Unified Electro-Gravity (UEG) Theory Applied to Spiral Galaxies

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The unified electro-gravity (UEG) theory, which has been successfully used for modeling elementary particles, as well as single and binary stars, is extended in this paper to model gravitation in spiral galaxies. A new UEG model would explain the "flat rotation curves" commonly observed in the spiral galaxies. The UEG theory is developed in a fundamentally different manner for a spiral galaxy, as compared to prior applications of the UEG theory to the elementary particle and single stars. This is because the spiral galaxy, unlike the elementary particles or single stars, is not spherically symmetric. The UEG constant γ, required in the new model to support the galaxies' flat rotation speeds, is estimated using measured data from a galaxy survey, as well as for a selected galaxy for illustration. The estimates are compared with the γ derived from a UEG model of elementary particles. The UEG model for the galaxy is shown to explain the empirical Tuly-Fisher Relationship (TFR), is consistent with the Modified Newtonian Dynamics (MOND), and is also independently supported by measured trends of galaxy thickness with surface brightness and rotation speed.

I. INTRODUCTION

Rotation curves of spiral galaxies [1] have been suspected not to confirm to gravitational forces due to galaxies' visible mass as per the Newton's law of gravitation, which is known to work well in our day-to-day experience on earth as well for planetary orbits in our solar system. In order to explain the observed rotation curves, it has been proposed and long believed that there is significant amount of invisible "dark matter" surrounding almost all spiral galaxies. There was no other existing theory which could explain the rotation behavior in a satisfactory manner, although modification of the laws of Newtonian dynamics has been proposed [2]. Recently, a new unified electro-gravity (UEG) theory is established, which has been successfully applied to model elementary particles [3, 4], where a new gravitational force, proportional to electromagnetic energy density, is introduced. This UEG theory has also been extended to model energy generation in single stars [5], which are spherically symmetric bodies like the elementary particles. However, the theory needed some basic modification when it was extended to model orbiting of a binary-star system [5], in order to accommodate the spherical asymmetry of the binary system. In this paper, the UEG theory would be applied to a spiral galaxy, which is another different non-spherical body. The energy density due to star lights in the galaxy would contribute to a new gravitational force, which could support the observed stellar rotation around the galaxy. A constant rotation speed beyond certain radial distance would require a $1/r$-dependent gravitational acceleration, in the given region. When the UEG theory is properly modified for the non-spherical structure of a spiral galaxy, the required $1/r$-dependent acceleration may result, although the stellar light radiation from the galaxy exhibit an approximate $1/r^2$ dependence, in the given region. This is possible, because the energy density of the actual light radiation may need to be redistributed, based on the physical asymmetry of the spiral galaxy. The UEG field may be defined in proportion to the redistributed, effective energy density, so that the field may satisfy certain basic requirements and be self-consistent when applied to general problems.

The required UEG constant γ of proportionality, between the UEG field and the associated effective energy density, may be deduced from the new UEG model using measured data from galaxy survey as well as data for selected individual galaxies. The results may be compared with the UEG constant deduced from a UEG theory of elementary particles, for validation or verification of the new UEG model. The functional trends established from the new UEG model may be compared, for validation of the model, with those from the empirical Tully-Fisher Relation (TFR) [6] and the Modified Newtonian Dynamics (MOND) model [2, 7]. The trends predicted from the UEG model would explicitly depend upon the spiral galaxy's aspect ratio (ratio of the scale lengths in radius and thickness), because the new model is formulated based on the spherical asymmetry of the galaxy. This is distinct from the the MOND model, where there may not be such definitive interrelation between the galaxy's aspect ratio and the rotation speed. The functional dependence of the galaxy's aspect ratio on the surface brightness and rotation velocity, as required for the UEG galaxy model to reproduce the rotation curves, may be compared with available measurements, for another independent validation of the basic UEG galaxy model.

The formulation of the force-field in the UEG model of a spiral galaxy, which is a non-spherical body, is expected to be distinct from that for an elementary particle or an isolated star [3, 5], which are spherical structures. The galaxy's UEG force field is defined in proportion to an effective distribution of energy density, not the actual energy density of stellar radiation as was the case for the spherical structures. The effective energy density is obtained by suitable redistribution of the galaxy's light radiation, in proportion to the distribution of the Newtonian gravitation potential of the galaxy. The divergence of the resulting UEG force field surrounding the galaxy would be equivalent to having a fictitious "dark-matter" distribution, which may be needed in order to explain the observed rotation behavior of the spiral galaxies, as well as formation and evolution of the galaxies, on the basis of the conventional Newtonian gravitation [8]. Beyond a sufficiently large radial distance from the galactic center, the galaxy would "look" like a point source with a spherically symmetric distribution of the Newtonian potential, and with a $1/r^2$ dependence of its light intensity. In this far region the radial UEG field would also be spherically symmetric, and therefore the field would be directly proportional to the $1/r^2$-dependent light's energy density, without any need for redistribution of the energy density as per the proposed model. This spherically symmetric, $1/r^2$-dependent radial UEG field in the far region is associated with zero field divergence, and therefore with no dark matter. In contrast, the region sufficiently close to the center would in general be associated with a strong divergent UEG field, and therefore with a heavy dark-matter distribution. This region of heavy dark-matter presence would at least include the smallest spherical region which encloses most of the galaxy's mass and light sources, and may extend much farther.

Section II presents the theoretical concepts and an analytical formulation of the theory. The results for flat rotation velocity deduced from the model are validated with measured data for a galaxy survey as well as for an individual galaxy, in sections III, IV. The Tully-Fisher Relation (TFR) and the Modified Newtonian Dynamics (MOND) model are studied in section V, in relation to the present UEG galaxy model, for further validation of the model. This is followed by discussion and general conclusion from the study.

II. THEORY

A. The Basic Concept

As per the UEG theory, there exists a new gravitational force-field which is dependent on the electromagnetic energy density. For a simple spherical body, the new UEG field at any particular location is directly proportional to the energy density at the given location, and is directed toward the center of the body [3–5]. Such simple, direct relationship between the UEG field and the energy density may not be valid for a general non-spherical structure, with a non-spherical light distribution. Certain additional conditions may be established for a UEG force-field, which could be implicit in, or consistent with, the simple relationship for a spherical structure. But, the additional conditions may have to be explicitly applied for a general non-spherical structure. The Newtonian gravitational field in a spiral galaxy structure is not directed radially toward the center of the galaxy at every location, unlike that of a spherical structure which is radially directed at every location. Assuming an ideal disk structure for a spiral galaxy, which is independent of the azimuth (ϕ) coordinate, the Newtonian gravitational field may be shown to consist of only the radial (r) and elevation (θ) components, with no ϕ-component. Like the Newtonian gravitation field, the UEG field for a spiral galaxy may not be required to be strictly radial in direction, at all general locations. And, the UEG field for the galaxy may ideally be directed along the galaxy's Newtonian gravitational field with the $r-$ and $\theta-$ components.

We may assume the UEG field to be energy-conservative, which is defined as the gradient of an associated potential function. The desired non-radial (θ-) component of the UEG field for a spiral galaxy, as discussed above, would require the potential function, and accordingly the field, to maintain a gradient or a derivative along the $\theta-$direction. In other words, the distribution of the potential or the field on a spherical surface would be non-uniform in the θ variable. Such spherical asymmetry in the galaxy's UEG field or potential is in distinct contrast to the UEG field or potential for a spherical body, which is uniform on any spherical surface. We may define the UEG field for a spiral galaxy, on any spherical surface of a given radius, to be distributed in proportion to the galaxy's Newtonian potential on the spherical surface. This would ensure the gradients of the UEG and Newtonian potentials in the $\theta-$direction, or equivalently the $\theta-$components of the respective fields, to be in proportion to each other at all points on the surface of the given radius. Further, the gradients of the two potentials in the radial ($r-$) direction, and therefore the $r-$components of the respective fields, may be assumed to be proportionate to each other, at least in terms of their general functional trends on a first-order basis. Accordingly, the essentially proportionate components of the two fields would ensure the UEG and the Newtonian fields to be directed approximately parallel to each other, at all locations, which may be desired as discussed earlier. The above model may be formulated by having the radial UEG field to be proportional to a suitable distribution of an effective energy density, with the UEG constant γ [3, 4] as the constant of proportionality. The effective energy density at any given location is defined by redistribution of the actual energy density of the galaxy's stellar radiation on a spherical surface passing through the location, in proportion to the galaxy's Newtonian potential on the spherical surface. The redistribution would maintain the total integral of the actual and effective energy densities on the spherical surface to be equal to each other, which is a definite measure of the equivalent UEG mass (dark-mass) enclosed inside the sphere.

An additional fundamental condition may need to be enforced in any general UEG field. It would be reasonable to require the total UEG force due to a general distribution of energy density, produced due to the general distribution of its associated sources internal to a massive body, to be zero. Otherwise, a non-zero total force produced by a general source internal to a particular body, acting upon the given body itself, would not be fundamentally sensible. The above UEG model, as specifically proposed for a spiral galaxy, may be verified to enforce this basic condition of having zero total force. The azimuthal (ϕ) symmetry we assumed for an ideal spiral galaxy would ensure the total force to be zero, as required. However, it may also be ensured that this condition of zero total force may not be evidently violated when the UEG model is extended for a more general structure.

It may be argued, that the definition of the UEG field for a spiral galaxy, as proposed above and implemented in the following section II B, is perhaps not the unique or best way to define the desired field. For example, the UEG field could have been defined in direct proportion to the actual energy density, and been non-radially directed along the galaxy's Newtonian gravitation field as may be desired. Such a UEG field may also be shown to satisfy the above required condition of having zero total force. The actual energy density due to the star light in a disk galaxy is ideally independent of the azimuthal angle ϕ. All components of the above alternate UEG field for the disk galaxy, expressed in direct proportion to the galaxy's ϕ-independent actual energy density, may also be shown to produce a total zero force when the field acts upon an ideal ϕ-independent mass distribution of the galaxy. However, this alternate model would evidently fail to produce the required zero total force for a general condition, if either the mass or the light distribution were ϕ-dependent.

Such an alternate model, or any other similar proposition which would lead to such evident invalidity when extended to a general situation, is rejected. Whereas, the original UEG field as proposed earlier and formulated in the following section II B, particularly when the mass distribution is maintained to be symmetric in the azimuth (ϕ) coordinate, may be verified to properly enforce the required condition of having zero total force, for any general distribution of the radiation energy density. This would be based on the symmetry that is maintained in the proposed re-distribution model of energy density, which would result in a $\phi-$independent effective energy density, in proportion to the Newtonian potential of the $\phi-$independent mass distribution, and therefore a proportionate $\phi-$independent UEG field. The $\phi-$independent field, acting upon the ideal $\phi-$independent mass distribution, would produce the required total zero force. On the other hand, if the mass distribution was not ideally symmetric in the ϕ coordinate (with or without a $\phi-$symmetry of the energy density), the required condition of zero total force is expected to be closely established. The proposed UEG field tries to closely mimic the Newtonian gravitational field of the given mass distribution, as discussed before. Therefore, like the zero total force which is guaranteed from the

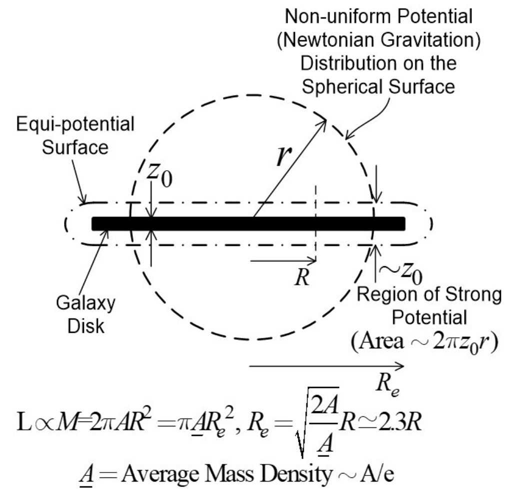

FIG. 1.

Newtonian gravitational field of any general mass distribution, acting upon its own mass distribution, the proposed UEG field would similarly establish the condition of zero total force, at least on a first order basis.

As suggested above, the proposed UEG model, as formulated in the following section (II B), may not be the most rigorous form of the UEG theory for general applications, even for the specific application to spiral galaxies. The model is intended as a first-order working hypothesis for the specific study of the flat rotation curves in spiral galaxies. However, the proposed UEG model is developed as a valuable theoretical framework, which satisfies expected fundamental conditions, ensures compatibility with all prior successful applications of the UEG theory in [3–5], while it foresees no evident contradiction for a general application. That is a significant scientific objective, to support future generalization of the UEG theory towards a rigorous and complete theory.

B. Analytical Model

The light radiation from a spherically distributed source, like a single isolated star for example, exhibits a $1/r^2$ dependence of its radiation energy density with radial distance r, external to the spherical source. Such $1/r^2$ dependence of radiation may also be seen for a non-spherical source, in an approximate form, outside of a spherical region of certain threshold radius. For a spiral galaxy, such a spherical region may be identified with a threshold radius equal to the galaxy's scale radius R. This means, the radiation of the galaxy establishes an approximate spherical symmetry beyond the radius R

A spherical source is defined by spherical equi-potential surfaces, which means all points on a spherical surface of radius r have the same potential. In contrast, the spiral galaxy may be represented as a thin disk of an average thickness z_0, with the z_0 much smaller than its disk radius $\sim R$. The equi-potential surfaces (as per Newtonian gravity) for the disk structure would be thin disk-like surfaces in the vicinity enclosing the source disk (see Fig.1). Such equi-potential surfaces exhibit spherical asymmetry inherent in the disk structure, and such asymmetry in the Newtonian potential distribution may effectively extend well beyond the scale radius R. This is unlike the light's energy density discussed above, which establishes a fairly spherical symmetry beyond the galaxy's scale radius.

Now, consider a spherical surface of radius r, with a common center as the disk galaxy, as shown in Fig.1. The distribution of the Newtonian gravitational potential on this surface would in general be non-uniform, with stronger potential values near the plane of the disk over a constant thickness $\sim z_0$ (independent of r), and weaker values in the rest of the spherical surface. As a first-order model, one may approximate the potential distribution to be uniform over its strong region of area $\sim 2\pi r z_0$ (Fig.1), and be negligible over the rest of the spherical surface. A uniform energy density $W_\mathcal{T}$ of light radiation over the surface may be redistributed in proportion to the potential distribution, as approximated above, resulting in a stronger effective energy density $W_{\mathcal{T}e}$ near the galaxy plane. The radial UEG force is proposed to be proportional to this effective energy density $W_{\mathcal{T}e}$, not the actual energy density $W_\mathcal{T}$. In accordance with the above principle, the two energy densities would in principle be equal if the potential was spherically symmetric, with a uniform value everywhere on the spherical surface of Fig.1.

$$\begin{gathered}[W_\mathcal{T}(r) \times 4\pi r^2] = [W_{\mathcal{T}e}(r) \times (\sim\ 2\pi r z_0)],\\ W_{\mathcal{T}e}(r) \propto \frac{r}{z_0} \times W_\mathcal{T}(r);\\ W_\mathcal{T}(r) \sim \frac{1}{r^2},\ W_{\mathcal{T}e}(r) \sim \frac{1}{r},\ r > R. \end{gathered} \tag{1}$$

The original energy density $W_\mathcal{T}$ with a $\sim 1/r^2$ dependence would transform into an effective energy density $W_{\mathcal{T}e}$ with a $\sim 1/r$ dependence on the galaxy plane.

The gravitational potential distribution would exhibit closer spherical symmetry as one approaches towards the center, resulting in the effective density $W_{\mathcal{T}e}$ to be close to the actual energy density $W_\mathcal{T}$ in the central region. Accordingly, as a first-order estimate, the effective and actual energy densities may be assumed to be equal to each other for $r < R$. Based on this assumption and the above modeling (1), the effective and actual energy densities may be expressed as follows.

$$\begin{gathered} W_{\mathcal{T}e}(r) = W_\mathcal{T}(r),\ r < R;\\ W_\mathcal{T}(r) = W_\mathcal{T}(r=R)\frac{R^2}{r^2},\\ W_{\mathcal{T}e}(r) = W_\mathcal{T}(r=R)\frac{R}{r},\ r > R. \end{gathered} \tag{2}$$

The energy density $W_\mathcal{T}$ for $r > R$ may be approximated using the total luminosity L and the speed of light c, and assuming that the total light radiates in a spherically symmetric manner in the region, as if it radiates from a point source at the galaxy center. The total luminosity may be expressed using the surface density μ, which may be modeled with an exponential profile with amplitude μ_0 and scale radius R.

$$\begin{gathered} W_\mathcal{T}(r) \simeq \frac{L}{4\pi r^2 c} = \frac{\mu_0 R^2}{2r^2 c},\ W_\mathcal{T}(r=R) \simeq \frac{\mu_0}{2c},\ \mu(r) = \mu_0 e^{-r/R},\\ L = \int_0^\infty \mu(r) 2\pi r dr = \int_0^\infty \mu_0 e^{-r/R} 2\pi r dr = 2\pi\mu_0 R^2. \end{gathered} \tag{3}$$

The approximate energy density $W_\mathcal{T}$ at $r = R$ can then be related to the light surface density μ at $r = R$, with $e/(2c)$ as the proportionality factor. For convenience of reference, the effective energy density function $W_{\mathcal{T}e}(r > R)$ may be defined proportional to an equivalent effective surface density function $\mu_e(r)$, with the same above factor $e/(2c)$ of proportionality. Using the relation (2) between the $W_{\mathcal{T}e}$ function and $W_\mathcal{T}(r = R)$ in the proposed definition, the effective surface density function μ_e may be related to the actual surface-density function μ.

$$\begin{gathered} W_\mathcal{T}(r=R) \simeq \frac{\mu_0}{2c} = \frac{e\mu(r=R)}{2c},\\ W_{\mathcal{T}e}(r > R) = \frac{e\mu_e(r)}{2c} = W_\mathcal{T}(r=R)\frac{R}{r}\\ \simeq \frac{e\mu(r=R)\times R}{2cr} = \frac{ea}{2cr},\\ a = \mu(r=R) \times R,\ \mu_e(r) = \frac{a}{r} = \frac{\mu(r=R)\times R}{r}. \end{gathered} \tag{4}$$

The effective surface density function $\mu_e(r)$ may be viewed as a $1/r$-functional fit to the actual surface surface density function $\mu(r)$, such that they are equal to each other at $r = R$. As mentioned above, the surface density function $\mu(r)$ is modeled as an exponential distribution with an amplitude μ_0 and a scale radius R. The amplitude a of the μ_e distribution may be related to the parameters μ_0 and R. Consequently, the total luminosity L in (3) may be expressed in terms of the parameters a and R.

$$\begin{gathered} \mu(r) = \mu_0 e^{-r/R};\ \mu(r=R) = \frac{a}{R} = \mu_0 e^{-1},\ \mu_0 = \frac{ea}{R},\\ L = 2\pi\mu_0 R^2 = 2\pi e a R. \end{gathered} \tag{5}$$

If the amplitude μ_0 is maintained to be approximately constant, then a would be proportional to R, or equivalently the luminosity L would be proportional to a^2. This may be the case for a large group of high surface brightness (HSB) galaxies, which were believed to confirm to the Freeman's Law [9] of having an approximately constant central brightness μ_0.

$$\begin{gathered} \mu_0 \sim \text{constant (Freeman's Law, HSB Galaxy)},\\ a \propto R,\ L \propto a^2. \end{gathered} \tag{6}$$

The radial UEG field $\overline{E}_{gu}$ may now be expressed proportional to the equivalent energy density $W_{\mathcal{T}e}$, with the constant of proportionality equal to the UEG constant γ. The potential function associated with the above radial field could be obtained by integrating the field in the radial variable r, from which the θ component of the field may also be derived (in principle) as the θ-derivative of the potential function. However, we are interested here only on the radial UEG field, which completely determines the orbital acceleration on the central plane of the galaxy, because the $\theta-$ component of the UEG field on this plane would be zero. The magnitude E_{gu} of the radial UEG field on the central galaxy plane would be equal to the orbital acceleration v^2/r. The E_{gu} (for $r > R$) is proportional to the effective surface density $\mu_e(r) = a/r$, having the same $1/r$ dependence as the orbital acceleration. Accordingly, the rotation velocity v would exhibit a "flat" behavior for $r > R$, with v^2 equal to the constant amplitude 'a'.

$$\begin{aligned}\bar{E}_{gu} &= -\hat{r}E_{gu} = -\hat{r}\gamma W_{\tau e} = -\hat{r}\frac{\gamma e\mu_e}{2c},\\ E_{gu}(r) &= \frac{\gamma e\mu_e(r)}{2c} = \frac{\gamma ea}{2cr} = \frac{v^2}{r},\\ v^2 &= \frac{\gamma ea}{2c},\ r > R. \end{aligned} \tag{7}$$

Combining (7,5), the luminosity L may be expressed in terms of the velocity v, radius R, and the UEG constant γ.

$$L = 2\pi eaR = \frac{4\pi Rv^2c}{\gamma},\ \gamma = \frac{4\pi Rv^2c}{L}. \tag{8}$$

Accordingly, the UEG constant γ may be estimated from (8) using measured values of the L, v and R, available from a galaxy survey [10]. Alternatively, the amplitude a for the effective surface density $\mu_e(r)$ may be estimated directly from a measured surface-brightness profile $\mu(r)$ for a selected individual galaxy, and then the γ be estimated using the a and the measured flat rotation velocity v, as per (7,4). The estimation directly using measured data of an individual galaxy would complement the estimation from the galaxy survey, providing an explicit illustration of the UEG model. However, the estimation using an averaged data from the galaxy survey can, in principle, be more reliable than that using data for individual galaxies. Inaccuracies from astronomical measurements of individual galaxy parameters, as well as uncertainty due to deviation of individual galaxy characteristics from any ideal theoretical assumptions, can often be significant. The resulting inaccuracy or uncertainty in the estimation of the γ is expected to be minimized by using an "average" or a central data point among a survey of large number of sample galaxies.

III. ESTIMATION OF γ USING MEASURED DATA FROM GALAXY SURVEY

We first estimate the γ based on (8), using an average data point from the I-band measurement of the galaxy survey [10]. As suggested above, the data point is located approximately at the statistical center of the survey samples.

$$\begin{gathered}\text{(I-band data):}\\ L = 10^{10.4}L_0 = 3.864\times 10^{36.4}W,\ v = 10^{5.2}\text{m/s},\\ R = 10^{0.5}\text{kpc} = 10^{0.5}\times 3.086\times 10^{19}\text{m},\\ \gamma(\text{I-band}) = \gamma_I = \frac{4\pi\times 3\times 3.086\times 10^{1.5}}{3.864}\\ = 0.95\times 10^3[(\text{ms}^{-2})/(\text{Jm}^{-3})].\end{gathered} \tag{9}$$

Similarly, we estimate the γ from the K-band measurement of [10]. Note that an effective radius, R_e, is provided in [10] for the K-band measurements. The effective radius, defined as the radius of a sphere that encloses half of the total luminosity, would be 1.678 times the scale radius R used in our modeling, assuming an exponential light profile.

$$\begin{gathered}\text{(K-band data)}:\\ L = 10^{10.8}L_0 = 3.864\times 10^{36.8}\text{W},\ v = 10^{5.2}\text{m/s},\\ R_e = 10^{0.6}\text{kpc} = 10^{0.6}\times 3.086\times 10^{19}\text{m},\ R = R_e/1.678\ ,\\ \gamma(\text{K-band}) = \gamma_K = \frac{4\pi\times 3\times 3.086\times 10^{1.2}}{3.864\times 1.678}\\ = 0.28\times 10^3[(\text{ms}^{-2})/(\text{Jm}^{-3})].\end{gathered} \tag{10}$$

Measurements in the K-band overestimates the luminosity and the energy density, leading to underestimation of the γ. On the other hand, measurements in the I-band underestimates the energy density, leading to overestimation of the γ. Accordingly, the above results estimate a useful range for the value of the γ, which is consistent with the value of the $\gamma = 0.6\times 10^3$ $(\text{ms}^{-2})/(\text{Jm}^{-3})$ deduced from the UEG model [3] of elementary particles.

$$\begin{gathered}0.28\times 10^3 < \gamma < 0.95\times 10^3[(\text{ms}^{-2})/(\text{Jm}^{-3})],\\ \gamma = 0.6\times 10^3[(\text{ms}^{-2})/(\text{Jm}^{-3})].\end{gathered} \tag{11}$$

The best estimate for γ is assumed to be the average of the two estimates in the $I-$ and $K-$ bands.

$$\gamma \simeq \frac{(\gamma_I+\gamma_K)}{2}{=}0.62\times 10^3[(\text{ms}^{-2})/(\text{Jm}^{-3})]. \tag{12}$$

The above estimate closely agrees with the γ from the particle model [3]. Considering that we used a first-order approximation in the UEG modeling of (1,2), such agreement is remarkable. This means that the ideal conditions we assumed in the first-order UEG modeling of (1,2) are remarkably valid for the central data point of [10] used in our estimation.

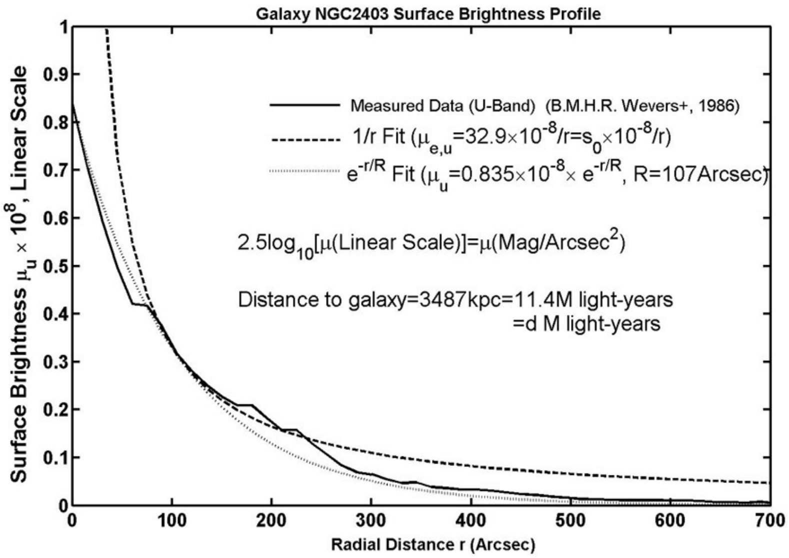

FIG. 2.

IV. ESTIMATION OF γ USING MEASURED DATA OF AN INDIVIDUAL GALAXY

Measured data for the surface brightness distribution $\mu(r)$ of a specific galaxy is first properly fitted with an exponential, and then an effective surface brightness distribution $\mu_e(r)$, as defined in (4). The data using mixed units, such as magnitude, arcsec, light-years, may be converted to suitable standard units. The μ_e distribution can then be related to the rotation velocity v using (7).

$$\begin{aligned}
W_{\tau e} &= \tfrac{e\mu_e}{2c} = \tfrac{s_0\times 6.61\times 10^{-13}}{r}\mathrm{J/m}^3,\\
&\text{UEG Acceleration}(\mathrm{m/s}^2) = E_{gu}\\
&= \gamma W_{\tau e} = \tfrac{\gamma s_0\times 6.61\times 10^{-13}}{r}\\
&= \tfrac{v^2\times 10^{10}}{r_m} = \tfrac{v^2}{r\times d\times 4.6\times 10^6};\ v(10^5\mathrm{m/s}),\\
&1(\text{lin-mag/arcsec}^2) = 1.46\times 10^4(\mathrm{W/m}^2),\\
&r(\text{arcsec})=\mathrm{r}\times\mathrm{d}\times 4.6\times 10^{16}(\mathrm{m})\\
&= r_m(\mathrm{m}),\ \text{at distance d(MLyr)}. \qquad (13)
\end{aligned}$$

The UEG constant γ is deduced using the amplitude a, or its equivalent parameter s_0, of the effective surface brightness distribution $\mu_e(r)$, the flat rotation velocity v and the distance d of the galaxy. Suitable correction factors may be needed to relate the K- and U-band measured magnitudes to a common reference of solar bolometric magnitude of 4.74. This assumes the solar magnitudes in the K- and U-bands are 3.28 and 5.56, respectively.

$$\begin{aligned}
\gamma &= \tfrac{v^2\times 10^7}{s_0 d\times 6.61\times 4.6} = \tfrac{v^2\times 10^6}{s_0 d\times 3.04}[(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})]\ \text{(Visible)},\\
\gamma &= \tfrac{\Delta_u\times v^2\times 10^6}{s_0 d\times 3.04}[(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})]\ \text{(U-Band)},\\
\gamma &= \tfrac{\Delta_k\times v^2\times 10^6}{s_0 d\times 3.04}[(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})]\ \text{(K-Band)};\\
&v(10^5\mathrm{m/s}),\ \mathrm{d(MLyr)},\\
&\Delta_k = 10^{(4.74-3.28)/2.5}\\
&= 3.84 = \text{K-Band correction factor},\\
&\Delta_u = 10^{(4.74-5.56)/2.5}\\
&= 0.47 = \text{U-Band correction factor}. \qquad (14)
\end{aligned}$$

Using the U-band (assumed $\simeq$ U'-band) surface-brightness data [11] for the galaxy NGC-2403, presented in Fig.2, we estimate the amplitude parameter $s_0 = 32.9$. This parameter, together with the galaxy's distance $d = 11.4$MLyr [12] and flat rotation velocity $v = 1.35\times 10^5$m/s [13], would provide an estimate for the $\gamma_u = 0.75\times 10^3$ $(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})$, using the above relation (14). Similarly, using the K-band data [14] for the same galaxy NGC-2403, presented in Fig.3, we estimate the amplitude parameter $s_0 = 430$. This would provide an estimate for the $\gamma_k = 0.47\times 10^3$ $(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})$, using (14). An average of these two estimates for the γ would lead to the best estimate for the $\gamma = 0.61\times 10^3$ $(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})$ from the available data for the galaxy NGC-2403. This is close to the $\gamma = 0.62\times 10^3$ $(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})$ deduced from the galaxy survey in (12) or the $\gamma = 0.60\times 10^3$ $(\mathrm{ms}^{-2})/(\mathrm{Jm}^{-3})$ from particle model [3]. Such remarkable agreement implies that any deviation from the basic model of (1-4) due to

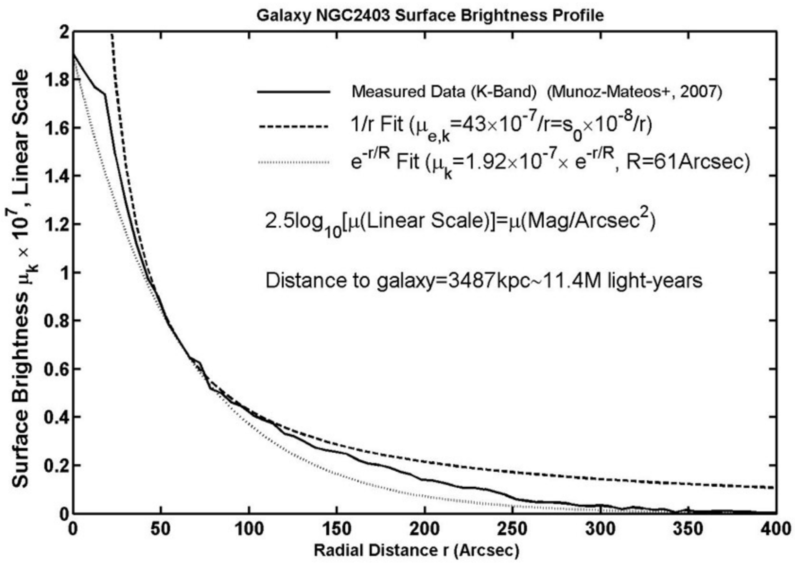

FIG. 3.

differences in the surface brightness μ_0 (see section V)) of the individual galaxy NGC-2403 from the "average" galaxy used in the estimation (12), is minimal. The $\mu_{0,k}$ are estimated to be roughly equal to 16.75 (mag/arcsec^2) in both cases ([10], Fig.3), which is consistent with the above expectation.

$$\begin{aligned} &\text{NGC-2403:}\\ &\gamma_u{=}0.75\times10^3(\text{ms}^{-2})/(\text{Jm}^{-3})\ \text{(U-Band)},\\ &\gamma_k{=}0.47\times10^3(\text{ms}^{-2})/(\text{Jm}^{-3})\text{(K-Band)},\\ &\gamma=(\gamma_v+\gamma_k)/2\\ &=0.61\times10^3(\text{ms}^{-2})/(\text{Jm}^{-3})\ \text{(Best Estimate)}. \end{aligned} \tag{15}$$

V. THE TULLY-FISHER RELATION (TFR) AND THE MODIFIED NEWTONIAN DYNAMICS (MOND) MODEL, DERIVED FROM THE UEG MODEL

Combining (5,7) and assuming an approximately constant μ_0, a Tully-Fisher Relation (TFR) [6] may be deduced, where the total luminosity L would be proportional to the fourth power of the flat rotation velocity v. As mentioned before, the above condition of an approximately constant μ_0 is satisfied by a large group of high surface brightness (HSB) galaxies that were believed to confirm to the Freeman's Law [9].

$$\begin{aligned} &L=2\pi\mu_0R^2=\frac{2\pi e^2a^2}{\mu_0}=\frac{8\pi v^4c^2}{\mu_0\gamma^2},\ \mu_0=\frac{ea}{R},\\ &L\propto v^4\ \text{(TFR)},\\ &\mu_0\sim\text{constant (Freeman's Law, HSB Galaxy)}. \end{aligned} \tag{16}$$

However, the Freeman's Law is no longer believed to strictly valid, and galaxies are measured to exhibit a broad range of amplitudes μ_0 covering variations among the HSB galaxies as well as extending to low surface brightness (LSB) galaxies with lower values of μ_0. For a general treatment to closely model the variation in the amplitude μ_0, we may introduce a new parameter α for fitting the $1/r$ profile of μ_e with the exponential profile of μ in (4). The unit reference value of α is expected to apply for an "average" HSB galaxy, as assumed in the basic model of (4) and in the estimations of (12,15). The μ_e may be adjusted to a smaller or larger value, relative to the $\mu(r=R)$, with a proportional adjustment of the parameter α, which would represent a smaller or large value of the UEG force, respectively, as per (7).

The variable factor α is accommodated in the gravitational potential model of (1,2), Fig.1, by recognizing the galaxy thickness z_0 to be an active variable, like the scale radius R or the surface brightness μ_0, for parametrization of galaxy characteristics. In the potential model of (1), an approximately uniform (spherically) potential would be established for all radial distances less than a variable threshold radius R_t, dependent on a variable thickness z_0, not less than the ideal fixed threshold radius $r=R$ assumed in (2). Accordingly, the effective energy density $W_{\tau e}$ would match with the actual energy density W_τ for all the radial distances less than the variable threshold radius, not the ideal reference threshold $r = R$ assumed in (2). Consequently, the $W_{\mathcal{T}e}(r = R)$ would no longer be equal to $W_{\mathcal{T}}(r = R)$ as ideally assumed in (2), but now be equal to $\alpha W_{\mathcal{T}}(r = R)$, with the variable factor α proportional to the normalized galaxy thickness R/z_0.

The model of (1,2) may be revised as follows, as explained above.

$$
\begin{gathered}
W_{\mathcal{T}e} \propto W_{\mathcal{T}} \times \frac{r}{z_0} = W_{\mathcal{T}} \times \frac{r}{R} \times \frac{R}{z_0};\ W_{\mathcal{T}e} \sim \frac{1}{r},\ W_{\mathcal{T}} \sim \frac{1}{r^2}.\\
W_{\mathcal{T}e}(r) \propto W_{\mathcal{T}}(r = R) \times \frac{R}{r} \times \frac{R}{z_0},\ r > R;\\
W_{\mathcal{T}e} = W_{\mathcal{T}},\ r < R_t \propto z_0. \qquad (17)
\end{gathered}
$$

Using the above revisions and (3), the relation (4) between the surface density μ and effective surface density μ_e, and the resulting expression for the luminosity L (5) using (7), may also be revised.

$$
\begin{gathered}
\mu_e(r) = \frac{a}{r} = \alpha \times \frac{\mu(r{=}R)R}{r},\ \alpha \propto \frac{R}{z_0},\\
\frac{a}{R} = \alpha \times \mu(r = R) = \alpha\mu_0 e^{-1},\ \mu_0 = \frac{ea}{\alpha R},\\
L = 2\pi\mu_0 R^2 = \frac{2\pi e^2 a^2}{\alpha^2\mu_0} = \frac{8\pi v^4 c^2}{\alpha^2\mu_0\gamma^2}. \qquad (18)
\end{gathered}
$$

The TFR (16), which was established based on the simple assumption of an approximately constant μ_0, would still be valid for a range of different surface brightness μ_0, if $\mu_0\alpha^2$ in (18) is approximately a constant. This condition, of having a larger value of the α for a lower μ_0, means there would be relatively more contribution from the UEG force as the surface brightness μ_0 reduces. This trend better represents observed characteristics among the HSB galaxies, extending to LSB galaxies as well. The higher UEG contribution for a lower surface brightness μ_0 would be equivalent to having relatively more "dark matter" contribution for a LSB galaxy [15], as per the current dark-matter paradigm.

$$
\begin{gathered}
L \propto v^4 \text{ (MOND, TFR)},\\
\mu_0\alpha^2 = \text{constant},\ \alpha \propto \frac{1}{\sqrt{\mu_0}};\ \alpha \propto \frac{R}{z_0},\ \mu_0 \propto (\frac{z_0}{R})^2,\\
\alpha \text{ (LSB Galaxy)} > \alpha \text{ (HSB Galaxy)} \sim 1,\\
\text{Dark Matter (LSB)} > \text{Dark Matter (HSB)},\\
\frac{z_0}{R}\text{(LSB)} < \frac{z_0}{R}\text{(HSB)}. \qquad (19)
\end{gathered}
$$

The above TFR of having the luminosity proportional to the fourth power of the velocity v, is also consistent with prediction from an alternate model using a modified Newtonian dynamics (MOND) [2, 7].

As derived in (17,18), the parameter α, which proportionately represents the equivalent distribution $W_{\mathcal{T}e}$ or μ_e, is proportional to the normalized galaxy scale R/z_0. Accordingly, the condition (19) of a constant factor $\mu_0\alpha^2$, required for the validity of the TFR or MOND, would be satisfied if the normalized scale parameter (z_0/R) is proportional to the square-root of the surface brightness μ_0. This general trend, of having the normalized galaxy thickness z_0/R to be smaller for a lower surface brightness μ_0, may seem to be a sensible characteristic. The specific required relationship between the galaxy thickness and the surface brightness may be compared and verified with the measured data in [16].

Using the above required relationship (19) between the μ_0 and the normalized scale z_0/R in (18,7) would translate to another galaxy scaling relationship between the absolute thickness z_0 (not normalized to R) and the flat rotation velocity v.

$$
\begin{gathered}
\mu_0 = \frac{ea}{\alpha R},\ v^2 = \frac{\gamma ea}{2c} = \frac{\gamma\mu_0\alpha R}{2c} = \frac{(\mu_0\alpha^2)R}{2c\alpha} \propto z_0,\\
\mu_0\alpha^2 = \text{constant},\ \alpha \propto \frac{R}{z_0}. \qquad (20)
\end{gathered}
$$

Accordingly, the galaxy thickness z_0 is required to be proportional to the square of the flat rotation velocity v. This required relationship is clearly verified from the measured data of [16]. It is significant to note that the above two required relations (a) between the galaxy normalized thickness z_0/R and the surface brightness μ_0, and (b) between the thickness z_0 and the flat rotation velocity v, are independently predicted from the UEG model of (17,18), based on the observed TFR (19,16), but could not have been anticipated either from the TFR of [6] or the MOND [2, 7]. Verification of the above predictions from [16] is a significant development, which strongly validates the new UEG model of (1,17), as applied to the non-spherical structure of a galaxy.

A. Refinement in the Tully-Fisher Relation

Some refinement in the above TFR (19) may be needed, in order to confirm to the measured data [6, 10] more accurately, where the luminosity seems to be proportional to a smaller exponent (than the ideal value of 4 in (19)) of the velocity v. This trend may be empirically established from (19) by having the factor $\mu_0\alpha^2$ to be weakly dependent on the velocity v (proportional to a relatively small exponent of v), instead of the ideal constant factor $\mu_0\alpha^2$ suggested above. This may be represented by suitable refinement in the required relation in (19) between the galaxy normalized thickness z_0/R and the surface brightness μ_0.

$$
\begin{gathered}
\mu_0\alpha^2 \sim v^b,\ 0 < b < 0.5;\\
L \sim v^{4-b} = v^d,\ 3.5 < d < 4\ . \qquad (21)
\end{gathered}
$$

However, this refined TFR does not confirm to the MOND, where the luminosity is definitively required to be proportional to the fourth power of the velocity v. It is not clear if the above refinement (21) is really fundamental or is simply due to selection bias in the measurements of [6, 10], resulting in a limited range in the data over which the exponent d is estimated with a smaller value $d < 4$.

The total luminosity and surface brightness profile are usually proportional to the total baryonic mass and its mass distribution, respectively, in which case the TFR would work as well if the luminosity is interchanged with the baryonic mass. The proportionality between the baryonic mass and the luminosity may not, however, strictly extend to all LSB galaxies, having smaller luminosity and rotation velocity. In this case, the measured data follow a TFR more accurately, if the total baryonic mass M_b is used in the relation (19,21), instead of the total luminosity L. The revised relation is referred to as the Baryonic Tully-Fisher Relation (BTFR) [17]. The baryonic mass M_b would be proportional either to the fourth power or to a smaller exponent of the velocity, if the baronic mass substitutes the luminosity in the TFR versions (19) or (21), respectively. The former version of the BTFR is consistent with MOND which, to fundamentally begin with, relates the baryonic mass to the fourth power of the velocity v.

The deviation from the original TFR may be partly attributed to the larger contribution to the rotation velocity v from the Newtonian gravity due to the proportionately larger regular mass (baryonic), in the lower-luminosity LSB galaxies. More significantly, the revised trend may be empirically accommodated by properly adjusting the parameter α in (18) to be dependent on both the surface brightness μ_0 and an equivalent baryonic surface mass density A_b of the galaxy. This would be consistent with the basic principles of the present UEG model in (1- 5,17), where the gravitational potential function that determines the redistribution of the energy density $W_\mathcal{T}$ into the effective density $W_{\mathcal{T}e}$ (see Fig.1) may be recognized to depend upon both the Newtonian gravitation (related to mass profile) as well as the UEG field due to the light profile of a galaxy. However, more specific physical explanation behind such an empirical trend, leading to the preference of the baryonic mass over the luminosity in the BTFR, is at this point unclear, and is beyond the scope of the present work.

$$\begin{aligned} M_b &= L \times \frac{M_b}{L} = \frac{8\pi v^4 c^2}{\alpha^2 \mu_0 \gamma^2} \times \frac{M_b}{L} \\ &= \frac{8\pi v^4 c^2}{\alpha^2 ({\mu_0}^2/A_b)\gamma^2}, \ \frac{M_b}{L} = \frac{A_b}{\mu_0}, \\ M_b &\propto v^4, \ \alpha^2 \times ({\mu_0}^2/A_b) = \text{constant}. \end{aligned} \tag{22}$$

Accordingly, for a given surface luminosity μ_0, a larger value of the baryonic mass density A_b is expected to result in a tighter confinement of the gravitational potential near the galaxy surface (smaller z_0), resulting in a larger α. The two refinements (21,22) may need to be studied together, which may be associated with interdependent and/or mutually compensating physical effects.

VI. CONCLUSION

The estimate of the UEG constant γ from measured data from a galaxy survey [10], based on the new UEG model, agrees well with an accurate value derived from the UEG model of elementary particles [3, 4]. This is based on a statistically average data point from the survey samples. Direct analysis of measured brightness profile and rotation curve of a specific selected galaxy is also illustrated to provide a similar estimate for the γ, that is consistent with the estimate from the galaxy survey. Further, the UEG galaxy model confirms to the TFR [6, 17] for varying range of galaxy amplitudes, and is consistent with results from a modified Newtonian dynamics (MOND) [2, 7] model. The required condition for the agreement between the UEG model, TFR and MOND is supported by measured relations of the galaxy thickness with the surface brightness and the rotation velocity [16], which may be considered as an independent validation of the UEG model. The above studies strongly support validity of the new UEG model, established for the non-spherical structure of a disk galaxy. The UEG theory is intended to serve as a theoretical substitute for the current "dark-matter" hypothesis.

The UEG theory, which has been successfully applied for elementary particles [3, 4] as well as single and binary stars [5], and is now supported as well for galaxy modeling, may provide a new unified theoretical paradigm for a broad range of physical concepts, covering both small and large size scales of nature, and spherically symmetric as well as asymmetric structures.

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