On the Ramanujan's Mock theta functions of tenth order: new possible mathematical developments and mathematical connections with some sectors of Particle Physics and Black Hole physics I

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Abstract

In the present research thesis, we have obtained various and interesting new possible mathematical developments concerning some Ramanujan's Mock theta functions of tenth order and mathematical connections with some sectors of Particle Physics and Black Hole physics

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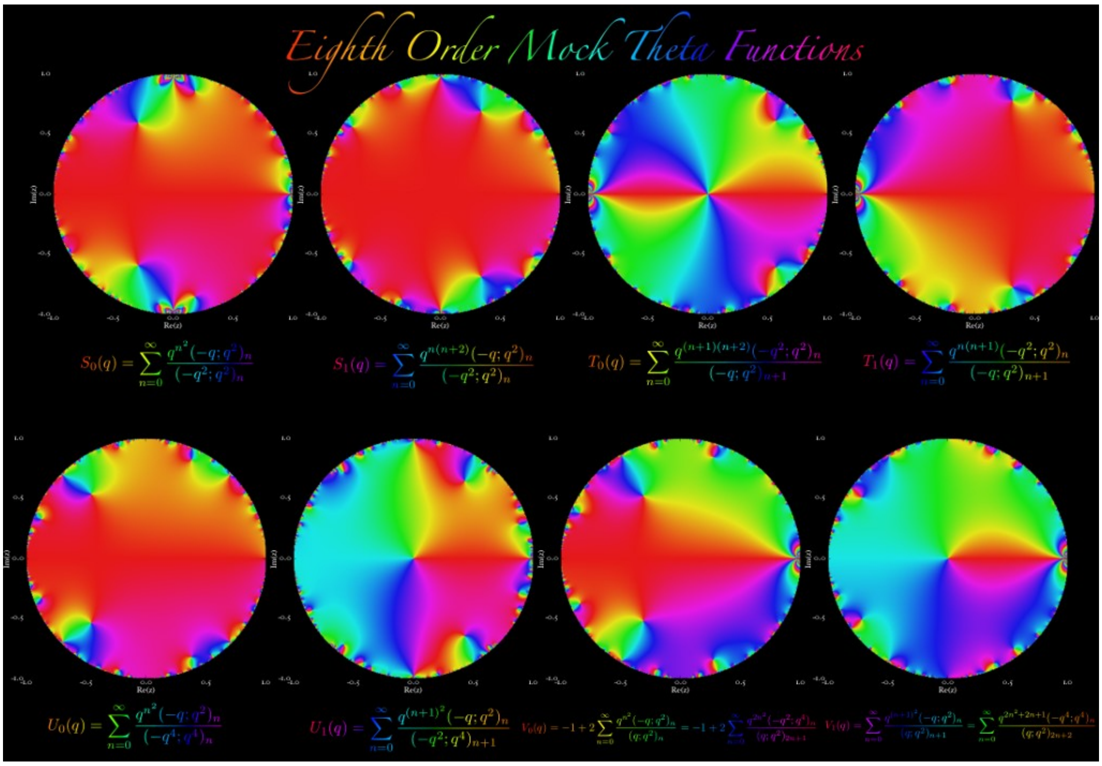
The Ramanujan's mathematical paradise

From:

Ramanujan Institute for Advanced Study in Mathematics
University of Madras, Chennai, India.

From:

http://owen.maresh.info/mocktheta.html



From:

Tenth Order Mock Theta Functions in Ramanujan's Lost Notebook II
Youn-Seo Choi - Advances in Mathematics 156, 180-285 (2000)

THEOREM 2.4.1. *Let q be fixed, $0 < |q| < 1$. Let a, b, and m be fixed integers with $b \neq 0$ and $m \geq 1$. Define*

$$F(z) = \frac{1}{f(-z^b q^a, -z^{-b} q^{m-a})}.$$

Then F is meromorphic for $z \neq 0$, with simple poles at all points z_0 such that $z_0^b = q^{km-a}$ for some integer k. The residue of $F(z)$ at such a point z_0 is

$$\frac{(-1)^{k+1} q^{mk(k-1)/2} z_0}{b(q^m; q^m)_\infty^3}.$$

LEMMA 2.5.1. *Let q and x be fixed, where $0 < |q| < 1$, and suppose that x is neither zero nor an integral power of q. Then $k(x, q)$ is the coefficient of z^0 in the Laurent series expansion of $B(z, x, q)$ in the annulus $|q| < |z| < 1$.*

Next, $R(P(z); q) = -x^{-1}q$ by (2.5.6) and the equalities

$$\frac{1}{2}\left(\frac{x^{-1}}{1-q^{-1}z} - \frac{qx^{-1}z^{-1}}{1-qz^{-1}}\right) = -\frac{1}{2}\left(\frac{x^{-1}q}{z-q} + \frac{x^{-1}q}{z-q}\right) = -\frac{x^{-1}q}{z-q}.$$

Also, $R(P(z); q^3) = -x^{-3}q^3$, by (2.5.6) and the equalities

$$\frac{1}{2}\left(\frac{x^{-3}}{1-q^{-3}z} - \frac{q^3x^{-3}z^{-1}}{1-q^3z^{-1}}\right) = -\frac{1}{2}\left(\frac{x^{-3}q^3}{z-q^3} + \frac{x^{-3}q^3}{z-q^3}\right) = -\frac{x^{-3}q^3}{z-q^3}.$$

We have that the algebraic sum, for x = 2, q = 0,5 and z = 0,8 is:

(((([-((2)^-3 * (0.5)^3))/((0.8-(0.5)^3))]-[-((2)^-1 * (0.5))/((0.8-0.5))])))

Input:

$$-\frac{\frac{0.5^3}{2^3}}{0.8-0.5^3} - -\frac{\frac{0.5}{2}}{0.8-0.5}$$

Open code

Result:

- More digits

0.810185185185185185185185185185185185185185185185185185185185...

Open code

Enlarge Data Customize A Plaintext Interactive

Repeating decimal:

$0.810\overline{185}$ (period 3)

And

Input:

$$2\left(-\frac{\frac{0.5^3}{2^3}}{0.8-0.5^3} - -\frac{\frac{0.5}{2}}{0.8-0.5}\right)$$

Open code

Result:

- More digits

1.620370370370370370370370370370370370370370370370370370370370370...

Open code

1,62037037037......

Enlarge Data Customize A Plaintext Interactive

Repeating decimal:

$1.62\overline{037}$ (period 3)

By Theorem 2.4.1 with $a=3$, $b=-1$, $m=2$, and $k=1$,

$$R(B(z);q)=\frac{(q^2;q^2)^3_\infty f(-x^2,-x^{-2}q^2)\, f(q,q^3)}{f(q,q^3)\, xf(-x^2,-x^{-2}q^2)}\;\frac{-q}{(q^2;q^2)^3_\infty}=-x^{-1}q,$$

and with $a=3$, $b=-1$, $k=0$, and $m=2$,

$$\begin{aligned}R(B(z);q^3)&=\frac{(q^2;q^2)^3_\infty f(-x^2q^2,-x^{-2})\, f(q,q^3)}{f(q,q^3)\, xf(-x^2,-x^{-2}q^2)}\;\frac{q^3}{(q^2;q^2)^3_\infty}\\&=\frac{(q^2;q^2)^3_\infty (-x^{-2})\, f(-x^2,-x^{-2}q^2)\, f(q,q^3)}{f(q,q^3)\, xf(-x^2,-x^{-2}q^2)}\;\frac{q^3}{(q^2;q^2)^3_\infty}\\&=-x^{-3}q^3.\end{aligned}$$

Thence, we have that the algebraic sum is:

[-((2)^-1 * (0.5)]+[-((2)^-3 * (0.5))^3]

Input:

$-\left(\frac{0.5}{2}-\left(\frac{0.5}{2^3}\right)^3\right)$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

-0.249755859375

Rational form:

$-\frac{1023}{4096}$
Open code

And that:

1/ [-((2)^-1 * (0.5)]+[-((2)^-3 * (0.5))^3]

Input:
$-\frac{1}{\frac{0.5}{2}-\left(\frac{0.5}{2^3}\right)^3}$
Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

-4.00391006842619745845552297165200391006842619745845552297...
Open code
This result -4,0039 is the minimal possible value of the mass of hypothetical DM particles, with minus sign.

Now, multiplying the algebraic sum for the value of Planck constant without exponent, we obtain:

6.626 * [-((2)^-1 * (0.5)]+[-((2)^-3 * (0.5))^3]

Input:
$6.626\left(-\left(\frac{0.5}{2}-\left(\frac{0.5}{2^3}\right)^3\right)\right)$
Open code

Enlarge Data Customize A Plaintext Interactive
Result:
-1.65488232421875 that in absolute value is:
1,6548823

Dividing, we have:

[-((2)^-1 * (0.5)] / [-((2)^-3 * (0.5))^3]

Input:

$$-\left(\frac{1}{2}\left(-\frac{0.5}{\left(\frac{0.5}{2^3}\right)^3}\right)\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

1024

The square root is:

Input:

$$\sqrt{-\left(\frac{1}{2}\left(-\frac{0.5}{\left(\frac{0.5}{2^3}\right)^3}\right)\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

32

Multiplying by 54, we have:

Input:

$$54\sqrt{-\left(\frac{1}{2}\left(-\frac{0.5}{\left(\frac{0.5}{2^3}\right)^3}\right)\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

1728

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We have that:

LEMMA 2.5.7. *For* $0 \leqslant m \leqslant 4$,

$$R(J_m(z); \zeta q) = -\frac{2\zeta^{m+1}q^2}{5} \qquad and \qquad R(J_m(z); \zeta q^3) = -\frac{2\zeta^{m+1}q^4}{5},$$

where $\zeta^5 = 1$ *and* $\zeta \neq 1$.

Proof. For $1 \leqslant m \leqslant 4$, by (2.5.15) and Theorem 2.4.1 with $a = -5$, $b = 5$, $m = 10$, and $k = 0$,

$$R(J_m(z); \zeta q)$$

$$= \frac{2q(\zeta q)^m (q^{10}; q^{10})_\infty^3 \, f(-(\zeta q)^5 q^{2m-5}, -(\zeta q)^{-5} q^{15-2m}) \, f((\zeta q)^5, (\zeta q)^{-5} q^{20})}{f(q^5, q^{15}) \, q^m f(-q^{2m}, -q^{10-2m})}$$

$$\times \frac{-\zeta q}{5(q^{10}; q^{10})_\infty^3}$$

$$= -\frac{2\zeta^{m+1}q^2}{5},$$

and by (2.5.15), Theorem 2.4.1 with $a = -5$, $b = 5$, $m = 10$ and $k = 1$, and (2.3.4) with $z = -q^{2m}$ and $q = q^{10/4}$,

$$R(J_m(z); \zeta q^3)$$

$$= \frac{2q(\zeta q^3)^m (q^{10}; q^{10})_\infty^3 \, f(-(\zeta q^3)^5 q^{2m-5}, -(\zeta q^3)^{-5} q^{15-2m}) \, f((\zeta q^3)^5, (\zeta q^3)^{-5} q^{20}}{f(q^5, q^{15}) \, q^m f(-q^{2m}, -q^{10-2m})}$$

$$\times \frac{\zeta q^3}{5(q^{10}; q^{10})_\infty^3}$$

$$= \frac{2q(\zeta q^3)^m (-q^{-2m}) \, f(-q^{2m}, -q^{10-2m}) \, f(q^{15}, q^5)}{f(q^5, q^{15}) \, q^m f(-q^{2m}, -q^{10-2m})} \, \frac{\zeta q^3}{5}$$

$$= -\frac{2\zeta^{m+1}q^4}{5},$$

We have, for $\zeta = 2$, $q = 0,5$ and $m = 3$:

-(2^4*2*0.5^2)/5

Input:

$$-\frac{1}{5}\left(2^4 \times 2 \times 0.5^2\right)$$

Open code

Result:

-1.6

And for $q = q^{10/4}$

-(2^4*2*(0.5^2.5)^4)/5

Input:

$$-\frac{1}{5}\left(2^4 \times 2\left(0.5^{2.5}\right)^4\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

-0.00625

Now:

-(2^4*2*0.5^2)/5-(2^4*2*(0.5^2.5)^4)/5

Input:

$$-\frac{1}{5}\left(2^4 \times 2 \times 0.5^2\right) - \frac{1}{5}\left(2^4 \times 2\left(0.5^{2.5}\right)^4\right)$$

Open code

Result:

-1.60625

1,60625

This result -1,60625 is very near to the value of the electric charge of electron.

We have that:

3 * ((-(2^4*2*0.5^2)/5-(2^4*2*(0.5^2.5)^4)/5))

Input:

$$3\left(-\frac{1}{5}\left(2^4\times 2\times 0.5^2\right)-\frac{1}{5}\left(2^4\times 2\left(0.5^{2.5}\right)^4\right)\right)$$

Open code

Result:

-4.81875

And

2.5849 * ((-(2^4*2*0.5^2)/5-(2^4*2*(0.5^2.5)^4)/5))

where 2.5849 is a Hausdorff dimension

Input interpretation:

$$2.5849\left(-\frac{1}{5}\left(2^4\times 2\times 0.5^2\right)-\frac{1}{5}\left(2^4\times 2\left(0.5^{2.5}\right)^4\right)\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

-4.151995625

This result -4,1519 is in the range of the mass of DM particle that is between 4 – 4.2 eV with minus sign.

Note that:

11 + 1/3*[2.5849 * ((-(2^4*2*0.5^2)/5-(2^4*2*(0.5^2.5)^4)/5))]^6

Input interpretation:

$$11+\frac{1}{3}\left(2.5849\left(-\frac{1}{5}\left(2^4\times 2\times 0.5^2\right)-\frac{1}{5}\left(2^4\times 2\left(0.5^{2.5}\right)^4\right)\right)\right)^6$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1718.7333847727611489655526313812700935850342114766438802082...

Open code

Repeating decimal:

- More digits

1718.733384772761148965552631381270093585034211476643880208...

(period 1)

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We have also:

((((11 + 1/3*[(1.3934+1.2083) * ((-(2^4*2*0.5^2)/5-(2^4*2*(0.5^2.5)^4)/5))]^6))))^1/3

where 1.3934 and 1.2083 are Hausdorff dimensions

Input interpretation:

$$\sqrt[3]{11+\frac{1}{3}\left((1.3934+1.2083)\left(-\frac{1}{5}\left(2^4\times2\times0.5^2\right)-\frac{1}{5}\left(2^4\times2\left(0.5^{2.5}\right)^4\right)\right)\right)^6}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.1337...

This result 12,1337 is very near to the value of black hole entropy 12,1904

Now, we have:

and for $m=0$, by (2.5.14), Theorem 2.4.1 with $a=-5$, $b=5$, $m=10$, and $k=0$ and (2.3.4) with $z=q^5$ and $q=q^5$,

$$R(J_0(z);\zeta q)$$
$$=\frac{2q^6(q^{10};q^{10})_\infty^3\ f((\zeta q)^5 q^{-10},(\zeta q)^{-5}q^{30})\ f(-(\zeta q)^5,\ -(\zeta q)^{-5}q^{10})}{f(q^5,q^{15})\ f(-q^5,\ -q^5)}$$
$$\times\frac{-\zeta q}{5(q^{10};q^{10})_\infty^3}$$
$$=-\frac{2\zeta q^2}{5},$$

and by Theorem 2.4.1 with $a=-5$, $b=5$, $m=10$, and $k=1$ and (2.3.4) with $z=-q^5$ and $q=q^{10/4}$,

$$R(J_0(z);\zeta q^3)$$
$$=\frac{2q^6(q^{10};q^{10})_\infty^3\ f((\zeta q^3)^5 q^{-10},(\zeta q^3)^{-5}q^{30})\ f(-(\zeta q^3)^5,\ -(\zeta q^3)^{-5}q^{10})}{f(q^5,q^{15})\ f(-q^5,\ -q^5)}$$
$$\times\frac{\zeta q^3}{5(q^{10};q^{10})_\infty^3}$$
$$=\frac{2q^6\ f(q^5,q^{15})(-q^{-5})\ f(-q^5,\ -q^5)}{f(q^5,q^{15})\ f(-q^5,\ -q^5)}\ \frac{\zeta q^3}{5}$$
$$=-\frac{2\zeta q^4}{5}.$$

We have, from the algebraic sum and for for ζ = 2, q = 0,5:

-((2*2(0.5^5)^2))/5 -((2*2(0.5^2.5)^4))/5

Input:

$$-\frac{1}{5}\left(2\times 2\left(0.5^5\right)^2\right)-\frac{1}{5}\left(2\times 2\left(0.5^{2.5}\right)^4\right)$$

Open code

Result:

−0.0015625

For:

(0.0508834375)^2/-(((-((2*2(0.5^5)^2))/5 -((2*2(0.5^2.5)^4))/5)))

where 0.0508834375 = 0.0814135/8 + 0.0814135/2 and 0.0814135 is a result of a Ramanujan Mock theta function

Input interpretation:

$$-\frac{0.0508834375^2}{-\frac{1}{5}\left(2\times 2\left(0.5^5\right)^2\right)-\frac{1}{5}\left(2\times 2\left(0.5^{2.5}\right)^4\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

1.6570394955625

Open code

1,6570394955625

This result 1,657039... is very near to the fourteenth root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 that is 1,65578...

We have:

(-16+12^2)+(2.5)/-(((-((2*2(0.5^5)^2))/5 -((2*2(0.5^2.5)^4))/5)))

where 2,5 is a Hausdorff dimension

Input:

$$\left(-16+12^2\right)+-\frac{2.5}{-\frac{1}{5}\left(2\times 2\left(0.5^5\right)^2\right)-\frac{1}{5}\left(2\times 2\left(0.5^{2.5}\right)^4\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

1728

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

(((0.48/(((((2*2(0.5^5)^2))/5 +((2*2(0.5^2.5)^4))/5)))^1/3

Input:

$$\frac{0.48}{\sqrt[3]{\frac{1}{5}\left(2\times 2\left(0.5^5\right)^2\right)+\frac{1}{5}\left(2\times 2\left(0.5^{2.5}\right)^4\right)}}$$

Open code

Result:

- Fewer digits
- More digits

4.136514604861216745777843647717152950897942289008848478379...

4.1365146048612167457778436477171529508979422890088484

Continued fraction:

- Linear form

$$4+\cfrac{1}{7+\cfrac{1}{3+\cfrac{1}{13+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$\frac{48\sqrt[3]{2}}{5\times 5^{2/3}}\approx 4.136514604861216745777843647717152950897942289008848478379$$

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$$\frac{5}{7}\pi\,\text{csch}^2\left(\frac{1443895}{2116091}\right)\approx 4.13651460486121682039$$

$$\frac{475 + 723\sqrt{\pi} + 91\pi + 66\pi^{3/2} + 6\pi^2}{190\pi} \approx 4.13651460486121674566503$$

This result 4,1365 is in the range of the mass of DM particle that is between 4 – 4.2 eV

((((((((0.48/(((((2*2(0.5^5)^2))/5 +((2*2(0.5^2.5)^4))/5)))^1/3))))))^1/3

Input:

$$\sqrt[3]{\frac{0.48}{\sqrt[3]{\frac{1}{5}\left(2\times 2\left(0.5^5\right)^2\right)+\frac{1}{5}\left(2\times 2\left(0.5^{2.5}\right)^4\right)}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

1.605258040875225834584751425314694535856735747245489259431...

1.605258040875225834584751425314694535856735747245489 2

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{135+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{2+\cfrac{1}{19+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$\frac{2(2003\, C_{HSM} + 110)}{6309\, C_{HSM} - 1210} \approx 1.605258040875225811093$$

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$$\sqrt{\frac{2}{5}}\sqrt[4]{\frac{2019247}{4739467}}\,\pi \approx 1.6052580408752225851840$$

$$\frac{1070740242\,\pi}{2095507135} \approx 1.60525804087522583447631$$

C_{HSM} is the Hafner-Sarnak-McCurley constant

This result 1,605258 is very near to the electric charge of positron

On the Dedekind Eta function and Ramanujan Mock Theta Functions

THEOREM 2.6.8. *For* $z \in \mathscr{H}$,

$$2\eta_{90}^3\,\eta_{180}^3\,\eta_{30,\,9}\,\eta_{180,\,72}\,\eta_{45,\,15}\,\eta_{180,\,60}^3\,\eta_{90,\,30}^5$$
$$= \eta_{30}^5\,\eta_{60}\,\eta_{60,\,18}\,\eta_{60,\,24}\,\eta_{90,\,30}^3 + \eta_{30}^5\,\eta_{60}\,\eta_{30,\,9}^2\,\eta_{60,\,24}\,\eta_{45,\,15}^2\,\eta_{180,\,60}. \qquad (2.6.3)$$

We have that:

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_1; z) = \frac{3 \cdot 20736}{12} = 5184 \geq ord(F_1; \infty), \qquad (2.6.4)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_2; z) = \frac{8 \cdot 20736}{12} = 13824 \geq ord(F_2; \infty), \qquad (2.6.6)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_3; z) = \frac{9 \cdot 20736}{12} = 15552 \geq ord(F_3; \infty), \qquad (2.6.8)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_4; z) = \frac{5 \cdot 20736}{12} = 8640 \geq ord(F_4; \infty), \qquad (2.6.10)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_5; z) = \frac{6 \cdot 20736}{12} = 10368 \geqslant ord(F_5; \infty), \qquad (2.7.2)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_6; z) = \frac{6 \cdot 20736}{12} = 10368 \geqslant ord(F_6; \infty), \qquad (2.7.4)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_7; z) = \frac{7 \cdot 20736}{12} = 12096 \geqslant ord(F_7; \infty), \qquad (2.7.6)$$

$$\sum_{z \in R} Ord_{\Gamma_1(180)}(F_8; z) = \frac{7 \cdot 20736}{12} = 12096 \geqslant ord(F_8; \infty), \qquad (2.7.8)$$

Thence:

$\frac{3 \times 20736}{12} = 5184$ $\quad \frac{8 \times 20736}{12} = 13824$ $\quad \frac{9 \times 20736}{12} = 15552$ $\quad \frac{5 \times 20736}{12} = 8640$

$\frac{6 \times 20736}{12} = 10368$ $\quad \frac{7 \times 20736}{12} = 12096$

We note that:

20736 / 1728 = 12;

and that:

5184 / 1728 = 3; 13824 / 1728 = 8; 15552 / 1728 = 9; 8640 / 1728 = 5;

10368 / 1728 = 6; 12096 / 1728 = 7.

Furthermore: $12^3 = 1728$ and

$(1728)^{1/15}$ = 1.643751829517225762308497936230979517383492589945475200411

Indeed:

Input:

$\sqrt[15]{1728}$

Result

- Approximate form

$2^{2/5}\sqrt[5]{3}$

Decimal approximation:

1.6437518295172257623084979362309795173834925899454752004 11...

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{20+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

$$2^{2/5}\sqrt[5]{3} \approx 1.643751829517225762308497936230979517383492589945475200411 02976$$

$$4\sqrt{\frac{3139915}{18593681}} \approx 1.6437518295172258 3645$$

$$\frac{e^{\frac{3}{4}+\frac{29}{2e}-4e+\frac{9}{4\pi}-\frac{13\pi}{4}}\pi^{8e-7}\cos^6(e\pi)}{\sin^{5/2}(e\pi)} \approx 1.64375182951722576 53822$$

Note that:

3 * 1.643751829517225762308497936230979517383492589945475200411 =

= 4.931255488551677286925493808692938552150477769836425601233

Possible closed forms:

$3 \times 2^{2/5} \sqrt[5]{3} \approx$
4.93125548855167728692549380869293855215047776983642560123308930

Enlarge Data Customize A Plaintext Interactive

$\text{root of } 26x^4 + 339x^3 - 2228x^2 + 244x - 3050 \text{ near } x = 4.93126 \approx$
4.93125548855167728622037

$\pi \text{ root of } 895x^3 - 70090x^2 + 84599x + 36438 \text{ near } x = 1.56967 \approx$
4.931255488551677286955884

This value 4,931255 is very near to the first value of upper bound dark photon energy range ($4.95 * 10^{16} – 5.4 * 10^{16}$) without multiplying the base and its exponent.

We have also that:

((1728)^1/15))*10^3 + (8^2+5^2) where 5 and 8 are Fibonacci's numbers,

Input:
$\sqrt[15]{1728} \times 10^3 + (8^2 + 5^2)$

Result:

$89 + 1000 \times 2^{2/5} \sqrt[5]{3}$

Decimal approximation:

- More digits

1732.751829517225762308497936230979517383492589945475200411...

Alternate form:

$\text{root of } x^5 - 445x^4 + 79210x^3 - 7049690x^2 + 313711205x - 12000005584059449 \text{ near } x = 1732.75$

Open code

Minimal polynomial:

$x^5 - 445x^4 + 79210x^3 - 7049690x^2 + 313711205x - 12000005584059449$

The result is: 1732.7518295... and is very near to the range of the mass of $f_0(1710)$ candidate glueball.

We obtain the following theta function identity from (2.6.3)

$$2\eta_{90}^3\,\eta_{180}^3\,\eta_{30,\,9}\eta_{180,\,72}\eta_{45,\,15}\eta_{180,\,60}^3\,\eta_{90,\,30}^5$$
$$=\eta_{30}^5\,\eta_{60}\eta_{60,\,18}\eta_{60,\,24}\eta_{90,\,30}^3+\eta_{30}^5\,\eta_{60}\eta_{30,\,9}^2\,\eta_{60,\,24}\eta_{45,\,15}^2\,\eta_{180,\,60}. \qquad (2.6.3)$$

We have the following Theorem (first identity):

THEOREM 2.6.12. *If* $|q|<1$, *then*

$$2\,\frac{(q^{30};q^{30})_\infty^2\,(q^{45};q^{45})_\infty\,(q^{60};q^{60})_\infty^3\;f(-q^{72},-q^{108})}{(q^{15};q^{15})_\infty\,(q^{180};q^{180})_\infty\;f(-q^{24},-q^{36})}$$
$$=\frac{(q^{30};q^{30})_\infty^4\;f(q^9,q^{21})\,f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2}$$
$$+(q^{30};q^{30})_\infty\,(q^{60};q^{60})_\infty\;f(-q^9,-q^{21})\,f(-q^{90},-q^{90}).$$

For the proof, we multiply the left hand side and the right hand side of (2.6.3) by

$$q^{-23/2}\,\frac{\eta_{45}^2\,\eta_{90}^3\,\eta_{180}}{\eta_{30}^5\,\eta_{60}}$$

We obtain:

$$2\eta_{90}^3\,\eta_{180}^3\,\eta_{30,\,9}\eta_{180,\,72}\eta_{45,\,15}\eta_{180,\,60}^3\,\eta_{90,\,30}^5 \;*\; q^{-23/2}\,\frac{\eta_{45}^2\,\eta_{90}^3\,\eta_{180}}{\eta_{30}^5\,\eta_{60}}$$

$$\eta_{30}^5\,\eta_{60}\eta_{60,\,18}\eta_{60,\,24}\eta_{90,\,30}^3+\eta_{30}^5\,\eta_{60}\eta_{30,\,9}^2\,\eta_{60,\,24}\eta_{45,\,15}^2\,\eta_{180,\,60} \;*\; q^{-23/2}\,\frac{\eta_{45}^2\,\eta_{90}^3\,\eta_{180}}{\eta_{30}^5\,\eta_{60}}$$

and apply the two equations:

Let $\mathscr{H}=\{z:\operatorname{Im}\,z>0\}$ For $z\in\mathscr{H}$, $q=e^{2\pi iz}$

$$\eta(nz):=\eta_n=e^{(\pi inz)/12}\prod_{m=1}^{\infty}(1-e^{2\pi imnz})=q^{n/24}(q^n;q^n)_\infty$$

$$\eta_{n,\,m}(z):=\eta_{n,\,m}\;=q^{P_2(m/n)\,n/2}\,\frac{f(-q^m,-q^{n-m})}{(q^n;q^n)_\infty}$$

We know that F_1 is equal to the difference between

$$2\eta_{90}^3\,\eta_{180}^3\,\eta_{30,\,9}\eta_{180,\,72}\eta_{45,\,15}\eta_{180,\,60}^3\,\eta_{90,\,30}^5$$

and

$$\eta_{30}^{5}\,\eta_{60}\,\eta_{60,\,18}\,\eta_{60,\,24}\,\eta_{90,\,30}^{3} + \eta_{30}^{5}\,\eta_{60}\,\eta_{30,\,9}^{2}\,\eta_{60,\,24}\,\eta_{45,\,15}^{2}\,\eta_{180,\,60}$$

Thence from (2.6.3), $F_1 = 5184$;

Indeed:

$$\frac{3 \times 20\,736}{12} = 5184$$

In conclusion, we have that:

$$5184 * \quad q^{-23/2}\,\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}}$$

$$= 2\eta_{90}^{3}\,\eta_{180}^{3}\,\eta_{30,\,9}\,\eta_{180,\,72}\,\eta_{45,\,15}\,\eta_{180,\,60}^{3}\,\eta_{90,\,30}^{5} \quad q^{-23/2}\,\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}} \quad +$$

$$- \quad \eta_{30}^{5}\,\eta_{60}\,\eta_{60,\,18}\,\eta_{60,\,24}\,\eta_{90,\,30}^{3} + \eta_{30}^{5}\,\eta_{60}\,\eta_{30,\,9}^{2}\,\eta_{60,\,24}\,\eta_{45,\,15}^{2}\,\eta_{180,\,60}, \quad q^{-23/2}\,\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}}$$

The same procedure is done for the other three identities. We get:

THEOREM 2.6.13. *If* $|q|<1$, *then*

$$q^{-1}\,\frac{f(-q^{6},-q^{24})\,f(-q^{9},-q^{21})^{2}\,f(-q^{15},-q^{45})}{f(-q^{18},-q^{42})^{2}\,f(-q^{30},-q^{60})^{4}}$$

$$\times\frac{\begin{pmatrix} f(-q,-q^{29})\,f(-q^{5},-q^{25})\,f(-q^{10},-q^{20}) \\ f(-q^{11},-q^{19})\,f(-q^{14},-q^{46})\,f(-q^{26},-q^{34}) \end{pmatrix}}{f(-q^{2},-q^{28})\,f(-q^{8},-q^{22})}$$

$$=q\,\frac{(q^{5};q^{5})_{\infty}\,(q^{10};q^{10})_{\infty}\,f(-q,-q^{4})(q^{60};q^{60})_{\infty}}{f(-q^{2},-q^{3})\,f(-q^{4},-q^{6})}$$

$$+\frac{(q^{30};q^{30})_{\infty}^{3}}{qf(q^{5},q^{15})\,f(q^{9},q^{21})}\left(\frac{f(q^{15},q^{15})\,f(-q^{60},-q^{120})\,f(-q^{24},-q^{36})}{f(-q^{6},-q^{24})\,f(-q^{12},-q^{48})}\right.$$

$$-q^{9}\,\frac{f(q^{5},q^{25})\,f(-q^{6},-q^{24})\,f(-q^{10},-q^{20})^{2}\,f(-q^{36},-q^{144})}{f(-q^{2},-q^{8})\,f(-q^{30},-q^{60})^{3}}$$

$$\left.-q\,\frac{f(q^{5},q^{25})\,f(-q^{84},-q^{96})}{f(-q^{4},-q^{26})}+q^{15}\,\frac{f(q^{5},q^{25})\,f(-q^{24},-q^{156})}{f(-q^{14},-q^{16})}\right).$$

Proof. Dividing both sides of (2.6.5) by

$$q^{(91/40)}\eta_{5}\eta_{10}^{3}\,\eta_{15}^{4}\,\eta_{20}^{2}\,\eta_{30}^{2}\,\eta_{60}^{2}\,\eta_{10,2}\eta_{10,3}\eta_{10,4}\eta_{30,2}\eta_{30,4}\eta_{30,8}\eta_{30,14}\eta_{60,6}\eta_{60,12}\eta_{60,18}^{2},$$

and applying (2.6.1) and (2.6.2), we have completed the proof.

The (2.6.5) is:

$$
\begin{aligned}
&\eta_1\eta_5\eta_{10}^3\eta_{12}\eta_{15}\eta_{20}^2\eta_{45}^3\eta_{90}^2\eta_{180}^2\eta_{60,6}\eta_{30,4}\eta_{30,14}\eta_{30,9}\eta_{20,6}\eta_{90,30}^2\eta_{180,60}^2\eta_{45,15}^3\\
&\quad=\eta_5^2\eta_{10}^3\eta_{15}^3\eta_{30}^2\eta_{45}\eta_{60}^3\eta_{180}^2\eta_{60,6}\eta_{60,12}\eta_{30,4}\eta_{30,14}\eta_{5,1}\eta_{60,18}^2\eta_{30,2}\eta_{30,8}\\
&\qquad\times\eta_{45,15}\eta_{60,20}^2\eta_{180,60}^2\\
&\qquad+\eta_{15}^4\eta_{20}^2\eta_{30}^7\eta_{60}^3\eta_{5,2}\eta_{30,9}\eta_{60,18}\eta_{30,2}\eta_{30,8}\eta_{10,4}\eta_{60,30}\eta_{30,5}\eta_{30,4}\eta_{30,14}\\
&\qquad\times\eta_{30,10}^2\eta_{15,5}\\
&\qquad-\eta_5\eta_{10}^3\eta_{15}^3\eta_{20}\eta_{30}^2\eta_{45}^2\eta_{60}^3\eta_{180}\eta_{30,9}\eta_{60,18}\eta_{30,2}\\
&\qquad\times\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{30,6}\eta_{180,36}\\
&\qquad\times\eta_{60,6}\eta_{60,12}\eta_{30,4}\eta_{30,14}\eta_{45,15}^2\eta_{60,20}\\
&\qquad-\eta_{10}^2\eta_{20}\eta_{30}^3\eta_{45}^4\eta_{60}^3\eta_{90}^2\eta_{180}\eta_{30,9}\eta_{60,18}\eta_{30,2}\\
&\qquad\times\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{180,84}\eta_{30,15}\\
&\qquad\times\eta_{60,6}\eta_{60,12}\eta_{10,2}\eta_{30,14}\eta_{15,5}\eta_{45,15}^4\eta_{60,20}\eta_{90,30}^2\\
&\qquad+\eta_{10}^2\eta_{20}\eta_{30}^3\eta_{45}^4\eta_{60}^3\eta_{90}^2\eta_{180}\eta_{30,9}\eta_{60,18}\\
&\qquad\times\eta_{30,2}\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{180,84}\eta_{30,15}\\
&\qquad\times\eta_{60,6}\eta_{60,12}\eta_{10,2}\eta_{30,4}\eta_{15,5}\eta_{45,15}^4\eta_{60,20}\eta_{90,30}^2\,. \qquad (2.6.5)
\end{aligned}
$$

We know that $F_2 = 13824$ is equal to:

$$
\begin{aligned}
&\eta_1\eta_5\eta_{10}^3\eta_{12}\eta_{15}\eta_{20}^2\eta_{45}^3\eta_{90}^2\eta_{180}^2\eta_{60,6}\eta_{30,4}\eta_{30,14}\eta_{30,9}\eta_{20,6}\eta_{90,30}^2\eta_{180,60}^2\eta_{45,15}^3 -\\
&\eta_5^2\eta_{10}^3\eta_{15}^3\eta_{30}^2\eta_{45}\eta_{60}^3\eta_{180}^2\eta_{60,6}\eta_{60,12}\eta_{30,4}\eta_{30,14}\eta_{5,1}\eta_{60,18}^2\eta_{30,2}\eta_{30,8}\\
&\times\eta_{45,15}\eta_{60,20}^2\eta_{180,60}^2\\
&+\eta_{15}^4\eta_{20}^2\eta_{30}^7\eta_{60}^3\eta_{5,2}\eta_{30,9}\eta_{60,18}\eta_{30,2}\eta_{30,8}\eta_{10,4}\eta_{60,30}\eta_{30,5}\eta_{30,4}\eta_{30,14}\\
&\times\eta_{30,10}^2\eta_{15,5}\\
&-\eta_5\eta_{10}^3\eta_{15}^3\eta_{20}\eta_{30}^2\eta_{45}^2\eta_{60}^3\eta_{180}\eta_{30,9}\eta_{60,18}\eta_{30,2}\\
&\times\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{30,6}\eta_{180,36}\\
&\times\eta_{60,6}\eta_{60,12}\eta_{30,4}\eta_{30,14}\eta_{45,15}^2\eta_{60,20}\\
&-\eta_{10}^2\eta_{20}\eta_{30}^3\eta_{45}^4\eta_{60}^3\eta_{90}^2\eta_{180}\eta_{30,9}\eta_{60,18}\eta_{30,2}\\
&\times\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{180,84}\eta_{30,15}\\
&\times\eta_{60,6}\eta_{60,12}\eta_{10,2}\eta_{30,14}\eta_{15,5}\eta_{45,15}^4\eta_{60,20}\eta_{90,30}^2\\
&+\eta_{10}^2\eta_{20}\eta_{30}^3\eta_{45}^4\eta_{60}^3\eta_{90}^2\eta_{180}\eta_{30,9}\eta_{60,18}\\
&\times\eta_{30,2}\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{180,84}\eta_{30,15}\\
&\times\eta_{60,6}\eta_{60,12}\eta_{10,2}\eta_{30,4}\eta_{15,5}\eta_{45,15}^4\eta_{60,20}\eta_{90,30}^2\,. \qquad (2.6.5)
\end{aligned}
$$

Thence, we have that:

$$13824 / q^{(91/40)}\eta_5\eta_{10}^3\eta_{15}^4\eta_{20}^2\eta_{30}^2\eta_{60}^2\eta_{10,2}\eta_{10,3}\eta_{10,4}\eta_{30,2}\eta_{30,4}\eta_{30,8}\eta_{30,14}\eta_{60,6}\eta_{60,12}\eta_{60,18}^2 =$$

$$(\eta_1\eta_5\eta_{10}^3\eta_{12}\eta_{15}\eta_{20}^2\eta_{45}^3\eta_{90}^2\eta_{180}^2\eta_{60,6}\eta_{30,4}\eta_{30,14}\eta_{30,9}\eta_{20,6}\eta_{90,30}^2\eta_{180,60}^2\eta_{45,15}^3 /$$

$$q^{(91/40)}\eta_5\eta_{10}^3\eta_{15}^4\eta_{20}^2\eta_{30}^2\eta_{60}^2\eta_{10,2}\eta_{10,3}\eta_{10,4}\eta_{30,2}\eta_{30,4}\eta_{30,8}\eta_{30,14}\eta_{60,6}\eta_{60,12}\eta_{60,18}^2) - ($$

$$\begin{aligned}
&\eta_5^2\eta_{10}^3\eta_{15}^3\eta_{30}^2\eta_{45}\eta_{60}^3\eta_{180}^2\eta_{60,6}\eta_{60,12}\eta_{30,4}\eta_{30,14}\eta_{5,1}\eta_{60,18}^2\eta_{30,2}\eta_{30,8}\\
&\times\eta_{45,15}\eta_{60,20}^2\eta_{180,60}^2\\
&+\eta_{15}^4\eta_{20}^2\eta_{30}^7\eta_{60}^3\eta_{5,2}\eta_{30,9}\eta_{60,18}\eta_{30,2}\eta_{30,8}\eta_{10,4}\eta_{60,30}\eta_{30,5}\eta_{30,4}\eta_{30,14}\\
&\times\eta_{30,10}^2\eta_{15,5}\\
&-\eta_5\eta_{10}^3\eta_{15}^3\eta_{20}\eta_{30}^2\eta_{45}^2\eta_{60}^3\eta_{180}\eta_{30,9}\eta_{60,18}\eta_{30,2}\\
&\times\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{30,6}\eta_{180,36}\\
&\times\eta_{60,6}\eta_{60,12}\eta_{30,4}\eta_{30,14}\eta_{45,15}^2\eta_{60,20}\\
&-\eta_{10}^2\eta_{20}\eta_{30}^3\eta_{45}^4\eta_{60}^3\eta_{90}^2\eta_{180}\eta_{30,9}\eta_{60,18}\eta_{30,2}\\
&\times\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{180,84}\eta_{30,15}\\
&\times\eta_{60,6}\eta_{60,12}\eta_{10,2}\eta_{30,14}\eta_{15,5}\eta_{45,15}^4\eta_{60,20}\eta_{90,30}^2\\
&+\eta_{10}^2\eta_{20}\eta_{30}^3\eta_{45}^4\eta_{60}^3\eta_{90}^2\eta_{180}\eta_{30,9}\eta_{60,18}\\
&\times\eta_{30,2}\eta_{30,8}\eta_{10,4}\eta_{10,3}\eta_{60,10}\eta_{180,84}\eta_{30,15}\\
&\times\eta_{60,6}\eta_{60,12}\eta_{10,2}\eta_{30,4}\eta_{15,5}\eta_{45,15}^4\eta_{60,20}\eta_{90,30}^2\,\cdot \qquad (2.6.5)
\end{aligned}$$

$$/\; q^{(91/40)}\eta_5\eta_{10}^3\eta_{15}^4\eta_{20}^2\eta_{30}^2\eta_{60}^2\eta_{10,2}\eta_{10,3}\eta_{10,4}\eta_{30,2}\eta_{30,4}\eta_{30,8}\eta_{30,14}\eta_{60,6}\eta_{60,12}\eta_{60,18}^2)$$

Now, we have:

THEOREM 2.6.14. *If* $|q| < 1$, *then*

$$\frac{(q^9;q^9)_\infty^3 (q^{30};q^{30})_\infty f(-q^3,-q^{15})}{(q^{18};q^{18})_\infty^3 f(-q^6,-q^{24})}$$

$$= -\frac{(q^{45};q^{45})_\infty (q^{90};q^{90})_\infty f(-q^{18},-q^{27})}{f(-q^{18},-q^{72}) f(-q^9,-q^{36})}$$

$$+q^9 \left(q^9 \frac{(q^{90};q^{90})_\infty^3 f(q^3,q^{87}) f(-q^{24},-q^{156})}{(q^{60};q^{60})_\infty f(q^{45},q^{135}) f(-q^{18},-q^{72}) f(q^{21},q^{69})} \right.$$

$$+\frac{(q^{90};q^{90})_\infty^3 f(q^{33},q^{57}) f(-q^{84},-q^{96})}{q^9 (q^{60};q^{60})_\infty f(q^{45},q^{135}) f(-q^{18},-q^{72}) f(q^{39},q^{51})}$$

$$+q^9 \frac{\begin{pmatrix} (q^{180};q^{180})_\infty^2 f(-q^9,-q^{21})^4 f(-q^{15},-q^{75}) \\ f(-q^{15},-q^{45}) f(-q^{36},-q^{54}) \end{pmatrix}}{(q^{30};q^{30})_\infty^6 (q^{60};q^{60})_\infty (q^{90};q^{90})_\infty f(-q^{18},-q^{42})^2}$$

$$\times (f(-q^{39},-q^{51}) f(-q^{45},-q^{45}) - q^3 f(-q^{21},-q^{69}) f(-q^{45},-q^{45})$$

$$+q^{12} f(-q^9,-q^{81}) f(-q^{15},-q^{75}))$$

$$-\frac{(q^{15};q^{15})_\infty (q^{60};q^{60})_\infty^3 f(-q^9,-q^{21})^2 f(-q^{36},-q^{54}) f(-q^{72},-q^{108})}{(q^{30};q^{30})_\infty^7 (q^{90};q^{90})_\infty (q^{180};q^{180})_\infty f(-q^{18},-q^{42})^2 f(-q^{24},-q^{36})}$$

$$\times (-f(-q^{15},-q^{75}) f(-q^{39},-q^{51})^2 f(-q^{21},-q^{69})^2$$

$$+q^9 f(-q^{45},-q^{45}) f(-q^9,-q^{81})^2 f(-q^{21},-q^{69})^2$$

$$+q^3 f(-q^{45},-q^{45}) f(-q^9,-q^{81})^2 f(-q^{39},-q^{51})^2)$$

$$+q^6 \frac{(q^{90};q^{90})_\infty^2 f(-q^9,-q^{21})^3 f(q^9,q^{21}) f(-q^{36},-q^{54})}{(q^{30};q^{30})_\infty^6 (q^{45};q^{45})_\infty f(-q^{18},-q^{42})^2}$$

$$\times (q^6 f(-q^{21},-q^{69}) f(-q^{15},-q^{75})^2 + f(-q^9,-q^{81}) f(-q^{45},-q^{45})^2$$

$$-q^3 f(-q^{39},-q^{51}) f(-q^{15},-q^{75})^2).$$

Dividing both sides of (2.6.7) by:

$$q^{(336/5)} \frac{\eta_3^2 \eta_6 \eta_{12} \eta_{15} \eta_{18}^2 \eta_{30}^7 \eta_{45} \eta_{60}^5 \eta_{15,3}^2 \eta_{30,6} \eta_{45,9} \eta_{60,18}^2 \eta_{60,24} \eta_{90,18}}{\eta_{90}^2 \eta_{180}}$$

From $F_3 = 15552$, we have that:

$$15552 \Big/ \left(q^{(336/5)} \frac{\eta_3^2 \eta_6 \eta_{12} \eta_{15} \eta_{18}^2 \eta_{30}^7 \eta_{45} \eta_{60}^5 \eta_{15,3}^2 \eta_{30,6} \eta_{45,9} \eta_{60,18}^2 \eta_{60,24} \eta_{90,18}}{\eta_{90}^2 \eta_{180}} \right) =$$

$$\Bigg(\begin{array}{l} \eta_3^2 \eta_9^3 \eta_{12} \eta_{15} \eta_{30}^4 \eta_{45} \eta_{60}^2 \eta_{90}^2 \eta_{180}^2 \eta_{15,3} \eta_{30,6} \eta_{30,12} \eta_{90,30}^4 \eta_{180,60}^3 \eta_{15,3} \eta_{60,18}^2 \\ \quad \times \eta_{60,24} \eta_{18,3} \eta_{90,18} \eta_{45,9} \end{array} \Big/$$

$$q^{(336/5)} \frac{\eta_3^2 \eta_6 \eta_{12} \eta_{15} \eta_{18}^2 \eta_{30}^7 \eta_{45} \eta_{60}^5 \eta_{15,3}^2 \eta_{30,6} \eta_{45,9} \eta_{60,18}^2 \eta_{60,24} \eta_{90,18}}{\eta_{90}^2 \eta_{180}} \Bigg) - \Bigg($$

$$\begin{aligned}
&-\eta_3 \eta_9 \eta_{15} \eta_{18}^3 \eta_{30}^5 \eta_{36} \eta_{45}^2 \eta_{60}^3 \eta_{180} \eta_{15,3}^2 \eta_{9,3} \eta_{18,6} \\
&\quad \times \eta_{36,12} \eta_{90,30}^2 \eta_{180,60}^2 \eta_{60,24} \eta_{45,18} \eta_{30,6} \eta_{60,18}^2 \\
&+ \eta_3 \eta_9 \eta_{12} \eta_{15} \eta_{18}^3 \eta_{30}^3 \eta_{45}^2 \eta_{60}^2 \eta_{90}^2 \eta_{180}^2 \eta_{15,3} \eta_{9,3} \\
&\times \eta_{18,6} \eta_{90,30}^4 \eta_{180,60}^2 \eta_{30,6} \eta_{45,9} \eta_{60,18}^2 \eta_{60,24} \\
&\times \eta_{90,18} \eta_{180,48} \eta_{180,12} \eta_{90,27} \eta_{90,21} \eta_{180,6} \eta_{180,24} \eta_{180,78} \eta_{90,33} \\
&+ \eta_3 \eta_9 \eta_{12} \eta_{15} \eta_{18}^3 \eta_{30}^3 \eta_{45}^2 \eta_{60}^2 \eta_{90}^2 \eta_{180}^2 \eta_{15,3} \eta_{9,3} \\
&\times \eta_{18,6} \eta_{90,30}^4 \eta_{180,60}^2 \eta_{30,6} \eta_{45,9} \eta_{60,18}^2 \\
&\times \eta_{60,24} \eta_{90,18} \eta_{180,48} \eta_{180,12} \eta_{90,27} \eta_{90,39} \eta_{180,66} \eta_{180,84} \eta_{180,42} \eta_{90,3} \\
&+ \eta_6^3 \eta_{12} \eta_{18}^2 \eta_{45} \eta_{60}^2 \eta_{90}^7 \eta_{180}^2 \eta_{15,3} \eta_{12,3}^2 \eta_{60,15} \\
&\times \eta_{90,30}^6 \eta_{180,60} \eta_{60,24} \eta_{30,9}^4 \eta_{90,15} \eta_{60,15} \\
&\times \eta_{90,36} \eta_{15,3} \eta_{30,6} \eta_{45,9} \eta_{90,18} (\eta_{90,39} \eta_{90,45} - \eta_{90,21} \eta_{90,45} + \eta_{90,9} \eta_{90,15}) \\
&+ \eta_6 \eta_9^2 \eta_{12} \eta_{15}^2 \eta_{18}^2 \eta_{45} \eta_{60}^2 \eta_{90}^5 \eta_{180}^2 \eta_{15,3} \\
&\times \eta_{9,3}^2 \eta_{90,30}^2 \eta_{180,60}^3 \eta_{30,9}^2 \eta_{90,36} \eta_{180,72} \eta_{15,3} \eta_{30,6} \eta_{45,9} \eta_{90,18} \\
&\times (\eta_{90,15} \eta_{90,39}^2 \eta_{90,21}^2 - \eta_{90,45} \eta_{90,9}^2 \eta_{90,21}^2 - \eta_{90,45} \eta_{90,9}^2 \eta_{90,39}^2) \\
&+ \eta_3^2 \eta_{15} \eta_{18}^3 \eta_{30}^3 \eta_{36} \eta_{90}^6 \eta_{180}^2 \eta_{15,3} \eta_{18,6} \eta_{36,12} \\
&\times \eta_{90,30}^2 \eta_{180,60}^3 \eta_{60,24} \eta_{30,9}^2 \eta_{60,18} \eta_{90,36} \\
&\times \eta_{15,3} \eta_{30,6} \eta_{45,9} \eta_{90,18} (\eta_{90,21} \eta_{90,15}^2 + \eta_{90,9} \eta_{90,45}^2 - \eta_{90,39} \eta_{90,15}^2).
\end{aligned}$$

$$\Big/ \; q^{(336/5)} \frac{\eta_3^2 \eta_6 \eta_{12} \eta_{15} \eta_{18}^2 \eta_{30}^7 \eta_{45} \eta_{60}^5 \eta_{15,3}^2 \eta_{30,6} \eta_{45,9} \eta_{60,18}^2 \eta_{60,24} \eta_{90,18}}{\eta_{90}^2 \eta_{180}} \Bigg)$$

We have:

THEOREM 2.6.15. *If* $|q| < 1$, *then*

$$\frac{(q^9;q^9)_\infty (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty f(-q^3,-q^{15})^2}{(q^{18};q^{18})_\infty^2 f(-q^6,-q^{24})}$$

$$\times \frac{(q^{30};q^{30})_\infty^4 f(-q^{18},-q^{42})^2}{f(-q^9,-q^{21})^2 f(-q^{15},-q^{45})}$$

$$= \frac{\begin{pmatrix} (q^{45};q^{45})_\infty (q^{60};q^{60})_\infty^3 f(q^9, q^{21}) \\ f(q^{36}, q^{54}) f(q^{72}, q^{108}) \end{pmatrix}}{(q^{15};q^{15})_\infty (q^{30};q^{30})_\infty^2 (q^{180};q^{180})_\infty f(-q^{24},-q^{36})}$$

$$\times (q^6 f(-q^{21},-q^{69}) f(-q^{15},-q^{75})^2$$

$$+ f(-q^9,-q^{81}) f(-q^{45},-q^{45})^2$$

$$- q^3 f(-q^{39},-q^{51}) f(-q^{15},-q^{75})^2)$$

$$- q^3 \frac{(q^{90};q^{90})_\infty^4 f(q^9,q^{21}) f(-q^9,-q^{21}) f(-q^{36},-q^{54})}{(q^{30};q^{30})_\infty^2 (q^{45};q^{45})_\infty}$$

$$\times (f(-q^{39},-q^{51}) f(-q^{45},-q^{45})$$

$$- q^3 f(-q^{21},-q^{69}) f(-q^{45},-q^{45})$$

$$+ q^{12} f(-q^9,-q^{81}) f(-q^{15},-q^{75}))$$

$$- q^9 \frac{(q^{15};q^{15})_\infty (q^{180};q^{180})_\infty^2 f(-q^9,-q^{21}) f(-q^{36},-q^{54})}{(q^{30};q^{30})_\infty^2 (q^{45};q^{45})_\infty (q^{90};q^{90})_\infty^2}$$

$$\times (-f(-q^{15},-q^{75}) f(-q^{39},-q^{51})^2 f(-q^{21},-q^{69})^2$$

$$+ q^9 f(-q^{45},-q^{45}) f(-q^9,-q^{81})^2 f(-q^{21},-q^{69})^2$$

$$+ q^3 f(-q^{45},-q^{45}) f(-q^9,-q^{81})^2 f(-q^{39},-q^{51})^2).$$

Dividing both sides of (2.6.9) for

$$q^{(169/40)} \frac{\eta_3 \eta_{15} \eta_{30}^4 \eta_{45} \eta_{60}^2 \eta_{15,3} \eta_{30,6} \eta_{30,9}^2 \eta_{60,24}}{\eta_{90}^5}$$

From $F_4 = 8640$, we have that:

$$8640 \Big/ q^{(169/40)} \frac{\eta_3\eta_{15}\eta_{30}^4\eta_{45}\eta_{60}^2\eta_{15,3}\eta_{30,6}\eta_{30,9}^2\eta_{60,24}}{\eta_{90}^5} =$$

$$\Big(\eta_9^2\eta_{30}^2\eta_{45}\eta_{60}^4\eta_{180}\eta_{15,3}\eta_{9,3}\eta_{90,30}^5\eta_{180,60}\eta_{60,24}\eta_{60,18}^2\eta_{18,3}^2 \Big/$$

$$q^{(169/40)} \frac{\eta_3\eta_{15}\eta_{30}^4\eta_{45}\eta_{60}^2\eta_{15,3}\eta_{30,6}\eta_{30,9}^2\eta_{60,24}}{\eta_{90}^5}\Big) - \Big($$

$$\eta_{45}^3\eta_{90}^2\eta_{180}^5\eta_{15,3}^2\eta_{15,6}\eta_{45,15}\eta_{90,30}^3\eta_{180,60}^5\eta_{30,6}\eta_{30,9}^3\eta_{90,36}$$

$$\times(\eta_{180,72}\eta_{90,21}\eta_{90,15}^2+\eta_{180,72}\eta_{90,9}\eta_{90,45}^2-\eta_{180,72}\eta_{90,39}\eta_{90,15}^2)$$

$$-\eta_{15}\eta_{30}^4\eta_{60}\eta_{90}^2\eta_{180}^2\eta_{15,3}^2\eta_{15,6}\eta_{45,15}\eta_{180,60}^2\eta_{30,6}\eta_{30,9}^2\eta_{60,24}\eta_{60,18}\eta_{90,36}$$

$$\times(\eta_{90,39}\eta_{90,45}-\eta_{90,21}\eta_{90,45}+\eta_{90,9}\eta_{90,15})$$

$$+\eta_{45}^3\eta_{90}^2\eta_{180}^5\eta_{15,3}^2\eta_{15,6}\eta_{45,15}^3\eta_{90,30}^3\eta_{180,60}^3\eta_{30,6}\eta_{30,9}^3\eta_{60,24}\eta_{90,36}$$

$$\times(\eta_{90,15}\eta_{90,39}^2\eta_{90,21}^2-\eta_{90,45}\eta_{90,9}^2\eta_{90,21}^2-\eta_{90,45}\eta_{90,9}^2\eta_{90,39}^2). \qquad (2.6.9)$$

$$\Big/ q^{(169/40)} \frac{\eta_3\eta_{15}\eta_{30}^4\eta_{45}\eta_{60}^2\eta_{15,3}\eta_{30,6}\eta_{30,9}^2\eta_{60,24}}{\eta_{90}^5}\Big)$$

We have:

THEOREM 2.7.5. *If* $|q| < 1$, *then*

$$2q^3\frac{(q^{30};q^{30})_\infty^2(q^{45};q^{45})_\infty(q^{60};q^{60})_\infty^3 f(-q^{36},-q^{144})}{(q^{15};q^{15})_\infty(q^{180};q^{180})_\infty f(-q^{12},-q^{48})}$$
$$=\frac{(q^{30};q^{30})_\infty^4 f(q^3,q^{27})f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2}$$
$$-(q^{30};q^{30})_\infty(q^{60};q^{60})_\infty f(-q^3,-q^{27})f(-q^{90},-q^{90}).$$

Dividing both sides of (2.7.1) for

$$q^{45/2}\frac{\eta_3\eta_6\eta_{12}\eta_{15}^2\eta_{60}^4\eta_{180}\eta_{30,3}\eta_{60,12}}{\eta_{30}^2}$$

From $F_{5,6} = 10368$, we have that:

$$10368 \;/\; q^{45/2}\frac{\eta_3\eta_6\eta_{12}\eta_{15}^2\eta_{60}^4\eta_{180}\eta_{30,3}\eta_{60,12}}{\eta_{30}^2} =$$

$$\Big(2\eta_6\eta_{12}\eta_{15}\eta_{45}^2\eta_{60}^5\eta_{180}^2\eta_{15,3}\eta_{15,6}\eta_{30,3}\eta_{45,15}\eta_{180,60}\eta_{180,36} \;/$$

$$q^{45/2}\frac{\eta_3\eta_6\eta_{12}\eta_{15}^2\eta_{60}^4\eta_{180}\eta_{30,3}\eta_{60,12}}{\eta_{30}^2} \Big) -$$

$$\eta_9\eta_{18}\eta_{30}^3\eta_{60}\eta_{90}\eta_{180}^5\eta_{9,3}\eta_{18,6}\eta_{60,6}\eta_{60,12}^2\eta_{60,24}\eta_{90,45}\eta_{180,60}^4$$

$$\Big(-\eta_6\eta_{12}\eta_{15}\eta_{45}^2\eta_{60}^5\eta_{180}^2\eta_{15,3}\eta_{15,6}\eta_{30,3}^2\eta_{45,15}^2\eta_{60,12}\eta_{180,90} \;/$$

$$q^{45/2}\frac{\eta_3\eta_6\eta_{12}\eta_{15}^2\eta_{60}^4\eta_{180}\eta_{30,3}\eta_{60,12}}{\eta_{30}^2} \Big)$$

We have:

THEOREM 2.7.6. *If* $|q|<1$, *then*

$$-q^{-2}\frac{f(-q^3,-q^{27})^2 f(-q^{12},-q^{18})f(-q^{15},-q^{45})}{f(-q^6,-q^{54})^2 f(-q^{30},-q^{60})^4}$$
$$\times\frac{\begin{pmatrix} f(-q^7,-q^{23})f(-q^{13},-q^{17})f(-q^5,-q^{25}) \\ f(-q^{10},-q^{20})f(-q^{22},-q^{38})f(-q^2,-q^{58})\end{pmatrix}}{f(-q^4,-q^{26})f(-q^{14},-q^{16})}$$
$$=\frac{(q^5;q^5)_\infty(q^{10};q^{10})_\infty f(-q^2,-q^3)}{f(-q,-q^4)f(-q^2,-q^8)}(q^{60};q^{60})_\infty$$
$$+\frac{(q^{30};q^{30})_\infty^3}{q^2 f(q^5,q^{15})f(q^3,q^{27})}\left(-\frac{f(q^5,q^{25})f(-q^{60},-q^{120})f(-q^8,-q^{52})}{f(-q^2,-q^{28})f(-q^4,-q^{56})}\right.$$
$$+q^2\frac{f(q^5,q^{25})f(-q^{60},-q^{120})f(-q^{28},-q^{32})}{f(-q^8,-q^{22})f(-q^{16},-q^{44})}$$
$$\left.-q^5\frac{f(q^{15},q^{15})f(-q^{48},-q^{132})}{f(-q^{12},-q^{18})}+q^{17}\frac{f(q^{15},q^{15})f(-q^{12},-q^{168})}{f(-q^{12},-q^{18})}\right).$$

Dividing both sides of (2.7.3) by

$$q^{679/60}\eta_1^2\eta_2^2\eta_{10}\eta_{15}\eta_{30}\eta_{60}^3\eta_{5,1}\eta_{10,2}\eta_{10,4}\eta_{15,5}\eta_{30,6}\eta_{60,6}^2\eta_{60,4}\eta_{60,16}$$

From $F_{5,6} = 10368$, we have that:

$$10368 / \; q^{679/60}\eta_1^2\eta_2^2\eta_{10}\eta_{15}\eta_{30}\eta_{60}^3\eta_{5,1}\eta_{10,2}\eta_{10,4}\eta_{15,5}\eta_{30,6}\eta_{60,6}^2\eta_{60,4}\eta_{60,16} =$$

$$= \Big(-\eta_1\eta_2\eta_3\eta_5^2\eta_6\eta_{10}^2\eta_{60}^4\eta_{3,1}\eta_{5,1}\eta_{10,2}^2\eta_{10,4}\eta_{10,3}\eta_{30,3}\eta_{30.6}\eta_{60,2}\eta_{60,4}\eta_{60,16}\eta_{60,22} \; /$$

$$q^{679/60}\eta_1^2\eta_2^2\eta_{10}\eta_{15}\eta_{30}\eta_{60}^3\eta_{5,1}\eta_{10,2}\eta_{10,4}\eta_{15,5}\eta_{30,6}\eta_{60,6}^2\eta_{60,4}\eta_{60,16} \Big) - \Big($$

$$\eta_1^2\eta_2^2\eta_5\eta_{10}\eta_{15}\eta_{30}\eta_{60}^4\eta_{5,2}\eta_{10,4}\eta_{15,5}\eta_{30,6}\eta_{60,6}^2\eta_{60,4}\eta_{60,16}$$
$$-\eta_3^2\eta_6^2\eta_{30}^4\eta_{60}^4\eta_{5,1}\eta_{10,4}\eta_{15,5}\eta_{30,6}\eta_{30,3}\eta_{30,15}\eta_{30,8}\eta_{30,12}\eta_{60,6}$$
$$\times\eta_{60,10}\eta_{60,8}\eta_{60,16}$$
$$+\eta_3^2\eta_6^3\eta_{30}^4\eta_{60}^2\eta_{180}\eta_{5,1}\eta_{10,4}\eta_{15,5}\eta_{3,1}^2\eta_{6,2}^2\eta_{30,6}\eta_{30,3}\eta_{30,15}\eta_{30,2}\eta_{30,12}$$
$$\times\eta_{60,6}\eta_{60,10}\eta_{60,28}\eta_{60,4}$$
$$-\eta_3^2\eta_6^2\eta_{90}^4\eta_{180}^4\eta_{3,1}\eta_{5,1}\eta_{6,2}^2\eta_{10,4}\eta_{15,5}\eta_{30,3}\eta_{30,6}\eta_{30,5}\eta_{30,2}\eta_{30,8}\eta_{60,6}$$
$$\times\eta_{60,30}\eta_{60,4}\eta_{60,16}\eta_{90,30}^4\eta_{180,48}\eta_{180,60}^3$$
$$+\eta_3^2\eta_6^2\eta_{90}^4\eta_{180}^4\eta_{3,1}^2\eta_{5,1}\eta_{6,2}^2\eta_{10,4}\eta_{15,5}\eta_{30,4}\eta_{30,6}\eta_{30,6}\eta_{30,2}\eta_{30,8}\eta_{60,6}$$
$$\times\eta_{60,4}\eta_{60,30}\eta_{60,16}\eta_{90,30}^4\eta_{180,12}\eta_{180,60}^3 \cdot \qquad (2.7.3) \qquad /$$

$$q^{679/60}\eta_1^2\eta_2^2\eta_{10}\eta_{15}\eta_{30}\eta_{60}^3\eta_{5,1}\eta_{10,2}\eta_{10,4}\eta_{15,5}\eta_{30,6}\eta_{60,6}^2\eta_{60,4}\eta_{60,16} \Big)$$

We have:

THEOREM 2.7.7. *If $|q| < 1$, then*

$$
\begin{aligned}
&-q^3 \frac{(q^3;q^3)_\infty (q^9;q^9)_\infty^2 (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty}{(q^6;q^6)_\infty (q^{18};q^{18})_\infty f(-q^{12},-q^{18})} \\
&\quad = \frac{(q^{45};q^{45})_\infty (q^{90};q^{90})_\infty f(-q^9,-q^{36})}{f(-q^{18},-q^{27}) f(-q^{36},-q^{54})} q^9 (q^{60};q^{60})_\infty \\
&\qquad + q^{18}\left(q^6 \frac{(q^{90};q^{90})_\infty^3 f(q^{21},q^{69}) f(-q^{12},-q^{168})}{f(q^{45},q^{135}) f(-q^{36},-q^{54}) f(q^{33},q^{57})} \right. \\
&\qquad \left. - \frac{(q^{90};q^{90})_\infty^3 f(q^{39},q^{51}) f(-q^{48},-q^{132})}{q^{18} f(q^{45},q^{135}) f(-q^{36},-q^{54}) f(q^3,q^{87})} \right) \\
&\qquad + \frac{f(-q^3,-q^{27})^2 f(-q^{15},-q^{45})}{(q^{30};q^{30})_\infty^4 f(-q^6,-q^{54})^2} \\
&\qquad \times \left\{ \frac{\begin{pmatrix} (q^{45};q^{45})_\infty (q^{60};q^{60})_\infty^3 f(-q^{15},-q^{75}) \\ f(-q^{18},-q^{72}) f(-q^{36},-q^{144}) \end{pmatrix}}{\begin{pmatrix} (q^{15};q^{15})_\infty (q^{30};q^{30})_\infty (q^{90};q^{90})_\infty^3 \\ (q^{180};q^{180})_\infty f(-q^{12},-q^{48}) \end{pmatrix}} \right. \\
&\quad \times (f(-q^{45},-q^{45}) f(-q^{33},-q^{57})^2 f(-q^{27},-q^{63})^2 \\
&\quad + q^{18} f(-q^{45},-q^{45}) f(-q^3,-q^{87})^2 f(-q^{27},-q^{63})^2 \\
&\quad - q^{21} f(-q^{15},-q^{75}) f(-q^3,-q^{87})^2 f(-q^{33},-q^{57})^2) \\
&\quad - q^6 \frac{f(-q^3,-q^{27}) f(q^3,q^{27}) f(-q^{15},-q^{75}) f(-q^{18},-q^{72})}{(q^{15};q^{15})_\infty^2} \\
&\quad \times (-q^9 f(-q^{33},-q^{57}) f(-q^{15},-q^{75})^2 \\
&\quad + f(-q^{27},-q^{63}) f(-q^{45},-q^{45})^2 \\
&\quad + q^{18} f(-q^3,-q^{87}) f(-q^{15},-q^{75})^2) \\
&\quad + q^{15} \frac{\begin{pmatrix} (q^{60};q^{60})_\infty f(-q^3,-q^{27})^2 f(-q^{15},-q^{75}) \\ f(-q^{18},-q^{72}) f(-q^{30},-q^{150}) \end{pmatrix}}{(q^{30};q^{30})_\infty^3} \\
&\quad \times (q^6 f(-q^{27},-q^{63}) f(-q^{15},-q^{75}) \\
&\quad - q^9 f(-q^3,-q^{87}) f(-q^{45},-q^{45}) \\
&\quad \left. + f(-q^{33},-q^{57}) f(-q^{45},-q^{45})) \right\}.
\end{aligned}
$$

Dividing both sides of (2.7.5) by

$$
q^{149/20} \eta_6 \eta_9^2 \eta_{15}^2 \eta_{18} \eta_{30}^3 \eta_{60}^3 \eta_{30,12} \eta_{45,18} \eta_{60,6}^2 \eta_{60,12} \eta_{90,36} \eta_{90,21} \eta_{90,39} \eta_{180,66} \eta_{180,6},
$$

From $F_{7,8} = 12096$, we have that:

$$12096 \Big/ \frac{q^{149/20}\eta_6\eta_9^2\eta_{15}^2\eta_{18}\eta_{30}^3\eta_{60}^3\eta_{30,12}\eta_{45,18}\eta_{60,6}^2}{\eta_{60,12}\eta_{90,36}\eta_{90,21}\eta_{90,39}\eta_{180,66}\eta_{180,6}} =$$

$$\Bigg(\frac{-\eta_9^4\eta_{15}^3\eta_{30}^2\eta_{60}^3\eta_{90}\eta_{180}\eta_{15,3}^2\eta_{15,6}\eta_{90,30}\eta_{180,60}\eta_{60,12}\eta_{60,6}^2\eta_{45,18}\eta_{90,36}}{\times\eta_{90,21}\eta_{90,39}\eta_{180,66}\eta_{180,6}} \Big/$$

$$\frac{q^{149/20}\eta_6\eta_9^2\eta_{15}^2\eta_{18}\eta_{30}^3\eta_{60}^3\eta_{30,12}\eta_{45,18}\eta_{60,6}^2}{\eta_{60,12}\eta_{90,36}\eta_{90,21}\eta_{90,39}\eta_{180,66}\eta_{180,6}} \Bigg) - \Bigg($$

$$\eta_6\eta_9^2\eta_{18}\eta_{30}^2\eta_{45}^3\eta_{60}^3\eta_{90}\eta_{180}\eta_{15,3}\eta_{45,15}^2\eta_{90,30}$$
$$\times\eta_{180,60}\eta_{45,9}\eta_{60,6}^2\eta_{60,12}\eta_{30,12}\eta_{90,21}\eta_{90,39}\eta_{180,6}\eta_{180,66}$$
$$+\eta_6\eta_9^2\eta_{18}\eta_{30}^2\eta_{45}^3\eta_{60}^3\eta_{90}\eta_{180}\eta_{15,3}\eta_{45,15}^2\eta_{90,30}$$
$$\times\eta_{60,6}^2\eta_{60,12}\eta_{30,12}\eta_{45,18}\eta_{90,33}\eta_{90,39}\eta_{180,42}\eta_{180,12}\eta_{180,6}$$
$$-\eta_6\eta_9^2\eta_{18}\eta_{30}^2\eta_{45}^3\eta_{60}^3\eta_{90}\eta_{180}\eta_{15,3}\eta_{45,15}^2\eta_{90,30}$$
$$\times\eta_{60,6}^2\eta_{60,12}\eta_{30,12}\eta_{45,18}\eta_{90,3}\eta_{90,21}\eta_{180,78}\eta_{180,48}\eta_{180,66}$$
$$+\eta_9^2\eta_{15}\eta_{18}^2\eta_{60}^3\eta_{90}^5\eta_{180}\eta_{15,3}\eta_{18,6}\eta_{180,45}\eta_{180,60}\eta_{30,12}\eta_{45,18}\eta_{90,36}\eta_{90,21}$$
$$\times\eta_{90,39}\eta_{180,66}\eta_{180,6}\eta_{30,3}^2\eta_{60,15}\eta_{90,15}\eta_{90,18}\eta_{180,36}(\eta_{90,45}\eta_{90,33}^2\eta_{90,27}^2$$
$$+\eta_{90,45}\eta_{90,3}^2\eta_{90,27}^2-\eta_{90,15}\eta_{90,3}^2\eta_{90,33}^2)$$
$$-\eta_6\eta_9\eta_{18}^2\eta_{30}\eta_{90}^7\eta_{180}^2\eta_{15,3}\eta_{36,9}\eta_{90,30}^2\eta_{180,60}^2\eta_{30,12}$$
$$\times\eta_{45,18}\eta_{90,36}\eta_{90,21}\eta_{90,39}$$
$$\times\eta_{180,66}\eta_{180,6}\eta_{60,12}\eta_{30,3}^2\eta_{60,15}\eta_{90,15}\eta_{60,6}\eta_{90,18}(-\eta_{90,15}^2\eta_{90,33}$$
$$+\eta_{90,45}^2\eta_{90,27}+\eta_{90,15}^2\eta_{90,3})$$
$$+\eta_6\eta_9^2\eta_{15}\eta_{18}\eta_{30}\eta_{60}^3\eta_{90}^4\eta_{180}\eta_{15,3}\eta_{60,15}\eta_{30,12}\eta_{45,18}\eta_{90,36}\eta_{90,21}\eta_{90,39}$$
$$\times\eta_{180,66}\eta_{180,6}\eta_{60,12}\eta_{30,3}^4\eta_{60,15}\eta_{90,15}\eta_{90,18}\eta_{180,30}(\eta_{90,15}\eta_{90,27}$$
$$-\eta_{90,45}\eta_{90,3}+\eta_{90,45}\eta_{90,33}). \qquad (2.7.5)$$

$$\Big/ \frac{q^{149/20}\eta_6\eta_9^2\eta_{15}^2\eta_{18}\eta_{30}^3\eta_{60}^3\eta_{30,12}\eta_{45,18}\eta_{60,6}^2}{\eta_{60,12}\eta_{90,36}\eta_{90,21}\eta_{90,39}\eta_{180,66}\eta_{180,6}} \Bigg)$$

We have that:

THEOREM 2.7.8. *If* $|q| < 1$, *then*

$$
\begin{aligned}
&q^4 \frac{(q^3;q^3)_\infty^2 (q^{18};q^{18})_\infty^2 (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty}{(q^6;q^6)_\infty^2 (q^9;q^9)_\infty f(-q^{12},-q^{18})} \\
&\quad - q^4 \frac{f(-q^3,-q^{27})^3 f(-q^{15},-q^{45})}{(q^{30};q^{30})_\infty^4 f(-q^6,-q^{54})^2} \\
&\qquad \times \Bigg\{ q^3 \frac{(q^{60};q^{60})_\infty f(-q^{15},-q^{75}) f(-q^{18},-q^{72}) f(-q^{30},-q^{150})}{(q^{30};q^{30})_\infty^2 (q^{90};q^{90})_\infty^3} \\
&\qquad \times (f(-q^{45},-q^{45}) f(-q^{33},-q^{57})^2 f(-q^{27},-q^{63})^2 \\
&+ q^{18} f(-q^{45},-q^{45}) f(-q^3,-q^{87})^2 f(-q^{27},-q^{63})^2 \\
&- q^{21} f(-q^{15},-q^{75}) f(-q^3,-q^{87})^2 f(-q^{33},-q^{57})^2) \\
&+ \frac{(q^{45};q^{45})_\infty (q^{60};q^{60})_\infty^3 f(-q^{18},-q^{72}) f(-q^{36},-q^{144})}{(q^{15};q^{15})_\infty (q^{30};q^{30})_\infty^2 (q^{180};q^{180})_\infty f(-q^{12},-q^{48})} \\
&\times (-q^9 f(-q^{33},-q^{57}) f(-q^{15},-q^{75})^2 \\
&+ f(-q^{27},-q^{63}) f(-q^{45},-q^{45})^2 \\
&+ q^{18} f(-q^3,-q^{87}) f(-q^{15},-q^{75})^2) \\
&- \frac{f(q^3,q^{27}) f(-q^{15},-q^{75})^2 f(-q^{18},-q^{72})}{(q^{15};q^{15})_\infty^2} \\
&\times (q^{12} f(-q^{27},-q^{63}) f(-q^{15},-q^{75}) \\
&- q^{15} f(-q^3,-q^{87}) f(-q^{45},-q^{45}) \\
&+ q^6 f(-q^{33},-q^{57}) f(-q^{45},-q^{45})) \Bigg\}.
\end{aligned}
$$

Dividing both sides of (2.7.7) by

$$q^{413/10} \frac{\eta_3^2 \eta_6^2 \eta_{15}^2 \eta_{30}^4 \eta_{60}^3 \eta_{12,3} \eta_{15,3}^4 \eta_{30,12} \eta_{60,6}^2 \eta_{60,12}}{\eta_{180}}$$

From $F_{7,8} = 12096$, we have that:

$$12096 / q^{413/10} \frac{\eta_3^2 \eta_6^2 \eta_{15}^2 \eta_{30}^4 \eta_{60}^3 \eta_{12,3} \eta_{15,3}^4 \eta_{30,12} \eta_{60,6}^2 \eta_{60,12}}{\eta_{180}} = ($$

$$\eta_3 \eta_6 \eta_{15}^3 \eta_{18}^2 \eta_{30}^4 \eta_{60}^3 \eta_{15,3}^5 \eta_{12,3}^2 \eta_{9,3} \eta_{15,6} \eta_{180,60} \eta_{60,6}^2 \eta_{60,12}$$

$$q^{413/10} \frac{\eta_3^2 \eta_6^2 \eta_{15}^2 \eta_{30}^4 \eta_{60}^3 \eta_{12,3} \eta_{15,3}^4 \eta_{30,12} \eta_{60,6}^2 \eta_{60,12}}{\eta_{180}}) - ($$

$$\begin{aligned}
&\eta_9^2 \eta_{12}^2 \eta_{15}^2 \eta_{90}^5 \eta_{180}^3 \eta_{15,3}^4 \eta_{12,3} \eta_{24,6}^2 \eta_{9,3}^2 \eta_{90,30} \\
&\times \eta_{180,60}^3 \eta_{30,12} \eta_{60,12} \eta_{60,15}^3 \eta_{90,15} \\
&\times \eta_{90,18} \eta_{30,3}^3 \eta_{180,30} (\eta_{90,45} \eta_{90,33}^2 \eta_{90,27}^2 + \eta_{90,45} \eta_{90,9}^2 \eta_{90,27}^2 \\
&\quad \eta_{90,15} \eta_{90,3}^2 \eta_{90,33}^2) \\
&+ \eta_3 \eta_6 \eta_9 \eta_{15} \eta_{18} \eta_{45} \eta_{60}^3 \eta_{90}^5 \eta_{15,3}^4 \eta_{12,3} \eta_{9,3} \\
&\times \eta_{18,6} \eta_{90,30} \eta_{180,60} \eta_{30,12} \eta_{30,3}^3 \eta_{60,15} \\
&\times \eta_{90,18} \eta_{180,36} (-\eta_{90,15}^2 \eta_{90,33} + \eta_{90,45}^2 \eta_{90,27} + \eta_{90,15} \eta_{90,3}) \\
&- \eta_6^4 \eta_{30}^4 \eta_{90}^5 \eta_{180} \eta_{15,3}^4 \eta_{12,3}^3 \eta_{180,60}^2 \eta_{30,12} \eta_{60,12} \\
&\times \eta_{30,3}^2 \eta_{60,6} \eta_{60,15} \eta_{90,15}^2 \eta_{90,18} \\
&\times (\eta_{90,15} \eta_{90,27} - \eta_{90,45} \eta_{90,3} + \eta_{90,45} \eta_{90,33}). \qquad (2.7.7)
\end{aligned}$$

$$/ q^{413/10} \frac{\eta_3^2 \eta_6^2 \eta_{15}^2 \eta_{30}^4 \eta_{60}^3 \eta_{12,3} \eta_{15,3}^4 \eta_{30,12} \eta_{60,6}^2 \eta_{60,12}}{\eta_{180}})$$

Example:

From eq. (2.6.3):

$$\begin{aligned}
&2\eta_{90}^3 \eta_{180}^3 \eta_{30,9} \eta_{180,72} \eta_{45,15} \eta_{180,60}^3 \eta_{90,30}^5 \\
&\quad = \eta_{30}^5 \eta_{60} \eta_{60,18} \eta_{60,24} \eta_{90,30}^3 + \eta_{30}^5 \eta_{60} \eta_{30,9}^2 \eta_{60,24} \eta_{45,15}^2 \eta_{180,60}. \qquad (2.6.3)
\end{aligned}$$

We have multiplied the left hand side and the right hand side of (2.6.3), i.e.

$$2\eta_{90}^{3}\,\eta_{180}^{3}\,\eta_{30,9}\,\eta_{180,72}\,\eta_{45,15}\,\eta_{180,60}^{3}\,\eta_{90,30}^{5}$$

$$\eta_{30}^{5}\,\eta_{60}\,\eta_{60,18}\,\eta_{60,24}\,\eta_{90,30}^{3}+\eta_{30}^{5}\,\eta_{60}\,\eta_{30,9}^{2}\,\eta_{60,24}\,\eta_{45,15}^{2}\,\eta_{180,60}.$$

by:

$$q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}}$$

By Lemma 2.6.5, $[\Gamma(1):\Gamma_1(180)]=20736$. Let F_1 denote the difference of the left and right sides of (2.6.3). Applying Theorem 2.6.3 (the valence formula), for a fundamental region R for $\Gamma_1(180)$, we deduce that, for F_1,

$$\sum_{z\in R} Ord_{\Gamma_1(180)}(F_1;z)=\frac{3\cdot 20736}{12}=5184\geqslant ord(F_1;\infty), \qquad (2.6.4)$$

Thence, we obtain the values of F_n (n = 1, 2, 3, 4, 5, 6, 7, 8) as follows in this example:

$$5184 \Big/ q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}} =$$

$$= \left(2\eta_{90}^{3}\,\eta_{180}^{3}\,\eta_{30,9}\,\eta_{180,72}\,\eta_{45,15}\,\eta_{180,60}^{3}\,\eta_{90,30}^{5} \Big/ q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}}\right) -$$

$$\left(\eta_{30}^{5}\,\eta_{60}\,\eta_{60,18}\,\eta_{60,24}\,\eta_{90,30}^{3}+\eta_{30}^{5}\,\eta_{60}\,\eta_{30,9}^{2}\,\eta_{60,24}\,\eta_{45,15}^{2}\,\eta_{180,60}. \Big/ q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}}\right)$$

which simplifies becomes:

$$5184 \Big/ q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}} = 1\Big/ q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}} * \Big(2\eta_{90}^{3}\,\eta_{180}^{3}\,\eta_{30,9}\,\eta_{180,72}\,\eta_{45,15}\,\eta_{180,60}^{3}\,\eta_{90,30}^{5} -$$

$$\eta_{30}^{5}\,\eta_{60}\,\eta_{60,18}\,\eta_{60,24}\,\eta_{90,30}^{3}+\eta_{30}^{5}\,\eta_{60}\,\eta_{30,9}^{2}\,\eta_{60,24}\,\eta_{45,15}^{2}\,\eta_{180,60}.\Big)$$

In conclusion:

$$1\Big/ q^{-23/2}\frac{\eta_{45}^{2}\,\eta_{90}^{3}\,\eta_{180}}{\eta_{30}^{5}\,\eta_{60}} *$$

$$5184 = 1/q^{-23/2} \frac{\eta_{45}^2 \eta_{90}^3 \eta_{180}}{\eta_{30}^5 \eta_{60}} \left(2\eta_{90}^3 \eta_{180}^3 \eta_{30,9} \eta_{180,72} \eta_{45,15} \eta_{180,60}^3 \eta_{90,30}^5 - \eta_{30}^5 \eta_{60} \eta_{60,18} \eta_{60,24} \eta_{90,30}^3 + \eta_{30}^5 \eta_{60} \eta_{30,9}^2 \eta_{60,24} \eta_{45,15}^2 \eta_{180,60}\right)$$

Now, we have:

$$2\eta_{90}^3 \eta_{180}^3 \eta_{30,9} \eta_{180,72} \eta_{45,15} \eta_{180,60}^3 \eta_{90,30}^5$$
$$= \eta_{30}^5 \eta_{60} \eta_{60,18} \eta_{60,24} \eta_{90,30}^3 + \eta_{30}^5 \eta_{60} \eta_{30,9}^2 \eta_{60,24} \eta_{45,15}^2 \eta_{180,60}$$

$$(F_1; z) = \frac{3 \cdot 20736}{12} = 5184$$

and

$$2\,\frac{(q^{30};q^{30})_\infty^2 (q^{45};q^{45})_\infty (q^{60};q^{60})_\infty^3 \, f(-q^{72},-q^{108})}{(q^{15};q^{15})_\infty (q^{180};q^{180})_\infty \, f(-q^{24},-q^{36})}$$
$$= \frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21}) \, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2}$$
$$+ (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21}) \, f(-q^{90},-q^{90})$$

that is:

$$\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21}) \, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2}$$
$$+ (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21}) \, f(-q^{90},-q^{90})$$

From (3 * 20736) / 12 = 5184, we have that 5184 * 12 = 20736 * 3 and

5184 * 12 / 1296 = 48; 62208 / 1296 = 48. Thence:

$$62208 \Big/ q^{-23/2} \frac{\eta_{45}^2 \eta_{90}^3 \eta_{180}}{\eta_{30}^5 \eta_{60}} = \frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21}) \, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21}) \, f(-q^{90},-q^{90})$$

where $q^{-23/2} \frac{\eta_{45}^2 \eta_{90}^3 \eta_{180}}{\eta_{30}^5 \eta_{60}} = 1296$

and:

$$\left(\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21}) \, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21}) \, f(-q^{90},-q^{90}) \right) = 48$$

Thence:

$$36 * \left(\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21})\, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21})\, f(-q^{90},-q^{90}) \right) = 1728$$

Note that 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729.

5184 / 1296 = 4; 9 * (5184 / 1296) = 36

$$9 * \left(5184 / \frac{q^{-23/2}\, \eta_{45}^2 \eta_{90}^3 \eta_{180}}{\eta_{30}^5 \eta_{60}}\right) = 36$$

62208 / 1296 = 48; 9 * (5184 / 1296) = 36.

Furthermore:

$$\sqrt[15]{36 * \left(\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21})\, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21})\, f(-q^{90},-q^{90}) \right)}$$
$$= 1.6437518 \ldots$$

$$2 \times \sqrt[3]{36 * \left(\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21})\, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21})\, f(-q^{90},-q^{90}) \right)}$$
$$= 24$$

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

1.643751829517225762308497936230979517383492589945475200411

$$\frac{1}{2}\left[\sqrt[15]{36 * \left(\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21})\, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21})\, f(-q^{90},-q^{90}) \right)} + \right.$$

$$+ \sqrt[16]{36 * \left(\frac{(q^{30};q^{30})_\infty^4 \, f(q^9,q^{21}) \, f(-q^{45},-q^{45})}{(q^{15};q^{15})_\infty^2} + (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty \, f(-q^9,-q^{21}) \, f(-q^{90},-q^{90}) \right)}] =$$

= 1.618615670181102435516227417163352011810811958816290019893

Indeed:

1/2 * [((((62208 / 1296) 9 * (5184 / 1296))))^1/16 + ((((62208 / 1296) 9 * (5184 / 1296))))^1/15))))]

Input:

$$\frac{1}{2}\left(\sqrt[16]{\frac{62\,208}{1296} \times 9 \times \frac{5184}{1296}} + \sqrt[15]{\frac{62\,208}{1296} \times 9 \times \frac{5184}{1296}}\right)$$

Open code

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Result:

- Approximate form
- Step-by-step solution

$$\frac{1}{2}\left(2^{3/8} \times 3^{3/16} + 2^{2/5} \sqrt[5]{3}\right)$$

Decimal approximation:

- More digits

1.618615670181102435516227417163352011810811958816290019893...

Open code

1.618615670181102435516227417163352011810811958816290019893

Alternate forms:

$$\frac{3^{3/16}}{2^{5/8}} + \frac{\sqrt[5]{3}}{2^{3/5}}$$

Open code

$$\frac{3^{3/16}\left(1 + \sqrt[40]{2} \sqrt[80]{3}\right)}{2^{5/8}}$$

$$\text{root of } 1\,125\,899\,906\,842\,624\,x^{80} - 6\,755\,399\,441\,055\,744\,x^{75} + 18\,999\,560\,927\,969\,280\,x^{70} - 33\,249\,231\,623\,946\,240\,x^{65} - 148\,434\,069\,749\,760\,x^{64} + 40\,522\,501\,041\,684\,480\,x^{60} - 862\,105\,077\,106\,606\,080\,x^{59} - 36\,470\,250\,937\,516\,032\,x^{55} - 63\,055\,126\,806\,354\,984\,960\,x^{54} + 25\,073\,297\,519\,542\,272\,x^{50} - 819\,369\,563\,241\,893\,068\,800\,x^{49} + 7\,827\,577\,896\,960\,x^{48} - 13\,432\,123\,671\,183\,360\,x^{45} - 3\,374\,567\,517\,567\,791\,923\,200\,x^{44} - 162\,923\,206\,347\,325\,440\,x^{43} + 5\,666\,677\,173\,780\,480\,x^{40} - 5\,580\,889\,105\,880\,926\,126\,080\,x^{39} + 32\,199\,776\,876\,321\,832\,960\,x^{38} - 1\,888\,892\,391\,260\,160\,x^{35} - 4\,140\,497\,702\,729\,117\,859\,840\,x^{34} - 540\,848\,566\,163\,252\,183\,040\,x^{33} - 206\,391\,214\,080\,x^{32} + 495\,834\,252\,705\,792\,x^{30} - 1\,440\,211\,855\,454\,219\,796\,480\,x^{29} + 1\,502\,565\,913\,250\,900\,213\,760\,x^{28} - 3\,051\,287\,708\,958\,720\,x^{27} - 101\,420\,642\,598\,912\,x^{25} - 233\,772\,701\,611\,504\,435\,200\,x^{24} - 883\,163\,826\,121\,628\,712\,960\,x^{23} - 224\,679\,926\,543\,155\,200\,x^{22} + 15\,846\,975\,406\,080\,x^{20} - 16\,755\,940\,501\,114\,060\,800\,x^{19} + 115\,227\,272\,119\,319\,101\,440\,x^{18} - 587\,549\,251\,716\,710\,400\,x^{17} + 2\,720\,977\,920\,x^{16} - 1\,828\,497\,162\,240\,x^{15} - 466\,134\,667\,451\,228\,160\,x^{14} - 2\,969\,021\,418\,242\,088\,960\,x^{13} - 97\,492\,325\,961\,139\,200\,x^{12} - 4\,456\,961\,832\,960\,x^{11} + 146\,932\,807\,680\,x^{10} - 3\,812\,563\,516\,078\,080\,x^{9} + 10\,475\,755\,128\,455\,040\,x^{8} - 797\,636\,990\,891\,520\,x^{7} + 3\,064\,161\,260\,160\,x^{6} - 7\,346\,640\,384\,x^{5} - 4\,449\,309\,082\,560\,x^{4} - 1\,717\,277\,189\,760\,x^{3} - 112\,495\,430\,880\,x^{2} - 2\,295\,825\,120\,x + 157\,837\,977 \text{ near } x = 1.61862$$

Minimal polynomial:

$$1\,125\,899\,906\,842\,624\,x^{80} - 6\,755\,399\,441\,055\,744\,x^{75} + 18\,999\,560\,927\,969\,280\,x^{70} - 33\,249\,231\,623\,946\,240\,x^{65} - 148\,434\,069\,749\,760\,x^{64} + 40\,522\,501\,041\,684\,480\,x^{60} - 862\,105\,077\,106\,606\,080\,x^{59} - 36\,470\,250\,937\,516\,032\,x^{55} - 63\,055\,126\,806\,354\,984\,960\,x^{54} + 25\,073\,297\,519\,542\,272\,x^{50} - 819\,369\,563\,241\,893\,068\,800\,x^{49} + 7\,827\,577\,896\,960\,x^{48} - 13\,432\,123\,671\,183\,360\,x^{45} - 3\,374\,567\,517\,567\,791\,923\,200\,x^{44} - 162\,923\,206\,347\,325\,440\,x^{43} + 5\,666\,677\,173\,780\,480\,x^{40} - 5\,580\,889\,105\,880\,926\,126\,080\,x^{39} + 32\,199\,776\,876\,321\,832\,960\,x^{38} - 1\,888\,892\,391\,260\,160\,x^{35} - 4\,140\,497\,702\,729\,117\,859\,840\,x^{34} - 540\,848\,566\,163\,252\,183\,040\,x^{33} - 206\,391\,214\,080\,x^{32} + 495\,834\,252\,705\,792\,x^{30} - 1\,440\,211\,855\,454\,219\,796\,480\,x^{29} + 1\,502\,565\,913\,250\,900\,213\,760\,x^{28} - 3\,051\,287\,708\,958\,720\,x^{27} - 101\,420\,642\,598\,912\,x^{25} - 233\,772\,701\,611\,504\,435\,200\,x^{24} - 883\,163\,826\,121\,628\,712\,960\,x^{23} - 224\,679\,926\,543\,155\,200\,x^{22} + 15\,846\,975\,406\,080\,x^{20} - 16\,755\,940\,501\,114\,060\,800\,x^{19} + 115\,227\,272\,119\,319\,101\,440\,x^{18} - 587\,549\,251\,716\,710\,400\,x^{17} + 2\,720\,977\,920\,x^{16} - 1\,828\,497\,162\,240\,x^{15} - 466\,134\,667\,451\,228\,160\,x^{14} - 2\,969\,021\,418\,242\,088\,960\,x^{13} - 97\,492\,325\,961\,139\,200\,x^{12} - 4\,456\,961\,832\,960\,x^{11} + 146\,932\,807\,680\,x^{10} - 3\,812\,563\,516\,078\,080\,x^{9} + 10\,475\,755\,128\,455\,040\,x^{8} - 797\,636\,990\,891\,520\,x^{7} + 3\,064\,161\,260\,160\,x^{6} - 7\,346\,640\,384\,x^{5} - 4\,449\,309\,082\,560\,x^{4} - 1\,717\,277\,189\,760\,x^{3} - 112\,495\,430\,880\,x^{2} - 2\,295\,825\,120\,x + 157\,837\,977$$

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{8+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$\frac{222\,645\,775\,\pi}{432\,136\,142} \approx 1.61861567018110243523041$$

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$$\sqrt{\frac{1}{13}\left(-4081+1787\,e+565\,\pi-3632\log(2)\right)} \approx 1.61861567018110242544 9$$

root of $865\,x^3+22\,920\,x^2-98\,833\,x+96\,256$ near $x = 1.61862$ $\approx$

1.6186156701811024355 22943

We note that the coefficients of the minimal polynomial, for example,

150256591325090021376 0, 414049770272911785984 0,

337456751756779192320 0, 558088910588092612608 0

are all numbers divisible two or three times by 1728. Indeed:

((((1502565913250900213760/1728)/1728))/1728))) = 291207340380

((((4140497702729117859840/1728)/1728))/1728))) = 802456193920

((((3374567517567791923200/1728))/1728))) = 1130135833804800

((((5580889105880926126080/1728)/1728)) = 1869028469637120.

The sum of the coefficients is:

(1502565913250900213760+4140497702729117859840 +3374567517567791923200 +5580889105880926126080)

Input:

1502565913250900213760 + 4140497702729117859840 + 3374567517567791923200 + 5580889105880926126080

Open code

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Result:

14598520239428736122880

Scientific notation:

$1.4598520239428736122880 \times 10^{22}$

Open code

14598520239428736122880

$1.4598520239428736122880 \times 10^{22}$

And the product is:

(1502565913250900213760 * 4140497702729117859840 * 3374567517567791923200 * 5580889105880926126080)

Input:

1502565913250900213760 × 4140497702729117859840 × 3374567517567791923200 × 5580889105880926126080

Open code

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Result:

117167616474961323468217387921205690619205359457893673086508 734029682181595883110400000

Decimal approximation:

- More digits

$1.1716761647496132346821738792120569061920535945789367... \times 10^{86}$

$1.1716761647496132346821738792120569061920535945789367 \times 10^{86}$

Note that:

(sqrt(27)) * (1.1716761647496132346821738792120569061920535945789367 × 10^86)

Input interpretation:

$\sqrt{27} \times 1.1716761647496132346821738792120569061920535945789367 \times 10^{86}$

Open code

- Result:
  More digits
  $6.0882079420913175646875538868200432704359268907628427... \times 10^{86}$

This result $6.08820794209... \times 10^{86}$ is practically equal to the value of Dark Matter entropy contained within the cosmic event horizon, that is $6 * 10^{86}$.

While:

2 * (1.1716761647496132346821738792120569061920535945789367 × 10^86)

Input interpretation:
$2 \times 1.1716761647496132346821738792120569061920535945789367 \times 10^{86}$
Open code

Result:
234 335 232 949 922 646 936 434 775 842 411 381 238 410 718 915 787 340 000 000 000 000 000 000 000 000 000 000 000 000
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Scientific notation:
$2.34335232949922646936434775842411381238410718915787734 \times 10^{86}$

$2.3433523294992264693643477584241138123841071891578733 \times 10^{86}$

This result $2.34335232949... \times 10^{86}$ is practically equal to the value of Relic Gravitons entropy contained within the cosmic event horizon, that is $2.3 * 10^{86}$

We note that:

(sqrt(2)) * (1.1716761647496132346821738792120569061920535945789367 × 10^86)

Input interpretation:
$\sqrt{2} \times 1.1716761647496132346821738792120569061920535945789367 \times 10^{86}$
Open code

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- Result:
  More digits
  $1.6570003228981959466092514871040511329212732711140786... \times 10^{86}$

$1.6570003228981959466092514871040511329212732711140786 \times 10^{86}$

and

(sqrt(1.8928)) * (1.1716761647496132346821738792120569061920535945789367 × 10^86)
Where 1.8928 is a Hausdorff dimension

Input interpretation:

$\sqrt{1.8928} \times 1.1716761647496132346821738792120569061920535945789367 \times 10^{86}$

Open code

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Result:

- Fewer digits
- More digits

$1.6119811494952986380358022394337548071347234255717938... \times 10^{86}$

1.611981149495298638035802239433754807134723425571 × 10^86

We have that:

(1.657000322898195946609251487104051132921273271114 0786 × 10^86 + 1.611981149495298638035802239433754807134723425571 × 10^86)*1/10^86 * 1/2.02

Where 2.02 is a Hausdorff dimension

Input interpretation:

$\left(1.6570003228981959466092514871040511329212732711140786 \times 10^{86} + 1.611981149495298638035802239433754807134723425571 \times 10^{86}\right) \times \frac{1}{10^{86}} \times \frac{1}{2.02}$

Open code

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Result:

- More digits

1.6183076596007398933886404586820821485425726222120830990099...

Open code

Repeating decimal:

$1.61830765960073989338864045868208214854257262221208\overline{30990}$

(period 4)

1.61830765960073989338864045868208214854257262221208309 90099

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{12+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

More

- $\frac{4396852645\,\pi}{8535533949} \approx 1.618307659600739893388664828$

  $\text{root of } 40598\,x^3 - 12488\,x^2 - 80376\,x - 9285 \text{ near } x = 1.61831 \approx$ 1.618307659600739893389 3924

  $\text{root of } 967\,x^4 - 4858\,x^3 + 749\,x^2 + 3599\,x + 6171 \text{ near } x = 1.61831 \approx$ 1.618307659600739893395016

This result 1.6183076596007398933886404.... is a very good approximation to the value of the golden ratio!

From the sum, we obtain:

((((14598520239428736122880)*(1.12915) + (14598520239428736122880)*(1.0933))))*1/2 * 1/10^22

Input interpretation:

$$(14598520239428736122880 \times 1.12915 + 14598520239428736122880 \times 1.0933) \times \frac{1}{2} \times \frac{1}{10^{22}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

1.6222240653059197298147328

Open code

1.6222240653059197298147328

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{267+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{8+\cfrac{1}{13+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{17+\cfrac{1}{3+\cfrac{1}{10+\cfrac{1}{9+\cfrac{1}{5+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}$$

Open code

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Possible closed forms:

- More

$$-\frac{-2-297\,e+e^2}{2\,(52-181\,e+93\,e^2)} \approx 1.62222406530591972975092$$

$$\frac{999\,627\,466\,\pi}{1\,935\,874\,563} \approx 1.622224065305919729897471$$

$$\frac{9}{7}\,\pi\,\sin^{-1}\left(\frac{17\,731\,679}{29\,944\,104}\right)^2 \approx 1.622224065305919730025$$

-

Multiplying the two results (sum and product), we obtain:

((((14598520239428736122880)*(1.171676164749613234682173879212056906192 0535945789367 * 10^86)*(1/33021.10)*1/4

Where 33021,10 is a result of a Ramanujan mock theta function (see our previous papers)

Input interpretation:

$$14\,598\,520\,239\,428\,736\,122\,880\,(1.1716761647496132346821738792120569061920535945789367\times10^{86})\times\frac{1}{33\,021.10}\times\frac{1}{4}$$

Open code

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Result:

- More digits

$1.294985494513619100368827008519668098268062354732581 3\ldots \times 10^{103}$

Open code

This result $1.294985494513619\ldots \times 10^{103}$ is practically equal to the value of the SMBHs entropy contained within cosmic event horizon.

We note that: 5184 = 1728 * 3

10368 = 2 * 5184 and 15552 = 3 * 5184

5184 * 1,6666666666666666666666666666667 = 8640 and

5184 * 2,3333333333333333333333333333333 = 12096.

We have that:

2,3333333333333333333333333333333 + 1,6666666666666666666666666666667 = 4

From:

TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY
Volume 354, Number 2, Pages 705-733 - S 0002-9947(01)02861-6
Article electronically published on September 21, 2001
TENTH ORDER MOCK THETA FUNCTIONS IN RAMANUJAN'S LOST NOTEBOOK (IV)
YOUN-SEO CHOI

We have:

Theorem 3.1 (Ramanujan's seventh tenth order mock theta function identity).

$$\int_0^\infty \frac{e^{-\pi nx^2}}{\cosh\frac{2\pi x}{\sqrt{5}}+\frac{1+\sqrt{5}}{4}}dx+\frac{1}{\sqrt{n}}e^{\frac{\pi}{5n}}\psi(-e^{-\frac{\pi}{n}})$$
$$=\sqrt{\frac{5+\sqrt{5}}{2}}e^{-\frac{\pi n}{5}}\phi(-e^{-\pi n})-\frac{\sqrt{5}+1}{2\sqrt{n}}e^{-\frac{\pi}{5n}}\phi(-e^{-\frac{\pi}{n}}).$$

Proof. Replacing ω, x, t, and θ by $5in$, $-5n$, $\frac{z}{\sqrt{5}}+1$, and $\theta'-i$, respectively, in the left side of (1.14), we find that

$$\int_{-\infty}^\infty \frac{e^{\pi i\omega t^2-2\pi xt}}{e^{2\pi t}-e^{2\pi i\theta}}dt=\frac{e^{5\pi n-2\pi}}{\sqrt{5}}\int_{-\infty}^\infty \frac{e^{-\pi nz^2}}{e^{\frac{2\pi z}{\sqrt{5}}}-e^{2\pi i\theta'}}dz. \tag{3.1}$$

Replacing θ' by $\frac{2}{5}$ in (3.1), and using (1.14), we have

$$\int_{-\infty}^\infty \frac{e^{-\pi nz^2}}{e^{\frac{2\pi z}{\sqrt{5}}}+e^{-\frac{\pi i}{5}}}dz=\sqrt{5}e^{\frac{4\pi n}{5}-\frac{4\pi i}{5}}\frac{F(\frac{2}{5},\frac{i}{5n})-5nF(2in,5in)}{5in\theta_{11}(2in,5in)}, \tag{3.2}$$

and replacing θ' by $\frac{3}{5}$ in (3.1), and using (1.14), we have

$$\int_{-\infty}^\infty \frac{e^{-\pi nz^2}}{e^{\frac{2\pi z}{\sqrt{5}}}+e^{\frac{\pi i}{5}}}dz=\sqrt{5}e^{\frac{9\pi n}{5}-\frac{6\pi i}{5}}\frac{F(\frac{3}{5},\frac{i}{5n})-5nF(3in,5in)}{5in\theta_{11}(3in,5in)}. \tag{3.3}$$

Using (3.2) and (3.3), we can easily derive that

$$\int_0^\infty \frac{e^{-\pi nx^2}}{\cosh\frac{2\pi x}{\sqrt{5}}+\frac{1+\sqrt{5}}{4}}dx=\frac{1}{2}\int_{-\infty}^\infty \frac{e^{-\pi nx^2}}{\cosh\frac{2\pi x}{\sqrt{5}}+\frac{1+\sqrt{5}}{4}}dx$$
$$=-\frac{1}{2}\int_{-\infty}^\infty \frac{e^{-\pi nx^2}}{i\sin\frac{\pi}{5}}\left(\frac{1}{1+e^{\frac{2\pi x}{\sqrt{5}}+\frac{\pi}{5}i}}-\frac{1}{1+e^{\frac{2\pi x}{\sqrt{5}}-\frac{\pi}{5}i}}\right)dx$$
$$=-\frac{1}{2i\sin\frac{\pi}{5}}\left(e^{-\frac{\pi}{5}i}\int_{-\infty}^\infty \frac{e^{-\pi nx^2}}{e^{\frac{2\pi x}{\sqrt{5}}}+e^{-\frac{\pi}{5}i}}dx-e^{\frac{\pi}{5}i}\int_{-\infty}^\infty \frac{e^{-\pi nx^2}}{e^{\frac{2\pi x}{\sqrt{5}}}+e^{\frac{\pi}{5}i}}dx\right)$$
$$=-\frac{\sqrt{5+\sqrt{5}}}{5n\sqrt{2}}\left(e^{-\frac{\pi n}{5}+\pi n}\frac{F(\frac{2}{5},\frac{i}{5n})-5nF(2in,5in)}{\theta_{11}(2in,5in)}\right.$$
$$\left.-e^{-\frac{\pi n}{5}+2\pi n}\frac{F(\frac{3}{5},\frac{i}{5n})-5nF(3in,5in)}{\theta_{11}(3in,5in)}\right). \tag{3.4}$$

We take:

$$\int_0^\infty \frac{e^{-\pi nx^2}}{\cosh\frac{2\pi x}{\sqrt{5}}+\frac{1+\sqrt{5}}{4}}dx=\frac{1}{2}\int_{-\infty}^\infty \frac{e^{-\pi nx^2}}{\cosh\frac{2\pi x}{\sqrt{5}}+\frac{1+\sqrt{5}}{4}}dx$$

We obtain, for n = 2 and x,[-1, 36.5]:

1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5]

Definite integral:

$$\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}x^2}{\frac{1}{4}(1+\sqrt{5})+\cosh\left(\frac{2\pi}{\sqrt{5}}\right)}dx = 1.65528$$

where 1,65528 is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164{,}2696$ i.e. 1,65578...

Now, we have that:

(((((((1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5])))))))^14.7789

Where 14,7789 is the result of the difference between the following two black hole entropies: 30.5963 – 15.8174 = 14.7789

Input interpretation:

$$\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}}{\cosh\left(2\times\frac{\pi}{\sqrt{5}}\right)+\frac{1}{4}(1+\sqrt{5})}x\,x\,dx\right)^{14.7789}$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function

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Result:

1716.72

Computation result:

$$\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}x\,x}{\cosh\left(\frac{2\pi}{\sqrt{5}}\right)+\frac{1}{4}(1+\sqrt{5})}dx\right)^{14.7789} = 1716.72$$

This result is very near to the mass of candidate glueball $f_0(1710)$ meson

We have also:

2*(((((((((((((1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5])))))))^14.7789))))))^1/3

Input interpretation:

$$2\sqrt[3]{\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}}{\cosh\left(2\times\frac{\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,x\,x\,dx\right)^{14.7789}}$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function

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Result:

23.9476

Computation result:

$$2\sqrt[3]{\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}\,x\,x}{\cosh\left(\frac{2\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,dx\right)^{14.7789}} = 23.9476$$

This value 23.9476 ≈ 24, is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

1/3 * ((((((((((((((1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5])))))))^14.7789))))))^1/3

Input interpretation:

$$\frac{1}{3}\sqrt[3]{\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}}{\cosh\left(2\times\frac{\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,x\,x\,dx\right)^{14.7789}}$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function

Result:

3.99127

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Computation result:

$$\frac{1}{3}\sqrt[3]{\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}\,x\,x}{\cosh\left(\frac{2\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,dx\right)^{14.7789}} = 3.99127$$

This result 3.99127 ≈ 4 is the minimal possible value of the mass of hypothetical DM particles

Furthermore:

1/((0.538+1.2683)*10^34) * (((((((((((((1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5])))))))^14.7789))))))^1/3

where 0.538 and 1.2683 are two Hausdorff dimensions, we obtain:

1/((0.538+1.2683)*10^34) * (((((((((((((1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5])))))))^14.7789))))))^1/3

Input interpretation:

$$\frac{1}{(0.538+1.2683)\times 10^{34}}\sqrt[3]{\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}}{\cosh\left(2\times\frac{\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,x\,x\,dx\right)^{14.7789}}$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function

Result:

6.62892×10^{-34}

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Computation result:

$$\frac{\sqrt[3]{\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}\,x\,x}{\cosh\left(\frac{2\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,dx\right)^{14.7789}}}{(0.538+1.2683)\,10^{34}}=6.62892\times 10^{-34}$$

Practically, the result is a number very near to the value of the Planck's constant $6.62607015\times 10^{-34}$

1 / (((((((1/(240-13) * (((((1/2 * integrate [(e^(-2Pi))) / (((cosh(2Pi/(sqrt(5)))) + (1+sqrt(5))/4))]x x,[-1, 36.5]))))))))

Input:

$$\frac{1}{\frac{1}{240-13}\left(\frac{1}{2}\int_{-1}^{36.5}\frac{e^{-2\pi}}{\cosh\left(2\times\frac{\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}\,x\,x\,dx\right)}$$

Open code

- $\cosh(x)$ is the hyperbolic cosine function

Result:

137.137

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Computation result:

$$\frac{1}{\frac{\int_{-1}^{36.5} \frac{e^{-2\pi x x}}{\cosh\left(\frac{2\pi}{\sqrt{5}}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)} dx}{(240-13)2}} = 137.137$$

This result 137,137 is very near to the inverse of fine-structure constant α , that is 137.035999084(21) and to the mean of the pion masses $\pi^{\pm}$: 139.57018(35) MeV/c^2 π^{0} : 134.9766(6) MeV/c^2 , that is 137,27342175

Now, we have that:

From:

George E. Andrews • Bruce C. Berndt
Ramanujan's Lost Notebook Part V - ©Springer International Publishing AG, part of Springer Nature 2018

$$\sqrt{\frac{5+\sqrt{5}}{2}} e^{-\pi n/5} \phi_{10}(-e^{-\pi n}) - \frac{\sqrt{5}+1}{2\sqrt{n}} e^{-\pi/(5n)} \phi_{10}(-e^{-\pi/n}). \quad (12.1.3)$$

For ϕ_{10} = 1 and n = 2, we have that:

[sqrt(((5+sqrt(5))/(2))) * e^(-2Pi/5) * (-e^(-2Pi))] - [((sqrt(5)+1))*(e^(-Pi/10))*(-e^(-Pi/2)) / ((2sqrt(2))]

Input:

$$\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-2\times\pi/5}\left(-e^{-2\pi}\right)-\left(\sqrt{5}+1\right) e^{-\pi/10}\left(-\frac{e^{-\pi/2}}{2\sqrt{2}}\right)$$

Open code

Exact result:

$$\frac{\left(1+\sqrt{5}\right) e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5}$$

Decimal approximation:

- More digits

0.172707845854182573433677777644474941480979480674859217827...

Open code

0.172707845854182573433677777644474941480979480674859217827

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Property:

$$-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5} + \frac{\left(1+\sqrt{5}\right) e^{-(3\pi)/5}}{2\sqrt{2}}$$ is a transcendental number

Alternate forms:

$$-\frac{e^{-(12\pi)/5}\left(2\sqrt{5+\sqrt{5}} - e^{(9\pi)/5} - \sqrt{5}\, e^{(9\pi)/5}\right)}{2\sqrt{2}}$$

Open code

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$$-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5} + \frac{e^{-(3\pi)/5}}{2\sqrt{2}} + \frac{1}{2}\sqrt{\frac{5}{2}}\, e^{-(3\pi)/5}$$

10^4 * ((((\[sqrt(((5+sqrt(5))/(2))) * e^(-2Pi/5) * (-e^(-2Pi))] - [((sqrt(5)+1))*(e^(-Pi/10))*(-e^(-Pi/2)) / ((2sqrt(2))]))))

Input:

$$10^4\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-2\times\pi/5}\left(-e^{-2\pi}\right) - \left(\sqrt{5}+1\right) e^{-\pi/10}\left(-\frac{e^{-\pi/2}}{2\sqrt{2}}\right)\right)$$

Open code

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Exact result:

$$10\,000\left(\frac{\left(1+\sqrt{5}\right) e^{-(3\pi)/5}}{2\sqrt{2}} - \sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5}\right)$$

Decimal approximation:

- More digits

1727.078458541825734336777776444749414809794806748592178276...

Open code

This result is very near to the mass of candidate glueball $f_0(1710)$ meson

Property:

$$10\,000\left(-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(12\pi)/5}+\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}\right)$$ is a transcendental number

[(([10^4*((((((sqrt(((5+sqrt(5))/(2))) * e^(-2Pi/5) * (-e^(-2Pi)))) - ((((sqrt(5)+1))*(e^(-Pi/10))*(-e^(-Pi/2)) / ((2sqrt(2))))))))]))]^1/3

Input:

$$\sqrt[3]{10^4\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-2\times\pi/5}\left(-e^{-2\pi}\right)-\left(\sqrt{5}+1\right)e^{-\pi/10}\left(-\frac{e^{-\pi/2}}{2\sqrt{2}}\right)\right)}$$

Open code

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Exact result:

$$10\sqrt[3]{10\left(\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(12\pi)/5}\right)}$$

Decimal approximation:

- More digits

11.997866422856682284752513408909613142550108325112222221726...

Open code

Property:

$$10\sqrt[3]{10\left(-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(12\pi)/5}+\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}\right)}$$ is a transcendental number

This result 11,9978 is very near to the value of black hole entropy 12,19

2 * [(([10^4*((((((sqrt(((5+sqrt(5))/(2))) * e^(-2Pi/5) * (-e^(-2Pi)))) - ((((sqrt(5)+1))*(e^(-Pi/10))*(-e^(-Pi/2)) / ((2sqrt(2))))))))]))]^1/3

Input:

$$2\sqrt[3]{10^4\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-2\times\pi/5}\left(-e^{-2\pi}\right)-\left(\sqrt{5}+1\right)e^{-\pi/10}\left(-\frac{e^{-\pi/2}}{2\sqrt{2}}\right)\right)}$$

Open code

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Exact result:

$$20\sqrt[3]{10\left(\frac{(1+\sqrt{5})\,e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(12\pi)/5}\right)}$$

Decimal approximation:

- More digits

23.995732845713364569505026817819226285100216650224444443452...

Open code

Property:

$$20\sqrt[3]{10\left(-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(12\pi)/5}+\frac{(1+\sqrt{5})\,e^{-(3\pi)/5}}{2\sqrt{2}}\right)}$$ is a transcendental number

Continued fraction:

- Linear form

$$23+\cfrac{1}{1+\cfrac{1}{233+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{25+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{20+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{20+\cfrac{1}{1+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}$$

Series representations:

$$2\sqrt[3]{10^4\left(\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\ e^{-(2\pi)/5}\right)(-1)\,e^{-2\pi}-\frac{\left(\sqrt{5}+1\right)\left(e^{-\pi/10}\left(-e^{-\pi/2}\right)\right)}{2\sqrt{2}}\right)} =$$

$$20\sqrt[3]{5}\left(\left(e^{-(12\pi)/5}\left(e^{(9\pi)/5}+e^{(9\pi)/5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.$$
$$2\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_1!\,k_2!}(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}$$
$$\left.\left.(2-z_0)^{k_1}\left(\frac{1}{2}\left(5+\sqrt{5}\right)-z_0\right)^{k_2}z_0^{-k_1-k_2}\right)\right)\Big/$$
$$\left.\left(\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/3)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

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$$2\sqrt[3]{10^4\left(\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\ e^{-(2\pi)/5}\right)(-1)\,e^{-2\pi}-\frac{\left(\sqrt{5}+1\right)\left(e^{-\pi/10}\left(-e^{-\pi/2}\right)\right)}{2\sqrt{2}}\right)} = 20\sqrt[3]{5}$$

$$\left(\left(e^{-(12\pi)/5}\left(e^{(9\pi)/5}+e^{(9\pi)/5}\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.$$
$$2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{1}{2}\left(5+\sqrt{5}\right)\right)}{2\pi}\right\rfloor\right)$$
$$\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_1!\,k_2!}(-1)^{k_1+k_2}(2-x)^{k_1}x^{-k_1-k_2}$$
$$\left.\left.\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(-x+\frac{1}{2}\left(5+\sqrt{5}\right)\right)^{k_2}\right)\right)\Big/$$
$$\left.\left(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}$$
$$(1/3)$$ for $(x\in\mathbb{R}$ and $x<0)$

$$2\sqrt[3]{10^4\left(\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(2\pi)/5}\right)(-1)\,e^{-2\pi}-\frac{\left(\sqrt{5}+1\right)\left(e^{-\pi/10}\left(-e^{-\pi/2}\right)\right)}{2\sqrt{2}}\right)} =$$

$$20\sqrt[3]{10}\left(-e^{-(12\pi)/5}\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{1}{2}\left(5+\sqrt{5}\right)-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2+1/2\left\lfloor\arg\left(\frac{1}{2}\left(5+\sqrt{5}\right)-z_0\right)/(2\pi)\right\rfloor}\right.$$
$$\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k\left(5+\sqrt{5}-2z_0\right)^k z_0^{-k}}{k!}+\left(e^{-(3\pi)/5}\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\right.$$
$$z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.$$
$$\left.\left.z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\bigg/$$
$$\left.\left(2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/3)$$

This value 23,99573 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

[(([10^4*((((((sqrt(((5+sqrt(5))/(2))) * e^(-2Pi/5) * (-e^(-2Pi)))) - ((((sqrt(5)+1))*(e^(-Pi/10))*(-e^(-Pi/2)) / ((2sqrt(2))))))))]))]^1/15

Input:

$$\sqrt[15]{10^4\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-2\times\pi/5}\left(-e^{-2\pi}\right)-\left(\sqrt{5}+1\right)e^{-\pi/10}\left(-\frac{e^{-\pi/2}}{2\sqrt{2}}\right)\right)}$$

Open code

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Exact result:

$$10^{4/15}\sqrt[15]{\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(12\pi)/5}}$$

Decimal approximation:

- More digits

1.643693374170899145754620920297174440767557825763625720665...

Open code

1.643693374170899145754620920297174440767557825763625720665

Property:

$$10^{4/15}\sqrt[15]{-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(12\pi)/5}+\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}}$$ is a transcendental number

Open code

Alternate form:

$$\sqrt[6]{2}\;5^{4/15}\,e^{-(4\pi)/25}\sqrt[15]{-2\sqrt{5+\sqrt{5}}+e^{(9\pi)/5}+\sqrt{5}\,e^{(9\pi)/5}}$$

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{9+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}}$$

All 15th roots of 10000 (((1 + sqrt(5)) e^(-(3 π)/5))/(2 sqrt(2)) - sqrt(1/2 (5 + sqrt(5))) e^(-(12 π)/5)):

- More roots
- More digits
- Polar form

$$10^{4/15}\sqrt[15]{\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(12\pi)/5}}\;e^{0}\approx 1.6437$$ (real, principal root)

Open code

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$$10^{4/15}\sqrt[15]{\frac{(1+\sqrt{5})\,e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(12\pi)/5}}\;e^{(2i\pi)/15}\approx 1.5016+0.6686\,i$$

Open code

$$10^{4/15}\sqrt[15]{\frac{(1+\sqrt{5})\,e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(12\pi)/5}}\;e^{(4i\pi)/15}\approx 1.0998+1.2215\,i$$

Open code

$$10^{4/15}\sqrt[15]{\frac{(1+\sqrt{5})\,e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(12\pi)/5}}\;e^{(2i\pi)/5}\approx 0.5079+1.5632\,i$$

Open code

$$10^{4/15}\sqrt[15]{\frac{(1+\sqrt{5})\,e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(12\pi)/5}}\;e^{(8i\pi)/15}\approx -0.17181+1.6347\,i$$

Series representations:

$$\sqrt[15]{10^4\left(\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\;e^{-(2\pi)/5}\right)(-1)\,e^{-2\pi}-\frac{\left(\sqrt{5}+1\right)\left(e^{-\pi/10}\left(-e^{-\pi/2}\right)\right)}{2\sqrt{2}}\right)}=$$

$$\sqrt[5]{2}\;5^{4/15}\left(\left(e^{-(12\pi)/5}\left(e^{(9\pi)/5}+e^{(9\pi)/5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-2\sqrt{z_0}^{\,2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_1!\,k_2!}(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{1}{2}\left(5+\sqrt{5}\right)-z_0\right)^{k_2} z_0^{-k_1-k_2}\right)\right)\Bigg/\left(\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

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$$\sqrt[15]{10^4\left(\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(2\pi)/5}\right)(-1)\,e^{-2\pi}-\frac{\left(\sqrt{5}+1\right)\left(e^{-\pi/10}\left(-e^{-\pi/2}\right)\right)}{2\sqrt{2}}\right)} = \sqrt[5]{2}\,5^{4/15} \left(\left(e^{-(12\pi)/5}\left(e^{(9\pi)/5}+e^{(9\pi)/5}\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} - 2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{1}{2}\left(5+\sqrt{5}\right)\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_1!\,k_2!}(-1)^{k_1+k_2}(2-x)^{k_1}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(-x+\frac{1}{2}\left(5+\sqrt{5}\right)\right)^{k_2}\right)\right)\Big/\left(\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)^\wedge(1/15) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\sqrt[15]{10^4\left(\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,e^{-(2\pi)/5}\right)(-1)\,e^{-2\pi}-\frac{\left(\sqrt{5}+1\right)\left(e^{-\pi/10}\left(-e^{-\pi/2}\right)\right)}{2\sqrt{2}}\right)} = 10^{4/15}\left(-e^{-(12\pi)/5}\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{1}{2}\left(5+\sqrt{5}\right)-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2+1/2\left\lfloor\arg\left(\frac{1}{2}\left(5+\sqrt{5}\right)-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k\left(5+\sqrt{5}-2z_0\right)^k z_0^{-k}}{k!}+\left(e^{-(3\pi)/5}\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\right)^\wedge(1/15)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\text{Re}(a) \text{ and } |\arg(z)|<\pi)$$

Open code

2 * [(([(((((((sqrt(((5+sqrt(5))/(2))) * e^(-2Pi/5) * (-e^(-2Pi)))) - ((((sqrt(5)+1))*(e^(-Pi/10))*(-e^(-Pi/2)) / ((2sqrt(2)))))))))]))]^1/8

Input:

$$2 \sqrt[8]{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-2\times\pi/5}\left(-e^{-2\pi}\right)-\left(\sqrt{5}+1\right)e^{-\pi/10}\left(-\frac{e^{-\pi/2}}{2\sqrt{2}}\right)}$$

Open code

Exact result:

$$2 \sqrt[8]{\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5}}$$

Decimal approximation:

- More digits

1.605809430491671437670995477965890462983757275092727699745...

Open code

1,605809430491671437670...

This result is very near to the electric charge of positron

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Property:

$$2 \sqrt[8]{-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5}+\frac{\left(1+\sqrt{5}\right)e^{-(3\pi)/5}}{2\sqrt{2}}}$$ is a transcendental number

Alternate forms:

$$2^{13/16}\, e^{-(3\pi)/10} \sqrt[8]{-2\sqrt{5+\sqrt{5}}+e^{(9\pi)/5}+\sqrt{5}\, e^{(9\pi)/5}}$$

Open code

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$$2 \sqrt[8]{\frac{1}{2}\sqrt{3+\sqrt{5}}\, e^{-(3\pi)/5}-\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\, e^{-(12\pi)/5}}$$

Now, we know that:

"where ω is a primitive cube root of unity (also equal to 1), and |q|<1 (q = 0.5). For a complex number q with |q|<1, |bc|<1, and any integer n, (n = 2)"

THEOREM 3.2. *If ω is any primitive third root of unity, then*

$$\frac{q^2}{1-\omega}\frac{(q^{10};q^{10})_\infty^3 f(\omega q,\omega^2 q^9) f(-\omega q^{12},-\omega^2 q^8)}{q^3 f(q^5,q^{15}) f(-q^4,-q^6) f(\omega q^7,\omega^2 q^3)}$$
$$+\frac{q^2}{1-\omega^2}\frac{(q^{10};q^{10})_\infty^3 f(\omega^2 q,\omega q^9) f(-\omega^2 q^{12},-\omega q^8)}{q^3 f(q^5,q^{15}) f(-q^4,-q^6) f(\omega^2 q^7,\omega q^3)}$$
$$=\frac{(q^{30};q^{30})_\infty^3}{q f(q^5,q^{15}) f(q^9,q^{21})}\Bigg(\frac{f(q^{15},q^{15}) f(-q^{60},-q^{120}) f(-q^{24},-q^{36})}{f(-q^6,-q^{24}) f(-q^{12},-q^{48})}$$
$$-q^9\frac{f(q^5,q^{25}) f(-q^6,-q^{24}) f(-q^{10},-q^{20})^2 f(-q^{36},-q^{144})}{f(-q^2,-q^8) f(-q^{30},-q^{60})^3}$$
$$-q\frac{f(q^5,q^{25}) f(-q^{84},-q^{96})}{f(-q^4,-q^{26})}+q^{15}\frac{f(q^5,q^{25}) f(-q^{24},-q^{156})}{f(-q^{14},-q^{16})}\Bigg).$$

Now, replacing q, x, and y by q^{10}, $-\omega^2 q^3$, and q^6, respectively, in Lemma 2.3.2, and applying (3.1), we find that

$$\frac{(q^{10};q^{10})_\infty^3 f(\omega q,\omega^2 q^9)}{f(-q^4,-q^6) f(\omega q^7,\omega^2 q^3)}=\sum_{n=-\infty}^{\infty}\frac{(-\omega^2 q^3)^n}{1-q^{10n+6}}. \tag{3.2}$$

Separating the right hand side of (3.2) according to the residue classes $n\equiv 0 \pmod 3$, $n\equiv 1 \pmod 3$, and $n\equiv 2 \pmod 3$, we find that the right hand side of (3.2) equals

$$\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n}}{1-q^{30n+6}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n \omega^2 q^{9n+3}}{1-q^{30n+16}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n \omega q^{9n+6}}{1-q^{30n+26}}. \tag{3.3}$$

By the definition of $f(a, b)$,

$$f(-\omega^2 q^8, -\omega q^{12}) = \sum_{n=-\infty}^{\infty} (-1)^n \omega^{2n} q^{10n^2-2n}. \qquad (3.4)$$

Separating the right hand side of (3.4) according to the residue classes $n \equiv 0 \pmod 3$, $n \equiv 1 \pmod 3$, and $n \equiv 2 \pmod 3$, we find that the right hand side of (3.4) equals

$$\sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2-6n} - \omega^2 q^8 \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+54n}$$
$$+ \omega q^{36} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+114n}. \qquad (3.5)$$

We have that:

(((0.5)^3))^2 / (((1-(0.5)^26)))

Input:

$$\frac{(0.5^3)^2}{1-0.5^{26}}$$

Open code

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Result:

- More digits

0.015625000232830647123316632558653243760067876578388759171...

0.015625000232830647123316632558653243760067876578388759171

(((0.5)^18))/(((1-(0.5)^66))) - ((0.5)^21))/(((1-(0.5)^76))) + (((0.5)^24))/(((1-(0.5)^86)))

Input:

$$\frac{0.5^{18}}{1-0.5^{66}} - \frac{0.5^{21}}{1-0.5^{76}} + \frac{0.5^{24}}{1-0.5^{86}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$3.3974647521972656250516924781676944393406147397968801... \times 10^{-6}$

3.3974647521972656250516924781676944393406147397968801 × 10^-6

[((0.5)^348)) - (((0.5)^8))] * [((0.5)^468)) + ((0.5)^36))] * [(((0.5)^588))]

Input:

$(0.5^{348} - 0.5^{8})(0.5^{468} + 0.5^{36}) \times 0.5^{588}$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-5.6110319334615117817668943542812693315348703286375 3... \times 10^{-191}$

-5.61103193346151178176689435428126933153487032863753 × 10^-191

Now, we have:

By (3.2), (3.3), and (3.5),

$$\frac{q^2}{1-\omega}\,\frac{(q^{10};q^{10})_\infty^3\, f(\omega q,\omega^2 q^9)\, f(-\omega q^{12},-\omega^2 q^8)}{q^3 f(q^5,q^{15})\, f(-q^4,-q^6)\, f(\omega q^7,\omega^2 q^3)}$$
$$=\frac{1}{(1-\omega)\,q f(q^5,q^{15})}$$
$$\times\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n}}{1-q^{30n+6}}\right.$$
$$\left.-\omega^2\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+3}}{1-q^{30n+16}}+\omega\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+6}}{1-q^{30n+26}}\right)$$
$$\times\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-6n}-\omega^2 q^8\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+54n}\right.$$
$$\left.+\omega q^{36}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+114n}\right). \qquad (3.6)$$

And, replacing ω by ω^2 in (3.6), we find that

$$\frac{q^2}{1-\omega^2}\,\frac{(q^{10};q^{10})_\infty^3\, f(\omega^2 q,\omega q^9)\, f(-\omega^2 q^{12},-\omega q^8)}{q^3 f(q^5,q^{15})\, f(-q^4,-q^6)\, f(\omega^2 q^7,\omega q^3)}$$
$$=\frac{1}{(1-\omega^2)\,q f(q^5,q^{15})}$$
$$\times\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n}}{1-q^{30n+6}}\right.$$
$$\left.-\omega\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+3}}{1-q^{30n+16}}+\omega^2\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+6}}{1-q^{30n+26}}\right)$$
$$\times\left(\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-6n}-\omega q^8\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+54n}\right.$$
$$\left.+\omega^2 q^{36}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+114n}\right). \qquad (3.7)$$

We obtain;

(0.01562500023283064712331663255865324376006787657838875917 1)*
(3.3974647521972656250516924781676944393406147397968801 × 10^-6) * (-
5.6110319334615117817668943542812693315348703286375 3 × 10^-191)

Input interpretation:

$$0.015625000232830647123316632558653243760067876578388759171\times 3.3974647521972656250516924781676944393406147397968801\times 10^{-6}\left(-\frac{5.61103193346151178176689435428126933153487032863753}{10^{191}}\right)$$

Open code

Result:

- More digits

−2.97863804710215932623514999227034272102028692628722... × 10^{-198}

-2.97863804710215932623514999227034272102028692628722 × 10^-198

ln -(-2.97863804710215932623514999227034272102028692628722 × 10^-198)

Input interpretation:

$$\log\left(-\left(-\frac{2.97863804710215932623514999227034272102028692628722}{10^{198}}\right)\right)$$

Open code

- log(x) is the natural logarithm

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Result:

- More digits

−454.820382247948083377918216905784715300326645800833623...

(454.820382247948083377918216905784715300326645800833623) * 4 - (64+27)

Input interpretation:

$454.820382247948083377918216905784715300326645800833623 \times 4 - (64 + 27)$

Open code

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Result:

1728.281528991792333511672867623138861201306583203334492

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

(454.820382247948083377918216905784715300326645800833623)^1/4

Input interpretation:

$\sqrt[4]{454.820382247948083377918216905784715300326645800833623}$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

4.618064344187036286996873127587103938982701774581011902...

This value 4,61806 is a good approximation to the first value of upper bound dark photon energy range ($4.95 * 10^{16} – 5.4 * 10^{16}$)

e * (454.820382247948083377918216905784715300326645800833623)^1/4

Input interpretation:

$e \sqrt[4]{454.820382247948083377918216905784715300326645800833623}$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

12.55320038945825860590648259697059615485432321002984392...

This result 12,5532 is very near to the value of black hole entropy 12,5664

(454.820382247948083377918216905784715300326645800833623)^1/12

Input interpretation:

$\sqrt[12]{454.820382247948083377918216905784715300326645800833623}$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

1.665277675156679903896500292688404515849064907076618444...

1,6652776751566799…

(454.820382247948083377918216905784715300326645800833623)^1/13

Input interpretation:

$\sqrt[13]{454.820382247948083377918216905784715300326645800833623}$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

1.6012134317778556182991284408618635171400896322263716 09...

1,601213431777855618299128 44…

This result is very near to the electric charge of positron

Now:

Adding (3.6) and (3.7), applying Lemma 2.3.2, and using (1.10) and Lemma 2.1.1(iv) several times, we find that

$$\frac{q^2}{1-\omega}\frac{(q^{10};q^{10})^3_\infty f(\omega q,\omega^2 q^9)f(-\omega q^{12},-\omega^2 q^8)}{q^3 f(q^5,q^{15})f(-q^4,-q^6)f(\omega q^7,\omega^2 q^3)}$$
$$+\frac{q^2}{1-\omega^2}\frac{(q^{10};q^{10})^3_\infty f(\omega^2 q,\omega q^9)f(-\omega^2 q^{12},-\omega q^8)}{q^3 f(q^5,q^{15})f(-q^4,-q^6)f(\omega^2 q^7,\omega q^3)}$$
$$=\frac{1}{qf(q^5,q^{15})}\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n}}{1-q^{30n+6}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-6n}\right.$$
$$-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+6}}{1-q^{30n+26}}q^8\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+54n}$$
$$\left.-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+3}}{1-q^{30n+16}}q^{36}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+114n}\right)$$
$$-\frac{1}{qf(q^5,q^{15})}\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n}}{1-q^{30n+6}}q^{36}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+114n}\right.$$
$$+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+3}}{1-q^{30n+16}}q^8\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+54n}$$
$$\left.+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{9n+6}}{1-q^{30n+26}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-6n}\right)$$

We obtain:

[(((0.5)^18))/(((1-(0.5)^66))*0.5^348))] - [(((0.5)^24))/(((1-(0.5)^86))*0.5^8*0.5^468))] - [(((0.5)^21))/(((1-(0.5)^76))*0.5^36*0.5^588))]

Input:

$$\frac{0.5^{18}}{1-0.5^{66}} \times 0.5^{348} - \frac{0.5^{24}}{1-0.5^{86}} \times 0.5^{8} \times 0.5^{468} - \frac{0.5^{21}}{1-0.5^{76}} \times 0.5^{36} \times 0.5^{588}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$6.653062250012735499949624738450733901370197811496998... \times 10^{-111}$

6.653062250012735499949624738450733901370197811496998 × 10^-111

[(((0.5)^18))/(((1-(0.5)^66)))*0.5^36*0.5^588)))] + [(((0.5)^21))/(((1-(0.5)^76)))*0.5^8*0.5^468)))] + [(((0.5)^24))/(((1-(0.5)^86)))*0.5^348)))]

Input:

$$\frac{0.5^{18}}{1-0.5^{66}} \times 0.5^{36} \times 0.5^{588} + \frac{0.5^{21}}{1-0.5^{76}} \times 0.5^{8} \times 0.5^{468} + \frac{0.5^{24}}{1-0.5^{86}} \times 0.5^{348}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.039540976564489921853040471503942851937571275327496... \times 10^{-112}$

1.039540976564489921853040471503942851937571275327496 × 10^-112

In conclusion, we obtain:

(6.653062250012735499949624738450733901370197811496998 × 10^-111) - (1.039540976564489921853040471503942851937571275327496 × 10^-112)

Input interpretation:

$$\frac{6.653062250012735499949624738450733901370197811496998}{10^{111}} - \frac{1.039540976564489921853040471503942851937571275327496}{10^{112}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$6.549108152356286507764320691300339616176440683964248... \times 10^{-111}$

6.549108152356286507764320691300339616176440683964248 × 10^-111

ln (6.549108152356286507764320691300339616176440683964248 × 10^-111)

Input interpretation:

$$\log\left(\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

−253.7076164419081224153007383187238695426770914323960 66...

(253.707616441908122415300738318723869542677091432396066)^1/4

(253.707616441908122415300738318723869542677091432396066)*7 – 48

Input interpretation:

253.707616441908122415300738318723869542677091432396066 × 7 – 48

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

1727.953315093356856907105168231067086798739640026772462

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

((((ln (6.549108152356286507764320691300339616176440683964248 × 10^-111))* -7)))^1/3

Input interpretation:

$$\sqrt[3]{\log\left(\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}\right)\times(-7)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

- $\log(x)$ is the natural logarithm

Result:

- More digits

12.1099917812085037064690769172919304470607120080687856 4...

This result 12,1099 is very near to the value of black hole entropy 12,1904

We have obtained three results:

1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111)^1/512

Input interpretation:

$$\frac{1}{\sqrt[512]{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.641355932529959771909920184724045124841777356025231886...

Possible closed forms:

- More

$\cot\left(\sec\left(\frac{52021793}{36535661}\right)\right) \approx 1.64135593252995976991 5$

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$$\frac{10\sqrt{\frac{18592145}{7084749}}}{\pi^2} \approx 1.64135593252995985871$$

$\text{root of } 22771x^3 - 4660x^2 - 55839x + 3515 \text{ near } x = 1.64136 \approx$ 1.64135593252995977191 6590

1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111)^1/529

Input interpretation:

$$\frac{1}{\sqrt[529]{\frac{6.5491081523562865077643206913003396161764406839642 48}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.6154257089203473228243887236513463899477216052564816 26...

Possible closed forms:

- More

root of $36x^4 - 771x^3 + 893x^2 + 331x + 140$ near $x = 1.61543$ ≈

1.615425708920347322872484

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$$\frac{8\left(\frac{4430110}{8127407}\right)^{3/4}}{\pi} \approx 1.61542570892034730 6932$$

$$\frac{7(-134+20\pi+85\pi^2)}{4(-21+262\pi+3\pi^2)} \approx 1.615425708920347322271311$$

1/ (6.5491081523562865077643206913003396161764406839642 48 × 10^-111)^1/496

Input interpretation:

$$\frac{1}{\sqrt[496]{\frac{6.5491081523562865077643206913003396161764406839642 48}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.6678031663926484364759689600283971427198960970175153 30...

Possible closed forms:

- More

$$\frac{3072036997\pi}{5786707362} \approx 1.66780316639264843649 4011$$

Enlarge Data Customize A Plaintext Interactive

π root of $69468x^3 - 107831x^2 + 38895x - 652$ near $x = 0.530878$ ≈

1.667803166392648436 51059

π root of $5374 x^4 + 1141 x^3 - 1365 x^2 + 733 x - 602$ near $x = 0.530878$ ≈
1.667803166392648436410203

1.641355932529959771909920184724045124841777356025231886
1.615425708920347322824388723651346389947721605256481626
1.667803166392648436475968960028397142719896097017515330

The mean of the three results is:

1.641528269280985177070092622801262885836465019433076280666

And also:

((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/503

Input interpretation:

$$\sqrt[503]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

1.655973245091590491200770258222461367389555756842924842...

((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/527

Input interpretation:

$$\sqrt[527]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

1.618368635959684971241450924028997416000002867630367029...

Results 1,6559732 and 1,6183686 that are practically equal to the to the fourteenth root of following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696$

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3 = 1164,269601267364$$

Indeed:

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1,65578 \ldots$$

and to the golden ratio
1.6180339887498948482045868343656381177203091798057628

We have also that:

(-2.9786380471021593262351499922703427210202869262872 2 × 10^-198) / (6.5491081523562865077643206913003396161764406839642 48 × 10^-111)

Input interpretation:

$$-\frac{\frac{2.97863804710215932623514999227034272102028692628722}{10^{198}}}{\frac{6.54910815235628650776432069130033961617644068396424 8}{10^{111}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-4.548158280193438135779621353873329932215921639096307\ldots \times 10^{-88}$

(6.5491081523562865077643206913003396161764406839642 48 × 10^-111) / (-2.9786380471021593262351499922703427210202869262872 2 × 10^-198)

Input interpretation:

$$-\frac{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}{\frac{2.978638047102159326235149992270342721020286926287 22}{10^{198}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-2.198692170311779273746304115472802102098918530968479\ldots \times 10^{87}$

-2.198692170311779273746304115472802102098918530968479 × 10^87

And

-2 * (-2.198692170311779273746304115472802102098918530968479 × 10^87)

4.397384340623558547492608230945604204197837061936958 × 10^87

We have also:

1/ (6.549108152356286507764320 × 10^-111) * (1/3.397464752197265625 05 × 10^-6)^2 * 1 / (((1/ (6.549108152356286507764320 × 10^-111)^1/512)))^5 * 1/(9 * 10^16)

Input interpretation:

$$\frac{1}{\frac{6.549108152356286507764320}{10^{111}}}\left(\frac{1}{3.39746475219726562505\times 10^{-6}}\right)^2\times\frac{1}{\left(\frac{1}{\sqrt[512]{\frac{6.549108152356286507764320}{10^{111}}}}\right)^5}\times\frac{1}{9\times 10^{16}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.2338193089728265976 8\ldots \times 10^{103}$

This result 1,233819 * 10^{103} is practically equal to the entropy of SMBHs contained within the cosmic event horizon

From the final result, we obtain:

6.653062250012735499949624738450733901370197811496998 × 10^-111

-12 + (((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/34

Input interpretation:

$$-12+\sqrt[34]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Result:

- More digits

1728.60619397830819290323585684201603079002678437175951...

1728.60619397830819290323585684201603079002678437175951

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729.

We have that:

((((((((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/34)))))))^1/3

Input interpretation:

$$\sqrt[3]{\sqrt[34]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.0291103298678357837270397805126521342664389814200554...

12.0291103298678357837270397805126521342664389814200554

This result 12,02911 is very near to the value of black hole entropy 12,1904

And:

1.8617* (6.549108152356286507764320691300339616176440683964248 × 10^-111))

Where 1,8617 is the following Hausdorff dimension:

$$\frac{\log(6)}{\log(1+\varphi)}$$

Input interpretation:

$$1.8617\times\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.219247464724169859150483583099384226343567962133624\ldots\times10^{-110}$

Open code

12.19247464724169859150483583099384226343567962133624 × 10^-111

This result 12,1924 is a sub-multiple practically equal to the value of black hole entropy 12,1904

We have that:

(((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/32

Input interpretation:

$$\sqrt[32]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

2774.88064323766275229368991768801400808027443666737067...

This result is very near to the rest mass of charmed Omega baryon 2765.9±2.0

We have that:

(((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/37

Input interpretation:

$$\sqrt[37]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

950.47570991177228779958262536093999255313280002202287 9...

This result is very near to the rest mass of Eta prime meson 957.66±0.24

We have that:

(((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/52

Input interpretation:

$$\sqrt[52]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

131.49812906616095548650162386229878831271358934238125 6...

(((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/51

Input interpretation:

$$\sqrt[51]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

144.699495328133073021566627628234398770891910034039812...

The results 131,498 and 144,699 are very near to the mass of the Pions. The mass of the Pion is both $\pi^{\pm}$: 139.57018(35) MeV/c^2 π^0 : 134.9766(6) MeV/c^2 and the mean is 137,273 MeV/c^2

Furthermore:

1/2 * (((((((((1/ (6.5491081523562865077643206913003396161764406839642 48 × 10^-111))))^1/52 + (((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/51)))))))

Input interpretation:

$$\frac{1}{2}\left(\sqrt[52]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}+\sqrt[51]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

138.098812197147014254034125745266593541802749688210534...

This result is very near to the mass of $\pi^{\pm}$: 139.57018(35) MeV/c^2 and to the inverse of the value of fine-structure constant that is 137.035999084

We have that:

((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/58

Input interpretation:

$$\sqrt[58]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

79.38181001509634009341224386887786978439921979818259 26...

((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/56

Input interpretation:

$$\sqrt[56]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

92.80431631711301189239885704149628386931227702517754 59...

((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/57

Input interpretation:

$$\sqrt[57]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

85.71354074119327701307626247993078731934235361598823 77...

The mass of the bosons W and Z is: W: 80.379±0.012 GeV/c^2

Z: 91.1876±0.0021 GeV/c^2 . The mean of two masses is 85,7833, value practically equal to the third result 85,7135.

We have that:

1 / (((((((1/2 * (((((((((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/52 + (((1/ (6.549108152356286507764320691300339616176440683964248 × 10^-111))))^1/51)))))))

Input interpretation:

$$1\Bigg/\left(\frac{1}{2}\left(\sqrt[52]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}+\sqrt[51]{\frac{1}{\frac{6.549108152356286507764320691300339616176440683964248}{10^{111}}}}\right)\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00724119189796086470082387806654247780051925131313488343...

This result is very near to the value of the fundamental Fine-structure constant:

α = 0.0072973525693(11).

From:

https://readingfeynman.org/2015/09/21/natural-units-plancks-constant-and-the-speed-of-light/

The fine-structure constant α is a *dimensionless* constant which also illustrates the underlying unity of Nature, but in a way that's *much more fascinating* than the two or three things the professor mentions. Indeed, it's quite incredible how this number (α = 0.00729735…, but you'll usually see it written as its *reciprocal*, which is a number that's close to 137.036…) links charge with the relative speeds, radii, and the mass of fundamental particles and, therefore, how this number also these concepts *with each other*.[2]

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = \frac{\mu_0}{4\pi}\frac{e^2 c}{\hbar} = \frac{k_e e^2}{\hbar c} = \frac{c\mu_0}{2R_K} = \frac{m_e c r_e}{\hbar}$$

$$\alpha = 0.0072973525693(11).$$

We have that:

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^{n+1} q^{10n^2+10n}}{1+\omega q^{20n+7}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega q^{90n^2+30n}}{1+\omega q^{60n+7}}$$
$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^2 q^{90n^2+90n+20}}{1+\omega q^{60n+27}} + \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+60}}{1+\omega q^{60n+47}},$$

and

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^{2(n+1)} q^{10n^2+10n}}{1+\omega^2 q^{20n+7}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^2 q^{90n^2+30n}}{1+\omega^2 q^{60n+7}}$$
$$- \sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega q^{90n^2+90n+20}}{1+\omega^2 q^{60n+27}} + \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2-150n+60}}{1+\omega^2 q^{60n+47}}.$$

We obtain:

(((0.5)^420))/(((1+(0.5)^127))-((0.5)^560))/(((1+(0.5)^147))+(((0.5)^720))/(((1+(0.5)^167)))

Input:

$$\frac{0.5^{420}}{1+0.5^{127}} - \frac{0.5^{560}}{1+0.5^{147}} + \frac{0.5^{720}}{1+0.5^{167}}$$

Open code

[2] See Appendix B

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

3.693191447114294312303572304998733375117736209874750... × 10^{-127}

3.693191447114294312303572304998733375117736209874750 × 10^-127

ln (3.693191447114294312303572304998733375117736209874750 × 10^-127)

Input interpretation:

$$\log\left(\frac{3.693191447114294312303572304998733375117736209874750}{10^{127}}\right)$$

Open code

- log(x) is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

-291.121815835177029955237491012434158591715934620526132...

-16 - 6 * ln (3.693191447114294312303572304998733375117736209874750 × 10^-127)

Input interpretation:

$$-16 - 6\log\left(\frac{3.693191447114294312303572304998733375117736209874750}{10^{127}}\right)$$

Open code

- log(x) is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1730.73089501106217973142494607460495155029560772315679...

This result is very near to the mass of candidate glueball f_0(1710) meson.

((((-16 - 6 * ln (3.693191447114294312303572304998733375117736209874750 × 10^-127))))^1/15

Input interpretation:

$$\sqrt[15]{-16-6\log\left(\frac{3.69319144711429431230357230499873337511773620987475 0}{10^{127}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.643924885320418906195555237864671662580504501414268 3503...

1,643924885320418906195...

((((-16 - 6 * ln (3.69319144711429431230357230499873337511773620987475 0 × 10^-127))))^1/3

Input interpretation:

$$\sqrt[3]{-16-6\log\left(\frac{3.69319144711429431230357230499873337511773620987475 0}{10^{127}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.006318189019429827580798043487081160539891529635877 25...

This result 12,0063 is very near to the value of black hole entropy 12,1904

And:

Input interpretation:

$$2\sqrt[3]{-16-6\log\left(\frac{3.69319144711429431230357230499873337511773620987475 0}{10^{127}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits
  24.01263637803885965516159608697416232107978305927175449...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^{n+1} q^{10n^2+10n-4}}{1+\omega q^{20n-3}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega q^{90n^2+30n-4}}{1+\omega q^{60n-3}}$$
$$-\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^2 q^{90n^2+90n+16}}{1+\omega q^{60n+17}} + \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+56}}{1+\omega q^{60n+37}},$$

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^{2(n+1)} q^{10n^2+10n-4}}{1+\omega^2 q^{20n-3}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega^2 q^{90n^2+30n-4}}{1+\omega^2 q^{60n-3}}$$
$$-\sum_{n=-\infty}^{\infty} \frac{(-1)^n \omega q^{90n^2+90n+16}}{1+\omega^2 q^{60n+17}} + \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+56}}{1+\omega^2 q^{60n+37}},$$

We obtain:

(((0.5)^416))/(((1+(0.5)^117)))-((0.5)^556))/(((1+(0.5)^137)))+(((0.5)^716))/(((1+(0.5)^157)))

Input:

$$\frac{0.5^{416}}{1+0.5^{117}} - \frac{0.5^{556}}{1+0.5^{137}} + \frac{0.5^{716}}{1+0.5^{157}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits
  $5.9091063153828708996857156879979733646589685494990 66... \times 10^{-126}$

5.909106315382870899685715687997973364658968549499066 × 10^-126

ln (5.909106315382870899685715687997973364658968549499066 × 10^-126)

Input interpretation:

$$\log\left(\frac{5.9091063153828708996857156879979733646589685494990 66}{10^{126}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

−288.34922711293724871756856252660145232542658768754 1112...

6 *ln (5.909106315382870899685715687997973364658968549499066 × 10^-126)

Input interpretation:

$$6\log\left(\frac{5.909106315382870899685715687997973364658968549499066}{10^{126}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

−1730.0953626776234923054113751596087139525595261252 4667...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson with minus sign.

((((-6 *ln (5.909106315382870899685715687997973364658968549499066 × 10^-126)))))^1/3

Input interpretation:

$$\sqrt[3]{-6\log\left(\frac{5.909106315382870899685715687997973364658968549499066}{10^{126}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.004848417375493798783558131789807193048702441679 00727...

This result 12,0048 is very near to the value of black hole entropy 12,1904

and

Input interpretation:

$$2 \sqrt[3]{-6 \log\left(\frac{5.909106315382870899685715687997973364658968549499066}{10^{126}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

24.00969683475098759756711626357961438609740488335801453...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

((((-6 *ln (5.909106315382870899685715687997973364658968549499066 × 10^-126)))))^1/15

Input interpretation:

$$\sqrt[15]{-6 \log\left(\frac{5.909106315382870899685715687997973364658968549499066}{10^{126}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.643884634637991799010380680241568015927441734200915650 1...

1,643884634637991799…

We have that:

2 * (3.693191447114294312303572304998733375117736209874750 × 10^-127)

Input interpretation:

$$2 \times \frac{3.693191447114294312303572304998733375117736209874750}{10^{127}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

$7.386382894228588624607144609997466750235472419749 5 \times 10^{-127}$

(7.3863828942285886246071446099974667502354724197495 × 10^-127)^1/59

Input interpretation:

$$\sqrt[59]{\frac{7.3863828942285886246071446099974667502354724197495}{10^{127}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.0072807615938503810119593782008573179639245195879322 55...

This result is very near to the value of the fundamental Fine-structure constant:

α = 0.0072973525693(11).

Indeed:

1/ (7.3863828942285886246071446099974667502354724197495 × 10^-127)^1/59

Input interpretation:

$$\frac{1}{\sqrt[59]{\frac{7.3863828942285886246071446099974667502354724197495}{10^{127}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

137.348268736699127931576699799686140581725950816450 9...

1 / (7.3863828942285886246071446099974667502354724197495 × 10^-127)^1/576

Input interpretation:

$$\frac{1}{\sqrt[576]{\frac{7.3863828942285886246071446099974667502354724197495}{10^{127}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.6556876791626702197559343802429740396673373902281171...

1 / (7.3863828942285886246071446099974667502354724197495 × 10^-127)^1/603

Input interpretation:

$$\frac{1}{\sqrt[603]{\frac{7.3863828942285886246071446099974667502354724197495}{10^{127}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.6187262639724336079629212292818340238564050007989530...

The results 1,6556876 and 1,6187262 are practically equal to the to the fourteenth root of following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$ i.e. 1,65578... and to the golden ratio 1.618033988749894848....

Now:

2 * (5.909106315382870899685715687997973364658968549499066 × 10^-126)

Input interpretation:

$$2 \times \frac{5.9091063153828708996857156879979733646589685494990 66}{10^{126}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.181821263076574179937143137599594672931793709899813... \times 10^{-125}$

(((1.181821263076574179937143137599594672931793709899813 × 10^-125)^2

Input interpretation:

$$\left(\frac{1.181821263076574179937143137599594672931793709899813}{10^{125}}\right)^2$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.396701497859909157098871132012517503917231956153569... \times 10^{-250}$

And

1/2 * (((((1.396701497859909157098871132012517503917231956153569 × 10^-250)^1/116 + (1.396701497859909157098871132012517503917231956153569 × 10^-250)^1/117))))

Input interpretation:

$$\frac{1}{2}\left(\sqrt[116]{\frac{1.396701497859909157098871132012517503917231956153569}{10^{250}}} + \sqrt[117]{\frac{1.396701497859909157098871132012517503917231956153569}{10^{250}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.007167716370461815970150274942573165419726390501142406 36...

This result is very near to the value of the fundamental Fine-structure constant:

α = 0.0072973525693(11).

1/ (1.396701497859909157098871132012517503917231956153569 × 10^-250)^1/1141

Input interpretation:

$$\frac{1}{\sqrt[1141]{\frac{1.396701497859909157098871132012517503917231956153569}{10^{250}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.655689425370692983842048450219875057365225161915625355...

1/ (1.396701497859909157098871132012517503917231956153569 × 10^-250)^1/1195

Input interpretation:

$$\frac{1}{\sqrt[1195]{\frac{1.396701497859909157098871132012517503917231956153569}{10^{250}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.618391524283211981574304379309044816353694209809456596...

The results 1,6556894 and 1,6183915 are practically equal to the to the fourteenth root of following Ramanujan's class invariant invariant $Q = \left(G_{505}/G_{101/5}\right)^3 =$ 1164,2696 i.e. 1,65578... and to the golden ratio 1.618033988749894848....

Now, we have:

$$\frac{1}{1-\omega}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega q^{90n^2+30n-4}}{1+\omega q^{60n-3}}+\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega^2 q^{90n^2+30n-4}}{1+\omega^2 q^{60n-3}}$$
$$=\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+30n-4}\left(\frac{-1}{1+\omega q^{60n-3}}+\frac{\omega^2}{1+\omega^2 q^{60n-3}}\right)$$
$$=-\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+30n-4}\frac{1-q^{120n-6}}{1+q^{180n-9}},$$

$$\frac{1}{1-\omega}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega^2 q^{90n^2+90n+16}}{1+\omega q^{60n+17}}+\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega q^{90n^2+90n+16}}{1+\omega^2 q^{60n+17}}$$
$$=\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+90n+16}\left(\frac{-\omega}{1+\omega q^{60n+17}}+\frac{\omega}{1+\omega^2 q^{60n+17}}\right)$$
$$=-\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+150n+33}\frac{1+q^{60n+17}}{1+q^{180n+51}},$$

$$\frac{1}{1-\omega}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+56}}{1+\omega q^{60n+37}}+\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+56}}{1+\omega^2 q^{60n+37}}$$
$$=\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+150n+56}\left(\frac{1+\omega}{1+\omega q^{60n+37}}+\frac{1}{1+\omega^2 q^{60n+37}}\right)$$
$$=\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+150n+56}\frac{1+q^{60n+37}}{1+q^{180n+111}},$$

$$\frac{1}{1-\omega}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega q^{90n^2+30n}}{1+\omega q^{60n+7}}+\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega^2 q^{90n^2+30n}}{1+\omega^2 q^{60n+7}}$$
$$=\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+30n}\left(\frac{-1}{1+\omega q^{60n+7}}+\frac{\omega^2}{1+\omega^2 q^{60n+7}}\right)$$
$$=-\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+30n}\frac{1-q^{120n+14}}{1+q^{180n+21}},$$

$$\frac{1}{1-\omega}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega^2q^{90n^2+90n+20}}{1+\omega q^{60n+27}}+\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\omega q^{90n^2+90n+20}}{1+\omega^2q^{60n+27}}$$
$$=\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+90n+20}\left(\frac{1+\omega^2}{1+\omega q^{60n+27}}+\frac{\omega}{1+\omega^2q^{60n+27}}\right)$$
$$=-\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+150n+47}\frac{1+q^{60n+27}}{1+q^{180n+81}},$$

and

$$\frac{1}{1-\omega}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+\omega q^{60n+47}}+\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+\omega^2q^{60n+47}}$$
$$=\frac{1}{1-\omega^2}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+150n+60}\left(\frac{1+\omega}{1+\omega q^{60n+47}}+\frac{1}{1+\omega^2q^{60n+47}}\right)$$
$$=\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+150n+60}\frac{1+q^{60n+47}}{1+q^{180n+141}}.$$

We have:

-(((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351)))

Input:

$$-0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-5.909106315382870899685715687997973400223112780836 16\ldots\times10^{-126}$

ln -(-(((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351)))

Input:

$$\log\left(-\left(-0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}\right)\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

-288.349...

((((-ln ((((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351))))))^1/11

Input:

$$\sqrt[11]{-\log\left(0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.673513...

1,673513…

And

Input:

$$\sqrt[12]{-\log\left(0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.603220...

1,603220...

(((((-ln ((((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351))))))^1/4

Input:

$$\sqrt[4]{-\log\left(0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

4.12078...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

((((6 * -ln ((((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351))))))^1/3

Input:

$$\sqrt[3]{6\times(-1)\log\left(0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.0048...

This result 12,0048 is very near to the value of black hole entropy 12,1904

And

Input:

$$2\sqrt[3]{6\times(-1)\log\left(0.5^{416}\times\frac{1-0.5^{234}}{1+0.5^{351}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

24.0097...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

((((6 * -ln ((((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351)))

Input:

$$6 \times (-1) \log\left(0.5^{416} \times \frac{1-0.5^{234}}{1+0.5^{351}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1730.10...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

1/2 * [(((((((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351)))))))^1/58] + [((((((0.5)^416))*(((1-(0.5)^234)))/(((1+(0.5)^351))))))^1/59]

Input:

$$\frac{1}{2}\left(\sqrt[58]{0.5^{416} \times \frac{1-0.5^{234}}{1+0.5^{351}}} + \sqrt[59]{0.5^{416} \times \frac{1-0.5^{234}}{1+0.5^{351}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00723721...

This result is very near to the value of the fundamental Fine-structure constant:

$\alpha = 0.0072973525693(11)$.

-(((0.5)^693))*(((1+(0.5)^137)))/(((1+(0.5)^411)))

Input:

$$-0.5^{693} \times \frac{1+0.5^{137}}{1+0.5^{411}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-2.4333972048578045740992926794689319798590491111 7308\ldots \times 10^{-209}$

1/2 * [((((((((0.5)^693))*(((1+(0.5)^137)))/(((1+(0.5)^411)))))))^1/98)))))+((((((0.5)^693))*(((1+(0.5)^137)))/(((1+(0.5)^411)))))))^1/97)))))]

Input:

$$\frac{1}{2}\left(\sqrt[98]{0.5^{693} \times \frac{1+0.5^{137}}{1+0.5^{411}}} + \sqrt[97]{0.5^{693} \times \frac{1+0.5^{137}}{1+0.5^{411}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00725193...

This result is very near to the value of the fundamental Fine-structure constant:

α = 0.0072973525693(11).

ln [(((0.5)^693))*(((1+(0.5)^137)))/(((1+(0.5)^411))]

Input:

$$\log\left(0.5^{693} \times \frac{1+0.5^{137}}{1+0.5^{411}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

-480.351...

(-(-480.351))^1/12

Input interpretation:
$\sqrt[12]{-(-480.351)}$
Open code

Enlarge Data Customize A Plaintext Interactive
Result:
- More digits

1.672874...

1,672874…

And

Input interpretation:
$\sqrt[13]{-(-480.351)}$
Open code

Enlarge Data Customize A Plaintext Interactive
Result:
- More digits

1.607954...

1,607954…

[(((-18/5 * ln (((0.5)^693))*(((1+(0.5)^137)))/(((1+(0.5)^411))))]

Input:
$-\frac{18}{5} \log(0.5^{693}) \times \frac{1+0.5^{137}}{1+0.5^{411}}$
Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive
Result:
- More digits

1729.26...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728, very near to the value 1729,26 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Series representations:

$$\frac{\left(\log\left(0.5^{693}\right)\left(1+0.5^{137}\right)\right)(-18)}{\left(1+0.5^{411}\right)5} = -7.2\, i\, \pi \left\lfloor \frac{\arg\left(2.4334\times 10^{-209}-x\right)}{2\pi} \right\rfloor - 3.6\log(x) + 3.6\sum_{k=1}^{\infty} \frac{(-1)^k \left(2.4334\times 10^{-209}-x\right)^k x^{-k}}{k} \quad \text{for } x<0$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{\left(\log\left(0.5^{693}\right)\left(1+0.5^{137}\right)\right)(-18)}{\left(1+0.5^{411}\right)5} = -3.6\left\lfloor \frac{\arg\left(2.4334\times 10^{-209}-z_0\right)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 3.6\log(z_0) - 3.6\left\lfloor \frac{\arg\left(2.4334\times 10^{-209}-z_0\right)}{2\pi} \right\rfloor \log(z_0) + 3.6\sum_{k=1}^{\infty} \frac{(-1)^k \left(2.4334\times 10^{-209}-z_0\right)^k z_0^{-k}}{k}$$

Open code

$$\frac{\left(\log\left(0.5^{693}\right)\left(1+0.5^{137}\right)\right)(-18)}{\left(1+0.5^{411}\right)5} = -7.2\, i\, \pi \left\lfloor -\frac{-\pi+\arg\left(\frac{2.4334\times 10^{-209}}{z_0}\right)+\arg(z_0)}{2\pi} \right\rfloor - 3.6\log(z_0) + 3.6\sum_{k=1}^{\infty} \frac{(-1)^k \left(2.4334\times 10^{-209}-z_0\right)^k z_0^{-k}}{k}$$

Integral representation:

$$\frac{\left(\log\left(0.5^{693}\right)\left(1+0.5^{137}\right)\right)(-18)}{\left(1+0.5^{411}\right)5} = -3.6\int_1^{2.4334\times 10^{-209}} \frac{1}{t}\, dt$$

[((((((((-18/5 * ln (((0.5)^693))*(((1+(0.5)^137)))/(((1+(0.5)^411)))))))))))]^1/3

Input:

$$\sqrt[3]{-\frac{18}{5}\log\left(0.5^{693}\right)\times\frac{1+0.5^{137}}{1+0.5^{411}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.0029...

This result 12,0029 is very near to the value of black hole entropy 12,1904

And

Input:

$$\frac{1}{3}\sqrt[3]{-\frac{18}{5}\log\left(0.5^{693}\right)\times\frac{1+0.5^{137}}{1+0.5^{411}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

4.00097...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

(((0.5)^716))*(((1+(0.5)^157)))/(((1+(0.5)^471)))

Input:

$$0.5^{716}\times\frac{1+0.5^{157}}{1+0.5^{471}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$2.900835519859557836174121713005223250221055917944221\ldots\times10^{-216}$

Open code

((((((0.5)^716))*(((1+(0.5)^157)))/(((1+(0.5)^471)))))))^1/101

Input:

$$\sqrt[101]{0.5^{716} \times \frac{1+0.5^{157}}{1+0.5^{471}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00734456...

This result is very near to the value of the fundamental Fine-structure constant:

α = 0.0072973525693(11).

-35/10 * ln((((((0.5)^716))*(((1+(0.5)^157)))/(((1+(0.5)^471)))))

Input:

$$-\frac{35}{10} \log\left(0.5^{716} \times \frac{1+0.5^{157}}{1+0.5^{471}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1737.03...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

$$\frac{1}{10} \log\left(\frac{0.5^{716}\,(1+0.5^{157})}{1+0.5^{471}}\right)(-35) = -7\,i\,\pi \left\lfloor \frac{\arg(2.90084\times 10^{-216} - x)}{2\,\pi} \right\rfloor - \frac{7\log(x)}{2} + \frac{7}{2}\sum_{k=1}^{\infty} \frac{(-1)^k\,(2.90084\times 10^{-216} - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{10}\log\left(\frac{0.5^{716}(1+0.5^{157})}{1+0.5^{471}}\right)(-35) = -\frac{7}{2}\left\lfloor\frac{\arg(2.90084\times10^{-216}-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-\frac{7\log(z_0)}{2} - \frac{7}{2}\left\lfloor\frac{\arg(2.90084\times10^{-216}-z_0)}{2\pi}\right\rfloor\log(z_0)+\frac{7}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(2.90084\times10^{-216}-z_0)^k z_0^{-k}}{k}$$

Open code

$$\frac{1}{10}\log\left(\frac{0.5^{716}(1+0.5^{157})}{1+0.5^{471}}\right)(-35) = -7 i\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{2.90084\times10^{-216}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor - \frac{7\log(z_0)}{2}+\frac{7}{2}\sum_{k=1}^{\infty}\frac{(-1)^k(2.90084\times10^{-216}-z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\frac{1}{10}\log\left(\frac{0.5^{716}(1+0.5^{157})}{1+0.5^{471}}\right)(-35) = -\frac{7}{2}\int_1^{2.90084\times10^{-216}}\frac{1}{t}\,dt$$

Open code

[((((((((((-35/10 * ln((((((0.5)^716))*(((1+(0.5)^157)))/(((1+(0.5)^471)))))))))))))]^1/3

Input:

$$\sqrt[3]{-\frac{35}{10}\log\left(0.5^{716}\times\frac{1+0.5^{157}}{1+0.5^{471}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.0209...

This result 12,0209 is very near to the value of black hole entropy 12,1904

And

Input:

$$\frac{1}{3}\sqrt[3]{-\frac{35}{10}\log\left(0.5^{716}\times\frac{1+0.5^{157}}{1+0.5^{471}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

4.00695...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

[(((((((((-ln((((((0.5)^716))*(((1+(0.5)^157)))/(((1+(0.5)^471)))))))))))))]^1/12

Input:

$$\sqrt[12]{-\log\left(0.5^{716}\times\frac{1+0.5^{157}}{1+0.5^{471}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.677432...

1,677432…

And

Input:

$$\sqrt[13]{-\log\left(0.5^{716}\times\frac{1+0.5^{157}}{1+0.5^{471}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.611998...

1,611998…

-(((0.5)^420))*(((1-(0.5)^254)))/(((1+(0.5)^381)))

Input:

$$-0.5^{420}\times\frac{1-0.5^{254}}{1+0.5^{381}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-3.693191447114294312303572304998733375139445488022 60\ldots\times10^{-127}$

-3.69319144711429431230357230499873337513944548802260 × 10^-127

1/2 * [[[(((((((0.5)^420))*(((1-(0.5)^254)))/(((1+(0.5)^381))))))^1/58 + (((((((0.5)^420))*(((1-(0.5)^254)))/(((1+(0.5)^381))))))^1/60]]]]

Input:

$$\frac{1}{2}\left(\sqrt[58]{0.5^{420}\times\frac{1-0.5^{254}}{1+0.5^{381}}}+\sqrt[60]{0.5^{420}\times\frac{1-0.5^{254}}{1+0.5^{381}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00721069...

This result is an approximation to the value of the fundamental Fine-structure constant: α = 0.0072973525693(11).

-6 * ln (((0.5)^420))*(((1-(0.5)^254)))/(((1+(0.5)^381)))

Input:

$$-6\log(0.5^{420})\times\frac{1-0.5^{254}}{1+0.5^{381}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1746.73...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

$$-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}} = -12 i \pi \left\lfloor \frac{\arg(3.69319\times10^{-127}-x)}{2\pi} \right\rfloor - 6\log(x) + 6\sum_{k=1}^{\infty} \frac{(-1)^k (3.69319\times10^{-127}-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}} = -6\left\lfloor \frac{\arg(3.69319\times10^{-127}-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 6\log(z_0) - 6\left\lfloor \frac{\arg(3.69319\times10^{-127}-z_0)}{2\pi} \right\rfloor \log(z_0) + 6\sum_{k=1}^{\infty} \frac{(-1)^k (3.69319\times10^{-127}-z_0)^k z_0^{-k}}{k}$$

Open code

$$-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}} = -12 i \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{3.69319\times10^{-127}}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor - 6\log(z_0) + 6\sum_{k=1}^{\infty} \frac{(-1)^k (3.69319\times10^{-127}-z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}} = -6\int_1^{3.69319\times10^{-127}} \frac{1}{t}\, dt$$

((((((-6 * ln (((0.5)^420))*(((1-(0.5)^254)))/(((1+(0.5)^381)))))))))^1/3

Input:

$$\sqrt[3]{-6\log(0.5^{420})\times\frac{1-0.5^{254}}{1+0.5^{381}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.0432...

This result 12,0432 is very near to the value of black hole entropy 12,1904

And

Input:

$$2\sqrt[3]{-6\log(0.5^{420})\times\frac{1-0.5^{254}}{1+0.5^{381}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

24.0864...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

$$2\sqrt[3]{-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}}} =$$
$$3.63424\left(-2\,i\,\pi\left\lfloor\frac{\arg(3.69319\times10^{-127}-x)}{2\,\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\,(3.69319\times10^{-127}-x)^k\,x^{-k}}{k}\right)\text{^}(1/3)\quad\text{for } x<0$$

Open code

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$$2\sqrt[3]{-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}}} = 3.63424\left(-\log(z_0) - \left\lfloor\frac{\arg(3.69319\times10^{-127}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right) + \sum_{k=1}^{\infty}\frac{(-1)^k (3.69319\times10^{-127}-z_0)^k z_0^{-k}}{k}\right)^{\wedge}(1/3)$$

Open code

$$2\sqrt[3]{-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}}} = 3.63424\left(-2 i\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{3.69319\times10^{-127}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor - \log(z_0) + \sum_{k=1}^{\infty}\frac{(-1)^k (3.69319\times10^{-127}-z_0)^k z_0^{-k}}{k}\right)^{\wedge}(1/3)$$

Integral representation:

$$2\sqrt[3]{-\frac{(6\log(0.5^{420}))(1-0.5^{254})}{1+0.5^{381}}} = 3.63424\sqrt[3]{-\int_1^{3.69319\times10^{-127}}\frac{1}{t}\,dt}$$

and

Input:

$$\frac{1}{3}\sqrt[3]{-6\log(0.5^{420})\times\frac{1-0.5^{254}}{1+0.5^{381}}}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

4.01440...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

((((((-ln (((0.5)^420))*(((1-(0.5)^254)))/(((1+(0.5)^381))))^1/11

Input:

$$\sqrt[11]{-\log(0.5^{420}) \times \frac{1-0.5^{254}}{1+0.5^{381}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.674969...

1,674969…

And

Input:

$$\sqrt[12]{-\log(0.5^{420}) \times \frac{1-0.5^{254}}{1+0.5^{381}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.604499...

1,604499...

-(((0.5)^707))*(((1+(0.5)^147)))/(((1+(0.5)^441)))

Input:

Input:

$$-0.5^{707} \times \frac{1+0.5^{147}}{1+0.5^{441}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-1.4852277861680936121211503170586743041131806383043\ldots \times 10^{-213}$

1/2 * [[[[(((((((((0.5)^707))*(((1+(0.5)^147)))/(((1+(0.5)^441)))))))^1/99 + ((((((((0.5)^707))*(((1+(0.5)^147)))/(((1+(0.5)^441)))))))^1/100]]]]

Input:

$$\frac{1}{2}\left(\sqrt[99]{0.5^{707}\times\frac{1+0.5^{147}}{1+0.5^{441}}}+\sqrt[100]{0.5^{707}\times\frac{1+0.5^{147}}{1+0.5^{441}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$0.00726277\ldots$

This result is a good approximation to the value of the fundamental Fine-structure constant: α = 0.0072973525693(11).

-18/5 * [ln (((0.5)^707))*(((1+(0.5)^147)))/(((1+(0.5)^441)))]

Input:

$$-\frac{18}{5}\left(\log(0.5^{707})\times\frac{1+0.5^{147}}{1+0.5^{441}}\right)$$

Open code

- log(x) is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1764.20\ldots$

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

$$\frac{(\log(0.5^{707})(1+0.5^{147}))(-18)}{(1+0.5^{441})5} = -7.2\, i\, \pi \left\lfloor \frac{\arg(1.48523\times 10^{-213} - x)}{2\pi} \right\rfloor - 3.6\log(x) + 3.6 \sum_{k=1}^{\infty} \frac{(-1)^k (1.48523\times 10^{-213} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Open code

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$$\frac{(\log(0.5^{707})(1+0.5^{147}))(-18)}{(1+0.5^{441})5} = -3.6 \left\lfloor \frac{\arg(1.48523\times 10^{-213} - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 3.6\log(z_0) - 3.6 \left\lfloor \frac{\arg(1.48523\times 10^{-213} - z_0)}{2\pi} \right\rfloor \log(z_0) + 3.6 \sum_{k=1}^{\infty} \frac{(-1)^k (1.48523\times 10^{-213} - z_0)^k z_0^{-k}}{k}$$

Open code

$$\frac{(\log(0.5^{707})(1+0.5^{147}))(-18)}{(1+0.5^{441})5} = -7.2\, i\, \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1.48523\times 10^{-213}}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor - 3.6\log(z_0) + 3.6 \sum_{k=1}^{\infty} \frac{(-1)^k (1.48523\times 10^{-213} - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\frac{(\log(0.5^{707})(1+0.5^{147}))(-18)}{(1+0.5^{441})5} = -3.6 \int_1^{1.48523\times 10^{-213}} \frac{1}{t}\, dt$$

((((((-18/5 * [ln (((0.5)^707))*(((1+(0.5)^147)))/(((1+(0.5)^441)))])))))^1/3

Input:

$$\sqrt[3]{-\frac{18}{5}\left(\log(0.5^{707}) \times \frac{1+0.5^{147}}{1+0.5^{441}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

12.0832...

This result 12.0832 is very near to the value of black hole entropy 12.1904

And

1/3 * (((((((-18/5 * [ln (((0.5)^707))*(((1+(0.5)^147)))/(((1+(0.5)^441)))]))))))^1/3

Input:

$$\frac{1}{3}\sqrt[3]{-\frac{18}{5}\left(\log(0.5^{707})\times\frac{1+0.5^{147}}{1+0.5^{441}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

4.02774...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

Series representations:

$$\frac{1}{3}\sqrt[3]{-\frac{18\left(\log(0.5^{707})\right)\left(1+0.5^{147}\right)}{5\left(1+0.5^{441}\right)}} = 0.510873\left(-2\,i\,\pi\left\lfloor\frac{\arg(1.48523\times10^{-213}-x)}{2\,\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\,(1.48523\times10^{-213}-x)^k\,x^{-k}}{k}\right)\hat{}\,(1/3)\quad\text{for } x<0$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{3}\sqrt[3]{-\frac{18\left(\log\left(0.5^{707}\right)\left(1+0.5^{147}\right)\right)}{5\left(1+0.5^{441}\right)}} = 0.510873\left(-\log(z_0) - \left\lfloor\frac{\arg\left(1.48523\times10^{-213}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right) + \sum_{k=1}^{\infty}\frac{(-1)^k\left(1.48523\times10^{-213}-z_0\right)^k z_0^{-k}}{k}\right)^\wedge(1/3)$$

Open code

$$\frac{1}{3}\sqrt[3]{-\frac{18\left(\log\left(0.5^{707}\right)\left(1+0.5^{147}\right)\right)}{5\left(1+0.5^{441}\right)}} = 0.510873\left(-2 i \pi\left\lfloor-\frac{-\pi+\arg\left(\frac{1.48523\times10^{-213}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor - \log(z_0) + \sum_{k=1}^{\infty}\frac{(-1)^k\left(1.48523\times10^{-213}-z_0\right)^k z_0^{-k}}{k}\right)^\wedge(1/3)$$

Integral representation:

$$\frac{1}{3}\sqrt[3]{-\frac{18\left(\log\left(0.5^{707}\right)\left(1+0.5^{147}\right)\right)}{5\left(1+0.5^{441}\right)}} = 0.510873\sqrt[3]{-\int_1^{1.48523\times10^{-213}}\frac{1}{t}\,dt}$$

[- ln (((0.5)^707))*(((1+(0.5)^147))/(((1+(0.5)^441)))])))))^1/12

Input:

$$\sqrt[12]{-\log\left(0.5^{707}\right)\times\frac{1+0.5^{147}}{1+0.5^{441}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.675665...

1.675665…

And

Input:

$$\sqrt[13]{-\log(0.5^{707}) \times \frac{1+0.5^{147}}{1+0.5^{441}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.610430...

1.610430...

(((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))

Input:

$$0.5^{720} \times \frac{1+0.5^{167}}{1+0.5^{501}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.8130221999122236476088260706282645313881599487052 24\ldots \times 10^{-217}$

1/2 * [((((((((((((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501))))))))^1/102))))) + (((((((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501))))))))^1/101))))]

Input:

$$\frac{1}{2}\left(\sqrt[102]{0.5^{720} \times \frac{1+0.5^{167}}{1+0.5^{501}}} + \sqrt[101]{0.5^{720} \times \frac{1+0.5^{167}}{1+0.5^{501}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00732302...

And

1 /   (((((1/2 * [(((((((((((((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501))))))))^1/102))))) + ((((((((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501))))))))^1/101))))]))))))

Input:

$$\frac{1}{\frac{1}{2}\left(\sqrt[102]{0.5^{720}\times\frac{1+0.5^{167}}{1+0.5^{501}}}+\sqrt[101]{0.5^{720}\times\frac{1+0.5^{167}}{1+0.5^{501}}}\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

136.556...

This result is practically equal to the value of the fundamental Fine-structure constant: α = 0.0072973525693(11).

ln (((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))

Input:

$$\log(0.5^{720})\times\frac{1+0.5^{167}}{1+0.5^{501}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

−499.066...

This result is very near to the rest mass of Kaon meson 497.614 with minus sign

Series representations:

$$\frac{\log(0.5^{720})(1+0.5^{167})}{1+0.5^{501}} = 2\,i\,\pi\left\lfloor\frac{\arg(1.81302\times10^{-217}-x)}{2\,\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k\,(1.81302\times10^{-217}-x)^k\,x^{-k}}{k} \quad \text{for } x<0$$

Open code

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$$\frac{\log(0.5^{720})(1+0.5^{167})}{1+0.5^{501}} = \left\lfloor \frac{\arg(1.81302\times10^{-217}-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(1.81302\times10^{-217}-z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.81302\times10^{-217}-z_0)^k z_0^{-k}}{k}$$

Open code

$$\frac{\log(0.5^{720})(1+0.5^{167})}{1+0.5^{501}} = 2 i \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1.81302\times10^{-217}}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.81302\times10^{-217}-z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function
- i is the imaginary unit
- More information

Integral representation:

$$\frac{\log(0.5^{720})(1+0.5^{167})}{1+0.5^{501}} = \int_1^{1.81302\times10^{-217}} \frac{1}{t}\,dt$$

-18-7/2 [ln (((0.5)^720))*(((1+(0.5)^167))/(((1+(0.5)^501)))]

Input:

$$-18 - \frac{7}{2}\left(\log(0.5^{720}) \times \frac{1+0.5^{167}}{1+0.5^{501}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1728.73...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Series representations:

$$-18 - \frac{(\log(0.5^{720})(1+0.5^{167}))\,7}{(1+0.5^{501})\,2} = -18 - 7\,i\,\pi \left\lfloor \frac{\arg(1.81302\times10^{-217} - x)}{2\,\pi} \right\rfloor - 3.5\log(x) + 3.5\sum_{k=1}^{\infty} \frac{(-1)^k\,(1.81302\times10^{-217} - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

Open code

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$$-18 - \frac{(\log(0.5^{720})(1+0.5^{167}))\,7}{(1+0.5^{501})\,2} = -18 - 3.5\left\lfloor \frac{\arg(1.81302\times10^{-217} - z_0)}{2\,\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 3.5\log(z_0) - 3.5\left\lfloor \frac{\arg(1.81302\times10^{-217} - z_0)}{2\,\pi} \right\rfloor \log(z_0) + 3.5\sum_{k=1}^{\infty} \frac{(-1)^k\,(1.81302\times10^{-217} - z_0)^k\,z_0^{-k}}{k}$$

Open code

$$-18 - \frac{(\log(0.5^{720})(1+0.5^{167}))\,7}{(1+0.5^{501})\,2} = -18 - 7\,i\,\pi \left\lfloor -\frac{-\pi + \arg\left(\frac{1.81302\times10^{-217}}{z_0}\right) + \arg(z_0)}{2\,\pi} \right\rfloor - 3.5\log(z_0) + 3.5\sum_{k=1}^{\infty} \frac{(-1)^k\,(1.81302\times10^{-217} - z_0)^k\,z_0^{-k}}{k}$$

Open code

- $\arg(z)$ is the complex argument
- $\lfloor x \rfloor$ is the floor function

- i is the imaginary unit
- More information

Integral representation:

$$-18-\frac{(\log(0.5^{720})(1+0.5^{167}))\,7}{(1+0.5^{501})\,2}=-18-3.5\int_1^{1.81302\times10^{-217}}\frac{1}{t}\,dt$$

(((((-18-7/2 [ln (((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))])))))^1/3

Input:

$$\sqrt[3]{-18-\frac{7}{2}\left(\log(0.5^{720})\times\frac{1+0.5^{167}}{1+0.5^{501}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

12.0017...

This result 12,0017 is very near to the value of black hole entropy 12,1904

And

2* (((((-18-7/2 [ln (((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))])))))^1/3

Input:

$$2\sqrt[3]{-18-\frac{7}{2}\left(\log(0.5^{720})\times\frac{1+0.5^{167}}{1+0.5^{501}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

24.0034...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

$$2\sqrt[3]{-18-\frac{\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)7}{\left(1+0.5^{501}\right)2}} = 2\left(-18-3.5\left(2\,i\,\pi\left\lfloor\frac{\arg\left(1.81302\times10^{-217}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1.81302\times10^{-217}-x\right)^k x^{-k}}{k}\right)\right)\hat{}\,(1/3) \quad \text{for } x<0$$

Open code

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$$2\sqrt[3]{-18-\frac{\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)7}{\left(1+0.5^{501}\right)2}} = 2\left(-18-3.5\left(\log(z_0)+\left\lfloor\frac{\arg\left(1.81302\times10^{-217}-z_0\right)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1.81302\times10^{-217}-z_0\right)^k z_0^{-k}}{k}\right)\right)\hat{}\,(1/3)$$

Open code

$$2\sqrt[3]{-18-\frac{\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)7}{\left(1+0.5^{501}\right)2}} = 2\left(-18-3.5\left(2\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{1.81302\times10^{-217}}{z_0}\right)+\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1.81302\times10^{-217}-z_0\right)^k z_0^{-k}}{k}\right)\right)\hat{}\,(1/3)$$

Integral representation:

$$2\sqrt[3]{-18-\frac{\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)7}{\left(1+0.5^{501}\right)2}} = 2\sqrt[3]{-18-3.5\int_1^{1.81302\times10^{-217}}\frac{1}{t}\,dt}$$

Open code

•

Furthermore

1/3 * ((((((-18-7/2 [ln (((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))])))))^1/3

Input:

$$\frac{1}{3}\sqrt[3]{-18-\frac{7}{2}\left(\log\left(0.5^{720}\right)\times\frac{1+0.5^{167}}{1+0.5^{501}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

4.00056...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

Series representations:

$$\frac{1}{3}\sqrt[3]{-18-\frac{7\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)}{2\left(1+0.5^{501}\right)}}=\frac{1}{3}\left(-18-3.5\left(2\,i\,\pi\left\lfloor\frac{\arg\left(1.81302\times10^{-217}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1.81302\times10^{-217}-x\right)^k x^{-k}}{k}\right)\right)\hat{}\,(1/3)\quad\text{for } x<0$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{3}\sqrt[3]{-18-\frac{7\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)}{2\left(1+0.5^{501}\right)}}=\frac{1}{3}\left(-18-3.5\left(\log(z_0)+\left\lfloor\frac{\arg\left(1.81302\times10^{-217}-z_0\right)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1.81302\times10^{-217}-z_0\right)^k z_0^{-k}}{k}\right)\right)\hat{}\,(1/3)$$

Open code

$$\frac{1}{3}\sqrt[3]{-18-\frac{7\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)}{2\left(1+0.5^{501}\right)}} = \frac{1}{3}\left(-18-3.5\left(2\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{1.81302\times10^{-217}}{z_0}\right)+\arg(z_0)}{2\,\pi}\right\rfloor + \log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1.81302\times10^{-217}-z_0\right)^k z_0^{-k}}{k}\right)\right)\hat{}\,(1/3)$$

Integral representation:

$$\frac{1}{3}\sqrt[3]{-18-\frac{7\left(\log\left(0.5^{720}\right)\left(1+0.5^{167}\right)\right)}{2\left(1+0.5^{501}\right)}} = \frac{1}{3}\sqrt[3]{-18-3.5\int_1^{1.81302\times10^{-217}}\frac{1}{t}\,dt}$$

Open code

(((((((-ln (((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))))))))^1/12

Input:

$$\sqrt[12]{-\log\left(0.5^{720}\right)\times\frac{1+0.5^{167}}{1+0.5^{501}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.678211...

1.678211...

And

(((((((-ln (((0.5)^720))*(((1+(0.5)^167)))/(((1+(0.5)^501)))))))))^1/13

Input:

$$\sqrt[13]{-\log\left(0.5^{720}\right)\times\frac{1+0.5^{167}}{1+0.5^{501}}}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

1.612689...

1.612689...

Now, we have:

Adding (3.26) and (3.27), dividing by 2, and using the six identities above, we find that

$$\begin{aligned}(q^{60};q^{60})_\infty \chi(q) = &\sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+30n-4} \frac{1-q^{120n-6}}{1+q^{180n-9}} \\ &- \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+33} \frac{1+q^{60n+17}}{1+q^{180n+51}} \\ &- \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+56} \frac{1+q^{60n+37}}{1+q^{180n+111}} \\ &+ \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+30n} \frac{1-q^{120n+14}}{1+q^{180n+21}} \\ &- \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+47} \frac{1+q^{60n+27}}{1+q^{180n+81}} \\ &- \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+60} \frac{1+q^{60n+47}}{1+q^{180n+141}} \\ &+ \frac{(q^5;q^5)_\infty (q^{10};q^{10})_\infty f(-q,-q^4)}{f(-q^2,-q^3)\, f(-q^4,-q^6)} q(q^{60};q^{60})_\infty \\ &+ \frac{q^2}{1-\omega} \frac{(q^{10};q^{10})^3_\infty f(\omega q,\omega^2 q^9)\, f(-\omega q^{12},-\omega^2 q^8)}{q^3 f(q^5,q^{15})\, f(-q^4,-q^6)\, f(\omega q^7,\omega^2 q^3)} \\ &+ \frac{q^2}{1-\omega^2} \frac{(q^{10};q^{10})^3_\infty f(\omega^2 q,\omega q^9)\, f(-\omega^2 q^{12},-\omega q^8)}{q^3 f(q^5,q^{15})\, f(-q^4,-q^6)\, f(\omega^2 q^7,\omega q^3)}. \end{aligned} \tag{3.28}$$

With the previously results of the six identities that we have calculated, we obtain:

(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)

Input interpretation:

$$\frac{5.909106315}{10^{126}} - \frac{2.433397204}{10^{209}} - \frac{2.900835519}{10^{216}} + \frac{3.693191447}{10^{127}} - \frac{1.485227786}{10^{213}} - \frac{1.813022199}{10^{217}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$6.27842545969999999999999999999999999999999999999999... \times 10^{-126}$

Open code

We note that, this first result is an infinitesimal sub-multiple of the length of a circle with radius equal to 1: 2π

colog [(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)]

Input interpretation:

$$-\log\left(\frac{5.909106315}{10^{126}} - \frac{2.433397204}{10^{209}} - \frac{2.900835519}{10^{216}} + \frac{3.693191447}{10^{127}} - \frac{1.485227786}{10^{213}} - \frac{1.813022199}{10^{217}}\right)$$

Open code

- log(x) is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

288.288602491...

Series representations:

$$-\log\left(\frac{5.90911}{10^{126}} - \frac{2.4334}{10^{209}} - \frac{2.90084}{10^{216}} + \frac{3.69319}{10^{127}} - \frac{1.48523}{10^{213}} - \frac{1.81302}{10^{217}}\right) = -2 i \pi \left\lfloor \frac{\arg(6.27843 \times 10^{-126} - x)}{2 \pi} \right\rfloor - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (6.27843 \times 10^{-126} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Open code

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$$-\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)= -\left\lfloor\frac{\arg(6.27843\times10^{-126}-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-\log(z_0)-\left\lfloor\frac{\arg(6.27843\times10^{-126}-z_0)}{2\pi}\right\rfloor\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\,(6.27843\times10^{-126}-z_0)^k\,z_0^{-k}}{k}$$

Open code

$$-\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)= -2i\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{6.27843\times10^{-126}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\,(6.27843\times10^{-126}-z_0)^k\,z_0^{-k}}{k}$$

Open code

- Integral representation:

$$-\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)= -\int_1^{6.27843\times10^{-126}}\frac{1}{t}\,dt$$

6 * colog [(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)]

Input interpretation:

$$6\left(-\log\left(\frac{5.909106315}{10^{126}}-\frac{2.433397204}{10^{209}}-\frac{2.900835519}{10^{216}}+\frac{3.693191447}{10^{127}}-\frac{1.485227786}{10^{213}}-\frac{1.813022199}{10^{217}}\right)\right)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

1729.73161495...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728, very near to the value 1729,7 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Series representations:

$$6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)= -12 i\pi\left\lfloor\frac{\arg(6.27843\times10^{-126}-x)}{2\pi}\right\rfloor-6\log(x)+6\sum_{k=1}^{\infty}\frac{(-1)^k(6.27843\times10^{-126}-x)^k x^{-k}}{k} \quad \text{for } x<0$$

Open code

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$$6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)= -6\left\lfloor\frac{\arg(6.27843\times10^{-126}-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-6\log(z_0)- 6\left\lfloor\frac{\arg(6.27843\times10^{-126}-z_0)}{2\pi}\right\rfloor\log(z_0)+6\sum_{k=1}^{\infty}\frac{(-1)^k(6.27843\times10^{-126}-z_0)^k z_0^{-k}}{k}$$

Open code

$$6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)= -12 i\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{6.27843\times10^{-126}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor- 6\log(z_0)+6\sum_{k=1}^{\infty}\frac{(-1)^k(6.27843\times10^{-126}-z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right) = -6\int_1^{6.27843\times10^{-126}}\frac{1}{t}\,dt$$

Open code

((((((((6 * colog [(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)]))))))))^1/3

Input interpretation:

$$\left(6\left(-\log\left(\frac{5.909106315}{10^{126}}-\frac{2.433397204}{10^{209}}-\frac{2.900835519}{10^{216}}+\frac{3.693191447}{10^{127}}-\frac{1.485227786}{10^{213}}-\frac{1.813022199}{10^{217}}\right)\right)\right)\wedge(1/3)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

12.00400702976...

This result 12,004 is very near to the value of black hole entropy 12,1904

2 * ((((((((6 * colog [(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)]))))))))^1/3

Input interpretation:

$$2\left(6\left(-\log\left(\frac{5.909106315}{10^{126}}-\frac{2.433397204}{10^{209}}-\frac{2.900835519}{10^{216}}+\frac{3.693191447}{10^{127}}-\frac{1.485227786}{10^{213}}-\frac{1.813022199}{10^{217}}\right)\right)\right)\wedge(1/3)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

24.00801405952...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

$$2\sqrt[3]{6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)} = 2\sqrt[3]{6}\left(-2 i\pi\left\lfloor\frac{\arg(6.27843\times10^{-126}-x)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k(6.27843\times10^{-126}-x)^k x^{-k}}{k}\right)\hat{}\,(1/3) \quad \text{for } x<0$$

Open code

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$$2\sqrt[3]{6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)} = 2\left(-6\left\lfloor\frac{\arg(6.27843\times10^{-126}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-6\left(\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(6.27843\times10^{-126}-z_0)^k z_0^{-k}}{k}\right)\right)\hat{}\,(1/3)$$

Open code

$$2\sqrt[3]{6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)} = 2\sqrt[3]{6}\left(-2 i\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{6.27843\times10^{-126}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k(6.27843\times10^{-126}-z_0)^k z_0^{-k}}{k}\right)\hat{}\,(1/3)$$

Integral representation:

$$2\sqrt[3]{6(-1)\log\left(\frac{5.90911}{10^{126}}-\frac{2.4334}{10^{209}}-\frac{2.90084}{10^{216}}+\frac{3.69319}{10^{127}}-\frac{1.48523}{10^{213}}-\frac{1.81302}{10^{217}}\right)} =$$

$$2\sqrt[3]{6}\sqrt[3]{-\int_1^{6.27843\times10^{-126}}\frac{1}{t}\,dt}$$

(((((((colog [(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)])))))^1/12

Input interpretation:

$$\left(-\log\left(\frac{5.909106315}{10^{126}}-\frac{2.433397204}{10^{209}}-\frac{2.900835519}{10^{216}}+\frac{3.693191447}{10^{127}}-\frac{1.485227786}{10^{213}}-\frac{1.813022199}{10^{217}}\right)\right)^{\wedge}(1/12)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

1.6031923163430...

1.6031923163430...

(((((((colog [(5.909106315 × 10^-126) + (-2.433397204 × 10^-209) + (-2.900835519 × 10^-216) + (3.693191447 × 10^-127) + (-1.485227786 × 10^-213) + (-1.813022199 × 10^-217)])))))^1/11

Input interpretation:

$$\left(-\log\left(\frac{5.909106315}{10^{126}}-\frac{2.433397204}{10^{209}}-\frac{2.900835519}{10^{216}}+\frac{3.693191447}{10^{127}}-\frac{1.485227786}{10^{213}}-\frac{1.813022199}{10^{217}}\right)\right)^{\wedge}(1/11)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.6734805893380...

1.6734805893380...

Now, we have:

(((((1/ (((((([(5.909106 × 10^-126) + (-2.433397 × 10^-209) + (-2.900835 × 10^-216) + (3.693191 × 10^-127) + (-1.485227 × 10^-213) + (-1.813022 × 10^-217)])))^1/5)))^(1.6031923^3) ((1/(2*0.61803398))

Input interpretation:

$$\left(\frac{1}{\sqrt[5]{\frac{5.909106}{10^{126}} - \frac{2.433397}{10^{209}} - \frac{2.900835}{10^{216}} + \frac{3.693191}{10^{127}} - \frac{1.485227}{10^{213}} - \frac{1.813022}{10^{217}}}}\right)^{1.6031923^3} \times \frac{1}{2\times 0.61803398}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.22658\ldots \times 10^{103}$

This value is practically equal to the entropy of SMBHs contained within the Cosmic Event Horizon

((((((((((((1/ (((((([(5.909106 × 10^-126) + (-2.433397 × 10^-209) + (-2.900835 × 10^-216) + (3.693191 × 10^-127) + (-1.485227 × 10^-213) + (-1.813022 × 10^-217)]))))))))))* 1/(1.2083*2)(0.6309^12)

where 1,2083 (Fibonacci word fractal 60°) and 0,6309 (Cantor set) are the following Hausdorff dimensions:

$$3\frac{\log(\varphi)}{\log\left(\frac{3+\sqrt{13}}{2}\right)}$$

$\log_3(2)$

Input interpretation:

$$\frac{1}{\frac{5.909106}{10^{126}} - \frac{2.433397}{10^{209}} - \frac{2.900835}{10^{216}} + \frac{3.693191}{10^{127}} - \frac{1.485227}{10^{213}} - \frac{1.813022}{10^{217}}} \times \frac{1}{1.2083 \times 2} \times 0.6309^{12}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$2.6210230491351396042136616770233230985388349551928564... \times 10^{122}$

Open code

This result is practically equal to the value of Cosmic Event Horizon

In conclusion we have obtained an interesting value of entropy of DM contained within the Cosmic Even Horizon, practically very near to the multiple of the length of a circle with radius equal to 1: 2π

((((((((((((1/ (((((([(5.909106 × 10^-126) + (-2.433397 × 10^-209) + (-2.900835 × 10^-216) + (3.693191 × 10^-127) + (-1.485227 × 10^-213) + (-1.813022 × 10^-217)])))))))^1/3)))^2.06 * (1.6087643^4)

Input interpretation:

$$\sqrt[3]{\frac{1}{\frac{5.909106}{10^{126}} - \frac{2.433397}{10^{209}} - \frac{2.900835}{10^{216}} + \frac{3.693191}{10^{127}} - \frac{1.485227}{10^{213}} - \frac{1.813022}{10^{217}}}}^{2.06} \times 1.6087643^4$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$6.28221... \times 10^{86}$

Where 2.06 is a Hausdorff dimension and 1.6087643 is given by: $(((775.4 + 775.49 + 782.65)/3))^{1/14}$ = 1,608764329....

Note that 775.4 775.49 and 782.65 are the rest mass of the Charged rho meson, Neutral rho meson and Omega meson.

The value reported in the table is $6 * 10^{86}$

((((((((((((1/ ((((((([(5.909106 × 10^-126) + (-2.433397 × 10^-209) + (-2.900835 × 10^-216) + (3.693191 × 10^-127) + (-1.485227 × 10^-213) + (-1.813022 × 10^-217)])))))))^1/3))))^(1.3057+0.6942)*0.6942

Where 1.3057 and 0.6942 are two Hausdorff dimensions (0.6942 the Asymmetric Cantor set and 1.3057 the Apollonian gasket

Input interpretation:

$$\sqrt[3]{\frac{1}{\frac{5.909106}{10^{126}} - \frac{2.433397}{10^{209}} - \frac{2.900835}{10^{216}} + \frac{3.693191}{10^{127}} - \frac{1.485227}{10^{213}} - \frac{1.813022}{10^{217}}}}^{1.3057+0.6942} \times 0.6942$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$2.02028... \times 10^{83}$

This result is very near to the values of entropy of Photons and Relic Neutrinos contained within the Cosmic Even Horizon: 2.03 * 10^{88} and 1.93 * 10^{88}

We have that:

$$
= -\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+33}}{1+q^{180n+51}} - \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+210n+93}}{1+q^{180n+111}}
$$

$$
+ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+21}} - \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+141}}
$$

$$
-\left\{ q^9 \frac{\left(\dfrac{(q^{180};q^{180})_\infty^2 f(-q^9,-q^{21})^4 f(-q^{15},-q^{75})}{f(-q^{15},-q^{45}) f(-q^{36},-q^{54})}\right)}{(q^{30};q^{30})_\infty^6 (q^{90};q^{90})_\infty f(-q^{18},-q^{42})^2} \right.
$$

$$
\times (f(-q^{39},-q^{51}) f(-q^{45},-q^{45})
$$

$$
-q^3 f(-q^{21},-q^{69}) f(-q^{45},-q^{45})
$$

$$
+q^{12} f(-q^9,-q^{81}) f(-q^{15},-q^{75}))
$$

$$
-\frac{(q^{15};q^{15})_\infty (q^{60};q^{60})_\infty^4 f(-q^9,-q^{21})^2 f(-q^{36},-q^{54}) f(-q^{72},-q^{108})}{(q^{30};q^{30})_\infty^7 (q^{90};q^{90})_\infty (q^{180};q^{180})_\infty f(-q^{18},-q^{42})^2 f(-q^{24},-q^{36})}
$$

$$
\times (-f(-q^{15},-q^{75}) f(-q^{39},-q^{51})^2 f(-q^{21},-q^{69})^2
$$

$$
+q^9 f(-q^{45},-q^{45}) f(-q^9,-q^{81})^2 f(-q^{21},-q^{69})^2
$$

$$
+q^3 f(-q^{45},-q^{45}) f(-q^9,-q^{81})^2 f(-q^{39},-q^{51})^2)
$$

$$
+q^6 \frac{(q^{60};q^{60})_\infty (q^{90};q^{90})_\infty^2 f(-q^9,-q^{21})^3 f(q^9,q^{21}) f(-q^{36},-q^{54})}{(q^{30};q^{30})_\infty^6 (q^{45};q^{45})_\infty f(-q^{18},-q^{42})^2}
$$

$$
\times (q^6 f(-q^{21},-q^{69}) f(-q^{15},-q^{75})^2 + f(-q^9,-q^{81}) f(-q^{45},-q^{45})^2
$$

$$
\left. -q^3 f(-q^{39},-q^{51}) f(-q^{15},-q^{75})^2) \right\}
$$

$$
-q \frac{(q^9;q^9)_\infty (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty f(-q^3,-q^{15})^2}{(q^{18};q^{18})_\infty^2 f(\quad q^6,\quad q^{24})}. \tag{3.33}
$$

From:

$$= -\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+33}}{1+q^{180n+51}} - \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+210n+93}}{1+q^{180n+111}}$$

$$+ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+21}} - \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+141}}$$

We obtain:

-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501)))

Input:

$$-\frac{0.5^{693}}{1+0.5^{411}} - \frac{0.5^{873}}{1+0.5^{471}} + \frac{0.5^{420}}{1+0.5^{381}} - \frac{0.5^{720}}{1+0.5^{501}}$$

Open code

Result:

- More digits

$3.6931914471142943123035723049987333751394454880226 04... \times 10^{-127}$

3.693191447114294312303572304998733375139445488022604 × 10^-127

This result is similar to that obtained at pag.65

6 colog [[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501))))))]]]

Input:

$$6\left(-\log\left(-\frac{0.5^{693}}{1+0.5^{411}} - \frac{0.5^{873}}{1+0.5^{471}} + \frac{0.5^{420}}{1+0.5^{381}} - \frac{0.5^{720}}{1+0.5^{501}}\right)\right)$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

1746.73...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

$$6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)=$$
$$-12\,i\,\pi\left\lfloor\frac{\arg(3.69319\times10^{-127}-x)}{2\,\pi}\right\rfloor-6\log(x)+6\sum_{k=1}^{\infty}\frac{(-1)^k\,(3.69319\times10^{-127}-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

Open code

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$$6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)=$$
$$-6\left\lfloor\frac{\arg(3.69319\times10^{-127}-z_0)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-6\log(z_0)-6\left\lfloor\frac{\arg(3.69319\times10^{-127}-z_0)}{2\,\pi}\right\rfloor\log(z_0)+6\sum_{k=1}^{\infty}\frac{(-1)^k\,(3.69319\times10^{-127}-z_0)^k\,z_0^{-k}}{k}$$

Open code

$$6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)=$$
$$-12\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{3.69319\times10^{-127}}{z_0}\right)+\arg(z_0)}{2\,\pi}\right\rfloor-6\log(z_0)+6\sum_{k=1}^{\infty}\frac{(-1)^k\,(3.69319\times10^{-127}-z_0)^k\,z_0^{-k}}{k}$$

Integral representation:

$$6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)=$$
$$-6\int_1^{3.69319\times10^{-127}}\frac{1}{t}\,dt$$

Open code

(((((6 colog [[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381))-(((0.5)^720))/(((1+(0.5)^501))))))]]])))))^1/3

Input:

$$\sqrt[3]{6\left(-\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

12.0432...

This result 12,0432 is very near to the value of black hole entropy 12,1904

And

Input:

$$2\sqrt[3]{6\left(-\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

24.0864...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

$$2\sqrt[3]{6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)}=$$
$$2\sqrt[3]{6}\left(-2\,i\,\pi\left\lfloor\frac{\arg(3.69319\times10^{-127}-x)}{2\,\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\,(3.69319\times10^{-127}-x)^k\,x^{-k}}{k}\right)^{\wedge}(1/3)\quad \text{for } x<0$$

Open code

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$$2\sqrt[3]{6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)} = 2\left(-6\left\lfloor\frac{\arg(3.69319\times10^{-127}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-6\left(\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(3.69319\times10^{-127}-z_0)^k z_0^{-k}}{k}\right)\right)\text{^}(1/3)$$

Open code

$$2\sqrt[3]{6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)} = 2\sqrt[3]{6}\left(-2i\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{3.69319\times10^{-127}}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k(3.69319\times10^{-127}-z_0)^k z_0^{-k}}{k}\right)\text{^}(1/3)$$

Integral representation:

$$2\sqrt[3]{6(-1)\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)} = 2\sqrt[3]{6}\sqrt[3]{-\int_1^{3.69319\times10^{-127}}\frac{1}{t}\,dt}$$

And

Input:

$$\frac{1}{6}\times2\sqrt[3]{6\left(-\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

4.01440...

This result is in the range of the mass of DM particle that is between 4 – 4.2 eV

(((((colog [[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501))))))]]])))))^1/11

Input:

$$\sqrt[11]{-\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

1.674969...

1.674969…

Input:

$$\sqrt[12]{-\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- More digits

1.604499...

1.604499…

Continued fraction:

- Linear form

For 1.674969…, we have:

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{13+\cfrac{1}{20+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{3+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

root of $4043x^3 + 9692x^2 - 17002x - 17712$ near $x = 1.67497$ ≈

1.674969084756767004409350

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$$\frac{6703315613\,\pi}{12572821359} \approx 1.674969084756767004409679$$

$$\log\left(\frac{1}{22}\left(-69 - 343\sqrt{2} - 118\,e + 34\,e^2 + 305\,\pi - 22\,\pi^2\right)\right) \approx$$

1.674969084756767004445235

Furthermore:

((((([[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501))))))]]])))))^1/58

Input:

$$\sqrt[58]{-\frac{0.5^{693}}{1+0.5^{411}} - \frac{0.5^{873}}{1+0.5^{471}} + \frac{0.5^{420}}{1+0.5^{381}} - \frac{0.5^{720}}{1+0.5^{501}}}$$

Open code

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Result:

- More digits

0.00660888...

1/2 * (((((((0.00660888 + ((((([[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501))))))]]])))))^1/60))))))))

Input interpretation:

$$\frac{1}{2}\left(0.00660888+\sqrt[60]{-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.00721069...

And

Input interpretation:

$$\frac{1}{\frac{1}{2}\left(0.00660888+\sqrt[60]{-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}}\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

138.683...

Results that are a good approximation to the value of the fundamental Fine-structure constant: $\alpha = 0.0072973525693(11)$ and to the reciprocal 137,0359990834....

Furthermore, we note that the rest masses of the Pions are 139.570 18 ± 0.000 35 and 134.976 6 ± 0.000 6, thence 138.683 is practically in the range of the values (the mean is 137,27339).

Now, we have:

To complete the proof, we need to modify the four sums in (3.33). Replacing n by $-n-1$ in the first, second, and fourth sums in (3.33), we find that

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+33}}{1+q^{180n+51}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+33}}{1+q^{180n+51}} \frac{q^{-180n-51}}{q^{-180n-51}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2-30n-18}}{1+q^{-180n-51}}$$

$$= -\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+210n+102}}{1+q^{180n+129}},$$

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+210n+93}}{1+q^{180n+111}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+210n+93}}{1+q^{180n+111}} \frac{q^{-180n-111}}{q^{-180n-111}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+30n-18}}{1+q^{-180n-111}}$$

$$= -\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+42}}{1+q^{180n+69}},$$

and

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+141}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+141}} \frac{q^{-180n-141}}{q^{-180n-141}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2-30n-81}}{1+q^{-180n-141}}$$

$$= -\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+210n+39}}{1+q^{180n+39}}.$$

And, replacing n by $-n$ in the third sum in (3.33), we find that

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+21}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+21}} \frac{q^{-180n-21}}{q^{-180n-21}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2-150n-21}}{1+q^{-180n-21}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{90n^2+150n-21}}{1+q^{180n-21}}.$$

Thence:

-(((0.5)^882))/(((1+(0.5)^489)))

Input:

$$-\frac{0.5^{882}}{1+0.5^{489}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-3.1013003229050298983324099477976554711342964641753 2\ldots \times 10^{-266}$

-3.10130032290502989833240994779765547113429646417532 × 10^-266

-(((0.5)^702))/(((1+(0.5)^429)))

Input:

$$-\frac{0.5^{702}}{1+0.5^{429}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-4.7527289157378995587876810145877577731621780159337 9\ldots \times 10^{-212}$

Open code

-4.75272891573789955878768101458775777316217801593379 × 10^-212

-(((0.5)^819))/(((1+(0.5)^399)))

Input:

$$-\frac{0.5^{819}}{1+0.5^{399}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-2.8604446676170939537384880783616358824538453984052 8\ldots \times 10^{-247}$

Open code

-2.86044466761709395373848807836163588245384539840528 × 10^-247

(((0.5)^639))/(((1+(0.5)^339)))

Input:

$$\frac{0.5^{639}}{1+0.5^{339}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$4.383618698016806079505386214282241665261617444248073... \times 10^{-193}$

4.383618698016806079505386214282241665261617444248073 × 10^-193

Now:

Using (1.10), (3.22), (3.33), the four identities above, Theorem 2.6.14, and Theorem 3.6, we deduce that

$$
\begin{aligned}
&(q^{60};q^{60})_\infty\left(X(q^9)-\frac{\omega\chi(\omega q)-\omega^2\chi(\omega^2 q)}{\omega-\omega^2}\right)\\
&=2(q^{60};q^{60})_\infty+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+42}}{1+q^{180n+69}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2-30n}}{1+q^{180n-21}}\\
&\quad+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+102}}{1+q^{180n+129}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+39}}\\
&\quad-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+102}}{1+q^{180n+129}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+42}}{1+q^{180n+69}}\\
&\quad-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n-21}}{1+q^{180n-21}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+39}}{1+q^{180n+39}}\\
&\quad-\frac{(q^{45};q^{45})_\infty(q^{90};q^{90})_\infty f(q^{18},\ q^{27})}{f(-q^{18},-q^{72})f(-q^9,-q^{36})}(q^{60};q^{60})_\infty\\
&\quad+q^9\left(q^9\frac{(q^{90};q^{90})_\infty^3 f(q^3,q^{87})f(-q^{24},-q^{156})}{f(q^{45},q^{135})f(-q^{18},-q^{72})f(q^{21},q^{69})}\right.\\
&\quad\left.+\frac{(q^{90};q^{90})_\infty^3 f(q^{33},q^{57})f(-q^{84},-q^{96})}{q^9 f(q^{45},q^{135})f(-q^{18},-q^{72})f(q^{39},q^{51})}\right)\\
&\quad+\left\{q^9\,\frac{\left(\dfrac{(q^{180};q^{180})_\infty^2 f(-q^9,-q^{21})^4 f(-q^{15},-q^{75})}{f(\ q^{15},\ q^{45})f(\ q^{36},\ q^{54})}\right)}{(q^{30};q^{30})_\infty^6(q^{90};q^{90})_\infty f(-q^{18},-q^{42})^2}\right.\\
&\quad\times(f(-q^{39},-q^{51})f(-q^{45},-q^{45})\\
&\quad-q^3 f(-q^{21},-q^{69})f(-q^{45},-q^{45})\\
&\quad+q^{12}f(-q^9,-q^{81})f(-q^{15},-q^{75}))\\
&\quad-\frac{(q^{15};q^{15})_\infty(q^{60};q^{60})_\infty^4 f(-q^9,-q^{21})^2 f(-q^{36},-q^{54})f(-q^{72},-q^{108})}{(q^{30};q^{30})_\infty^7(q^{90};q^{90})_\infty(q^{180};q^{180})_\infty f(-q^{18},-q^{42})^2 f(-q^{24},-q^{36})}\\
&\quad\times(-f(-q^{15},-q^{75})f(-q^{39},-q^{51})^2 f(-q^{21},-q^{69})^2\\
&\quad+q^9 f(-q^{45},-q^{45})f(-q^9,-q^{81})^2 f(-q^{21},-q^{69})^2\\
&\quad+q^3 f(\ q^{45},\ q^{45})f(\ q^9,\ q^{81})^2 f(\ q^{39},\ q^{51})^2)\\
&\quad+q^6\frac{(q^{60};q^{60})_\infty(q^{90};q^{90})_\infty^2 f(-q^9,-q^{21})^3 f(q^9,q^{21})f(-q^{36},-q^{54})}{(q^{30};q^{30})_\infty^6(q^{45};q^{45})_\infty f(-q^{18},-q^{42})^2}
\end{aligned}
$$

$$\times(q^6 f(-q^{21},-q^{69}) f(-q^{15},-q^{75})^2$$
$$+ f(-q^9,-q^{81}) f(-q^{45},-q^{45})^2$$
$$- q^3 f(-q^{39},-q^{51}) f(-q^{15},-q^{75})^2)\}$$
$$+ q\frac{(q^9;q^9)_\infty (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty f(-q^3,-q^{15})^2}{(q^{18};q^{18})^2_\infty f(-q^6,-q^{24})}$$
$$= 2(q^{60};q^{60})_\infty - 2\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+30n}$$
$$+ \frac{(q^9;q^9)^3_\infty (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty f(-q^3,-q^{15})}{(q^{18};q^{18})^3_\infty f(\quad q^6,\quad q^{24})}$$
$$+ q\frac{(q^9;q^9)_\infty (q^{30};q^{30})_\infty (q^{60};q^{60})_\infty f(-q^3,-q^{15})^2}{(q^{18};q^{18})^2_\infty f(\quad q^6,\quad q^{24})}$$
$$= \frac{(q^3;q^3)_\infty (q^{60};q^{60})_\infty f(q,q^3) f(-q^{12},-q^{18})}{(q^6;q^6)_\infty f(q^3,q^9)}. \tag{3.34}$$

Dividing both sides of (3.34) by $(q^{60};q^{60})_\infty$, we complete the proof of Theorem 3.7.

Thence:

(-3.1013003229050298983324099477 × 10^-266) + (-4.7527289157378995587876810145 × 10^-212) + (-2.8604446676170939537384880783 × 10^-247) + (-4.38361869801680607950538621428 × 10^-193)

Input interpretation:

$$-\frac{3.1013003229050298983324099477}{10^{266}} - \frac{4.7527289157378995587876810145}{10^{212}} - \frac{2.8604446676170939537384880783}{10^{247}} - \frac{4.38361869801680607950538621428}{10^{193}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-4.38361869801680607998065910585378995587876810145000\ldots \times 10^{-193}$

-4.3836186980168060799806591058537899558787681014500 × 10^-193

Now, from the two last results:

3.693191447114294312303572304998733375139445488022604 × 10^-127

-4.3836186980168060799806591058537899558787681014500 × 10^-193

We obtain:

(-4.3836186980168060799806591058537899558787681014500 × 10^-193) * (3.693191447114294312303572304998733375139445488022604 × 10^-127)

Input interpretation:

$$-\frac{4.3836186980168060799806591058537899558787681014500}{10^{193}} \times \frac{3.693191447114294312303572304998733375139445488022604}{10^{127}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-1.61895430829259667613330799829031994534117111686034... \times 10^{-319}$

-1.61895430829259667613330799829031994534117111686034 × 10^-319

Repeating decimal:

- Fewer digits

$-1.6189543082925966761333079982903199453411711168603426434339029753024093613079233248658212431765175800 \times 10^{-319}$

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Or, changing the sign:

1.61895430829259667613330799829031994534117111686034 × 10^-319

1 / (1.6189543082925966761333079982903199453411711686034 × 10^-319) =

= 6.176826577981891428544642652705262921550208162512914 × 10^318

≈6,17682658 * 10^{318} that is equal to

0,6176826577981891428544642652705262921550208162512914 × 10^{319}

Note that:

(1.6189543082925966761333079982903199453411711686034 × 10^-319) * 10^319

Input interpretation:

$$\frac{1.6189543082925966761333079982903199453411711686034}{10^{319}} \times 10^{319}$$

Open code

Result:

1.6189543082925966761333079982903199453411711686034

1.6189543082925966761333079982903199453411711686034

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{23+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{15+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$\frac{320978994\pi}{622862081} \approx 1.61895430829259667574 74$

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π root of $6291x^4 - 2125x^3 + 20x^2 + 935x - 640$ near $x = 0.515329$ $\approx$
1.61895430829259667617 6633

$\frac{12307 C_{Th} + 6}{7300 C_{Th} + 263} \approx 1.6189543082925966796219$

1/ 1.6189543082925966761333079982903199453417111686034

0.61768265779818914285446426527052629215502081625129 1413893

Continued fraction:

- Linear form

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{23+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{15+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$\frac{176383423\pi}{897102839} \approx 0.61768265779818914 30200$

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π root of $10810x^3 + 57715x^2 - 25442x + 2689$ near $x = 0.196614$ $\approx$
0.61768265779818914238400

$\frac{1064 e\, e! - 3948 + 1029 e - 135 e^2}{6060 e} \approx 0.6176826577981891428541 5219$

•

Note that the previous result

1.6189543082925966761333079982903199453411711116 × 10^-319 isn't a continuous fraction.

Note that the third Ramanujan's identity is:

$$X(q^3) - \frac{\omega\chi(\omega q^{1/3}) - \omega^2\chi(\omega^2 q^{1/3})}{\omega - \omega^2}$$
$$= \frac{1 + q^{1/3} + q^{3/3} + q^{6/3} + \cdots}{1 + q + q^3 + q^6 + \cdots} \frac{1 - q^4 - q^6 + q^{18} + \cdots}{(1+q)(1+q^2)(1+q^3)\cdots}, \quad (1.5)$$

where ω is a primitive cube root of unity, and $|q| < 1$.

For q = 0.5 we obtain:

((((((((1-(0.5)^4 - (0.5)^6 + (0.5)^18))) / (((1+(0.5)^1) (1+(0.5)^2) (1+ (0.5)^3))))))))

Input:

$$\frac{1 - 0.5^4 - 0.5^6 + 0.5^{18}}{(1 + 0.5^1)(1 + 0.5^2)(1 + 0.5^3)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.437038845486111111111111111111111111111111111111111...

((((((((1+(0.5)^(1/3)+(0.5)^(3/3) + (0.5)^(6/3))) / (((1+(0.5)^1 + (0.5)^3 + (0.5)^6)))))))) * 0.43703884548611111111111111111111111111111

Input interpretation:

$$\frac{1 + \sqrt[3]{0.5} + 0.5^{3/3} + 0.5^{6/3}}{1 + 0.5^1 + 0.5^3 + 0.5^6} \times 0.43703884548611111111111111111111111111111111111111$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

0.677605145074897997170343332955656071289412607521449502653...

0.6776051450748979971703433329556560712894126075214495

Continued fraction:

- Linear form

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$\sinh\left(\cos\left(\frac{160859335}{22444514}\right)\right) \approx 0.67760514507489799715950 4$

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$\frac{1513521825\,\pi}{7017167861} \approx 0.677605145074897997177 9871$

root of $9269x^4 + 144x^3 - 64x^2 - 4341x + 972$ near $x = 0.677605$ $\approx$

0.67760514507489799717076242

•

From this result, we have:

1+0.6776051450748979971703433329556560712894126075214495 =

1.6776051450748979971703433329556560712894126075214495

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$\frac{5\,189\,092\,989\,\pi}{9\,717\,433\,486} \approx 1.677605145074897997163003$$

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$$-\frac{3\left(114+250\,\pi+49\,\pi^2\right)}{129-1221\,\pi+125\,\pi^2} \approx 1.677605145074897997162588$$

$$\frac{1\,228\,651}{225\,309\,\pi} - \frac{795\,\pi}{42\,916} \approx 1.6776051450748979971795008$$

A result very near to that obtained (see pag. 93):

[[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501))))))]]]

Input:

$$-\frac{0.5^{693}}{1+0.5^{411}} - \frac{0.5^{873}}{1+0.5^{471}} + \frac{0.5^{420}}{1+0.5^{381}} - \frac{0.5^{720}}{1+0.5^{501}}$$

Open code

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Result:

- More digits

$3.693191447114294312303572304998733375139445488022604\ldots \times 10^{-127}$

(((((colog [[[-(((0.5)^693))/(((1+(0.5)^411))) - ((0.5)^873))/(((1+(0.5)^471))) + (((0.5)^420))/(((1+(0.5)^381)))-(((0.5)^720))/(((1+(0.5)^501))))))]]])))))^1/11

Input:

$$\sqrt[11]{-\log\left(-\frac{0.5^{693}}{1+0.5^{411}}-\frac{0.5^{873}}{1+0.5^{471}}+\frac{0.5^{420}}{1+0.5^{381}}-\frac{0.5^{720}}{1+0.5^{501}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

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Result:

- Fewer digits
- More digits

1.674969084756767004411934242214153714745693988006634079343...

1.6749690847567670044119342422141537147456939880066340

Indeed:

1.67760514 ≈ 1.67496908

Now, we have:

$$\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n}}{1-q^{30n+2}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n \omega q^{3n+1}}{1-q^{30n+12}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n \omega^2 q^{3n+2}}{1-q^{30n+22}}. \qquad (4.2)$$

$$\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-18n}-\omega^2\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+42n+4}$$

$$+\omega\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+102n+28}. \qquad (4.4)$$

That that must be multiplied between them:

$$\frac{q\omega}{1-\omega}\,\frac{(q^{10};q^{10})_\infty^3\, f(\omega q^3,\omega^2 q^7)\, f(-\omega^2 q^4,-\omega q^{16})}{q^3 f(q^5,q^{15})\, f(-q^2,-q^8)\, f(\omega q,\omega^2 q^9)}$$

$$+\frac{q\omega^2}{1-\omega^2}\,\frac{(q^{10};q^{10})_\infty^3\, f(\omega^2 q^3,\omega q^7)\, f(-\omega q^4,-\omega^2 q^{16})}{q^3 f(q^5,q^{15})\, f(-q^2,-q^8)\, f(\omega^2 q,\omega q^9)}$$

$$=-\frac{1}{q^2 f(q^5,q^{15})}\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n}}{1-q^{30n+2}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-18n}\right.$$

$$+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n+1}}{1-q^{30n+12}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+42n+4}$$

$$\left.+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n+2}}{1-q^{30n+22}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+102n+28}\right)$$

$$+\frac{1}{q^2 f(q^5,q^{15})}\left(-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n}}{1-q^{30n+2}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+42n+4}\right.$$

$$-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n+1}}{1-q^{30n+12}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2+102n+28}$$

$$\left.+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3n+2}}{1-q^{30n+22}}\sum_{n=-\infty}^{\infty}(-1)^n q^{90n^2-18n}\right)$$

Thence:

$$= -\frac{1}{q^2 f(q^5, q^{15})} \left(\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{3n}}{1-q^{30n+2}} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2-18n} \right.$$
$$+ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{3n+1}}{1-q^{30n+12}} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+42n+4}$$
$$\left. + \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{3n+2}}{1-q^{30n+22}} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+102n+28} \right)$$
$$+ \frac{1}{q^2 f(q^5, q^{15})} \left(-\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{3n}}{1-q^{30n+2}} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+42n+4} \right.$$
$$- \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{3n+1}}{1-q^{30n+12}} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+102n+28}$$
$$\left. + \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{3n+2}}{1-q^{30n+22}} \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2-18n} \right)$$

(((0.5)^6))/(((1-(0.5)^62))) - ((0.5)^7))/(((1-(0.5)^72)) + (((0.5)^8))/(((1-(0.5)^82)))

Input:

$$\frac{0.5^6}{1-0.5^{62}} - \frac{0.5^7}{1-0.5^{72}} + \frac{0.5^8}{1-0.5^{82}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

0.01171875000000000003386478235585662247974319545883959431...

0.01171875000000000003386478235585662247974319545883959431

Rational form:

$$\frac{3}{256}$$

(((0.5)^324)) - ((0.5)^448)) + (((0.5)^592))

Input:

$$0.5^{324} - 0.5^{448} + 0.5^{592}$$

Open code

Result:

- More digits

$2.9260477216826239544724058836616728560593074840633905... \times 10^{-98}$

2.9260477216826239544724058836616728560593074840633905 × 10^-98

We have that:

[(2.9260477216826239544724058836616728560593074840633905 × 10^-98) * (0.011718750000000000033864782355856622479743195458839594 31)]

Input interpretation:

$2.9260477216826239544724058836616728560593074840633905 \times 10^{-98} \times 0.01171875000000000003386478235585662247974319545883959431$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$3.4289621738468249476382503374923448067816224665430 99... \times 10^{-100}$

3.4289621738468249476382503374923448067816224665430 99 × 10^-100

((1/ [(2.9260477216826239544724058836616728560593074840633905 × 10^-98) + (0.011718750000000000033864782355856622479743195458839594 31)]))^1/9

Result:

- More digits

1.6389614762961056192571609792944141178403917707183038 2849...

1.6389614762961056192571609792944141178403917707183038 2849

Or

((1/ [(2.92604772168262395447 24 × 10^-98) + (0.01171875)]))^1/9

Input interpretation:

$$\sqrt[9]{\frac{1}{2.9260477216826239544724 \times 10^{-98} + 0.01171875}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.6389615...

1.6389614762961056193097861158248092155040466795124423

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{14+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{12+\cfrac{1}{15+\cfrac{1}{34+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$\frac{2^{8/9}}{\sqrt[9]{3}} \approx 1.638961476296105619309786115824809215504046679512442300064$$

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$$\frac{\sqrt[3]{-5070+701\,e+573\,\pi+2197\log(2)}}{6^{2/3}} \approx 1.638961476296105617897 6$$

$$\frac{1\,112\,998\,553\,\pi}{2\,133\,416\,879} \approx 1.638961476296105619204 72$$

Now:

(((0.5)^6))/(((1-(0.5)^62)))*0.5^(324) + ((0.5)^7))/(((1-(0.5)^72)))*0.5^(448) + (((0.5)^8))/(((1-(0.5)^82)))*0.5^(592)

Input:

$$\frac{0.5^6}{1-0.5^{62}}\times 0.5^{324}+\frac{0.5^7}{1-0.5^{72}}\times 0.5^{448}+\frac{0.5^8}{1-0.5^{82}}\times 0.5^{592}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$4.5719495651290999298545177234227893159715138745757 20... \times 10^{-100}$

4.57194956512909992985451772342278931597151387457 5720 × 10^-100

(((0.5)^6))/(((1-(0.5)^62)))*0.5^(448) - ((0.5)^7))/(((1-(0.5)^72)))*0.5^(592) - (((0.5)^8))/(((1-(0.5)^82)))*0.5^(324)

Input:

$$\frac{0.5^6}{1-0.5^{62}} \times 0.5^{448} - \frac{0.5^7}{1-0.5^{72}} \times 0.5^{592} - \frac{0.5^8}{1-0.5^{82}} \times 0.5^{324}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$-1.1429873912822749822157837846695935745517739468093 2... \times 10^{-100}$

(4.57194956512909992985451772342278931597151387457 5720 × 10^-100) - (((0.5)^6))/(((1-(0.5)^62)))*0.5^(448) - ((0.5)^7))/(((1-(0.5)^72)))*0.5^(592) - (((0.5)^8))/(((1-(0.5)^82)))*0.5^(324)

Input interpretation:

$$\frac{4.57194956512909992985451772342278931597151387457 5720}{10^{100}} - \frac{0.5^6}{1-0.5^{62}} \times 0.5^{448} - \frac{0.5^7}{1-0.5^{72}} \times 0.5^{592} - \frac{0.5^8}{1-0.5^{82}} \times 0.5^{324}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$3.4289621738468249476387339387531957409897958568820 98... \times 10^{-100}$

3.42896217384682494763873393875319574098979585688 2098 × 10^-100

We note that:

colog (3.428962173846824947638250337492344806781622466543099 × 10^-100)^1/11

Input interpretation:

$$\sqrt[11]{-\log\left(\frac{3.428962173846824947638250337492344806781622466543099}{10^{100}}\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.638834221398318836015032087808036066228269997623819403...

1.638834221398318836015032087808036066228269997623819403

Continued fraction:

- Linear form

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{13 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{165 + \cfrac{1}{1 + \cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}$$

Open code

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Possible closed forms:

- More

$$\frac{315\,778\,321\,\pi}{605\,336\,916} \approx 1.63883422139831883\textcolor{gray}{56860}$$

$$\pi \ \text{root of } 6x^3 + 63\,784\,x^2 + 50\,577\,x - 43\,742 \text{ near } x = 0.521657 \approx$$

1.6388342213983188360125751

$$\frac{-701\, e\, e! - 9475 + 3747\, e + 1075\, e^2}{120\, e} \approx 1.638834221398318836024469$$

7.6 * colog (3.42896217384682494763825033749234480678162246654 3099 × 10^-100)

Input interpretation:

$$7.6\left(-\log\left(\frac{3.428962173846824947638250337492344806781622466543099}{10^{100}}\right)\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1740.60...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

- More

$$7.6(-1)\log\left(\frac{3.4289621738468249476382503374923448067816224665430990000}{10^{100}}\right) = 7.6\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k(-0.99999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999657103782615317505236174966250765519321837753345690100000)^k$$

Open code

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$$7.6 (-1) \log\left(\frac{3.428962173846824947638250337492344806781622466543099 0000}{10^{100}}\right) = -15.2 i \pi \left\lfloor \frac{1}{2\pi} \arg(3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - x) \right\rfloor - 7.6 \log(x) + 7.6 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - x)^k x^{-k} \quad \text{for } x < 0$$

Open code

$$7.6 (-1) \log\left(\frac{3.4289621738468249476382503374923448067816224665430990000}{10^{100}}\right) = -7.6 \left\lfloor \frac{1}{2\pi} \arg(3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - z_0) \right\rfloor \log\left(\frac{1}{z_0}\right) - 7.6 \log(z_0) - 7.6 \left\lfloor \frac{1}{2\pi} \arg(3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - z_0) \right\rfloor \log(z_0) + 7.6 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - z_0)^k z_0^{-k}$$

Integral representation:

$$7.6 (-1) \log\left(\frac{3.4289621738468249476382503374923448067816224665430990000}{10^{100}}\right) = -7.6 \int_1^{3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100}} \frac{1}{t} \, dt$$

Open code

((((7.6 * colog (3.428962173846824947638250337492344806781622466543099 × 10^-100))))^1/3

Input interpretation:

$$\sqrt[3]{7.6 \left(-\log\left(\frac{3.428962173846824947638250337492344806781622466543099}{10^{100}}\right)\right)}$$

Open code

- log(x) is the natural logarithm

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Result:

- More digits

12.0291...

This result 12,0291 is very near to the value of black hole entropy 12,1904

2 * ((((7.6 * colog (3.428962173846824947638250337492344806781622466543099 × 10^-100))))^1/3

Input interpretation:

$$2\sqrt[3]{7.6\left(-\log\left(\frac{3.428962173846824947638250337492344806781622466543099}{10^{100}}\right)\right)}$$

Open code

- log(x) is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

24.0582...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

- More

$$2\left(7.6(-1)\log\left(\frac{3.4289621738468249476382503374923448067816224665430990000}{10^{100}}\right)\right)^{\wedge}(1/3) = 3.93219\left(\sum_{k=1}^{\infty}\frac{1}{k}(-1)^k(-0.99999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999999657103782615317505236174966250765519321837753345690100000)^k\right)^{\wedge}(1/3)$$

Open code

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$$2 (7.6 (-1) \log(3.4289621738468249476382503374923448067816224665430990000 / 10^{100})) \wedge (1/3) = 3.93219 \left(-2 i \pi \left\lfloor \frac{1}{2 \pi} \arg(3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - x) \right\rfloor - \log(x) + \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - x)^k x^{-k}\right) \wedge (1/3)$$

for $x < 0$

Open code

$$2 \left(7.6 (-1) \log\left(\frac{3.4289621738468249476382503374923448067816224665430990000}{10^{100}}\right)\right) \wedge (1/3) = 3.93219 \left(-\log(z_0) - \left\lfloor \frac{1}{2 \pi} \arg(3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - z_0) \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100} - z_0)^k z_0^{-k}\right) \wedge (1/3)$$

Integral representation:

$$2 \left(7.6 (-1) \log\left(\frac{3.4289621738468249476382503374923448067816224665430990000}{10^{100}}\right)\right) \wedge (1/3) = 3.93219 \sqrt[3]{-\int_1^{3.4289621738468249476382503374923448067816224665430990000 \times 10^{-100}} \frac{1}{t} \, dt}$$

Now, we have:

Adding (4.16) and (4.17), dividing by 2, and applying Lemma 2.1.1(iv), (2.5.12), (2.5.4) and (1.10), we find that

$$
\begin{aligned}
&(q^{60};q^{60})_\infty\,\chi(q^9)\\
&\quad=-\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1+q^{180n+33}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+123}}\right)\\
&\qquad-\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1+q^{180n+93}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+3}}\right)\\
&\qquad-b_2(q^9)\,q^9(q^{60};q^{60})_\infty-q^{18}(B(-q^{-12},q^{18},q^{45})+B(-q^{48},q^{18},q^{45}))
\end{aligned}
$$

$$
\begin{aligned}
&=-\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1-q^{180n+33}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1-q^{180n+123}}\right)\\
&\quad-\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1-q^{180n+93}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1-q^{180n+3}}\right)\\
&\quad+\frac{(q^{45};q^{45})_\infty\,(q^{90};q^{90})_\infty\,f(-q^9,-q^{36})}{f(-q^{18},-q^{27})\,f(-q^{36},-q^{54})}\,q^9(q^{60};q^{60})_\infty\\
&\quad+q^{18}\left(q^6\frac{(q^{90};q^{90})_\infty^3\,f(q^{21},q^{69})\,f(-q^{12},-q^{168})}{f(q^{45},q^{135})\,f(-q^{36},-q^{54})\,f(q^{33},q^{57})}\right.\\
&\quad\left.-\frac{(q^{90};q^{90})_\infty^3\,f(q^{39},q^{51})\,f(-q^{48},-q^{132})}{q^{18}f(q^{45},q^{135})\,f(-q^{36},-q^{54})\,f(q^{3},q^{87})}\right).
\end{aligned}
\tag{4.18}
$$

Thence:

$$
\begin{aligned}
&=-\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1-q^{180n+33}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1-q^{180n+123}}\right)\\
&\quad-\left(\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1\ \ q^{180n+93}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1\ \ q^{180n+3}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&(q^{60};q^{60})_\infty\,(X(q)-2)\\
&=\frac{2q}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega^{2(n+1)}q^{10n^2+10n-3}}{1+\omega^2q^{20n+1}}\\
&\quad+\frac{2}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega^{2(n+1)}q^{10n^2+10n-1}}{1+\omega^2q^{20n+11}}\\
&\quad-\frac{(q^5;q^5)_\infty\,(q^{10};q^{10})_\infty\,f(-q^2,-q^3)}{f(-q,-q^4)\,f(-q^2,-q^8)}\,q(q^{60};q^{60})_\infty\\
&\quad-\frac{2q\omega^2}{1-\omega^2}\,\frac{(q^{10};q^{10})_\infty^3\,f(\omega^2q^3,\omega q^7)\,f(-\omega^2q^{16},-\omega q^4)}{q^3f(q^5,q^{15})\,f(-q^2,-q^8)\,f(\omega^2q,\omega q^9)}.
\end{aligned}
\tag{4.21}
$$

Thence:

$$
\begin{aligned}
&=\frac{2q}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega^{2(n+1)}q^{10n^2+10n-3}}{1+\omega^2q^{20n+1}}\\
&\quad+\frac{2}{1-\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega^{2(n+1)}q^{10n^2+10n-1}}{1+\omega^2q^{20n+11}}
\end{aligned}
$$

And:

$$
\begin{aligned}
&\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega^{2(n+1)}q^{10n^2+10n-1}}{1+\omega^2q^{20n+11}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega^2q^{90n^2+30n-1}}{1+\omega^2q^{60n+11}}\\
&\quad-\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,\omega q^{90n^2+90n+19}}{1+\omega^2q^{60n+31}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n\,q^{90n^2+150n+59}}{1+\omega^2q^{60n+51}}.
\end{aligned}
$$

And:

$$= -\sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+30n-3} \frac{1-q^{120n+2}}{1+q^{180n+3}},$$

$$= -\sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+38} \frac{1+q^{60n+21}}{1+q^{180n+63}},$$

$$= \sum_{n=-\infty}^{\infty} (\quad 1)^n q^{90n^2+150n+57} \frac{1+q^{60n+41}}{1+q^{180n+123}},$$

$$= -\sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+30n-1} \frac{1-q^{120n+22}}{1+q^{180n+33}}.$$

$$= -\sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+50} \frac{1+q^{60n+31}}{1+q^{180n+93}},$$

$$= \sum_{n=-\infty}^{\infty} (-1)^n q^{90n^2+150n+59} \frac{1+q^{60n+51}}{1+q^{180n+153}}.$$

After, we have:

$$(q^{60};q^{60})_\infty\, q^2\, \frac{X(\omega q)-X(\omega^2 q)}{\omega-\omega^2}$$

$$= -\frac{q^2}{\omega-\omega^2}\Bigg(\sum_{n=-\infty}^{\infty} (-1)^n\, \omega q^{90n^2+30n-2}\, \frac{1-\omega^2 q^{120n+2}}{1+q^{180n+3}}$$

$$-\sum_{n=-\infty}^{\infty} (-1)^n\, \omega^2 q^{90n^2+30n-2}\, \frac{1-\omega q^{120n+2}}{1+q^{180n+3}}\Bigg)$$

$$+\frac{q^2}{\omega-\omega^2}\Bigg(\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+150n+39}\, \frac{1+q^{60n+21}}{1+q^{180n+63}}$$

$$-\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+150n+39}\, \frac{1+q^{60n+21}}{1+q^{180n+63}}\Bigg)$$

$$+\frac{q^2}{\omega-\omega^2}\Bigg(\sum_{n=-\infty}^{\infty} (-1)^n\, \omega q^{90n^2+150n+58}\, \frac{1+\omega^2 q^{60n+41}}{1+q^{180n+123}}$$

$$-\sum_{n=-\infty}^{\infty} (-1)^n\, \omega^2 q^{90n^2+150n+58}\, \frac{1+\omega q^{60n+41}}{1+q^{180n+123}}\Bigg)$$

$$-\frac{q^2}{\omega-\omega^2}\Bigg(\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+30n}\, \frac{1-\omega q^{120n+22}}{1+q^{180n+33}}$$

$$-\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+30n}\, \frac{1-\omega^2 q^{120n+22}}{1+q^{180n+33}}\Bigg)$$

$$+\frac{q^2}{\omega-\omega^2}\Bigg(\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+150n+51}\, \frac{1+\omega q^{60n+31}}{1+q^{180n+93}}$$

$$-\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+150n+51}\, \frac{1+\omega^2 q^{60n+31}}{1+q^{180n+93}}\Bigg)$$

$$+\frac{q^2}{\omega-\omega^2}\Bigg(\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+150n+60}\, \frac{1+q^{60n+51}}{1+q^{180n+153}}$$

$$-\sum_{n=-\infty}^{\infty} (-1)^n\, q^{90n^2+150n+60}\, \frac{1+q^{60n+51}}{1+q^{180n+153}}\Bigg)$$

After:

$$(q^{60};q^{60})_\infty\left(\chi(q^9)+q^2\,\frac{X(\omega q)-X(\omega^2 q)}{\omega-\omega^2}\right)$$

$$=-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1+q^{180n+33}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+123}}$$

$$-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1+q^{180n+93}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+3}}$$

$$-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+3}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+123}}$$

$$+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1+q^{180n+33}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1+q^{180n+93}}$$

$$+\frac{(q^{45};q^{45})_\infty(q^{90};q^{90})_\infty\,f(-q^9,-q^{36})}{f(-q^{18},-q^{27})\,f(-q^{36},-q^{54})}\,q^9(q^{60};q^{60})_\infty$$

$$+q^{18}\left(q^6\,\frac{(q^{90};q^{90})^3_\infty\,f(q^{21},q^{69})\,f(-q^{12},-q^{168})}{f(q^{45},q^{135})\,f(-q^{36},-q^{54})\,f(q^{33},q^{57})}\right.$$

$$\left.-\frac{(q^{90};q^{90})^3_\infty\,f(q^{39},q^{51})\,f(-q^{48},-q^{132})}{q^{18}\,f(q^{45},q^{135})\,f(-q^{36},-q^{54})\,f(q^3,q^{87})}\right)$$

$$+\frac{f(-q^3,-q^{27})^2\,f(-q^{15},-q^{45})}{(q^{30};q^{30})^4_\infty\,f(-q^6,-q^{54})^2}$$

$$\times\left\{\frac{(q^{45};q^{45})_\infty(q^{60};q^{60})^3_\infty\,f(-q^{15},-q^{75})\,f(-q^{18},-q^{72})\,f(-q^{36},-q^{144})}{(q^{15};q^{15})_\infty(q^{30};q^{30})_\infty(q^{90};q^{90})^3_\infty(q^{180};q^{180})_\infty\,f(-q^{12},-q^{48})}\right.$$

$$\times\left(f(-q^{45},-q^{45})\,f(-q^{33},-q^{57})^2\,f(-q^{27},-q^{63})^2\right.$$

$$
\begin{aligned}
&+q^{18} f(-q^{45}, -q^{45}) f(-q^{3}, -q^{87})^2 f(-q^{27}, -q^{63})^2 \\
&-q^{21} f(-q^{15}, -q^{75}) f(-q^{3}, -q^{87})^2 f(-q^{33}, -q^{57})^2) \\
&-q^{6} \frac{f(-q^{3}, -q^{27}) f(q^{3}, q^{27}) f(-q^{15}, -q^{75}) f(-q^{18}, -q^{72})}{(q^{15}; q^{15})_\infty^2} \\
&\times(-q^{9} f(-q^{33}, -q^{57}) f(-q^{15}, -q^{75})^2 \\
&+f(-q^{27}, -q^{63}) f(-q^{45}, -q^{45})^2 \\
&+q^{18} f(-q^{3}, -q^{87}) f(-q^{15}, -q^{75})^2) \\
&+q^{15} \frac{(q^{60}; q^{60})_\infty f(-q^{3}, -q^{27})^2 f(-q^{15}, -q^{75}) f(-q^{18}, -q^{72})}{(q^{30}; q^{30})_\infty^3} \\
&\times f(-q^{30}, -q^{150})(q^{6} f(-q^{27}, -q^{63}) f(-q^{15}, -q^{75}) \\
&-q^{9} f(-q^{3}, -q^{87}) f(-q^{45}, -q^{45}) \\
&+f(-q^{33}, -q^{57}) f(-q^{45}, -q^{45}))\} \\
&-q^{4} \frac{(q^{3}; q^{3})_\infty^2 (q^{18}; q^{18})_\infty^2 (q^{30}; q^{30})_\infty (q^{60}; q^{60})_\infty}{(q^{6}; q^{6})_\infty^2 (q^{9}; q^{9})_\infty f(-q^{12}, -q^{18})} \\
=&-q^{3}(q^{60}; q^{60})_\infty \left(\frac{(q^{3}; q^{3})_\infty (q^{9}; q^{9})_\infty^2 (q^{30}; q^{30})_\infty}{(q^{6}; q^{6})_\infty (q^{18}; q^{18})_\infty f(-q^{12}, -q^{18})}\right. \\
&\left.+q \frac{(q^{3}; q^{3})_\infty^2 (q^{18}; q^{18})_\infty^2 (q^{30}; q^{30})_\infty}{(q^{6}; q^{6})_\infty^2 (q^{9}; q^{9})_\infty f(-q^{12}, -q^{18})}\right) \\
=&-q^{3}(q^{60}; q^{60})_\infty \frac{(q^{3}; q^{3})_\infty f(q, q^{3}) f(-q^{6}, -q^{24})}{(q^{6}; q^{6})_\infty f(q^{3}, q^{9})}. \qquad (4.26)
\end{aligned}
$$

Thence:

$$(q^{60};q^{60})_\infty\left(\chi(q^9)+q^2\frac{X(\omega q)-X(\omega^2 q)}{\omega-\omega^2}\right)$$

$$=-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1+q^{180n+33}}-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+123}}$$

$$-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1+q^{180n+93}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+3}}$$

$$-\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+30n}}{1+q^{180n+3}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+60}}{1+q^{180n+123}}$$

$$+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+150n+24}}{1+q^{180n+33}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{90n^2+210n+84}}{1+q^{180n+93}}$$

In conclusion, we have:

[[[-(((0.5)^684))/(((1+(0.5)^393))) - ((0.5)^720))/(((1+(0.5)^483))) - (((0.5)^864))/(((1+(0.5)^453))+(((0.5)^420))/(((1+(0.5)^363))))))]]]

Input:

$$-\frac{0.5^{684}}{1+0.5^{393}}-\frac{0.5^{720}}{1+0.5^{483}}-\frac{0.5^{864}}{1+0.5^{453}}+\frac{0.5^{420}}{1+0.5^{363}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$3.6931914471142943123035723049987333751394454880226 04\ldots\times 10^{-127}$

3.69319144711429431230357230499873337513944548802260 4 × 10^-127

-[[[-(((0.5)^420))/(((1+(0.5)^363))) + ((0.5)^720))/(((1+(0.5)^483)))+(((0.5)^684))/(((1+(0.5)^393)))+(((0.5)^864))/(((1+(0.5)^453))))))]]]

Input:

$$-\left(-\frac{0.5^{420}}{1+0.5^{363}}+\frac{0.5^{720}}{1+0.5^{483}}+\frac{0.5^{684}}{1+0.5^{393}}+\frac{0.5^{864}}{1+0.5^{453}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$3.693191447114294312303572304998733375139445488022604\ldots \times 10^{-127}$

3.693191447114294312303572304998733375139445488022604 × 10^-127

We have already obtained (see pag.101) a similar result:

3.693191447114294312303572304998733375139445488022604 × 10^-127

We have that:

3.693191447114294312303572304998733375139445488022604 × 10^-127 + 3.693191447114294312303572304998733375139445488022604 × 10^-127

Input interpretation:

$$\frac{3.693191447114294312303572304998733375139445488022604}{10^{127}} + \frac{3.693191447114294312303572304998733375139445488022604}{10^{127}}$$

Open code

Result:

- More digits

$7.386382894228588624607144609997466750278890976045207\ldots \times 10^{-127}$

7.386382894228588624607144609997466750278890976045207 × 10^-127

(7.386382894228588624607144609997466750278890976045207 × 10^-127)^1/(17/2) + (7.386382894228588624607144609997466750278890976045207 × 10^-127)^1/8

Input interpretation:

$$\sqrt[\frac{17}{2}]{\frac{7.386382894228588624607144609997466750278890976045207}{10^{127}}} + \sqrt[8]{\frac{7.386382894228588624607144609997466750278890976045207}{10^{127}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$1.619964891245422580605314142285742263194948140348942 5... \times 10^{-15}$

1.6199648912454225806053141422857422631949481403489425 × 10^-15

From this formula, we can to obtain a value very near to the electric charge of positron:

Input interpretation:

$$\left(\sqrt[\frac{17}{2}]{\frac{7.3863828942285886246071446099974667502788909760452 07}{10^{127}}} + \sqrt[8]{\frac{7.3863828942285886246071446099974667502788909760452 07}{10^{127}}}\right) \times \frac{1}{10^4}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

$1.619964891245422580605314142285742263194948140348942 5... \times 10^{-19}$

1.6199648912454225806053141422857422631949481403489425 × 10^-19

and to the golden ratio:

Input interpretation:

$$\left(\sqrt[\frac{17}{2}]{\frac{7.3863828942285886246071446099974667502788909760452 07}{10^{127}}} + \sqrt[8]{\frac{7.3863828942285886246071446099974667502788909760452 07}{10^{127}}}\right) \times 10^{15}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Result:

- More digits

1.6199648912454225806053141422857422631949481403489425...

1.6199648912454225806053141422857422631949481403489425

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{36+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

- More

$$-e^{-4-1/e+e-7/\pi+63\pi}\,\pi^{-63-46e}\csc^8(e\pi)\sec^{43}(e\pi)\approx 1.6199648912454225\,63071$$

$$\frac{-5e^{\pi}+777\pi+485\log(\pi)+240\log(2\pi)-456\tan^{-1}(\pi)}{1695}\approx 1.6199648912454225806\,019694$$

$$\frac{1}{20}\sqrt[3]{\frac{866947}{23560485}}\,\pi^4\approx 1.619964891245422\,64436$$

Or:

(7.386382894228588624607144609997466750278890976045207 × 10^-127)^0.12519477

Input interpretation:

$$\left(\frac{7.386382894228588624607144609997466750278890976045207}{10^{127}}\right)^{0.12519477}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.61803\ldots \times 10^{-16}$

1.6180328386907909951925686060756835751998892273 × 10^-16

(7.3863828942285886246071446099974667502788909760452O7 × 10^-127)^0.12519477 * 10^16

Input interpretation:

$$\left(\frac{7.3863828942285886246071446099974667502788909760452O7}{10^{127}}\right)^{0.12519477} \times 10^{16}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

1.6180328386907909951925686060756835751998892273904630 10875...

1.6180328386907909951925686060756835751998892273904630

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{51+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive
Possible closed forms:

- More

$\frac{2}{7}\pi\tanh^{-1}\left(\frac{4566617}{5235146}\right)^2 \approx 1.61803283869079099507030$

$\left(\frac{20830391}{50456154}\right)^{3/4}\pi \approx 1.61803283869079100053$

$\frac{857063128\pi}{1664084413} \approx 1.61803283869079099524625$

•

This value is practically a very good approximation to the golden ratio!

Input:

$\frac{1}{2}\left(\sqrt{5}+1\right)$

Open code

Enlarge Data Customize A Plaintext Interactive
Decimal approximation:

- More digits

1.6180339887498948482045868343656381177203091798057628621354...

Open code

1.618033988749894848204586834365638117720309179805762862135

Possible closed forms:

- More

$\phi \approx 1.61803398874989484820458683436563811772030917980576286213544862$

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$\Phi + 1 \approx$
$1.61803398874989484820458683436563811772030917980576286213544862$

$\frac{1}{\Phi} \approx 1.61803398874989484820458683436563811772030917980576286213544862$

•

We now discuss the assertion in (14.2.2). L. Dragonette [127] first proved that if

$$f_3(q) =: \sum_{n=0}^{\infty} a(n)q^n, \tag{14.3.1}$$

then

$$a(n) = \sum_{k=1}^{[\sqrt{n}]} \frac{(-1)^{[(k+1)/2]} A_{2k}(n - k(1+(-1)^k)/4) \sinh\left(\frac{\pi}{k}\sqrt{\frac{n}{6} - \frac{1}{144}}\right)}{\sqrt{k(n - \frac{1}{24})}} + O(n^{\frac{1}{2}+\epsilon}), \tag{14.3.2}$$

for each $\epsilon > 0$, where $A_k(n)$ denotes the same sum that appears in the Hardy–Ramanujan–Rademacher formula for the partition function $p(n)$, i.e.,

$$A_k(n) = \sum_{\substack{h \pmod{k} \\ (h,k)=1}} \omega_{h,k} e^{-2\pi i h n/k}, \tag{14.3.3}$$

where $\omega_{h,k}$ is a certain 24kth root of unity. See [17, pp. 70–71] for a more

For n = 200, ih = 1, k = 13 and ω = 1, we obtain:

e^(((-2Pi*200)/13))

Input:

$e^{1/13(-2\pi\times 200)}$

Open code

Enlarge Data Customize A Plaintext Interactive

Exact result:

$e^{-(400\pi)/13}$

Decimal approximation:

- More digits

$1.0451752393602095105741300174119854767282393511712679... \times 10^{-42}$

Open code

Property:

$e^{-(400\pi)/13}$ is a transcendental number

Open code

200 * e^(((-2Pi*200)/13)) * ((((((((sinh(((Pi/13 ((sqrt(200/6 - 1/144)))))))))) * ((((((((1/((sqrt((13*(200-1/24))))))))))

Input:

$$200\, e^{1/13(-2\pi\times 200)}\left(\sinh\left(\frac{\pi}{13}\sqrt{\frac{200}{6}-\frac{1}{144}}\right)\times\frac{1}{\sqrt{13\left(200-\frac{1}{24}\right)}}\right)$$

Open code

- $\sinh(x)$ is the hyperbolic sine function

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Exact result:

$$400\sqrt{\frac{6}{62\,387}}\, e^{-(400\pi)/13}\sinh\left(\frac{\sqrt{4799}\,\pi}{156}\right)$$

Decimal approximation:

- More digits

$$7.764275431658444846078520960340891990261814974611327 3\ldots\times 10^{-42}$$

Open code

7.7642754316584448460785209603408919902618149746113273 × 10^-42

Continued fraction:

- Linear form

$$\cfrac{1}{128\,795\,018\,775\,679\,956\,128\,357\,908\,865\,686\,531\,486\,706+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{11+\cfrac{1}{\ldots}}}}}}$$

Series representations:

- More

$$\frac{\left(200\, e^{-(2\pi\, 200)/13}\right)\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}=400\sqrt{\frac{6}{62\,387}}\, e^{-(400\pi)/13}\sum_{k=0}^{\infty}\frac{156^{-1-2k}\times 4799^{1/2+k}\,\pi^{1+2k}}{(1+2k)!}$$

Open code

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$$\frac{\left(200\, e^{-(2\pi\, 200)/13}\right)\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}=800\sqrt{\frac{6}{62\,387}}\, e^{-(400\pi)/13}\sum_{k=0}^{\infty}I_{1+2k}\left(\frac{\sqrt{4799}\,\pi}{156}\right)$$

Open code

$$\frac{\left(200\, e^{-(2\pi\, 200)/13}\right) \sinh\left(\frac{1}{13}\,\pi \sqrt{\frac{200}{6} - \frac{1}{144}}\right)}{\sqrt{13\left(200 - \frac{1}{24}\right)}} = 400\, i \sqrt{\frac{6}{62\,387}}\, e^{-(400\pi)/13} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{156}\left(-78\, i + \sqrt{4799}\right)\pi\right)^{2k}}{(2k)!}$$

Integral representations:

$$\frac{\left(200\, e^{-(2\pi\, 200)/13}\right) \sinh\left(\frac{1}{13}\,\pi \sqrt{\frac{200}{6} - \frac{1}{144}}\right)}{\sqrt{13\left(200 - \frac{1}{24}\right)}} = \frac{100}{13} \sqrt{\frac{2}{39}}\, e^{-(400\pi)/13}\, \pi \int_0^1 \cosh\left(\frac{1}{156} \sqrt{4799}\, \pi\, t\right) dt$$

Open code

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$$\frac{\left(200\, e^{-(2\pi\, 200)/13}\right) \sinh\left(\frac{1}{13}\,\pi \sqrt{\frac{200}{6} - \frac{1}{144}}\right)}{\sqrt{13\left(200 - \frac{1}{24}\right)}} = -\frac{25}{13}\, i\, e^{-(400\pi)/13} \sqrt{\frac{2\pi}{39}} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{(4799\pi^2)/(97\,344\, s)+s}}{s^{3/2}}\, ds \quad \text{for } \gamma > 0$$

1/[200 * e^(((-2Pi*200)/13)) * ((((((((sinh(((Pi/13 ((sqrt(200/6 - 1/144)))))))) * ((((((((1/((sqrt((13*(200-1/24)))))))))]

Input:

$$\frac{1}{200\, e^{1/13\,(-2\pi\times 200)} \left(\sinh\left(\frac{\pi}{13} \sqrt{\frac{200}{6} - \frac{1}{144}}\right) \times \frac{1}{\sqrt{13\left(200 - \frac{1}{24}\right)}}\right)}$$

Open code

- $\sinh(x)$ is the hyperbolic sine function

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Exact result:

$$\frac{1}{400}\sqrt{\frac{62387}{6}}\, e^{(400\pi)/13}\operatorname{csch}\left(\frac{\sqrt{4799}\,\pi}{156}\right)$$

- $\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

- More digits

$1.2879501877567995612835790886568653148670674488913636\ldots \times 10^{41}$

Open code

Continued fraction:

- Linear form

$$128795018775679956128357908865686531486706 + \cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{11+\cfrac{1}{\ldots}}}}}$$

Series representations:

- More

$$\frac{1}{\frac{\left(200\, e^{-(2\pi\, 200)/13}\right)\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}} = \frac{62387\sqrt{\frac{39}{2}}\, e^{(400\pi)/13}\sum_{k=-\infty}^{\infty}\frac{(-1)^k}{4799+24336k^2}}{100\pi}$$

Open code

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$$\frac{1}{\frac{\left(200\, e^{-(2\pi\, 200)/13}\right)\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}} = \frac{1}{200}\sqrt{\frac{62387}{6}}\, e^{-1/156\left(-4800+\sqrt{4799}\right)\pi}\sum_{k=0}^{\infty} e^{-1/78\sqrt{4799}\,k\,\pi}$$

Open code

$$\frac{1}{\frac{\left(200\, e^{-(2\pi\, 200)/13}\right)\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}} = -\frac{1}{200}\sqrt{\frac{62387}{6}}\, e^{(400\pi)/13}\sum_{k=1}^{\infty} q^{-1+2k}$$

for $q = e^{\left(\sqrt{4799}\,\pi\right)/156}$

Open code

- More information

Integral representations:

$$\frac{1}{\frac{200 e^{-(2\pi\, 200)/13}\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}} = \frac{13\, e^{(400\pi)/13}\sqrt{\frac{62\,387}{24}}}{200\pi\sqrt{\frac{4799}{144}}\int_0^1 \cosh\left(\frac{1}{13}\pi t\sqrt{\frac{4799}{144}}\right)dt}$$

Integral representations:

$$\frac{1}{\frac{200 e^{-(2\pi\, 200)/13}\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}} = \frac{13\, e^{(400\pi)/13}\sqrt{\frac{62\,387}{24}}}{200\pi\sqrt{\frac{4799}{144}}\int_0^1 \cosh\left(\frac{1}{13}\pi t\sqrt{\frac{4799}{144}}\right)dt}$$

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$$\frac{1}{\frac{200 e^{-(2\pi\, 200)/13}\sinh\left(\frac{1}{13}\pi\sqrt{\frac{200}{6}-\frac{1}{144}}\right)}{\sqrt{13\left(200-\frac{1}{24}\right)}}} = \frac{13\, e^{(400\pi)/13}\, i\sqrt{\frac{62\,387}{24}}}{50\sqrt{\frac{4799}{144}}\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s+\left(\pi^2\sqrt{\frac{4799}{144}}\right)^2/(676 s)}}{s^{3/2}}\,ds} \quad \text{for } \gamma > 0$$

Note that, the square of the result is:

(1.2879501877567995612835790886568653148670674488913636 × 10^41)^2

Input interpretation:

$$(1.2879501877567995612835790886568653148670674488913636\times 10^{41})^2$$

Open code

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Result:

- More digits

1.6588156861427752425261528704888163029227154012609944... × 10^{82}
Open code

This result is a multiple of a golden number 1,658815…

Indeed renormalizing the exponent (by multiplication of the inverse of the 10^{82}), we obtain:

1/10^82 * (1.2879501877567995612835790886568653148670674488913636 × 10^41)^2

Input interpretation:

$$\frac{1}{10^{82}}\left(1.2879501877567995612835790886568653148670674488913636\times 10^{41}\right)^2$$

Open code

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Result:

- More digits
1.658815686142775242526152870488816302922715401260994498255...

1.6588156861427752425266...

With conclusive observation, if we take some results concerning the "golden numbers", i.e.:

1,638834 1,6389614 1,6439248 1,6548823 1,65528 1,6556876 1,6559732

1,65881568 1,66527 1,673513 1,677432

we note that are very near to the fourteenth root of following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3 = 1164{,}269601267364$$

Indeed:

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots$$

Indeed:

(1.638834 +1.6389614 +1.6439248 +1.6548823 +1.65528 +1.6556876 +1.6559732 +1.65881568 +1.66527 +1.673513 +1.677432)/11 =

= 1.65623399818181818181818....

that is the mean value

Continued fraction:

- Linear form

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{60+\cfrac{1}{8+\cfrac{1}{5+\cfrac{1}{29+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

Open code

Possible closed forms:

- More

$$\log\left(\frac{1}{12}\left(81-159\sqrt{2}+376e-220e^2+38\pi+70\pi^2\right)\right)\approx$$

1.65623399818181818135047

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$$\frac{1851008899\pi}{3511047331}\approx 1.65623399818181818108037$$

$$\pi\ \text{root of } 388x^5-647x^4+311x^3-37x^2-960x+505 \text{ near } x=0.527196\approx$$

1.65623399818181818163112

Continued fraction:

- Linear form

$$4+\cfrac{1}{1+\cfrac{1}{30+\cfrac{1}{1+\cfrac{1}{19+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{15+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{8+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}$$

Open code

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Possible closed forms:

- More

$$\frac{5553026697\pi}{3511047331} \approx 4.968701994545454545424111$$

$$\text{root of } 36x^5 - 222x^4 + 95x^3 + 491x^2 + 306x + 990 \text{ near } x = 4.9687 \approx 4.968701994545454545460113$$

$$\frac{840\pi\pi! - 9381 - 1590\pi + 7373\pi^2}{4956\pi} \approx 4.968701994545454545447602$$

And from the mean value, we obtain:

1.6562339981818... * 3 = 4.96870199454545....

where 4.96870199454545.... is very near to the first value of upper bound dark photon energy range $4.95 * 10^{16}$.

Appendix A

From:

Phenomenological consequences of superfluid dark matter with baryon-phonon coupling
Lasha Berezhiani
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Benoit Famaey

Universite de Strasbourg, CNRS UMR 7550, Observatoire astronomique de Strasbourg,
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Justin Khoury
Center for Particle Cosmology, Department of Physics and Astronomy,
University of Pennsylvania, Philadelphia PA 19104, USA
(Dated: November 17, 2017)

Using (22) this translates to an upper bound on the mass of the DM particle:

$$m \lesssim 4.2 \left(\frac{\sigma/m}{\mathrm{cm}^2/\mathrm{g}} \right)^{1/4} \ \mathrm{eV}\,. \tag{24}$$

Smaller and less massive galaxies result in a somewhat weaker bound.

The bound (24) on the DM particle mass is the main result of this Section. It shows that for values of σ/m satisfying the merging-cluster bound $\sim 1\,\mathrm{cm}^2/\mathrm{g}$ [85–88], m must be somewhat below 4 eV. The dependence on the cross section is rather weak, however, scaling as the 1/4 power. It should be mentioned that the upper bound (24) would be somewhat tighter had we assumed a $\rho \propto r^{-2}$ transition density profile outside the superfluid core, instead of $\rho \propto r^{-3}$.

From:

Received: September 7, 2007 - Accepted: October 28, 2007 - Published: November 9, 2007 - **Three-dimensional AdS gravity and extremal CFTs at c = 8m**
Spyros D. Avramis, Alex Kehagias and Constantina Mattheopoulou

m	L_0	d	S	S_{BH}
3	1	196883	12.1904	12.5664
	2	21296876	16.8741	17.7715
	3	842609326	20.5520	21.7656
4	2/3	139503	11.8458	11.8477
	5/3	69193488	18.0524	18.7328
	8/3	6928824200	22.6589	23.6954
5	1/3	20619	9.9340	9.3664
	4/3	86645620	18.2773	18.7328
	7/3	24157197490	23.9078	24.7812

m	L_0	d	S	S_{BH}
6	1	42987519	17.5764	17.7715
	2	40448921875	24.4233	25.1327
	3	8463511703277	29.7668	30.7812
7	2/3	7402775	15.8174	15.6730
	5/3	33934039437	24.2477	24.7812
	8/3	16953652012291	30.4615	31.3460
8	1/3	278511	12.5372	11.8477
	4/3	13996384631	23.3621	23.6954
	7/3	19400406113385	30.5963	31.3460

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of m and L_0.

From:

Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES** - HADES Collaboration

A B S T R A C T

We present a search for the e^+e^- decay of a hypothetical dark photon, also named U vector boson, in inclusive dielectron spectra measured by HADES in the p(3.5 GeV) + p, Nb reactions, as well as the Ar (1.756 GeV/u) + KCl reaction. An upper limit on the kinetic mixing parameter squared ϵ^2 at 90% CL has been obtained for the mass range $M_U = 0.02$–0.55 GeV/c^2 and is compared with the present world data set. For masses 0.03–0.1 GeV/c^2, the limit has been lowered with respect to previous results, allowing now to exclude a large part of the parameter region favored by the muon $g-2$ anomaly. Furthermore, an improved upper limit on the branching ratio of 2.3×10^{-6} has been set on the helicity-suppressed direct decay of the eta meson, $\eta \to e^+e^-$, at 90% CL.

olution. The upper frame of Fig. 2(a) shows the mass resolution obtained from a GEANT3-based Monte Carlo of e^+e^- decays detected in the HADES detector. The calculated peak width increases gradually with pair mass from about 15 MeV (fwhm) in the π^0 region to about 30 MeV at the η mass of 0.55 GeV/c^2.

The present analysis is based on the raw dilepton mass spectra, exhibited in Fig. 2(b), i.e. spectra not corrected for efficiency and acceptance. The low invariant-mass region of the spectra ($M_{ee} <$ 0.13 GeV/c^2) is dominated by π^0 Dalitz decays, at intermediate masses (0.13 GeV/$c^2 < M_{ee} <$ 0.55 GeV/c^2), η and Δ Dalitz decays prevail, and the high-mass region is populated mostly by low-energy tails of vector-meson decays [34,35]. However, as the electromagnetic decay branching ratios decrease with increasing particle mass, resulting in low sensitivity, we restrict our search to $M_U <$ 0.6 GeV/c^2.

6. Summary and outlook

Searching for a narrow resonance in dielectron spectra measured with HADES in the reactions p(at 3.5 GeV) + p, Nb, as well as Ar(at 1.756 GeV/u) + KCl we have established an upper limit at 90% CL on the mixing $\epsilon^2 = \alpha'/\alpha$ of a hypothetical dark photon U in the mass range $M_U = 0.02$–0.6 GeV/c^2. Our UL sets a tighter constraint than the recent WASA search at low masses excluding to a large extent the parameter space preferred by the muon $g-2$ anomaly. At higher masses, already surveyed by the recent KLOE-2 search, our analysis provides complementary information. We have thus covered for the first time in one and the same experiment a rather broad mass range. In addition, we have reduced the UL on the direct decay $\eta \rightarrow e^+e^-$ by a factor 2.5 with respect to the known limit to 2.3×10^{-6}. In future experiments at the FAIR facility we expect to be able to increase our sensitivity by up to one order of magnitude.

From the values of the masses 0,02 0,55 and 0,6 GeV/c^2, we obtain the following values in energy:

$1{,}8 * 10^{15}$ GeV - $4{,}95 * 10^{16}$ - $5{,}4 * 10^{16}$

Appendix B

From:
https://readingfeynman.org/tag/fine-structure-constant/

The fine-structure constant

I wrote that the 'set' of equations $c = \hbar = k_B = G = 1$ gave us Planck units for *most* of our SI base units. It turns out that these four equations do *not* lead to a 'natural' unit for electric charge. We need to equate a fifth constant with one to get that. That fifth constant is *Coulomb's constant* (often denoted as k_e) and, yes, it's the constant that appears in Coulomb's Law indeed, as well as in some other pretty fundamental equations in electromagnetics, such as the field caused by a point charge q: E =

$q/4\pi\varepsilon_0 r^2$. Hence, $k_e = 1/4\pi\varepsilon_0$. So if we equate k_e with one, then ε_0 will, obviously, be equal to $\varepsilon_0 = 1/4\pi$.

To make a long story short, adding this fifth equation to our set of five also gives us a Planck charge, and I'll give you its value: it's about 1.8755×10^{-18} C. As I mentioned that the elementary charge is 1 e ≈ 1.6022×10^{-19} C, it's easy to that the Planck charge corresponds to some 11.7 times the charge of the proton. In fact, let's be somewhat more precise and round, once again, to four digits after the decimal point: the q_P/e ratio is about 11.7062. Conversely, we can also say that the elementary charge as expressed in Planck units, is about 1/11.7062 ≈ 0.08542455. In fact, we'll use that ratio in a moment in some other calculation, so please jot it down.

0.08542455? That's a bit of a weird number, isn't it? You're right. And trying to write it in terms of the charge of a *u* or *d* quark doesn't make it any better. Also, note that the first four significant digits (8542) correspond to the first four significant digits *after the decimal point* of our ε_0 constant. So what's the physical significance here? Some other limit of quantum theory?

Frankly, I did not find anything on that, but the obvious thing to do is to relate is to what is referred to as the fine-structure constant, which is denoted by α. This physical constant is dimensionless, and can be defined in various ways, but all of them are some kind of ratio of a bunch of these physical constants we've been talking about:

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} = \frac{\mu_0}{4\pi}\frac{e^2 c}{\hbar} = \frac{k_e e^2}{\hbar c} = \frac{c\mu_0}{2R_K} = \frac{m_e c r_e}{\hbar}$$

The only constants you have *not* seen before are μ_0, R_K and, perhaps, r_e as well as m_e. However, these can be defined as a function of the constants that you *did* see before:

1. The μ_0 constant is the so-called magnetic constant. It's something similar as ε_0 and it's referred to as the *magnetic* permeability of the vacuum. So it's just like the (electric) *permittivity of the vacuum* (i.e. the electric constant ε_0) and the only reason why you haven't heard of this before is because we haven't discussed magnetic fields so far. In any case, you know that the electric and magnetic force are part and parcel of the same phenomenon (i.e. the *electromagnetic* interaction between charged particles) and, hence, they are closely related. To be precise, $\mu_0 = 1/\varepsilon_0 c^2$. That shows the first and second expression for α are, effectively, fully equivalent.
2. Now, from the definition of $k_e = 1/4\pi\varepsilon_0$, it's easy to see how those two expressions are, in turn, equivalent with the third expression for α.
3. The R_K constant is the so-called *von Klitzing* constant, but don't worry about it: it's, quite simply, equal to $R_K = h/e^2$. Hene, substituting (and don't forget that $h = 2\pi\hbar$) will demonstrate the equivalence of the fourth expression for α.
4. Finally, the r_e factor is the classical electron radius, which is usually written as a function of m_e, i.e. the electron mass: $r_e = e^2/4\pi\varepsilon_0 m_e c^2$. This very same equation implies that $r_e m_e = e^2/4\pi\varepsilon_0 c^2$. So… Yes. It's all the same really.

Let's calculate its (rounded) value in the old units first, using the third expression:

- The e^2 constant is (roughly) equal to $(1.6022\times10^{-19}\ C)^2 = 2.5670\times10^{-38}\ C^2$. Coulomb's constant $k_e = 1/4\pi\varepsilon_0$ is about $8.9876\times10^9\ N\cdot m^2/C^2$. Hence, the numerator $e^2k_e \approx 23.0715\times10^{-29}\ N\cdot m^2$.
- The (rounded) denominator is $\hbar c = (1.05457\times10^{-34}\ N\cdot m\cdot s)(2.998\times10^8\ m/s) = 3.162\times10^{-26}\ N\cdot m^2$.
- Hence, we get $\alpha = k_e e^2/\hbar c \approx 7.297\times10^{-3} = 0.007297$.

Note that this number is, effectively, *dimensionless*. Now, the interesting thing is that if we calculate α using Planck units, we get an e^2 constant that is (roughly) equal to $0.08542455^2 = \dots\ 0.007297$

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[4] **viXra:1906.0253** *submitted on 2019-06-15 02:09:19*, (9 unique-IP downloads)
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Authors: Michele Nardelli, Antonio Nardelli
Category: Mathematical Physics

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