Physical Mechanism underlying “Einstein’s Spooky-action-at-a-distance” and the nature of Quantum Entanglement

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The delayed-choice entanglement swapping experiments, both in space and time, are casually explained at a single quantum level by using the ‘wave-particle non-dualistic interpretation of quantum mechanics’. In order to achieve this, the actual mechanisms involved in the Wheeler’s delayed-choice experiment and Einstein’s spooky-action-at-a-distance are uncovered from the quantum formalism. The continuity in the motion of any individual quantum particle, due to the constants of motion, is responsible for the outcomes of Wheeler’s delayed-choice experiment. The purpose for the existence of spooky action in Nature is to strictly maintain the conservation laws in absence of exchange-interactions. The presence of a casual structure in the entanglement swapping is shown by detailed analysis of the experimental results presented in the papers, “X-S. Ma et al., Nature. Phys. 8, 480, (2012)” and “E. Megidish et al., Phys. Rev. Lett. 110, 210403 (2013)”, at the level of individual quantum events. These experiments are directly confirming the wave-particle non-duality.

I. INTRODUCTION

The quantum entanglement is a natural consequence of the quantum formalism whose existence in Nature was experimentally confirmed [1–8], thanks to Bell's inequalities [9]. Nature is indeed quantum mechanically spooky. Its existence was first figured out in the EPR paper [10] and a little later by Schrödinger [11]. Any measurement of a physical property on one quantum particle has an instantaneous influence on the measurement outcomes of its entangled partner, which is space-like separated to a great distance. No known physical carriers of Nature seem to be responsible for this instantaneous influence, since such carriers carry energy and momentum and hence bound to the Cosmic speed limit in accordance with the special theory of relativity. Einstein called this, "spooky action-at-a-distance". Such an effect was already pointed out by him for the case of single particle 'wavefunction collapse', at the 1927 Solvay Conference.

Another known quantum phenomenon is the entanglement swapping, where two well-separated quantum particles can be made to become entangled even though they have never interacted and shared any common past [12]. In order to achieve this, two pairs of polarization entangled photons are produced and one photon from each pair is sent to Alice and Bob, respectively. The remaining two photons are sent to Victor. Now, Alice's and Bob's photons can be made to become entangled by simply projecting Victor's photons onto an entangled state. Here, an interesting aspect is that this entanglement swapping can be delayed. First, Alice and Bob measure the polarization states of the received photons in their own basis. At later time, Victor randomly chooses either entangled or uncorrelated basis to project his photons. Victor's future choice of measurement basis seems to dictate the already recorded polarization states of Alice and Bob whether to become entangled or not. This kind of correlations is known as entanglement swapping in space [13–15].

The entanglement swapping can also be achieved in time [16] as follow: A polarization entangled pair of photons is produced at some initial time. The polarization state of the first photon of the pair is measured by Alice in her own basis. The remaining second photon, along with the third photon belonging to another entangled pair of photons produced at a later time are sent to Victor. Bob receives the remaining fourth photon and measures its polarization state in his basis. However, Victor can randomly project the second and third photons either onto a Bell state or a separable state. At the time of measurement of the first photon's polarization state, the fourth photon was not yet created. But, it was found that the first and the last photons exhibiting correlations between their measured polarization states, as though Bob's later measurements decided the outcomes of already recorded results of Alice's measurements. These correlations of entanglement swapping seem to violate our common sense casual experience, i.e., the 'present' is simply a resultant outcome of already happened and no more existing 'past'.

The work reported here is to provide a casual explanation of the Wheeler's delayed-choice experiment [18] and also the mechanism involved in the spooky action-at-a-distance [10]. The emergence of causality is a naturally consequence of wave-particle non-duality. The quantum mechanical conservation laws are shown to be responsible for the existence of spooky action, an influence occurring in the absence of exchange-interactions and hence unbounded to the Cosmic speed limit. And then, the casual structure of delayed-choice entanglement swapping experiments at a single quantum level is brought into light. All these are done by making use of the non-dualistic interpretation of quantum formalism proposed recently [19, 20].

The present article is organized as follows: Section-II contains a brief summary of non-dualistic interpretation of quantum mechanics which is needed in the following sections. An explanation of the casual mechanism involved in the Wheeler's delayed-choice experiment, which is considered in the context of Young's double-slit experiment, is presented in the Section-III. In Section-IV, a mathematical analysis is carried out to expose the physical mechanism underlying the spooky-action-at-a-distance. In Sections V and VI, a casual explanation is given for the delayed-choice entanglement swapping, both in space and time, for an individual quantum event and some experimental results available in the literature are analyzed in detail as examples. Section-VII contains the discussions and conclusions.

II. BRIEF SUMMARY OF THE NON-DUALISTIC INTERPRETATION OF QUANTUM MECHANICS

Some crucial points of non-dualistic interpretation [19, 20] which are needed to explain Wheeler's delayed-choice experiment [18], Einstein's spooky action-at-a-distance [10] and entanglement swapping experiments [12–16], are given below;

The entire space in which the Universe dwells is recognized as an infinite dimensional complex vector space (CVS) which is in general tensor-products of Hilbert spaces of quantum entities. The three dimensional Euclidean space (R^3ES) arises 'effectively' within the quantum formalism due to the inner-product interaction and also due to the equality of quantum mechanical and classical times. The Schrödinger's wavefunction is shown to be an *instantaneous resonant spacial mode* (IRSM) in which its resonant quantum particle flies akin to the case of a test particle moving in the curved space-time of general theory of relativity.

Any state vector or equivalently the IRSM, $|\psi>$, obeying the Schrödinger wave equation and belongs to CVS can be visualized with respect to R^3ES by attaching a complex vector $|\mathbf{r}><\mathbf{r}|\psi>$ at every eigenvalue, $\mathbf{r}=\{x,y,z\}$, of the position operator $\hat{\mathbf{r}}$. The set, $\{\mathbf{r}\}$, of all possible position eigenvalues form the R^3ES. Obviously, one has

$$|\psi>=\int d\mathbf{r}|\mathbf{r}><\mathbf{r}|\psi> \tag{1}$$

The quantum particle will be present at some eigenvalue $\mathbf{r_p}$, corresponding to the vector $|\mathbf{r_p}><\mathbf{r_p}|\psi>$. The particle naturally enters into this position eigenstate, $|\mathbf{r_p}><\mathbf{r_p}|\psi>$, in such a way that its absolute phase is the same as that of $|\psi>$, i.e., phase$\{<\mathbf{r_p}|\psi>\}$ = phase$\{|\psi>\}$.

This picture of a particle flying in its own IRSM is unlike any classical wave, though the IRSM is obeying the Schrödinger's wave equation. The intensity of a classical wave in R^3ES is proportional to the square of its amplitude. But, such an intensity can't be claimed for the IRSM. If the particle is going to end up on a detector screen, then a dual vector, $<\psi|$, gets excited in that screen and interacts with IRSM as $<\psi|\psi>$. The particle moving in its IRSM will interact at some location in the region of $<\psi|\psi>$. This inner-product interaction can be found within the quantum formalism. Let the state, $|\psi>$, gets scattered into some other state, $|\psi'>$, at the screen. This process can be described by associating an operator, $\hat{O}=|\psi'><\psi|$, such that,

$$\hat{O}|\psi>=<\psi|\psi>|\psi'> \tag{2}$$

Therefore, if the scattered state is discarded or it is a null-state, then the particle must have interacted or got absorbed at some location in the region of inner-product, $<\psi|\psi>$.

Since the Schrödinger's equation is a partial differential equation, it's necessary to impose boundary conditions to IRSM. Consider a free particle confined in a one-dimensional box of length 'L' along X-axis, i.e., $0\le x\le L$. The dual modes excited at the boundaries interact with the IRSM, $|\psi>$, such that

$$<\psi|\psi>|_{x=0}=<\psi|\psi>|_{x=L}=0 \tag{3}$$

which in turn implies $<x|\psi>|_{x=0}=<x|\psi>|_{x=L}=0$. Also, one can see that

$$<\psi|\psi>=\int_0^L dx<\psi|x><x|\psi>=\int_0^L dx \quad |<x|\psi>|^2\neq\infty \tag{4}$$

The above integral must converge in order to have any physical interpretation. This is a well-known Born's rule. Here the aim is just to show that it naturally emerges in the non-dualistic interpretation.

In the case of an unbounded free particle, the initial boundary condition is a point in the CVS where the momentum got originated and remains unaltered as long as the particle sustains with the same momentum. The final boundary condition depends on where the particle will end up and need not be a fixed boundary condition and can be changed before the arrival of the particle there. If the particle's momentum changes suddenly, then the corresponding IRSM disappears completely and a new IRSM of the modified momentum appears instantaneously in the entire space. The new IRSM's origin lie at the spatial point where its resonant particle gained new momentum. In other words, the old origin disappears and a new origin appears at the same instant, irrespective of the separation between them. This property simply follows from the Hamiltonian eigenvalue equation along with boundary conditions imposed on the eigenstates (here, it's important to keep in mind that the Hamiltonian and momentum operators commutes for a force-free particle; otherwise, the energy changing interaction should be considered which is the actual and general situation).

When the IRSM, $|\psi>$, encounters a space spanned by discrete eigenstates, $|a_i>$; $i=1,2,3,\cdots$, of an operator, $\hat{A}$, then the particle enters into one of the eigenstate, say $|a_p>$, having the minimum phase when compared to all other remaining eigenstates. All empty eigenstates will be present ontologically along with the particle state. During the detection, the particle will be naturally found in $|a_p>$ with an eigenvalue a_p since, empty states have nothing to contribute. The IRSM can be expressed as

$$|\psi>=\sum_i|a_i><a_i|\psi> \tag{5}$$

which interacts with its excited dual-mode, $<\psi|$, in the detector as

$$<\psi|\psi>=\sum_i<\psi|a_i><a_i|\psi>\xrightarrow{\text{Observation}}|<a_p|\psi>|^2 \tag{6}$$

yielding the eigenvalue a_p. This is the underlying physical mechanism of the 'wave function collapse' advocated in the Copenhagen interpretation [17]. Repeating the detection procedure on several particle states having different initial phases yields different eigenvalues and in the limit of infinite particles, the relative frequency of detection (RFD) coincides with $| < a_i|\psi > |^2$, i.e.,

$$< \psi|\psi >= \sum_i < \psi|a_i >< a_i|\psi >= \sum_i | < a_i|\psi > |^2 = 1 \tag{7}$$

which is the well-known Born's rule. Therefore,

$$\mathrm{RFD} = \mathrm{Born's\ rule}$$

and hence, quantum mechanics is not a probabilistic theory. It can be described at a single quantum level which, anyhow, statistically yields Born's rule for a large number of identical particles. It's like classical mechanics but in the CVS. The unavailability of the information about the absolute phase of the IRSM due to the inner-product interaction forces experiments to observe only RFD. Here, it's worth recollecting the Born's Probabilistic Interpretation [17]:

"*The wave function determines only the probability that a particle which brings with itself energy and momentum takes a path; but no energy and no momentum pertains to the wave*"

Notice that the above statement is in exact agreement with the spirit of wave-particle non-duality, where the Schrödinger wave function is shown to be an IRSM, except for the notion of probability.

Now, consider the case in Eq. (1),

$$|\psi >= \int d\mathbf{r}|\mathbf{r} >< \mathbf{r}|\psi >$$

where, the particle will enter into a state, $|\mathbf{r_p} >< \mathbf{r_p}|\psi >$, whose phase is exactly same as $|\psi >$. Therefore, the interaction of IRSM with its excited dual, $< \psi|$, in an apparatus is

$$< \psi|\psi >= \int d\mathbf{r} < \psi|\mathbf{r} >< \mathbf{r}|\psi > \xrightarrow{\text{Observation}} | < \mathbf{r_p}|\psi > |^2 \tag{8}$$

because, except the particle state $|\mathbf{r_p} >< \mathbf{r_p}|\psi >$, the remaining ones, $|\mathbf{r} >< \mathbf{r}|\psi >$, are empty.

III. CAUSALITY IN WHEELER'S DELAYED-CHOICE EXPERIMENT

Consider the Young's double-slit experiment as given in Fig. 1 which can be used to study the Wheeler's delayed-choice experiment [18].

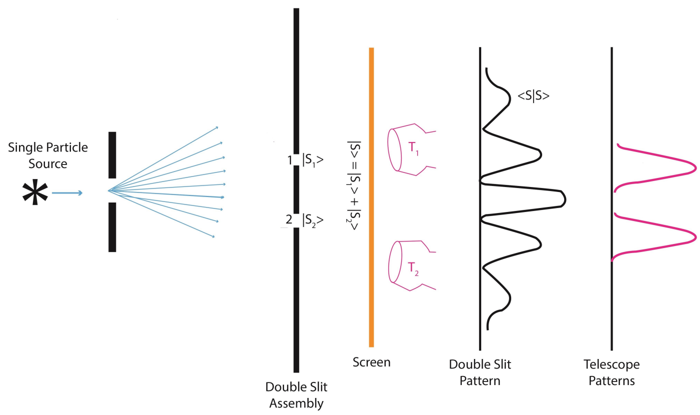

FIG. 1. **Diagram of single-particle Young's double-slit experiment**: A source shoots single particles, one at a time, towards a double-slit assembly. 1 and 2 represents two slits through which the state vectors $|S_1 >$ and $|S_2 >$ emerge out and get superposed as $|S >= |S_1 > +|S_2 >$. Immediately behind the screen, two telescopes, T_1 and T_2, are placed such that all the particles passing through 1 and 2 reach T_1 and T_2, respectively. After collecting a large number of individual particles, the observed particle distribution patterns on the screen and telescopes are also shown in the same diagram.

A monochromatic single-photon source shoots individual photons, one at a time, towards the screen such that every photon is emitted only after the registration of the previous photon. The IRSM of a photon, $|S>$, excited through the double-slit is

$$|S>=|S_1>+|S_2> \tag{9}$$

where, $|S_1>$ and $|S_2>$ are the IRSMs through the slits 1 and 2, respectively. The excited dual-mode in the detector screen, $<S|$, interacts with the IRSM according to the inner-product as given below:

$$<S|S>=<S_1|S_1>+<S_2|S_2>+<S_1|S_2>+<S_2|S_1> \tag{10}$$

The moment a photon appears at the source, its IRSM resonantly appears in the entire space and also the inner-product interaction occurs on the screen instantly. Depending on the initial phase of the state vector, the photon will fly either through slit 1 or 2 to the screen and contributes a point to $<S|S>$. The entire IRSM disappears once the momentum of photon changes either due absorption or scattering at the screen. The next photon appears at the source along with its IRSM, but with different initial phase and hence lands at some different location of the interaction region, $<S|S>$. After a large collection of photons hitting randomly at different positions, an interference pattern emerges on the screen which is actually the construction of the function $|<\mathbf{r}|S>|^2$ with individual but random points of photon hits.

Note that, if the detector screen is such that its dual vector space can distinguish $|S_1>$ and $|S_2>$, i.e., $<S_1|S_2>=0$, then the interference disappears and two clump patterns occur.

Now, consider the Wheeler's delayed-choice situation [18, 21, 22]: While a photon is in mid-flight between the double-slit and screen, the screen is replaced quickly by twin telescopes T_1 and T_2, which are tightly focused on slits 1 and 2, respectively, i.e., $\hat{T}_1|S_2>=0=\hat{T}_2|S_1>$; where, $\hat{T}_1$ and $\hat{T}_2$ are operators corresponding to the vector space of the telescopes, T_1 and T_2, respectively. Now, one has,

$$\begin{aligned}(\hat{T}_1+\hat{T}_2)|S> &= (\hat{T}_1+\hat{T}_2)(|S_1>+|S_2>) = \hat{T}_1|S_1>+\hat{T}_2|S_2> \\ &= |\tilde{S}_1>+|\tilde{S}_2>\equiv|\tilde{S}>\end{aligned} \tag{11}$$

The old IRSM, $|S>$ is replaced by the new IRSM $|\tilde{S}>$, but both of their origins remain unchanged i.e., same for both $|S>$ and $|\tilde{S}>$. This can also be seen as follows: Solve the Schrödinger's wave equation and obtain two solutions, $|S>$ and $|\tilde{S}>$, for the screen and the twin-telescope configurations. These two solutions will have the same initial boundary condition but differ by the final boundary condition. The initial boundary condition is unaffected as long as the momentum of the photon is unaffected but, the final boundary conditions can be changed suddenly (or even randomly) before the detection of the photon (see Section-II). The solution also changes suddenly from $|S>$ to $|\tilde{S}>$. Whatever be the position of photon in $|S>$ during the replacement, it continues to fly from there through $|\tilde{S}>$ i.e., the photon's motion is always continues though the IRSM itself changes suddenly. The continuity in photon's motion is governed by its conserved properties. Therefore, the observed photon distribution at telescopes is given by

$$<\tilde{S}|\tilde{S}>=<\tilde{S}_1|\tilde{S}_1>+<\tilde{S}_2|\tilde{S}_2> \tag{12}$$

which corresponds to two clump patterns, one at each telescope. Therefore, the *causality is preserved*, which plays a crucial role in the entanglement swapping experiments [12–16].

IV. EINSTEIN'S SPOOKY ACTION-AT-A-DISTANCE

The spooky action-at-a-distance exists in Nature in order to maintain the conservation laws even in the absence of exchange-interactions, as it will be shown in the following:

Let's consider two independent free particles 1 and 2, flying in their IRSMs $|S'_1>$ and $|S'_2>$, respectively. The tensor-product of the two states describes the joint-state obeying

$$(\hat{P}'_1+\hat{P}'_2)|S'_1>|S'_2>\equiv(\hat{P}'_1+\hat{P}'_2)|S'_{12}>>=0 \tag{13}$$

where, $\hat{P}'_1$ and $\hat{P}'_2$ are momentum operators of the particles 1 and 2 and their sum is chosen to be zero for convenience. When $|S_{12}>>\in\mathbf{H}_1\otimes\mathbf{H}_2$, is represented in the space spanned by the continuous eigenstates $|\mathbf{r}>$ of position operator $\hat{\mathbf{r}}$, both particle states are super-imposed on each other and can independently co-exist in the same region of R^3ES spanned by the set of eigenvalues, $\{\mathbf{r}\}$, of $\hat{\mathbf{r}}$; here, $\mathbf{H}_1$ and $\mathbf{H}_2$ are the Hilbert space of particle 1 and 2, respectively.

The super-imposed joint-state can be visualized with respect to R^3ES as,

$$
\begin{aligned}
|S_{12}' >> &= \int_{\mathbf{r}_{1,0}} d\mathbf{r}|\mathbf{r} >< \mathbf{r}|S_1' > \otimes \int_{\mathbf{r}_{2,0}} d\mathbf{r}|\mathbf{r} >< \mathbf{r}|S_2' > \\
&= \left(\int_{\mathbf{r}_{1,0}} d\mathbf{r} \otimes \int_{\mathbf{r}_{2,0}} d\mathbf{r} \right) \{ (|\mathbf{r} >< \mathbf{r}|S_1' >) \otimes (|\mathbf{r} >< \mathbf{r}|S_2' >) \}
\end{aligned} \tag{14}
$$

where, $\mathbf{r}_{1,0}$ and $\mathbf{r}_{2,0}$ are the initial boundary conditions on the IRSMs of particle 1 and 2, respectively. Note that, the classical waves, like, two ripples on a water surface, never behave this way. Also notice that, this unentangled two-particle IRSM has two origins.

Now, consider the EPR case: These two particles come together, interact for a brief time and fly apart [10] i.e.,

$$
|S_1' > |S_2' > \xrightarrow{\text{Brief Interaction}} |S_1 > |S_2 > \tag{15}
$$

obeying

$$
(\hat{P}_1 + \hat{P}_2)|S_{12} >>= 0 \tag{16}
$$

where, $\hat{P}_1$ and $\hat{P}_2$ are the new momentum operators of the particles 1 and 2, such that the total conserved momentum is

$$
\hat{P}_1' + \hat{P}_2' = \hat{P}_1 + \hat{P}_2 = 0 \tag{17}
$$

Also,

$$
(\hat{P}_1' + \hat{P}_2')|S_1' > |S_2' > \xrightarrow{\text{Brief Interaction}} (\hat{P}_1 + \hat{P}_2)|S_1 > |S_2 > \tag{18}
$$

According to the present non-dualistic interpretation, since the momentum of individual particles changes during the moment of interaction, the earlier IRSM $|S_{12}' >>$, disappears and a new IRSM, $|S_{12} >>$, with the origin at the spatial point of interaction, appears. Akin to the Eq. (14), $|S_{12} >>$ can be represented in position basis as

$$
\begin{aligned}
|S_{12} >> &= \int_0^\infty d\mathbf{r}|\mathbf{r} >< \mathbf{r}|S_1 > \otimes \int_0^\infty d\mathbf{r}|\mathbf{r} >< \mathbf{r}|S_2 > \\
&= \left(\int_0^\infty d\mathbf{r} \otimes \int_0^\infty d\mathbf{r} \right) \{ (|\mathbf{r} >< \mathbf{r}|S_1 >) \otimes (|\mathbf{r} >< \mathbf{r}|S_2 >) \}
\end{aligned} \tag{19}
$$

Unlike in Eq. (14), the lower limit for both integrals above are same, denoting the spatial point of interaction where the particles established the conservation law given in Eq. (16). This initial condition can be expressed as a constraint on the position eigenvalues, $\mathbf{r}_{\mathbf{p_1}}$ and $\mathbf{r}_{\mathbf{p_2}}$ of particles 1 and 2 as $\mathbf{r}_{\mathbf{p_1}} + \mathbf{r}_{\mathbf{p_2}} = 0$. The particles 1 and 2 fly in the super-imposed IRSM $|S_1 > \otimes |S_2 >$ which couple to each other to obey Eq. (16) at every eigenvalue $\mathbf{r}$. Any projection measurement on the state $|S_1 >$ at some $\mathbf{r'}$ instantly affects $|S_2 >$ at the same $\mathbf{r'}$ and hence everywhere. Most importantly, notice that, this entangled two-particle IRSM has only one origin.

Suppose that the particle 1 encounters a detector screen. Then the IRSM $|S_{12} >>$ (but not $|S >_1$ alone) interacts with its dual as

$$
<< S_{12}|S_{12} >> \rightarrow | < \mathbf{r}_{\mathbf{p_1}}|S_1 > |^2 . | < -\mathbf{r}_{\mathbf{p_2}}|S_2 > |^2 \tag{20}
$$

The original IRSM $|S_{12} >>$ completely disappears leaving the particle 1 in the position eigenvalue $\mathbf{r}_{\mathbf{p_1}}$. It's entangled partner acquires the correlated eigenvalue $-\mathbf{r}_{\mathbf{p_2}}$ but, its new IRSM is still having the same origin as that of $|S_{12} >>$.

Now, consider another situation: If the state $|S_1 >$ splits into two components during some physical process, i.e.,

$$
|S_1 > \rightarrow |S_{1,a} > + |S_{1,b} >
$$

where, a and b stand for two components, then in reality it's not $|S_1 >$ alone splits, but the entangled state $|S_{12} >>$ itself, i.e.,

$$
|S_{12} >>= |S_{1,a} > |S_{2,a} > + |S_{1,b} > |S_{2,b} > \tag{21}
$$

such that Eq. (16) holds for each component independently. This can be explicitly shown to be happening by using the Dopfer's experiment [23] given in Fig. 2. A source emits an entangled pair of particles with zero total momentum.

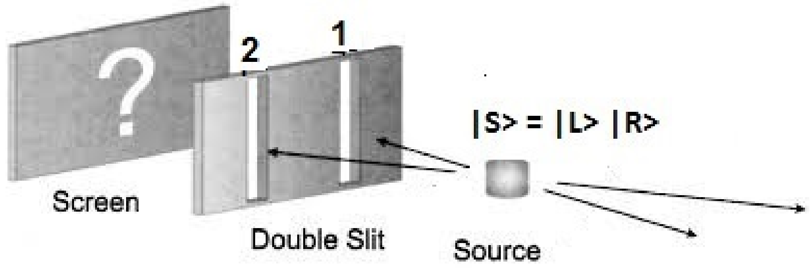

FIG. 2. **Principle involved in Dopfer's experiment**: The moment a pair of momentum-entangled particles is created at the source, their IRSM, $|S>>$ $(= |L>|R>)$ gets excited in the entire space and hence through the double-slit. The IRSM, $|S'>>$, through the double-slit interacts with its dual $<<S'|$ in the screen. The left particle hits the screen in the region of interaction $<<S'|S'>>$. Whether all such left particles exhibits interference or not can be decided by an appropriate measurement on the right particles.

Let us call the two particles as left and right particles such that their momentum entangled IRSM, $|S>>=|L>|R>$, obeys

$$(\hat{P}_L + \hat{P}_R)|S>>= 0 \tag{22}$$

where, $|L>$ and $|R>$ are IRSMs of left and right particles and $\hat{P}_L$ and $\hat{P}_R$ are their momentum operators, respectively.

The entangled IRSM $|S>>$, with origin lying at the source, gets projected through the double-slit onto the screen as $|S'>>$:

$$|S'>>=|S'_1>>+|S'_2>>=|L_1>|R_1>+|L_2>|R_2> \tag{23}$$

such that

$$(\hat{P}_L + \hat{P}_R)|S'>>= 0 \tag{24}$$

By invoking the following mapping,

$$|L_1>\to|a>_1, |R_1>\to|b>_2, |L_2>\to|a'>_1, |R_2>\to|b'>_2 \text{ and } |S'>>\to|\psi>$$

one gets

$$|\psi>=|a>_1|b>_2+|a'>_1|b'>_2 \tag{25}$$

which is in agreement with the state considered in [23]. Here, the states, $|a>_1$ and $|a'>_1$ corresponds to particle 1 (left particle) going through first and second slits, respectively. The states, $|b>_2$ and $|b'>_2$ belongs to particle 2 (right particle). If particle 2 is found in $|b>_2$ ($|b'>_2$), then the particle 1 will be found in $|a>_1$ ($|a'>_1$), since they are in anti-parallel momentum entangled state.

Though Eq. (23) and Eq. (25) yield the same experimental outcomes as shown above by explicitly mapping them into each other, the way Eq. (23) is interpreted in non-duality is quite different. There is only one $|S>>$ in the entire space with one origin at the source, such that both the left and right particles move independently in it. $|L>$ and $|R>$ are super-imposed on each other such that the Eq. (22) holds at every eigenvalue of the position operator (see Eq. (19)). Therefore, any projection measurement of the right particle influences the state of the left particle. This influence is instantaneous due to the nature of IRSM, but not due to any physical carries. This is the actual mechanism behind the spooky action and its nature will become more transparent from the explicit examples considered in the following sections.

When the IRSM, $|S'>>$, interacts with its excited dual in the screen:

$$\begin{aligned} <<S'|S'>>= & <L_1|L_1><R_1|R_1>+<L_2|L_2><R_2|R_2> \\ & +<L_1|L_2><R_1|R_2>+<L_2|L_1><R_2|R_1> \end{aligned} \tag{26}$$

As mentioned in the Section-II, the left particle flying in the IRSM, $|S' >>$, hits the screen at some location in the region of interaction $<< S'|S' >>$.

Now consider the case of discrete basis spanning a two-dimensional vector space: Alice makes a measurement on particle 1 in his basis $|P_1^{A+} >$ and $|P_1^{A-} >$ and Bob, on particle 2 using his basis, $|P_2^{B+} >$ and $|P_2^{B-} >$, respectively. Therefore, one can write,

$$|S_{12} >>=< P_1^{A+}|S_{12} >> |P_1^{A+} > + < P_1^{A-}|S_{12} >> |P_1^{A-} > \tag{27}$$

and

$$|S_{12} >>=< P_2^{B+}|S_{12} >> |P_2^{B+} > + < P_2^{B-}|S_{12} >> |P_2^{B-} > \tag{28}$$

If Alice finds particle 1 in $|P_1^{A+} >$, which depends on the initial phase of $|S_{12} >>$, then the state gained by particle 2, due to spooky action, is $< P_1^{A+}|S_{12} >>= |P_2^{A_1+} >$. But, Alice would have found the particle $|P_1^{A-} >$ as well. Then the state of particle 2 is $< P_1^{A-}|S_{12} >>= |P_2^{A_1-} >$. This can be written as a single equation:

$$|P_2^{A_1\pm} >< P_1^{A\pm}|S_{12} >>\equiv |P_2^{A_1\pm} > \tag{29}$$

Now, Bob will measure $|P_2^{A_1\pm} >$ in his basis

$$|P_2^{A_1\pm} >=< P_2^{B+}|P_2^{A_1\pm} > |P_2^{B+} > + < P_2^{B-}|P_2^{A_1\pm} > |P_2^{B-} > \tag{30}$$

which yields him a RFD

$$C_2^{B_2\pm}(A_1\pm) = | < P_2^{B\pm}|P_2^{A_1\pm} > |^2 \tag{31}$$

If Bob would have measured the particle 2, then the RFD of Alice for particle 1 is given by

$$C_1^{A_1\pm}(B_2\pm) = | < P_1^{A\pm}|P_1^{B_1\pm} > |^2 \tag{32}$$

If Alice and Bob blocks all measurement basis except one, say, $|P_1^{A+} >$ and $|P_2^{B+} >$, then one can have

$$C_2^{B_2+}(A_1+) = C_1^{A_1+}(B_2+) \tag{33}$$

Suppressing the plus sign and particle numbers, the above can be written as

$$C^B(A) = C^A(B) = | < P^A|P^B > |^2 \equiv C(A, B) \tag{34}$$

which is the Malus law, the heart for violation of Bell's inequalities [1–9].

V. DELAYED CHOICE ENTANGLEMENT SWAPPING IN SPACE

The delayed choice entanglement swapping experiment is carried as follows: Initially, two EPR sources produce two pairs of polarization entangled photons in an anti-symmetric singlet state. One photon from the first pair is sent to Alice and one from the second pair to Bob. They measure the polarization state of their photons in their own basis. The remaining two photons are sent to Victor who will subject them either to Bell State Measurement (BSM) or Separable State Measurement (SSM) (see Ref. [15] and references therein). It was experimentally observed that the future choice of Victor's measurements on photons 2 and 3 seems to decide the nature of the joint-state of photons 1 and 4 (which are no more except as a registered data), i.e., whether they are entangled or not.

According to the present non-dualistic picture, it is the measurement of Alice and Bob casually determines the outcome of Victor but not vice versa. This is shown by making use of the mechanisms from both the Wheeler's delayed choice experiment and the spooky action-at-a-distance as follows:

Let $|\psi_{12} >>$ and $|\psi_{34} >>$ be the two entangled states of two pairs of photons. Photon 1 in $|\psi_{12} >>$ is send to Alice and photon 4 in $|\psi_{34} >>$ is sent to Bob. Photons 2 and 3 are send to Victor.

Let $|P_1^{A+} >$ and $|P_1^{A-} >$ be Alice's measurement basis for photon 1 and $|P_4^{B+} >$ and $|P_4^{B-} >$ are that of Bob for photon 4. Then the entangled states $|\psi_{12} >>$ and $|\psi_{34} >>$ can be expressed in Alice's and Bob's basis, respectively:

$$|\psi_{12} >>=< P_1^{A+}|\psi_{12} >> |P_1^{A+} > + < P_1^{A-}|\psi_{12} >> |P_1^{A-} > \tag{35}$$

and

$$|\psi_{34} >>=< P_4^{B+}|\psi_{34} >> |P_4^{B+} > + < P_4^{B-}|\psi_{34} >> |P_4^{B-} > \tag{36}$$

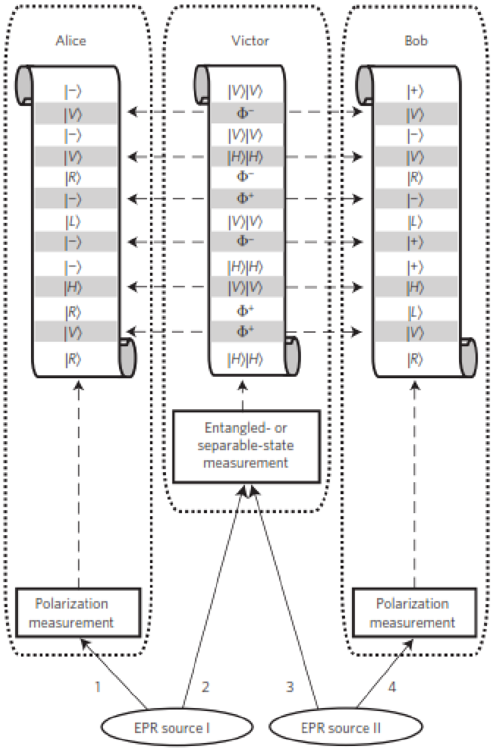

FIG. 3. **X-S. Ma et al., Experimental delayed-choice entanglement swapping**: Two pairs of polarization entangled photons are produced in Bell singlet states, $|\psi_{12}^{-} >$ and $|\psi_{34}^{-} >$, by the EPR sources 1 and 2. Photons 1 and 4 are sent to Alice and Bob respectively, who measure the polarization in their basis. After receiving the Photons 2 and 3, Victor randomly projects them either onto Bell State or Separable State Measurement in the future time with respect to the registration of the polarization states of photons 1 and 4 by Alice and Bob, respectively.

If Alice observes the photon 1 in the state $|P_1^{A+} >$, then the photon 2 is thrown into the state $< P_1^{A+}|\psi_{12} >>$ due to spooky action. Alice could have as well observed the photon 1 in $|P_1^{A-} >$. Therefore, the state of photon 2 depends on Alice's observation on photon 1 and similarly, that of photon 3 on Bob's measurement. The state of photon 2 after Alice's measurement can be written as

$$< P_1^{A\pm}|\psi_{12} >>= |P_2^{A_1\pm} > \tag{37}$$

where, the state $|P_2^{A_1+} >$ should be read as 'the state acquired by photon 2 when Alice observes photon 1 in the state $|P_1^{A+} >$. Similarly, the state of photon 3 after Bob's measurement is

$$< P_4^{B\pm}|\psi_{34} >>= |P_3^{B_4\pm} > \tag{38}$$

Therefore, the joint state of photon 2 and 3 encountered by Victor is

$$|P_2^{A_1\pm} > |P_3^{B_4\pm} >\equiv |P_2^{A_1\pm}; P_3^{B_4\pm} >> \tag{39}$$

If $|V_{23}^{+} >>$ and $|V_{23}^{-} >>$ are Victor's basis, then

$$|P_2^{A_1\pm}; P_3^{B_4\pm} >>=<< V_{23}^{+}|P_2^{A_1\pm}; P_3^{B_4\pm} >> |V_{23}^{+} >> + << V_{23}^{-}|P_2^{A_1\pm}; P_3^{B_4\pm} >> |V_{23}^{-} >>$$

which yields him a RFD:

$$C_{V_{23}}^{\pm}(A_1\pm; B_4\pm) = | << V_{23}^{\pm}|P_2^{A_1\pm}; P_3^{B_4\pm} >> |^2 \tag{40}$$

which clearly depends on already recorded measurements of Alice and Bob on photon 1 and 4, respectively. Further, Victor can change his basis, from $|V_{23}^{\pm} >>$ to some other basis, $|V'^{\pm}_{23} >>$, rather instantaneously and randomly like in the case of Wheeler's delayed choice experiment [18, 21, 22]. As one can easily observe from the above derivation, the entanglement swapping experiment is equivalent to two independent sequential measurements: (i) Alice and Bob prepare photons 2 and 3 in some initial state by performing measurements on photons 1 and 4, (ii) Photons 2 and 3 are subjected to Wheeler's delayed-choice experiment by Victor.

Now, I will show below that the Eq. (40) is in exact agreement with the experimental results found in Ref. [15]. Matching the notations used here with that of Ref. [15]:

$$|\psi_{ij} >>\sim |\psi_{ij}^{-} >>= \frac{1}{\sqrt{2}}(|H >_i |V >_j - |V >_i |H >_j)$$

where, $(i, j) = (1, 2)$ and $(3, 4)$, $|\psi_{ij}^{-} >>$ is the polarization entangled singlet state; $|H >$ and $|V >$ are the horizontal and vertical polarization states of a photon, respectively.

$$|V_{23}^{\pm} >>\sim |\phi_{23}^{\pm} >>= \frac{1}{\sqrt{2}}(|H >_2 |H >_3 \pm |V >_2 |V >_3)$$

$$|V'^{+}_{23} >>\sim |H >_2 |H >_3 \quad ; \quad |V'^{-}_{23} >>\sim |V >_2 |V >_3$$

In the above, $(|\phi_{23}^{+} >>, |\phi_{23}^{+} >>)$ and $(|H >_2 |H >_3, |V >_2 |V >_3)$ corresponds to BSM and SSM, respectively. The three basis sets in which Alice and Bob can measure the polarization state of photons 1 and 4 are $(|H >, |V >), (|R >, |L >)$ and $(|+ >, |- >)$; where,

$$|R >= \frac{1}{\sqrt{2}}(|H > +i|V >) \quad ; \quad |L >= \frac{1}{\sqrt{2}}(|H > -i|V >) \quad ; \quad |\pm >= \frac{1}{\sqrt{2}}(|H > \pm |V >)$$

and $|R > /|L >$ and $|\pm >$ stands for circular and linear polarization states.

The following matrix elements will be useful during the calculations:

$$< H|\pm >= \frac{1}{\sqrt{2}} \quad ; \quad < V|\pm >= \pm\frac{1}{\sqrt{2}} \quad ; \quad < H|R >=< H|L >= \frac{1}{\sqrt{2}} \quad ; \quad < V|R >= i\frac{1}{\sqrt{2}} \quad ; \quad < V|L >= -i\frac{1}{\sqrt{2}}$$

Given below is an explicit calculation for the result presented in TABLE I, when Alice and Bob find photons 1 and 4 in the states $|V >_1$ and $|V >_4$, respectively. Their measurements leave photons 2 and 3 in $|H >_2$ and $|H >_3$ via the spooky action-at-a-distance, i.e.,

$$< V_1|\psi_{12}^{-} >>= -\frac{1}{\sqrt{2}}|H >_2 \quad ; \quad < V_4|\psi_{34}^{-} >>= \frac{1}{\sqrt{2}}|H >_3$$

where, the polarization states $|P >_i$ and $|P_i >$ are treated as the same, i.e., $|P_i >\equiv |P >_i$ for notational convenience. However, it should be noted that the origins of IRSMs of photon 2 and 3 are still remaining unaffected at their respective sources, after the measurements on photons 1 and 4. If Victor subjects photon 2 and 3 to BSM or SSM, then he will find the following correlations:

$$-\frac{1}{2} << \phi_{23}^{+}|H >_2 |H >_3= -\frac{1}{2\sqrt{2}} \quad ; \quad -\frac{1}{2} << \phi_{23}^{-}|H >_2 |H >_3= -\frac{1}{2\sqrt{2}}$$

All possible future outcomes of Victor for BSM or SSM, when Alice and Bob find photons 1 and 4 in the basis $(|H >, |V >), (|R >, |L >)$ or $(|+ >, |- >)$ are listed in TABLES I, II and III, respectively. Some care is needed when considering the results in the tables with respect to the nature of doing the experiment as pointed out below:

Alice and Bob measure photons 1 and 4 in their independently chosen basis, which leave photons 2 and 3 in free states due to the spooky action, respectively. The dimensionality of the direct-product states of photons 1 and 4 and photons 2 and 3 are four. But Victor's basis, either BSM or SSM, span the same two-dimensional vector space. Therefore, in general, the total number of events seen by Alice and Bob together is twice to that seen by Victor. For example, consider the Table I: The sum of RFD for BSM is 1/2 and for SSM is also 1/2. But, the states of BSM and SSM are linearly dependent and lives in the same vector space. Therefore, half of the photons seen by both Alice and Bob are lost in Victor's basis. However, in the experimental situation [15], BSM and SSM are done mutually exclusively. It means that they can be regarded as two independent two-dimensional spaces. Moreover, the entangled partners are detected by coincidence measurements. Therefore, one has a situation where, every event detected by Victor will have a corresponding events detected by both Alice and Bob with respect to coincidence measurements. This can also be seen from the Table I: Victor's total RFD in BSM and SSM $= 1/2 + 1/2 = 1$.

Again consider the same Table I: According to the present non-dualistic interpretation, whenever Alice and Bob find photons 1 and 4 in the states $|V>_1$ and $|V>_4$, then Victor will surely be seeing photons 2 and 3 either in $|\phi^+_{23}>>$ or in $|\phi^-_{23}>>$ with RFD 1/8, during BSM. The absolute phase associated with the joint-state of photons 2 and 3 decides whether they will end up in $|\phi^+_{23}>>$ or in $|\phi^-_{23}>>$. Exactly the same will repeat when photons are found in $|H>_1$ and $|H>_4$. Therefore, it's clear that the measurement results of Alice and Bob are determining the future events of Victor. However, if one does a statistical analysis of a large number of collected coincidence events, then some strange inference become possible, because, all the coincidence events predicted for Victor can be grouped with respect to $|\phi^+_{23}>>$ and $|\phi^-_{23}>>$ with the following implication:

$$|\phi^+_{23}>> \Longrightarrow \frac{1}{\sqrt{2}}(|H>_1|H>_4 + |V>_1|V>_4)$$

$$|\phi^-_{23}>> \Longrightarrow \frac{1}{\sqrt{2}}(|H>_1|H>_4 - |V>_1|V>_4)$$

The above statistically found experimental states are suggesting that somehow Victor's future choice of measuring photons 2 and 3 in BSM is a backward-time causation to entangle the states of photons 1 and 4, which are no more existing except as a registered data. This kind of situation is said to be entanglement swapping in space. Similarly,

$$|V>_2|V>_3 \Longrightarrow |H>_1|H>_4$$

and

$$|H>_2|H>_3 \Longrightarrow |V>_1|V>_4$$

which are separable states and entanglement is said to be not swapped. Here lies the crucial difference between the statistical and non-dualistic interpretations. As already mentioned above, it is the measurements of Alice and Bob deciding the future outcome of Victor at a single quantum level. All other possible correlations computed using the non-duality (TABLES I, II and III) are in exact agreement with the results found in Ref. [15]; also see Fig. 3. So, the conclusion is that it's important to know the quantum phenomenon at a single quantum level during the statistical analysis of an experimental data in order to gain a deeper insight into the non-paradoxical reality of the quantum world.

TABLE I. Casual Relation Between the Observed States of Photons 1 & 4 and 2 , 3

S.No	Alice's Basis Photon-1	State of Photon-2	Bob's Basis Photon-4	State of Photon-3	Joint State of Photons 2 & 3	Victor's Basis Photons 2 , 3	Correlation seen by Victor	Victor's RFD
1	$\|V>_1$	$-\frac{1}{\sqrt{2}}\|H>_2$	$\|V>_4$	$\frac{1}{\sqrt{2}}\|H>_3$	$-\frac{1}{2}\|H>_2\|H>_3$	$\|\phi^+_{23}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|\phi^-_{23}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|H>_2\|H>_3$	$-\frac{1}{2}$	$\frac{1}{4}$
						$\|V>_2\|V>_3$	0	0
2	$\|H>_1$	$\frac{1}{\sqrt{2}}\|V>_2$	$\|H>_4$	$-\frac{1}{\sqrt{2}}\|V>_3$	$-\frac{1}{2}\|V>_2\|V>_3$	$\|\phi^+_{23}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|\phi^-_{23}>>$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|H>_2\|H>_3$	0	0
						$\|V>_2\|V>_3$	$-\frac{1}{2}$	$\frac{1}{4}$
3	$\|H>_1$	$\frac{1}{\sqrt{2}}\|V>_2$	$\|V>_4$	$\frac{1}{\sqrt{2}}\|H>_3$	$\frac{1}{2}\|V>_2\|H>_3$	$\|\phi^+_{23}>>$	0	0
						$\|\phi^-_{23}>>$	0	0
						$\|H>_2\|H>_3$	0	0
						$\|V>_2\|V>_3$	0	0
4	$\|V>_1$	$-\frac{1}{\sqrt{2}}\|H>_2$	$\|H>_4$	$-\frac{1}{\sqrt{2}}\|V>_3$	$\frac{1}{2}\|H>_2\|V>_3$	$\|\phi^+_{23}>>$	0	0
						$\|\phi^-_{23}>>$	0	0
						$\|H>_2\|H>_3$	0	0
						$\|V>_2\|V>_3$	0	0

TABLE II. Casual Relation Between the Observed States of Photons 1 & 4 and 2 , 3

S.No	Alice's Basis Photon-1	State of Photon-2	Bob's Basis Photon-4	State of Photon-3	Joint State of Photons 2 & 3	Victor's Basis Photons 2 , 3	Correlation seen by Victor	Victor's RFD
1	$\|L>_1$	$-i\frac{1}{\sqrt{2}}\|R>_2$	$\|L>_4$	$i\frac{1}{\sqrt{2}}\|R>_3$	$\frac{1}{2}\|R>_2\|R>_3$	$\|\phi_{23}^{+}>>$	0	0
						$\|\phi_{23}^{-}>>$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|H>_2\|H>_3$	$\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
2	$\|R>_1$	$i\frac{1}{\sqrt{2}}\|L>_2$	$\|R>_4$	$-i\frac{1}{\sqrt{2}}\|L>_3$	$\frac{1}{2}\|L>_2\|L>_3$	$\|\phi_{23}^{+}>>$	0	0
						$\|\phi_{23}^{-}>>$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|H>_2\|H>_3$	$\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
3	$\|R>_1$	$i\frac{1}{\sqrt{2}}\|L>_2$	$\|L>_4$	$i\frac{1}{\sqrt{2}}\|R>_3$	$-\frac{1}{2}\|L>_2\|R>_3$	$\|\phi_{23}^{+}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|\phi_{23}^{-}>>$	0	0
						$\|H>_2\|H>_3$	$\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
4	$\|L>_1$	$-i\frac{1}{\sqrt{2}}\|R>_2$	$\|R>_4$	$-i\frac{1}{\sqrt{2}}\|L>_3$	$-\frac{1}{2}\|R>_2\|L>_3$	$\|\phi_{23}^{+}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|\phi_{23}^{-}>>$	0	0
						$\|H>_2\|H>_3$	$\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$

TABLE III. Casual Relation Between the Observed States of Photons 1 & 4 and 2 , 3

S.No	Alice's Basis Photon-1	State of Photon-2	Bob's Basis Photon-4	State of Photon-3	Joint State of Photons 2 & 3	Victor's Basis Photons 2 , 3	Correlation seen by Victor	Victor's RFD
1	$\|->_1$	$\frac{1}{\sqrt{2}}\|+>_2$	$\|->_4$	$-\frac{1}{\sqrt{2}}\|+>_3$	$-\frac{1}{2}\|+>_2\|+>_3$	$\|\phi_{23}^{+}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|\phi_{23}^{-}>>$	0	0
						$\|H>_2\|H>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
2	$\|+>_1$	$-\frac{1}{\sqrt{2}}\|->_2$	$\|+>_4$	$\frac{1}{\sqrt{2}}\|->_3$	$-\frac{1}{2}\|->_2\|->_3$	$\|\phi_{23}^{+}>>$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|\phi_{23}^{-}>>$	0	0
						$\|H>_2\|H>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
3	$\|+>_1$	$-\frac{1}{\sqrt{2}}\|->_2$	$\|->_4$	$-\frac{1}{\sqrt{2}}\|+>_3$	$\frac{1}{2}\|->_2\|+>_3$	$\|\phi_{23}^{+}>>$	0	0
						$\|\phi_{23}^{-}>>$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|H>_2\|H>_3$	$\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$
4	$\|->_1$	$\frac{1}{\sqrt{2}}\|+>_2$	$\|+>_4$	$\frac{1}{\sqrt{2}}\|->_3$	$\frac{1}{2}\|+>_2\|->_3$	$\|\phi_{23}^{+}>>$	0	0
						$\|\phi_{23}^{-}>>$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{8}$
						$\|H>_2\|H>_3$	$\frac{1}{4}$	$\frac{1}{16}$
						$\|V>_2\|V>_3$	$-\frac{1}{4}$	$\frac{1}{16}$

VI. DELAYED CHOICE ENTANGLEMENT SWAPPING IN TIME

In the same manner as in Section-V, the delayed choice entanglement swapping in time [16] can also be explained casually (see Fig. 4).

First, an entangled pair of photons, in the state $|\psi_{12}>>$, is created and photon 1 is sent to Alice who makes a measurement in her basis.

$$|\psi_{12}>>=<P_1^{A+}|\psi_{12}>>|P_1^{A+}>+<P_1^{A-}|\psi_{12}>>|P_1^{A-}> \tag{41}$$

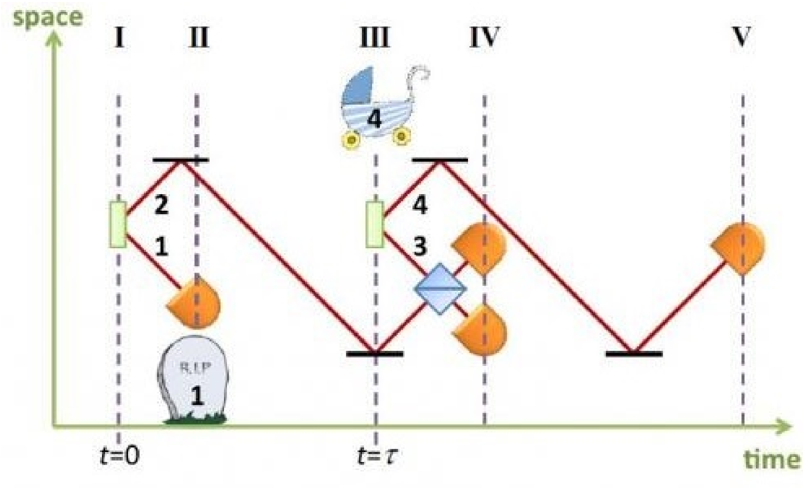

FIG. 4. **E. Magadish et al., Entanglement Swapping between Photons that have Never Coexisted**: A pair of entangled photons in Bell state, $|\psi_{12}^{-}>>$, is produced by an EPR source at time $t=0$. Alice measures the polarization state of photon 1 her own basis. Another pair of entangled photons in $|\psi_{34}^{-}>>$ is produced by the EPR source at time $t=\tau$. The photons 2,3 and 4 are subjected to either Bell state measurement or separable state measurement by Victor, leaving the photon 4 in some polarization state which will be measured in Bob's basis at a later time.

As a consequence of spooky action, photon 2 is thrown into a definite state depending on the out come of Alice measurement, given by

$$|P_2^{A_1\pm}>=<P_1^{A\pm}|\psi_{12}>> \tag{42}$$

At a later time, another entangles pair of photons in the state $|\psi_{34}>>$ is created and photon 3 along with the photon 2 is send to Victor for joint measurement. The joint state encountered by Victor is

$$|P_2^{A_1\pm}>|\phi_{34}>>\equiv|P_2^{A_1\pm};\phi_{34}>>> \tag{43}$$

Now, Victor makes measurement on photon 2 and 3 as

$$|P_2^{A_1\pm};\psi_{34}>>>=<<V_{23}^{+}|P_2^{A_1\pm};\psi_{34}>>>|V_{23}^{+}>>+<<V_{23}^{-}|P_2^{A_1\pm};\psi_{34}>>>|V_{23}^{-}>> \tag{44}$$

and finds them either in one of the basis which determines the state of the photon 4,

$$|P_4^{V_{23}\pm};A_1\pm>=<<V_{23}^{\pm}|P_2^{A_1\pm};\psi_{34}>>> \tag{45}$$

and Bob finally measures this state in his basis

$$|P_4^{V_{23}\pm};A_1\pm>=<P_4^{B+}|P_4^{V_{23}\pm};A_1\pm>|P_4^{B+}>+<P_4^{B-}|P_4^{V_{23}\pm};A_1\pm>|P_4^{B-}> \tag{46}$$

and calculates the RFD as,

$$C_4^{B_4\pm}(A_1\pm:V_{23}\pm)=|<P_4^{B\pm}|P_4^{V_{23}\pm};A_1\pm>|^2 \tag{47}$$

which clearly depends on in which state Alice has seen the photon 1 and Victor has seen the photons 2 and 3. Note that, both Victor and Bob are free to chose their basis randomly and Eq. (47) is in exact agreement with the results found in Ref. [16]. Some explicit calculations done akin to the Section-V are listed in TABLE IV. The non-dualistic interpretation predicts that whatever be the state of photon 1 found by Alice, the photon 4 will also be found in the same state by Bob. In other words, Bob's measurement outcomes are predetermined by that of Alice.

VII. DISCUSSIONS AND CONCLUSIONS

In conclusion, using the wave-particle non-dualistic interpretation, the emergence of causality is shown due to the continuity in the motion of a quantum particle. This aspect was explained in detail by considering the Wheeler's delayed-choice experiment in the context of Young's double-slit experiment. Also, it was shown that Nature is making

TABLE IV. Causal Relation between the Observed States of Photons 1 and 4

S.No	Alice's Basis Photon-1	State of Photon-2	Joint State of Photons 2 & 3 , 4	Victor's Basis for Photons 2 , 3	State of Photon-4
1	$\|V>_1$	$-\frac{1}{\sqrt{2}}\|H>_2$	$-\frac{1}{\sqrt{2}}\|H>_2 \|\psi^-_{34}>$	$\|\phi^+_{23}>>$	$-\frac{1}{2}\|V>_4$
				$\|\phi^-_{23}>>$	$-\frac{1}{2}\|V>_4$
				$\|H>_2 \|H>_3$	$-\frac{1}{2}\|V>_4$
				$\|V>_2 \|V>_3$	0
2	$\|H>_1$	$\frac{1}{\sqrt{2}}\|V>_2$	$\frac{1}{\sqrt{2}}\|V>_2 \|\psi^-_{34}>$	$\|\phi^+_{23}>>$	$-\frac{1}{2}\|H>_4$
				$\|\phi^-_{23}>>$	$\frac{1}{2}\|H>_4$
				$\|H>_2 \|H>_3$	0
				$\|V>_2 \|V>_3$	$-\frac{1}{2}\|H>_4$

use of Einstein's spooky-action-at-a-distance in order to strictly maintain the conservation laws even in the absence of exchange-interactions without touching the Cosmic speed limit.

Merely interpreting the square of the norm of Schrödinger's wavefunction as a probability corresponding to the observed statistics of an experimental data may lead to counter-intuitive inferences which need not exist in the casual and deterministic quantum world. This aspect was successfully explained in the case of delayed-choice entanglement swapping experiments, both in space and time, at a single quantum level without sacrificing the causality. Alice's and Bob's measurements determine the future outcomes of Victor's experiment but not the other way around in the case of entangled swapping in space. In the case of entanglement swapping in time, Bob's future measurement outcomes depend on Alice's results followed by Victor's measurements. It's important to have a quantum phenomenon at a single quantum level while statistically analyzing the experimental data in order to avoid any misleading inferences about quantum mechanics. Particularly, when the probability interpretation is applied to the case of a single-particle's single-event, unusual non-existing paradoxes, like 'measurement problem', retrocasual effects etc., pop out of nowhere.

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