

Name : Viktor Kalaj  
Email : vkalaj@gmail.com

So I will start the 2 - page proof by stating an equation :

$$\zeta(s) = \frac{2^s}{(-2+2^s)\Gamma(s)} \int_0^{\infty} \frac{t^{-1+s}}{1+e^t} dt \text{ for } \operatorname{Re}(s) > 0$$

(Eq .1) ( If you would like to see the demonstration of the validity of this Eq .1 ,  
it can be found at : <https://www.wolframalpha.com/input/?i=Riemann+zeta+function> ) . Scroll down and go to the section entitled  
"integral representations" and click on the button "more" at the top right -  
hand side of the box . Then scroll down the list ;  
see the equation thats valid on  $\operatorname{Re}(s) > 0$  . ) Please let me know if  
the equations as stated can be viewed easily ;  
if they can ' t , I will rewrite the equations ;  
just email me .

This is the Riemann Zeta Function , valid on the critical strip . Assume an arbitrary zero ,  
 $\alpha$  , of the Riemann Zeta Function above , lies anywhere on the critical strip  
(  $\alpha = a + bi$  , where "a" and "b" belong to the reals , and "i" is the imaginary number ) . We  
set for the above Zeta Function , on left - hand side of the equation ,  $s = \alpha$  . Therefore :

$$\zeta(\alpha) = 0$$

(Eq .2)

Since the left side of the above Eq .1 - Eq .2 is equal to zero , we see the following :

$$0 = \frac{2^\alpha}{(-2+2^\alpha)\Gamma(\alpha)} \int_0^{\infty} \frac{t^{-1+\alpha}}{1+e^t} dt$$

(Eq .3)

We take the absolute value of both sides of Eq . 3 :

$$0 = \left| \frac{2^\alpha}{(-2 + 2^\alpha) \Gamma(\alpha)} \right| \left| \int_0^\infty \frac{t^{-1+\alpha}}{1 + e^t} dt \right|$$

(Eq . 4)

We note immediately that :

$$\left| \frac{2^\alpha}{(-2 + 2^\alpha) \Gamma(\alpha)} \right| > 0$$

(for all  $\alpha$ )

(Eq . 5)

We divide both sides of eq. (4) by the left - hand side of inequality (5) , and we get :

$$0 = \left| \int_0^\infty \frac{t^{-1+\alpha}}{1 + e^t} dt \right|$$

(Eq . 6)

Since eq. 6 above is complex for  $\alpha = a + bi$ , we expand eq. 6 above by complex expansion. Moreover, the expression underneath the integrand is equal to zero. Making use of these two concepts, we conclude :

$$0 = \left| \int_0^\infty \frac{t^{-1+\alpha}}{1 + e^t} dt \right| \text{ is equal to the following :}$$

$$\left| \int_0^{\infty} \frac{t^{-1+a} (\cos (b \ln (t)) + i \sin (b \ln (t)))}{1 + e^t} dt \right| = 0$$

(by using complex expansion of  $t^{-1+a}$ ,  
resulting in  $t^{-1+a} = t^{-1+a} (\cos (b \ln (t)) + i \sin (b \ln (t)))$ )

(Eq .7)

$$\left| \int_0^{\infty} \frac{t^{-1+a} \cos (b \ln (t))}{1 + e^t} dt + i \int_0^{\infty} \frac{t^{-1+a} \sin (b \ln (t))}{1 + e^t} dt \right| = 0$$

(Eq .8)

Also note how in Eq.7 we have "a", like in the start of the paper. In Eq.8, we have "α", where  $\alpha = a + bi$ . So, we continue with the consequence of Eq.7 : Consequently, since the integral is a linear function, we "distribute" over Eq .7 - Eq .8 ; we then take the absolute value of Eq .8 :

$$\left( \int_0^{\infty} \frac{t^{-1+a} \cos (b \ln (t))}{1 + e^t} dt \right)^2 + \left( \int_0^{\infty} \frac{t^{-1+a} \sin (b \ln (t))}{1 + e^t} dt \right)^2 = 0 \text{ (Eq .9)}$$

Since Eq .9 is equal to zero,  
and is also the sum of two non - negative numbers, we get the following from Eq .9 :  
( $p^2 = 0$  means that  $p = 0$  if  $p$  is a real number, like the squared integral in Eq .10)

$$\left( \int_0^{\infty} \frac{t^{-1+a} \cos (b \ln (t))}{1 + e^t} dt \right)^2 = 0 \quad \text{(Eq .10)}$$

Then :

$$\int_0^{\infty} \frac{t^{-1+a} \cos (b \ln (t))}{1+e^t} dt = 0 \quad (\text{Eq. 11})$$

Then :

$$\int_0^{\infty} \frac{t^{-1+a} \cos (b \ln (t))}{1+e^t} dt = 0 \quad \text{is equal to the following :}$$

$$\int_0^{\infty} \frac{t^{-1+a} \cos \left( (2) \left( \frac{1}{2} \right) b \ln (t) \right)}{1+e^t} dt = 0 \quad (\text{Eq. 12})$$

(Note the inner function of Cosine function in Eq. 12)

By trigonometric identity :  $\cos (2x) = (\cos (x))^2 - (\sin (x))^2$  implies that :

$$\cos \left( (2) \left( \frac{1}{2} \right) b \ln (t) \right) = \left( \cos \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2 - \left( \sin \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2$$

then :

$$(\text{Eq. 13}) \quad 0 = \int_0^{\infty} \frac{t^{-1+a} \cos \left( (2) \left( \frac{1}{2} \right) b \ln (t) \right)}{1+e^t} dt =$$

$$(\text{Eq. 14}) \quad \int_0^{\infty} \frac{1}{1+e^t} \left( t^{-1+a} \left( \cos \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2 - \left( \sin \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2 \right) dt =$$

$$\int_0^{\infty} \frac{t^{-1+a} \left( \cos \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2}{1+e^t} dt - \int_0^{\infty} \frac{t^{-1+a} \left( \sin \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2}{1+e^t} dt \quad (\text{Eq. 15})$$

So, by Eq. 13 and Eq. 15, we see :

$$(\text{Eq. 16}) \quad \int_0^{\infty} \frac{t^{-1+a} \left( \cos \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2}{1+e^t} dt -$$

$$\int_0^{\infty} \frac{t^{-1+a} \left( \sin \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2}{1+e^t} dt = 0$$

$$\text{Then : } \int_0^{\infty} \frac{t^{-1+a} \left( \cos \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2}{1+e^t} dt = \int_0^{\infty} \frac{t^{-1+a} \left( \sin \left( \left( \frac{1}{2} \right) b \ln (t) \right) \right)^2}{1+e^t} dt$$

(Eq. 17)

The above Eq. 17 will be contradicted in a bit. For now, consider the following :

We see that :

$$1 + e^t \leq e^t + e^t \text{ for all } t \in [0, \infty). \text{ Then,}$$

by multiplying both sides of  $1 + e^t \leq e^t + e^t$  by  $\frac{1}{(1 + e^t)(e^t + e^t)}$ ,

$$\frac{1}{e^t + e^t} \leq \frac{1}{1 + e^t} \text{ for all } t \in [0, \infty). \text{ Then,}$$

since  $(\cos((1/2) b \ln(t)))^2 \geq 0$  for all "b" and for all "t",

we can multiply through the inequality (both sides)  $\frac{1}{e^t + e^t} \leq$

$\frac{1}{1 + e^t}$  by  $(\cos((1/2) b \ln(t)))^2$ , without changing the direction of

the inequality (since there 's no negative values for these  $\frac{1}{e^t + e^t}$ ,

$\frac{1}{1 + e^t}$ ,  $(\cos((1/2) b \ln(t)))^2$ ). We see :

$$\frac{1}{e^t + e^t} \leq \frac{1}{1 + e^t} \text{ implies } \frac{(\cos((1/2) b \ln(t)))^2}{e^t + e^t} \leq \frac{(\cos((1/2) b \ln(t)))^2}{1 + e^t},$$

and therefore since  $t^{-1+a} \geq 0$  for all  $t \in [0, \infty)$ , and for  $0 < a < 1/2$ , we now see :

(Eq. 18)

Then :

$$\frac{(\cos((1/2) b \ln(t)))^2}{e^t + e^t} \leq \frac{(\cos((1/2) b \ln(t)))^2}{1 + e^t} \text{ implies :}$$

$$\frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t + e^t} \leq \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} \text{ for all } t \in [0, \infty),$$

because  $t^{-1+a}$  is a non - negative number,

we can multiply through without having a periodic change in inequality.

(Eq. 18 b)

which implies :

$$\int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t + e^t} dt = \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{2 e^t} dt =$$

$$(1/2) \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t} dt \leq \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt$$

(note the denominator in these 3 expressions)

(Eq .19)

In explaining the immediate above lines in Eq. 19, we see that by multiplying on both sides of the inequality  $\frac{1}{e^t + e^t} \leq \frac{1}{1 + e^t}$  by non - negative functions like  $t^{-1+a}$ ,  $(\cos((1/2) b \ln(t)))^2$ , etc.,

does not change the direction of the inequality. Moreover, we multiply through the inequality by these functions, and come to a conclusion :

From Eq. 19 we see :

$$(1/2) \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t} dt =$$

$$(1/2) ((1/4) \Gamma(a - ib) + (1/4) \Gamma(a + ib) + (1/2) \Gamma(a)) \text{ (Eq .20)}$$

(Note :  $\Gamma(a)$  is the Euler - Gamma function)

(If you want an easy way to see this, put the left - hand side of the Eq .20 inequality above in Mathematica). Also note how since the left - hand side of the inequality of Eq .20 is a real number, then so if the right - hand side of Eq .20 a real number.

Now, in Eq .20 immediately above, we note :

$$0 \leq (1/2) \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t}, \text{ (Eq .21)}$$

since the value of the function underneath the integrand is greater than, or equal to, zero ; then, the integral from 0 to infinity of a non - negative, non - constant function is greater than zero. But the function

in Eq .21 is bounded also above by the right - hand side of Eq. 20. So, the function behaves well and doesn ' t diverge to infinity. Moreover :

We see the following :

$$0 \leq (1/2) \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t} dt \leq \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt \quad (\text{Eq. 22})$$

Now note :

$$\int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt \leq \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t} dt \quad (\text{Eq. 23})$$

To see this, note :

$$\frac{1}{1 + e^t} \leq \frac{1}{e^t} \text{ implies,}$$

now by multiplying both sides with  $(\cos((1/2) b \ln(t)))^2$  and  $t^{-1+a}$ , the following :

$$\frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} \leq \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t}$$

so that by integrating both sides, we get :

$$\int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt \leq \int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{e^t} dt \quad (\text{Eq. 24})$$

By integrating the right -

hand side of the immediate above inequality (use Mathematica if you need), we get :

$$\int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt \leq ((1/4) \Gamma(a - ib) + (1/4) \Gamma(a + ib) + (1/2) \Gamma(a)) \quad (\text{Eq. 25})$$

Then :

$$(1/2) ((1/4) \Gamma(a - ib) + (1/4) \Gamma(a + ib) + (1/2) \Gamma(a)) \leq$$

$$\int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt \leq$$

$$((1/4) \Gamma(a - ib) + (1/4) \Gamma(a + ib) + (1/2) \Gamma(a)) \quad (\text{Eq. 26})$$

Of pivotal importance is seeing this Eq. 26,

where the lower bound of the immediate above inequality

was found in (Eq .20) and the upper bound was found in Eq .25. Now,  
 if we find the lower and upper bounds of  $\int_0^{\infty} \frac{t^{-1+a} (\text{Sin} ((1/2) b \text{Ln} (t)))^2}{1 + e^t}$ ,  
 as seen and described from Eq .13 to Eq .17, we get :

$$(1/2) ((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \leq$$

$$\int_0^{\infty} \frac{t^{-1+a} (\text{Sin} ((1/2) b \text{Ln} (t)))^2}{1 + e^t} dt \leq$$

$$((1/4) \Gamma (a - ib) + (1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \text{ (Eq. 27)}$$

(We find the upper and lower bound of

$$\int_0^{\infty} \frac{t^{-1+a} (\text{Sin} ((1/2) b \text{Ln} (t)))^2}{1 + e^t} dt \text{ for the immediate above Eq .27 by the same methods}$$

as for  $\int_0^{\infty} \frac{t^{-1+a} (\text{Cos} ((1/2) b \text{Ln} (t)))^2}{1 + e^t} dt$  over the course of the paper. )

We conclude :

$$(1/2) ((1/4) \Gamma (a - ib) + (1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \leq$$

$$\int_0^{\infty} \frac{t^{-1+a} (\text{Cos} ((1/2) b \text{Ln} (t)))^2}{1 + e^t} dt \leq$$

$$((1/4) \Gamma (a - ib) + (1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \text{ (Eq. 26)}$$

and

$$(1/2) ((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \leq$$

$$\int_0^{\infty} \frac{t^{-1+a} (\text{Sin} ((1/2) b \text{Ln} (t)))^2}{1 + e^t} dt \leq$$

$$((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \text{ (Eq. 27)}$$



(The method for proof of Eq. 27 above is the same for Eq. 26 that we demonstrated earlier

in the paper, but in this case it is for  $\int_0^{\infty} \frac{t^{-1+a} (\sin ((1/2) b \ln (t)))^2}{1 + e^t} dt$ ).

So, now, consider the conclusion that we reached in Eq. 17 :

$$\int_0^{\infty} \frac{t^{-1+a} (\cos ((1/2) b \ln (t)))^2}{1 + e^t} dt = \int_0^{\infty} \frac{t^{-1+a} (\sin ((1/2) b \ln (t)))^2}{1 + e^t} dt \quad (\text{Eq. 28})$$

We arrived at this conclusion of Eq. 28

deductively from the first statement in the paper; namely, that  $\alpha = a + bi$  is an arbitrary zero that lies anywhere in the critical strip, even on or off the critical line. Now, in order to contradict Eq. 28 (seen prior in Eq. 17) :

Since :

$$(1/2) ((1/4) \Gamma (a - ib) + (1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \leq$$

$$\int_0^{\infty} \frac{t^{-1+a} (\cos ((1/2) b \ln (t)))^2}{1 + e^t} dt \leq$$

$$((1/4) \Gamma (a - ib) + (1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \quad (\text{Eq. 26})$$

$$(1/2) ((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \leq$$

$$\int_0^{\infty} \frac{t^{-1+a} (\sin ((1/2) b \ln (t)))^2}{1 + e^t} dt \leq$$

$$((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \quad (\text{Eq. 27})$$

$$\text{and, since } \int_0^{\infty} \frac{t^{-1+a} (\cos ((1/2) b \ln (t)))^2}{1 + e^t} dt =$$

$$\int_0^{\infty} \frac{t^{-1+a} (\sin ((1/2) b \ln (t)))^2}{1 + e^t} dt \text{ from (Eq. 28), as seen prior in Eq. 17, then :}$$

Substitute  $\int_0^\infty \frac{t^{-1+a} (\cos ((1/2) b \ln (t)))^2}{1 + e^t}$   
 $dt$  instead of  $\int_0^\infty \frac{t^{-1+a} (\sin ((1/2) b \ln (t)))^2}{1 + e^t} dt$  into Eq .27,

since these two functions are equal according to our deductive approach culminating in Eq .17 :

$$\begin{aligned} & (1/2) ((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \leq \\ & \int_0^\infty \frac{t^{-1+a} (\cos ((1/2) b \ln (t)))^2}{1 + e^t} dt \leq \\ & ((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \text{ (Eq .29)} \end{aligned}$$

$$(1/2) ((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \text{ (Eq .29 a)}$$

$$((-1/4) \Gamma (a - ib) + (-1/4) \Gamma (a + ib) + (1/2) \Gamma (a) ) \text{ (Eq .29 b)}$$

When  $b = 0$  for (Eq .29 a) and when  $b = 0$  for (Eq .29 b) ,  
 then the left - hand side of of Eq .29 above is equal to zero ,  
 and the right - hand side of Eq. 29 is equal to zero . Therefore ,  
 since both sides of the inequality of Eq .29 have  $b = 0$  ,

then the function in between them,  $\int_0^\infty \frac{t^{-1+a} (\cos ((1/2) b \ln (t)))^2}{1 + e^t} dt$ , has  $b = 0$  :

$$0 \leq \int_0^\infty \frac{t^{-1+a} (\cos ((1/2) 0 \ln (t)))^2}{1 + e^t} dt \leq 0 \text{ (Eq .30)}$$

which, by the "Pinch" theorem from Mathematical Analysis ,

implies :  $\int_0^\infty \frac{t^{-1+a} (\cos ((1/2) 0 \ln (t)))^2}{1 + e^t} dt = 0$  which is equal to :

$$\int_0^\infty \frac{t^{-1+a}}{1 + e^t} dt = 0 \text{ (Eq. 31)}$$

The above Eq. 31 is easily seen to yield a positive ,  
 finite value for the integral , and we see this since :

$$\begin{aligned} & \frac{1}{e^t + e^t} \leq \frac{1}{1 + e^t} \leq \frac{1}{e^t} \Rightarrow \\ & \frac{t^{-1+a}}{e^t + e^t} \leq \frac{t^{-1+a}}{1 + e^t} \leq \frac{t^{-1+a}}{e^t} \Rightarrow \left( \text{Since } \frac{1}{e^t + e^t} = \frac{1}{2 e^t} = \frac{1}{2} \left( \frac{1}{e^t} \right) \right) \int_0^\infty \frac{t^{-1+a}}{e^t + e^t} dt \leq \\ & \int_0^\infty \frac{t^{-1+a}}{1 + e^t} dt \text{ (Eq. 32) , which implies :} \end{aligned}$$

$$\int_0^{\infty} \frac{t^{-1+a}}{e^t + e^{-t}} dt \leq \int_0^{\infty} \frac{t^{-1+a}}{1 + e^t} dt \Rightarrow \frac{1}{2} \Gamma(a) \leq \int_0^{\infty} \frac{t^{-1+a}}{1 + e^t} dt \text{ since } \int_0^{\infty} \frac{t^{-1+a}}{e^t + e^{-t}} dt = \frac{1}{2} \Gamma(a) \text{ (use Mathematica if the integration is too hard);}$$

$\Gamma(a)$  is the gamma function). So, since  $\frac{1}{2} \Gamma(a)$  is strictly greater than zero for all  $0 < a < 1$  (like we said we were working on this interval from the very first few lines of the paper on Eq .1 and Eq .2), and since  $\frac{1}{2} \Gamma(a) \leq \int_0^{\infty} \frac{t^{-1+a}}{1 + e^t}$ , then the value of the integral in Eq. 31 is greater than zero for all "a", arriving at a contradiction (also note that in general,  $\frac{1}{2} \Gamma(a) \leq \int_0^{\infty} \frac{t^{-1+a}}{1 + e^t} \leq \Gamma(a)$ , so the integral in Eq .31 doesn' t diverge). Therefore, in Eq. 17, where it states that

$$\int_0^{\infty} \frac{t^{-1+a} (\cos((1/2) b \ln(t)))^2}{1 + e^t} dt = \int_0^{\infty} \frac{t^{-1+a} (\sin((1/2) b \ln(t)))^2}{1 + e^t} dt,$$

we see they are in fact,

unequal (read Eq. 27 - Eq .31 carefully to see contradiction). Since we have a deductive argument starting from Eq .1 without any errors, then, this contradiction of Eq .17 by the argument leads us to conclude that the arbitrary zero from Eq .1 - Eq .2, seen as  $\alpha = a + bi$  (as an assumption), and lying hypothetically anywhere in the critical strip, does not exist.

I will be further researching on what exactly is the cause of this anomaly in the critical strip; obviously, there can be no zeros in the critical strip with my technique. However, we see by other methods that are involved in the field of the riemann hypothesis analysis that zeros are supposed to exist on the critical line. So, if you like this paper, please donate to my paypal account; any amount will be more than helpful. My email for the paypal account is vkalaj@gmail.com, and my name, as stated at the top of the paper, is Viktor Kalaj.