \varphi) \end{aligned} \tag{13}$$

where G_{ren} is the renormalized gravitational constant and $f(t)_{error}$ is the environmental or theoretical error.

4. Gravitational action in the presence of environmental or theoretical errors

A scalar-tensor theory of gravity (STG), first proposed by Brans and Dicke [66], was inspired by Dirac's suggestion that the gravitational constant G_N varies with time [67]. In the Scalar-tensor theories of gravity, the gravitational action can be written:

$$S = \int d^n x \sqrt{-g} \left(-\frac{R}{16\pi G_N} f(\phi) + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) + F[\phi, g_{\mu\nu}] \right) \text{ [72]} \tag{14}$$

What characterizes different STG models is the specific choice of f(φ), V (φ) and F [φ, $g_{\mu\nu}$], a local scalar function of φ, $g_{\mu\nu}$ and their derivatives. The coefficient of the Ricci scalar R in conventional General Relativity (GR) is proportional to the inverse of Newton's constant G_N [66-72]. In scalar-tensor theories, then, where this coefficient is replaced by some function of a field which can vary throughout space-time, the "strength" of gravity (as measured by the local value of Newton's constant G_N) will be different from place to place and time to time [66-72].

Following our previous paper [45], we propose a scalar-tensor gravity where the scalar field is the environmental or theoretical error $f(t)_{error}$ given by equation. (13). In this proposition, using the results of **Appendix A,** the variation of gravity due to environmental or theoretical errors is given by:

$$\begin{aligned} \delta_G S &= \int d^n x \sqrt{-g} \left(-\frac{R}{16\pi G_N} F(f(t)_{error}) + \frac{1}{2} g^{00} (\partial_0 f(t)_{error})(\partial_0 f(t)_{error}) \right) \\ &= \int d^n x \sqrt{-g} \left(-\frac{R}{16\pi \delta G_N} F(f(t)_{error}) \right) \end{aligned}, \tag{15}$$

where

$$(\partial_0 f(t)_{error})(\partial_0 f(t)_{error}) = 4a_G^2 A_G^2 \cos^2(a_G t + \varphi), \qquad \text{(vanish by equation.12),} \tag{16}$$

and

$$F(f(t)_{error}) = \frac{1}{|f(t)_{error}| + \delta\left(f(t)_{error}\right)}, \delta\left(f(t)_{error}\right) = \sum_{zeros} \frac{\delta(t - t_{zeros})}{\left(\dfrac{df(t)_{error}}{dx}\right)_{t=t_{zeros}}} \tag{17}$$

In equation.17, $\delta(t-t_{zeros})$ is the delta function and t_{zeros} are the zeros of error function $f(t)_{error}$.

Without any loss of generality, we assume that the variation of the gravitational constant $\delta G_N(t)^{+(error)}$ is defined by the absolute value of the function $|f(t)_{error}|$ [45], and is calculated in Newtonian gauge (see **Appendix B)**:

$$\delta G_N(t)^{+(error)} = G_N\,|\,f(t)_{error}| = 2A_G G_N\,|\sin(a_G t+\varphi)| = \frac{\delta g(t)^2}{M_{Pl}^2}, \tag{18}$$

where

$$\delta g(t)^2 = 2g^2 A_G\,|\sin(a_G t+\varphi)|, \tag{19}$$

$$G_N = \frac{g^2}{M_{Pl}^2}. \tag{20}$$

In equation18, the variation of gravitational constant δG_N is absorbed by the dimensionless gravitational coupling δg, given equation.19. This differs from our earlier publication [45] where g was considered as a constant, and the variation of the gravitational constant δG_N was inversely proportional to the variation of the square Plank mass δM_{Pl}^2 .

From action (15), we obtain the gravitational action in the presence of environmental or theoretical errors:

$$S' = S_{EH} + \delta_G S\,, \tag{21}$$

where

$$S_{EH} = \int d^n x\sqrt{-g}\left(-\frac{R}{16\pi G_N}\right), \tag{22}$$

S_{HE} is the Einstein-Hilbert action; $\delta_G S$ and $f(t)_{error}$ are as in equation 13.

The zeros of the error function $f(\theta(t))_{error} : \theta(t) = a_G t+\varphi$ (equation13) are calculated as follows:

If $f(\theta(t))_{error} = 2A_G \sin\theta(t)_{error} = A_G\,\dfrac{e^{i\theta(t)_{error}} - e^{-i\theta(t)_{error}}}{i} = 0$, (23)

then

$$e^{i\theta(t)_{error}} = e^{-i\theta(t)_{error}} \text{ or } e^{2i\theta(t)_{error}} = 1 = e^{2k\pi i}, k = 0, \pm 1, \pm 2, \ldots \tag{24}$$

Hence $2i\theta(t)_{error} = 2k\pi i$ and $\theta_{error}^{zeros}(t) = k\pi = 0, \pm\pi, \pm 2\pi, \ldots$, i.e. the latter are all zeros and real.

Now, we calculate the action.15 at zeros $\theta(t) = \theta_{error}^{zeros}(t)$:

$$\delta_G S = \left[\int d^n x\sqrt{-g}\left(-\frac{R}{16\pi G_N}F(f(t)_{error})\right)\right]_{\theta=\theta^{(zero)}} = 0\ , \tag{25}$$

where

$F(f(t)_{error})=0$,at zeros $\theta_{error}(t)=\theta_{error}^{(zeros)}(t)$ (by equation.17), (26)

and $\delta(\theta(t)-\theta(t)_{zeros})=\infty,$ at zeros . (27)

Using equations.13, 17-19, and the interactions of gravitons with matter in the presence of environmental or theoretical errors, it can be written:

$$\delta_G \Im_h^{\text{int}} = -16\pi\delta G_N(t)^+ h_{\mu\nu}T^{\mu\nu}, \tag{28}$$

where $h_{\mu\nu}$ is the graviton, and $T^{\mu\nu}$ is the energy-momentum tensor. The requirement is that non-renormalizabele terms be suppressed by the inverse powers of Planck mass (equation 20).

5. Kaluza-Klein gravity in the presence of Newton's constant variation δG_N in (4+1) - space-time dimensions

Following, D. Bailint , A.Love (1987) [52], an effective action for the four-dimensional theory may be derived from the action for five-dimensional Einstein gravity in the presence of environmental or theoretical errors. Following equations15-20, we have the five-dimensional action:

$$\delta_G S_g = \int d^5x\,|\det\tilde{g}\,|^{1/2}\left(-\frac{\tilde{R}}{16\pi\tilde{G}_N}F(t)_{error}\right) = \int d^5x\,|\det\tilde{g}\,|^{1/2}\left(-\frac{\tilde{R}}{16\pi\delta\tilde{G}_N(t)^+}\right) \tag{38}$$

$\tilde{R}$; is the five-dimensional curvature scalar, $F(t)_{error}$ is given by equation.17 and $\tilde{G}_N$, $\delta\tilde{G}_N(t)^+$ are the gravitational constant for five dimensions and its variation, respectively.

After compactification, substituting the ansatz of ref [52] for $\tilde{g}_{AB}$ to equation.38**,** and integrating over the extra spatial coordinate θ**,** gives the following effective four-dimensional action in the presence of environmental or theoretical errors for time scales *t<<P$_G$=5,8yr*:

$$\delta_G S = -\frac{2\pi R_c}{16\pi\delta\tilde{G}_N(t)^+}\int d^5x\,|\det g\,|^{1/2}\,R - \frac{\delta\xi^2\tilde{g}_{55}}{4}\left(\frac{2\pi R_c}{16\pi\delta\tilde{G}_N(t)^+}\right)$$
$$\times\int d^5x\,|\det g\,|^{1/2}\,F_{\mu\nu}F^{\mu\nu}, \tag{39}$$

R_c: the radius of the compact manifold as in equation.31; R: the four-dimensional curvature scalar, and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{40}$$

A_μ is the Abelian gauge field and ξ is the scale factor.

Now, the variation of the four-dimensional gravitational constant δG_N becomes:

$$\delta G_N(t)^+ = \frac{\delta\tilde{G}_N(t)^+}{2\pi R_c}\quad. \tag{41}$$

The standard normalization for the gauge field [52], requires the following condition:

$$\delta\xi^2(t) = \frac{16\pi\delta G_N(t)^+}{\tilde{g}_{55}} = \frac{\delta\kappa^2(t)}{R_c^2} \quad , \tag{42}$$

where

$$\delta\kappa^2(t) = 16\pi\delta G_N(t)^+ . \tag{43}$$

Then, the effective four-dimensional action in the presence of environmental or theoretical errors for time scales $t<<P_G=5{,}8yr$ is given by:

$$\delta_G S = -\frac{1}{16\pi\delta G_N(t)^+}\int d^5x\,|\det g|^{1/2}\;R - \frac{1}{4}\int d^5x\,|\det g|^{1/2}\;F_{\mu\nu}F^{\mu\nu} \tag{44}$$

5.*1. The electron's unit charge from the fifth dimension*

Based on Ref. [52], the Fourier expansion of the five-dimensional scalar field $\Phi(x,\theta)$ on the compact manifold S^1 is as follows:

$$\phi(x,\theta) = \sum_{n=-\infty}^{\infty}\phi^n(x)e^{in\theta} \ . \tag{45}$$

The Klein-Gordon (KG) equation,

$$\left(\Box_x - \frac{1}{R_c^2}\frac{\partial^2}{\partial\theta^2}\right)\phi(x,\theta) = 0, \qquad , \tag{46}$$

$$\Box_x = g^{\mu\nu}\partial_\mu\partial_\nu = \partial^\mu\partial_\nu$$

then gives the equations for the components:

$$\left(\Box_x + M_n^2\right)\phi^n(x) = 0 \ , \tag{47}$$

where

$$M_n^2 = \frac{n^2}{R_c^2}, \qquad n = 0, \pm1, \pm2, \pm3, ... \tag{48}$$

All scalar particles have $\boldsymbol{n} \neq 0$. From (48), this means that they all have masses at the Planck scale M_{pl}, whereas the familiar particles have very small masses at the Plank scale. In this way Klein explained (for the first time) the quantization of electric charge [51]. (Note also that charge conjugation is just parity transformation y → −y in the fifth dimension) [73].

5.2. On the variations of electron's unit charge in the presence of environmental or theoretical errors in five-dimensional space-time

Following Ref. [52], we apply the coordinate transformation in the presence of environmental or theoretical errors,

$$\theta \to \theta' = \theta + \delta\xi(t)\varepsilon(x) \tag{49}$$

to the $\phi(x,\theta)$ as in equation.(45). We have:

$$\phi(x) \to \exp\left(i\delta\xi(t)\varepsilon(x)\right)\phi(x) \tag{50}$$

Here, $\delta\xi(t)$, given by equation 42, is treated as a constant for time scales $t << P_G = 5,8yr$.

The Abelian gauge field transforms in the manner:

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \varepsilon \, [52] \quad . \tag{51}$$

This means that, by applying equation 42, $\phi(x)$ has the variation of the unit electric charge $\delta \boldsymbol{e}$ due to the presence of environmental or theoretical errors:

$$\delta e(t) = -\delta\xi(t) = \frac{\delta\kappa(t)}{R_c} \ , \tag{52}$$

Thus, the variation of electric charge $\delta e^2(t)$ is quantized in units of $\delta\kappa(t)/R_c$. Using equation.43, the variation of the unit electric charge δe^2 can be written as:

$$\delta e^2(t) = \frac{16\pi\delta G_N(t)^+}{R_c^2} \ . \tag{53}$$

Inserting equations 13 to 53, and using equation.6, the variation of elementary electric charge can be expressed as follows:

$$\delta e^2(t) = 2A_G e^2 \, |\sin(a_G t + \varphi)| \, . \tag{54}$$

By using equation. (54), we obtain the time-depended unit of charge of the electron in the presence of environmental or theoretical errors,

$$e(t) = e + e\sqrt{2A_G \, |\sin(a_G t + \varphi)|} \ , \tag{55}$$

where $A_G = 10^{-4}, \varphi = 80.9 \, deg, a_G = 2\pi / P_G, P_G = 5.899 \, yr.$ [1], and e is the electron unit charge.

For the 4-dimensional long-range sector of a 5-dimensional KK-system, we suppose that the radius of compact dimension, R_c, tends to zero, while discarding from the theory all those harmonic components with mass n/R_c, where zero mode is dominant [82]. This is not a satisfactory limit (in fact, the Maxwell gauge coupling e^2 grows without limit [83]), and may lead to inconsistencies (Duff et al 1986 [84]). At sufficiently low energy scale, the Kaluza-Klein theory is consistent, since it has an infinite tower of massive KK-states [84].

7. The extreme black hole electron

A.Einstein, et.al (1938) [94] showed that, if elementary particles are treated as singularities in space-time, it is unnecessary to postulate geodesic motion as part of general relativity. Carter (1968) [95] showed that the magnetic moment of such an object would match that of an electron. More recently, the idea that elementary particles might behave like black holes has been explored by S. W. Hawking, Abdus Salam, and G.'t Hooft [96, 97, 98]. Calculations showed that the black hole electron should be 'super-extremal'; that is, it should display a naked singularity. In 1994, M.F Duff showed that the Kaluza-Klein string states are extreme black holes [73]. Following the latter work, the field equations that stem from the variation of action (44) have electrically-charged black hole solutions as follows [99, 100, 101, 102]:

$$ds^2 = -\Delta_+ \Delta_-^{-1/2} dt^2 + \Delta_+^{-1} \Delta_-^{1/2} dr^2 + dr^2 \Delta_-^{3/2} d\Omega^2 \ , \quad (56)$$

$$e^{2\phi} = \Delta_-^{\sqrt{3}}, e^{-\sqrt{3}\phi} F = \left(r_+ r_-\right)^{1/2} V_2 \quad , \quad (57)$$

where $\Delta_\pm = 1 - r_\pm/r$, and V_2 is the volume form on 2-sphere S^2. The variation of electric charge δe and mass m is related to $r_\pm$ as follows:

$$\begin{aligned} &\sqrt{2\delta k \delta e} / 4\pi = \left(r_+ r_-\right)^{1/2}, \\ &\delta k^2 m / 4\pi = 2r_+ - r_- \end{aligned} \quad (58)$$

An event horizon, $r_+ \geq r_-$, implies the bound:

$$\sqrt{2\delta km} \geq \delta e \quad (59)$$

In the extreme limit $r_+ = r_-$, the line element becomes:

$$ds^2 = -\Delta_+ \Delta_-^{-1/2} dt^2 + \Delta_+^{-1} \Delta_-^{1/2} dr^2 + dr^2 \Delta_-^{3/2} d\Omega^2 \ , \quad (60)$$

and the bound (59) is saturated, yielding exactly the same variation of charge (δe) to mass (m) ratio (53) as the massive KK-states. M.F Duff [73] showed that this is no coincidence: the massive KK-states are extreme black holes. The field equations stem from the variation of action (44), have magnetically-charged black hole solution with the same metric [73], with magnetic charge variation, δg, given by:

$$\sqrt{2\delta k \delta g} / 4\pi = \left(r_+ r_-\right)^{1/2}. \quad (61)$$

In the extreme limit $r_+ = r_-$, Equation (61) is the Kaluza-Klein monopole [101, 103, 104].

The four-dimensional monopole exhibits a curvature singularity at $r = r_-$. Considering the metric $\tilde{g}_{\mu\nu} = e^{\sqrt{3}\varphi} g_{\mu\nu}$, the magnetic monopole line element is:

$$\begin{aligned} &d\tilde{s}^2 = -\Delta_-^{-1} dt^2 + \Delta_-^{-2} dr^2 + dr^2 d\Omega^2 = (1 + r_- / r) dt^2 + (1 + 2r_- / r) dr^2 + dr^2 d\Omega, \\ &\Delta_-^{-1} = (1 + r_- / r), \Delta_-^{-2} = (1 + 2r_- / r), \end{aligned} \quad (62)$$

at large r. The curvature singularity at $r = r_-$ has now disappeared. The significance of this metric is that it is the one that couples to the worldline of an electrically-charged point particle [73,105, 106] at large distance scale. As in [73], the way the theory accommodates this requirement is that, when expressed in terms of the metric $e^{\sqrt{3}\varphi} g_{\mu\nu}$ that couples to the worldline of the particle, the elementary solutions are singular and the solitonic solutions are non-singular; when expressed in terms of the dual metric $e^{-\sqrt{3}\varphi} g_{\mu\nu}$, it is the other way around [105, 106].

8. Higher-dimensional space-time

There are convincing arguments that D=11 is the maximum dimension in which we can consistently formulate a supergravity theory and describe the Standard Model (SM) of particle physics [83]. If the internal space is a 7-dimensional sphere S^7, the isometry group is *SO(8)*, and the coupling constant, e^2, is $e^2 = 4/3m^2 \kappa^2$. All gauge bosons appearing in the consistent ansatz for pure gravity in D = 11 will also appear in the consistent ansatz for supergravity, but with a normalization differing by a factor of 2, which preserves the space-time supersymmetry: $g^2(sugar) = (1/4)g^2(gravity)$ [84].

Furthermore, heterotic strings theories [107-110] use the 26-dimensional bosonic string for the left-movers Left-N_L, and 10-dimensional superstrings for the right-movers Right-N_R. The remaining 16 right-moving degrees of freedom required by N=1 supersymmetry are internal, and come from 16-dimensional, even, self–dual lattice constraints by modular invariance [107-110]. There are only two such lattices, corresponding to the weight lattices of lie groups $E_8 \times E_8$ and $SO(32)$ [107-110]. Following [73], we consider the four-dimensional heterotic string obtained by toroidal compactification. At a generic point in the moduli space of vacuum configurations, the unbroken gauge symmetry is $U(1)^{28}$ and the low energy effective field theory is described by the N = 4 supergravity ([110], see Table.1), coupled to the 22-abelian vector multiples [73].

Theory	Spin=0	Spin=1/2	Spin=1	Spin=3/2	Spin=2
N=4	2	4	6	4	1

Table 1. Particle content of N = 4 supergravity.

Duff et al. [73] worked with the Schwarz-Sen [111,112] O(6,22;Z) invariant spectrum of elementary electrically-charged massive Right-N_R = 1/2, Left-N_L = 1 states of this four-dimensional heterotic string. They showed that the zero spin states correspond to the extreme limits of the Kaluza-Klein black hole solutions of Section 7 here.

The bosonic sector is given by:

$$\delta_\kappa S = \frac{1}{2\delta\kappa^2}\int d^4x\sqrt{-G}e^{-\Phi}(R_G + G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{12}G^{\mu\lambda}G^{\mu\tau}G^{\rho\sigma}H_{\mu\nu\rho}H_{\lambda\tau\sigma}$$
$$-\frac{1}{4}G^{\mu\lambda}G^{\nu\tau}F^a_{\mu\nu}(\mathrm{LML})\mathrm{F}^b_{\lambda\tau} + \frac{1}{8}G^{\mu\lambda}Tr(\partial_\mu ML\partial_\nu ML)) \quad , \qquad (63)$$

where $F_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a$, and $H_{\mu\nu\rho} = (\partial_\mu B_{\nu\rho} + 2A_\mu{}^a L_{ab} F_{\nu\rho}{}^b) + permutations$. Here, Φ is the D = 4 dilaton, and R_G is the scalar curvature formed from the string metric $G_{\mu\nu}$, related to the canonical metric $g_{\mu\nu}$ by $G_{\mu\nu} \equiv e^{\Phi} g_{\mu\nu}$. $B_{\mu\nu}$ is the 2-form which couples to the string world sheet, and $A_\mu{}^a$ $(a = 1, ..., 28)$ are the abelian gauge fields. M is a symmetric 28×28-dimensional matrix of scalar fields satisfying $MLM = L$, where L is the invariant metric on O(6, 22):

$$L = \begin{pmatrix} 0 & I_6 & 0 \\ I_6 & 0 & 0 \\ 0 & 0 & -I_{16} \end{pmatrix} \qquad (64)$$

The action is invariant under the O(6, 22) transformations:

$$M \to \Omega M \Omega\, T,\ A_\mu{}^a \to \Omega^a{}_b A_\mu{}^b,\ G_{\mu\nu} \to G_{\mu\nu},\ B_{\mu\nu} \to B_{\mu\nu},\ \Phi \to \Phi, \qquad (65)$$

where Ω is an O(6, 22) matrix satisfying $\Omega TL\Omega = L$ [73].

Action (24) can be consistently truncated by keeping the metric $g_{\mu\nu}$, just one field strength F^1, and one scalar field φ via the ansatz $\Phi = \varphi/\sqrt{3}$ and $M_{11} = e^{2\varphi/\sqrt{3}} = M^{-1}{}_{77}$ [73]. All other diagonal components of M are set equal to unity, and all non-diagonal components are set to zero. In this way, Equation (24) reduces to (44). This yields the electric and magnetic Kaluza-Klein monopoles [73].

Now, the purely electrically-charged solution with charge δe is expected to correspond to an elementary string excitation. Using Ref [73] for the mass of the state and our Section 7, we find that $m^2 = \delta e^2/16\pi\delta G_N$, which coincides with Equation (59) in the extreme limit.

9. Testing the number of space-time dimensions through the proposed P_G=5,9yr-repeated Millikan's oil drop experiments

The oil drop experiment was performed originally by Robert A. Millikan in 1909[74] to measure the charge of a single electron. The experiment apparatus (Figure.2) consists of an atomizer which sprays tiny oil droplets and of a short focal distance telescope, by means of which the droplets can be viewed. There are two plates, one of positive and one of negative charge, above and below the bottom chamber. A dc supply is attached to the plates. Some of the oil drops fall through the hole in the upper plate. The bottom chamber is illuminated with X-rays that cause the air to ionize. As the droplets move through the air, electrons accumulate over the droplets and negative charge is acquired. With the help of the dc supply a voltage is applied. The speed of droplet motion can be controlled by altering the voltage applied on the plates [74-78].

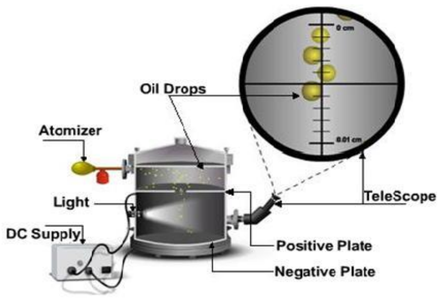


Fig. 2. Design of the Millikan oil-drop experiment for determining the electric charge of the electron.

By adjusting the applied voltage, a droplet can be suspended in the air. Millikan observed one drop after another, varying the voltage and noting the effect. After many repetitions, he concluded that charge could assume only certain fixed values. He repeated the experiment for many droplets and confirmed that the charges were all multiples of some fundamental value and calculated it to be 1.5924(17) $\times 10^{-19}$ C, within one percent of the currently accepted value of 1.602176487(40) $\times 10^{-19}$ C. He proposed that this was the charge of a single electron [74-78]. Millikan's paper [74], presented a complete summary of data on 58 drops studied over 60 consecutive days. Mathematically, Millikan started with the following equation:

$$v_1 / v_2 = mg / Fe - mg \quad . \tag{66}$$

With appropriate substitutions, the equation takes the following form:

$$e_n = (4/3)\pi(9\mu/2)^{3/2}\left\{1/g(\sigma-\delta)\right\}^{1/2}(v_1+v_2)v_1^{1/2}/F... \tag{67}$$

Including the correction from Stokes' law gives the equation:

$$v_1 = 2/9ga^2(\sigma-\delta)\mu\left\{1+A/\alpha\right\} \tag{68}$$

Combining equations (57) and (58) gives the value of *e*:

$$e(1+A/\alpha)^{3/2} = e_n \tag{69}$$

where v_1:speed of descent of the drop under gravity; v_2: speed of ascent of the drop in the electric field; mg: force of gravity; *F*: electric field; e_n: frictional charge on the drop; μ ;coefficient of viscosity of air; s density of the oil; d: density of air; *a*: radius of the drop; l: mean free path of a gas molecule; and *A* : correction term constant. The mean value obtained with this method was reported to be e =4.774 ±0.009×10^{-10} esu. At this stage, it is important to note that Millikan, based on his guiding assumptions, expected the value of e_n to be an integral multiple of *e*, where *n* = 1, 2, 3, . . . Apparently Millikan discarded values that did not turn out to be integral multiples [78]. Note that there is a larger gap between the values 2.2x10^{-19} C and 2.9x10^{-19} C than between the other points that define the first five gaps (increments). We cluster the first six values (e_1 to e_6) together by averaging that group, and we assign that group to integer 1 (1 unit of charge). The next significant gap occurs between 3.7x10^{-19} C and 4.5x10^{-19} C, so we average the values between 2.9x10^{-19} C and 3.7 x 10^{-19} C into the second cluster and assign them to integer 2 (2 units of charge) [78].

More recently, T Lee et al., at the Stanford Linear Accelerator (SLAC), carried out the largest search for fractional electric charge elementary particles using automated Millikan's oil drop experiments. No evidence for such particles was found [113].

Since the electron unit charge is given by Equation (75), we propose using an automated Millikan's oil drop experiment (similar to [113]) over the duration of 5.9 years, equal to the LOD oscillation (P_G=5,899yr) of the Newtonian constant of gravitation G_N, as detailed in Tables 2,3.

Lateral E field, vertical airflow apparatus
Drop generation rate 1 Hz (limited by drop to drop hydrodynamic cross interactions)
Fluid - Dow Corning silicon oil
Number of drops - 211,2 million
Mass –702,8 milligrams
Duration - 72 months

Table 2. Proposed experimental run utilizing automated Millikan apparatus.

Charge magnitude < 40e
Drop trajectory Chi Squared fit > 10^{-3}
Charge consistency of the positions <0.03e
Minimum drop to drop separation >0.62 mm

Table 3. Charge measurements to be valid.

Substituting Equation 55 to 69, we find the variation of electron fundamental charge measured by repeated Millikan's oil drop experiments with time scale equally to P_G=5,899yr:

$$\begin{aligned} e(t)(1+A/\alpha)^{3/2} &= (e+e\sqrt{A_G\,|\sin(a_G t+\varphi)|})(1+A/\alpha)^{3/2} \\ &= e_n\left(1+\sqrt{2A_G\,|\sin(a_G t+\varphi)|}\right) = e_n(t) \end{aligned} \tag{70}$$

for 4+D-dimensional space-time;

$$e(1+A/\alpha)^{3/2} = e_n \tag{71}$$

for 4-dimensional space-time,

with $A_G = 10^{-4}, \varphi = 80.9\ deg, a_G = 2\pi / P_G, P_G = 5.899\ yr.$ (Anderson et al. 2015a) [1], and e being of the currently accepted value of 1.602176487(40) $\times 10^{-19}$ C of electron charge [78].

The comparison between the values of fundamental electric charge *e* with the 5.9 year LOD oscillation cycle, predicted by 4-dimensional space-time model (Equation 71) and the set of *e(t)* values predicted by the 4+D-dimensional space-time model (Equation 70) is calculated in Table 1, and shown in Fig 3.

t (years)	*e* (Coulomb) predicted by the 4+D-dimensional space-time model (Equation (70)	*e* (Coulomb) predicted by 4-dimensions (Equation 71)
0	1.62469×10^{-19}	$1.602176487(40)\times10^{-19}$
1	1.61997×10^{-19}	$1.602176487(40)\times10^{-19}$
2	1.61633×10^{-19}	$1.602176487(40)\times10^{-19}$
3	1.62477×10^{-19}	$1.602176487(40)\times10^{-19}$
4	1.61933×10^{-19}	$1.602176487(40)\times10^{-19}$
5	1.61719×10^{-19}	$1.602176487(40)\times10^{-19}$
6	1.62482×10^{-19}	$1.602176487(40)\times10^{-19}$

Table.4. Fundamental electric charge *e* predicted by 4+D-dimensional space-time model (Equation 70), and 4-dimensional space-time model (Equation. 71), with the 5.9 year LOD oscillation cycle. The first column is the time decimal in years.

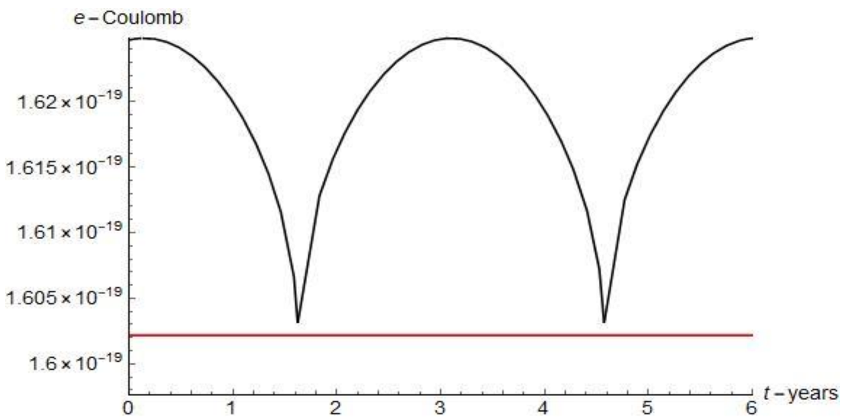


Fig. 3 Comparison between the values of fundamental electric charge *e* predicted by 4-dimensional space-time, and the set of *e(t)* values predicted by the 4+D-dimensional space-time model with the 5.9 year LOD oscillation cycle. The red curve shows the values of *e* predicted by the 4-dimensional space-time Equation (71); the black curve is the theoretical variation of *e(t)* predicted by 4+D-dimensional model (70).

For any given drop, there is approximately 0.2% of uncertainty about its radius, which contributes to the relative error of electric charge q [113-116]. The absolute error of q thus increases linearly with q. Since the absolute error of q must be kept to the order of 0.03 e, Table 3 restricts the dataset to drops with q < 9.5 (thus keeping this contribution error <0.02 e). The overall charge distribution is such that only a small percentage of drops have q values outside this range [113-116]. The drop distribution in units of electron charge (e) predicted by the 4+D-dimensional space-time model is given by:

$$N_{drop}(n,t) = N_{\text{drop}(0)} \exp\left(-e_n^2(t)/e^2\right) , \tag{72}$$

where $N_{\text{drop}(0)} = 10^6$ is the initial number of drops, and $e_n^2(\text{t})$ is given by Equation (70). *n* is the number of units of electron charge (e) in a given drop. For any given drop, the units of the electron charge (e) have the same variation due to the same environmental or theoretical errors. This is called collective variation. Using the results of **Appendix C**, Equation (70) becomes:

$$e_n^2(t) = n^2 e^2 + n e^2 \sin\left(a_G t + \varphi\right). \tag{73}$$

Using Equation (73), the drop distribution in units of electron charge (e) that was predicted by the 4+D-dimensional space-time model becomes:

$$\begin{aligned} N_{drop}(n,t) &= N_{drop}(\text{n}) \exp\left(-n \sin\left(a_G t + \varphi\right)\right) \\ &= N_{drop}(n) - n N_{drop}(\text{n}) \sin\left(a_G t + \varphi\right) = N_{drop}(n) - \delta N_{drop}(n,t) \end{aligned} , \tag{74}$$

where

$$N_{drop}(n) = N_{\text{drop}(0)} \exp\left(-e_n^2/e^2\right) = N_{\text{drop}(0)} \exp\left(-n^2\right) , \tag{75}$$

$$\delta N_{drop}^{(collective)}(n,t) = n N_{drop}(\text{n}) \sin\left(a_G t + \varphi\right). \tag{76}$$

$N_{drop}(n)$ is the Gaussian drop distribution in units of electron charge (e) predicted by the 4-dimensional space-time model, and $\delta N_{drop}(n,t)$ is the collective variation in drop distribution due to the presence of D-compact space dimensions. The drop distribution, N(*n*), predicted by the 4-dimensional space-time model (Equation 75),and the N(*n*,*t*) values, predicted by the 4+D-dimensional space-time model (Equation 74) with the 5.9 year LOD oscillation cycle, are calculated in Tables 5(a),5(b) and shown in Figures 4(a), and 4(b).

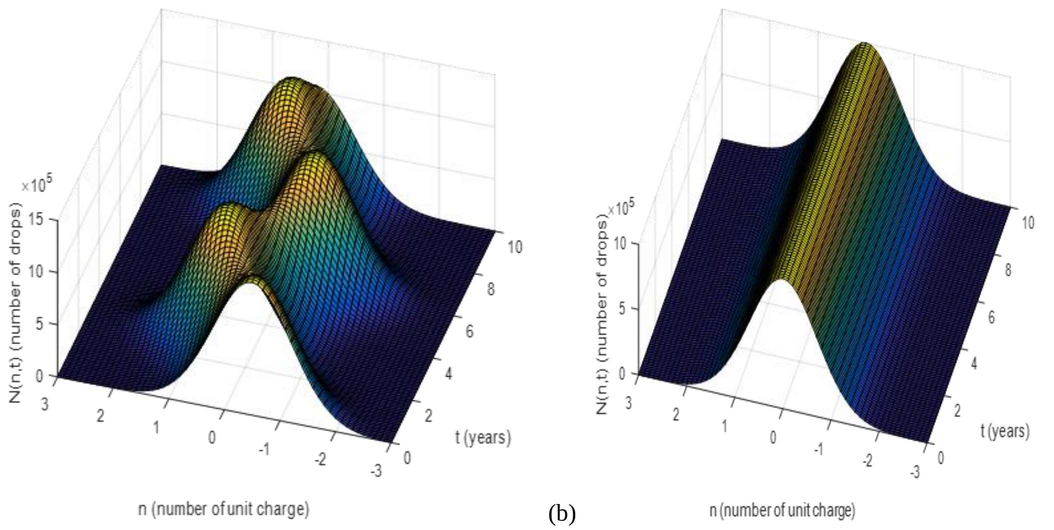
n (number of unit charges) / t (years)	0	1	2	3	4	5	6
-3	0.0024	0.0008	0.0000	0.0000	0.0000	0.0005	0.0025
-2	0.1320	0.0629	0.0084	0.0025	0.0058	0.0441	0.1350
-1	0.9875	0.6816	0.2491	0.1361	0.2073	0.5706	0.9987
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.1370	0.1986	0.5434	0.9945	0.6528	0.2372	0.1355
2	0.0025	0.0053	0.0400	0.1339	0.0577	0.0076	0.0025
3	0.0000	0.0000	0.0004	0.0024	0.0007	0.0000	0.0000

(a)

n (number of unit charges) / t years)	0	1	2	3	4	5	6
-3	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-2	0.0183	0.0183	0.0183	0.0183	0.0183	0.0183	0.0183
-1	0.3679	0.3679	0.3679	0.3679	0.3679	0.3679	0.3679
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.3679	0.3679	0.3679	0.3679	0.3679	0.3679	0.3679
2	0.0183	0.0183	0.0183	0.0183	0.0183	0.0183	0.0183
3	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

(b)

Table.5. (b);The drop distribution N(*n*), predicted by the 4-dimensional space-time model (Equation 75), and (a); the N(*n*,*t*) values, predicted by the 4+D-dimensional space-time model (Equation 74), with the 5.9 year LOD oscillation cycle.



(a) (b)

Figure 4. Comparison between, (a) the drop distribution N(*n*) predicted by 4-dimensional space-time (Equation 75), and, (b) the set of N(*n,t*) values predicted by the 4+D- dimensional space-time model (74), with the 5.9 year LOD oscillation cycle.

10. Discussion

In 2014, UK's Royal Society hosted a conference titled "*The Newtonian Constant of Gravitation: a constant too difficult to measure?*" [79]. The conference aimed to resolve the problem of large discrepancy between recent G_N values [80].

A reasonable explanation for the discrepancy of G_N measurements is that this is due to some still unknown physical cause [80,45,46]. In 2015, Anderson et al. [1], analyzed the measurement results and claimed that the recent values of G_N varied sinusoidally with a period of about 5.9 years. They proposed that one possible reason for this variation was the activity of the Earth's core. Schlamminger et al. corrected the acquisition time of these measurement results but did not find any remarkable correlation [46]. In 2017, Parra proposed that the temporal variation of G_N was potentially caused by the sun's dragging effect [81]. These hypotheses can be neither confirmed nor refuted at present, since the precision of G_N measurement is low. G_N measurements of higher precision, obtained by more methods, are, therefore, required.

Following L. Iorio [3], the time-variation of G investigated here contrasts with virtually all the theoretical models that predict a G(t) varying over typically cosmological timescales. The boundaries of variation of G reported in the current literature (of the order of 10^{-13}-10^{-14} yr^{-1}: Williams, Turyshev & Boggs 2004; Muller & Biskupek 2007; Pitjev & Pitjeva 2013; Pitjeva & Pitjev 2013) may not be applicable to the present case, since this range of variation was inferred from least-square reductions of planetary and lunar positional data though modeling δG(t) as a secular trend [3].

It is possible that the oscillation of the gravitational constant G_N given by Equation (13) is an artifact of unrecognized large systematic errors of measurement. If this is the case, the connection with compact D-dimensions is improbable.

Future work can focus on the local flatness of the metric tensor $g_{\mu\nu}$ within the GR framework. We argue that an artifact of unrecognized large systematic errors of the G_N measurement is improbable [117]: the Taylor series expansion of the metric ($g_{00} =1-2G_N m/r$) around the origin, O, incurs no error. However, for other points, there is an associated truncation error, and this must be accounted for [118,119]. So, the true metric $g_{00}(x)$ can be written as the Taylor series approximation $g_{00}(x)|$ plus a truncation error term: $g_{00}(x) = g_{00}(x) + E_{00}(x)$:

$$E_{00}(x)|_p = \frac{1}{(n+1)!}\sum_{k_1,..k_{n+1}} \partial_{k_{n+1}}..\partial_{k_{n+1}} g_{00}(c)|_p\ (x^0 - x^0_\zeta)^{k_{n+1}}...(x^0 - x^0_\zeta)^{k_1}$$
$$= \frac{m}{r(n+1)!} G_N \sum_{k_1,..k_{n+1}} \partial_{k_{n+1}}..\partial_{k_{n+1}} f(c)_{error}|_p\ (x^0 - x^0_\zeta)^{k_{n+1}}...(x^0 - x^0_\zeta)^{k_1} \quad , \qquad (77)$$

where for some c strictly between x^0 and x^0_ζ that belong to tangent spaces $T_p(M)$ of the manifold M ,and

$$\delta G_N(x^0)|_p = \frac{G_N}{(n+1)!}\sum_{k_1,..k_{n+1}} \partial_{k_{n+1}}..\partial_{k_{n+1}} f(c)_{error}|_p\ (x^0 - x^0_\zeta)^{k_{n+1}}...(x^0 - x^0_\zeta)^{k_1} \qquad (78)$$

is the local variation of G_N, and $f(c)_{error}$ is any real function with bound n+1-derivatives in the tangent space $T_p(M)$ of the manifold M. The right side of Equation (78) approaches zero as n tents to infinity. It follows that G_N is constant as should be as measure indirect from the light curves of type Ia supernovae [120], and by using gravitational wave observations of binary neutron stars [121]. We replace C^∞ of the manifold M by C^{n+2} in all tangent spaces $T_p(M)$ of the manifold M [117]. For our purposes the degree of differentiability of a manifold C^{n+2}, $n+2\geq 6$ is not crucial; we will always assume that any manifold is as differentiable as necessary for the application under consideration [122]. Mining that the reported difficulties of the local measure of Newton constant G_N [79] is a property of the gravitational field rather than an artifact error [117]. The implications of such a novel gravitational property to the differential manifold is under investigation [117].

At energies well below the Planck energy, the massive Kaluza-Klein states are extreme black holes which can describe stable elementary particles [73]. Based on this interpretation, we find that the variation of electric charge $\delta e^2(t)$ is quantized in units of $16\pi\delta G(t)/R$, and is proportional to the sinusoidal variation of δG_N with a period of about 5.9 years (Table1; Figure 3). We also observe that the changes of electron charge due to the presence of a D-space dimension should be about 10^{-2}, with a period of about 5.9 years (as shown by the red curve in Figure 3). In the case of four space-time dimensions, the electron charge is constant and perfectly fits a straight line (black curve in Figure 3). The collective magnetic field that follows from Equation (55) is vanished by Equation (12). Furthermore, the adiabatic variation of electric charge (55) does not give any atomic transition (for details see [123]).

In the proposed automated Millikan's oil-drop experiment over 5.9 years (see above), we observe that the drop distribution N(n) predicted by the 4-dimensional space-time model (Equation; Fig. 4.b) is a symmetrical Gaussian distribution. The drop distribution N(n,t) predicted by 4+D-dimensional model (Equation; Fig.4. b) is symmetrical when the variation of gravitational constant δG(t) is positive or negative, and it asymmetrical when δG is changes sing for positive to negative and via versa (Fig. 4.b). A similar, flipped asymmetric charge histogram has been observed by the SLAC experimental run utilizing automated Millikan apparatus numbers **2** and **3** (Fig. 5. c), 5.b) with change from positive to negative δG(t) (see Fig.1). A symmetrical Gaussian charge histogram has been observed in the SLAC experimental run utilizing automated Millikan apparatus numbers **4** and **1** as shown (Fig. 5.a, 5.d), with positive and negative δG respectively (see Fig.1). As long as we do not know what sets the charge distribution for a particular drop generator used in such experiments [113-116]. It is remarkable that those charge distributions as shown (Fig. 5.a, b, c, d) may be the first event of synchronous variations at G and e. Of course, a synchronous variation at G and e may also be accidental, due to the low time scale of SLAC results. An experimental run utilizing automated Millikan apparatus for the duration of 72 months, is, therefore, required.

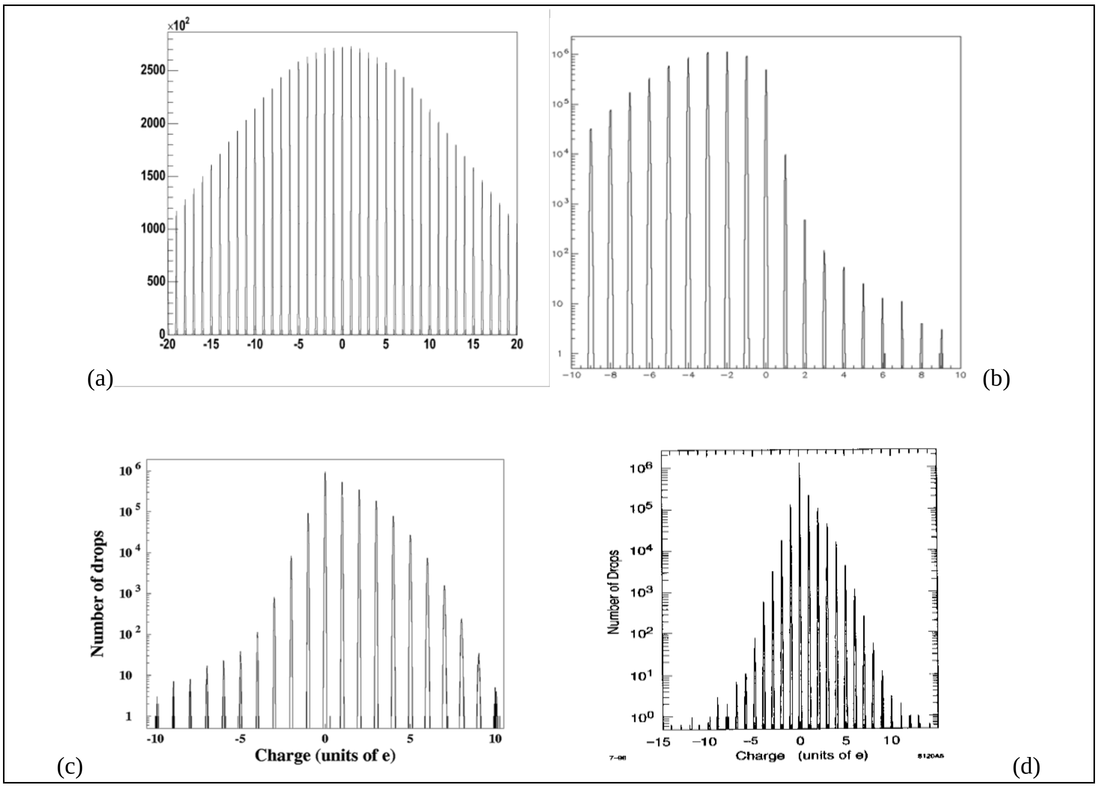


Fig.5. Charge histogram of the SLAC experimental run utilizing automated Millikan apparatus numbers **4:** (a) (2004-2007; duration: 36 months [116]); **3:** (b) (2000-2001; duration: 8 months [115]), **2:** (c) (1999; duration: 5 months [114]); **1:** (d) (1995: duration: 9 months [113]).

The coupling between e and G is a result of the 4+ D-dimensional theories and, therefore, a result congruent with the unification project. That this coupling and it resulting effects are absent from the 4-dimensional theory may be simply because the latter misses the unification try. Whatever the reason behind the variability of the G value, if there is a coupling, which justifies the unification try, it should have an effect, which could become measurable by the experiment proposed here, of by other experiments.

It is still possible, of course that deviations of G result from systematic errors of measurement rather than periodicity. Synchronous deviations at G and e, however, would be a significant finding, whatever rule those follow.

11. Conclusions

- At energies well below the Planck energy, the massive Kaluza-Klein states are extreme black holes, which can describe stable elementary particles [73].

- Based on this interpretation, we show that, if the observed harmonic pattern of the laboratory-measured values of the G_N is due to some environmental or theoretical errors, these errors must also affect the elementary electric charge, *e*.
- We calculated the values of fundamental electric charge *e* predicted by 4 and 4+D-dimensional space-time models.

- We find that, in the case of 4+D space-time dimensions, the fundamental electric charge (e) values oscillate with the 5.9 year LOD oscillation cycle, while in the case of 4 space-time dimensions. the fundamental electric charge (e) is constant.

- We also propose using an automated Millikan's oil drop experiment over the duration of 5.9 years to discriminate between 4 and 4+D space-time dimensions.

- We find that the collective drop distribution predicted by the 4+D-dimensional space-time models is very close to the Stanford Linear Accelerator results. At present, we do not know what sets the charge distribution observed in these experiments. It is remarkable that this charge distribution, observed in the experimental run utilizing automated Millikan apparatus numbers at SLAC, may be the first confirmation of synchronous variation at G and e.

  Since the proposed Millikan's oil drop experiment over 5.9 years would not give any information about the constants of the proportionality between the coupling constant charge g^2 and the Newton constant G_N, and is crucially dependent on the field content of the higher-dimensional theory, it is improbable to discriminate between one and seven compact extra dimensions through this experiment. However, the proposed experiment may reveal whether there exist one or more compact extra dimensions in our space-time.

Acknowledgments

The author would like to thank Stelios Tsilioukas for helpful discussions.

Appendix A. Following Ref. [119], the typical error length scale of the patch containing a point P should be $\varepsilon(t)=\sqrt{F(\mathrm{t})_{error}}$, where $F(\mathrm{t})_{error}$ is given by Equation (17). Let the coordinates of the patch be x^μ and let the coordinates of P be $x^\mu{}_*$. We define a new set of coordinates y^μ by:

$$x^\mu = x^\mu_* + \varepsilon(t) y^\mu \qquad \text{(A.1)}$$

Then,

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = \varepsilon(t)^2 g_{\mu\nu}(x_* + \varepsilon(t)\,\mathrm{y})dy^\mu dy^\nu \qquad \text{(A.2)}$$

We define the conformal metric $d\tilde{s}^2$ by:

$$d\tilde{s}^2 = g_{\mu\nu}(x_* + \varepsilon(t)\,\mathrm{y})dy^\mu dy^\nu = \tilde{g}_{\mu\nu}(\mathrm{y}, \varepsilon(t))dy^\mu dy^\nu \qquad \text{(A.3)}$$

In both coordinate systems, the geometry of the patch is described by the metric and the boundary of the patch. In the original x^μ coordinates, only the boundary depends on $\varepsilon(t)$, whereas in the conformal coordinates y^μ, the boundary is fixed but the metric depends on $\varepsilon(t)=\sqrt{F(\mathrm{t})_{error}}$. From the above it is easy to see that, at point P,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}, \tilde{g}_{\mu\nu,a} = g_{\mu\nu,\alpha}\sqrt{F(\mathrm{t})_{error}}, \tilde{g}_{\mu\nu,\alpha\beta} = F(\mathrm{t})_{error}\, g_{\mu\nu,\alpha\beta},$$
$$\tilde{R}_{\mu\nu\alpha\beta} = F(\mathrm{t})_{error} R_{\mu\nu\alpha\beta}, \tilde{R} = g^{\mu\nu} g^{\alpha\beta} \tilde{R}_{\mu\nu\alpha\beta} = F(\mathrm{t})_{error}\, g^{\mu\nu} g^{\alpha\beta} R_{\mu\nu\alpha\beta} = F(\mathrm{t})_{error} R \tag{A.4}$$

where $R = g^{\mu\nu} g^{\alpha\beta} R_{\mu\nu\alpha\beta}$,and the partial derivatives on the left are with respect to *y* and those on the right are with respect to *x*. By using equations (A.4), the gravitational action can be written as follows:

$$\int d^n x\sqrt{-\tilde{g}}\left(-\frac{\tilde{R}}{16\pi G_N}\right) = \int d^n x\sqrt{-g}\left(-\frac{R}{16\pi G_N}F(t)_{error}\right), \tag{A.5}$$

where $\sqrt{-\tilde{g}} = \sqrt{-g}$ follows from equations A.4. Action A.5 is the variation of gravity due to environmental or theoretical errors, as given by Equation. (15) in the limit of vanishing error kinetic terms $(\partial_0 f(t)_{error})^2$.

Appendix B. In the Newtonian gauge, Action.A.5 becomes:

$$\int d^n x\sqrt{-g}\left(-\frac{(\nabla\Phi)^2}{16\pi G_N}F(t)_{error} - \Phi\delta\rho_m\right) . \tag{B.1}$$

To this equation, we have added the gravitational coupling to non-relativistic matter $\frac{1}{2}\delta g^{00} T_{00}$. By varying with respect to Φ, we see that the usual Poisson equation for the gravitational potential is modified by the error term $F(t)_{error}$. The Poisson equation now reads:

$$\nabla^2\tilde{\Phi} = \frac{1}{2\mathrm{M}_{pl}^2 \mathrm{F(t)}_{error}}\delta\rho_m \tag{B.2}$$

By comparison with $\left(\nabla^2\Phi = \rho_m / 2M_{pl}^2\right)$, we finally obtain the relation between measured gravitational constant, $G_N = 1/M_{pl}^2$, and its measured variation in the presence of environmental or theoretical errors in the Newtonian gauge:

$$\delta G_N(\mathrm{t})_{error} = \frac{1}{2\mathrm{M}_{pl}^2 \mathrm{F(t)}_{error}} = G_N\, \mathrm{f(t)}_{error} \tag{B,3}$$

Appendix C. As explained in [13-116], the drop radius is determined from the horizontal terminal velocity V_x of integer charge particles. The measurement of e_n does is independent of the drop density and, also, of the gravitational force on the drop [113-116]. We consider the time-depended values of $e_n(t) = ne(t), (n = 0, \pm 1, \pm 2, \ldots)$ given by equation. (70). Following [113-116], we expect a time-depended sharp peak of V_x (t) measured values at $(n = 0, \pm 1, \pm 2. \ldots)$. For any given drop, the units of the electron charge (e) have the same variation, due to the same environmental or theoretical errors. This is a collective variation. As long as we do not know what sets the charge distribution for a particular drop generator [113-116], we consider the theoretical collective drop population. Let j drops, of radius r, density ρ, and time depended collective charges $e_{jn}(t) = je_n(t), (j = 1, 2 \ldots, N_c)$, falling in air through a horizontal

electric field of strength E. N_c is the number of drops that enclosed at least one-unit charge. Then, as shown in [113-116], for 1 million drops, N_c should be 10^4-10^5. This implies that the collective number of drops, $N_{c,}$ is about 1%-10% of the total drop number N.
Now let us examine the square averages of equation. (68) for j collective drops:

$$\bar{v}_{jx}{}^2(t) = \left(\frac{\bar{e}_{jn}{}^2(t)'}{36\pi^2\eta^2}\right)\left(\frac{E}{r}\right)^2, \tag{C.1}$$

where $\bar{e}_{jn}^2(t)' = \bar{e}_{jn}^2 + \delta\bar{e}_{jn}^2(t) = \frac{1}{N_c}\sum_j e_{jn}^2(t)' = \frac{1}{N_c}\left(\sum_j j^2\right)e_n^2(t)' \geq \frac{j^2}{N_c}e_n^2(t)'$, (C.2)

and $\bar{e}_{jn}^2 = \frac{1}{N_c}\sum_j e_{jn}^2 = \frac{n^2e^2}{N_c}\left(\sum_j j^2\right)e^2 \geq \frac{n^2e^2j^2}{N_c}$, (C.3)

$$\delta\bar{e}_{jn}^2(t) = \frac{1}{n}\sum_n \delta e_{jn}^2(t) = \frac{j^2}{n}\left(\sum_n n^2\right)\delta e^2(t) \geq nj^2\delta e^2(t), \tag{C.4}$$

$$\sum_n n^2 \geq n^2 \quad , \sum_j j^2 \geq j^2, \tag{C.5}$$

where $\bar{e}_{jn}{}^2(t)'$ are the square averages of j collective drops over time-depended drop charges; $\bar{e}_{jn}{}^2$ are the square averages of j collective drops over drop charges; $\delta\bar{e}_{jn}^2(t)$ are the square averages of j collective drops over the variation of n unit charges, $\delta e^2(t)$ is the square of the variation of unit charge (e), given by equation.(54). For equations (C.1), (C.2), (C.3), and (54), we have:

$$e_n^2(t)' = \mathrm{n}^2\mathrm{e}^2 + 2N_cA_G\,\mathrm{ne}^2\sin(a_Gt+\varphi) = \mathrm{n}^2\mathrm{e}^2 + \mathrm{ne}^2\sin(a_Gt+\varphi), \tag{C.6}$$

where $N_c = 10^4$ [113-116], and $A_G = 10^{-4}$ [1], implying that $2N_cA_G = O(1)$.

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