PROOF OF FERMAT'S LAST THEOREM

* To prove that where $n > 2$ $a^n + b^n \neq c^n$

First of all, let us investigate the relationship between a, b and c.

If we use values of a, b, c such that:
$a + b = c$, what will be the corresponding outcome?

* Solution

$(a+b)^n = c^n$

By the binomial theorem we have:

$$a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots b^n = c^n$$

which only have solution when $n = 1$
but for $n > 1$ no solution.

* If we use also value of a, b, c such that;
$a + b < c$, then we have that:

Solution

$(a+b)^n < c^n$

$$a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots b^n < c^n$$

which can not be, since $a^n + b^n = c^n$

$\therefore$ For for corresponding solution to the equation
$a^n + b^n = c^n$ where $n > 1$
we shall use values of these form $a + b > c$

Solution

$(a+b)^n > c^n$

$$a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots b^n > c^n$$

$a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 \dots b^n > a^n + b^n$; $a^n + b^n = c^n$

$\therefore {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 > 0$ which is true.

So that for the equation $a^n + b^n = c^n$ to have solutions where $n > 1$ we shall use values of these form $a + b > c$

* We know that $a + b > c$ Is known as the triangular inequality, which means: $a^n + b^n = c^n$ can be represented ~~by~~ the three sides of a triangle.
Since a, b and c have different values, then we know that the resulting triangle shall be a scalene triangle.

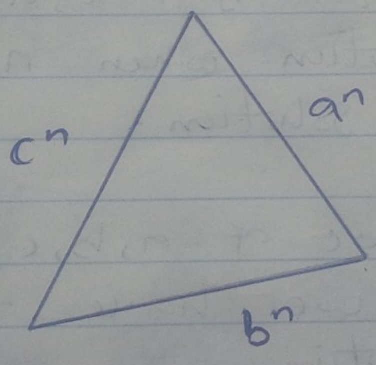


Fig 1.0

For $n = 2$

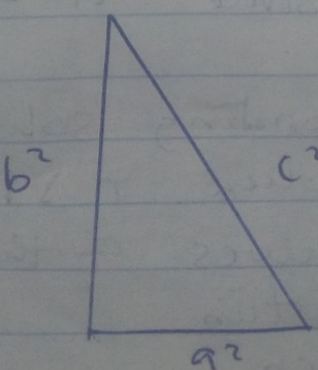


Fig 1.[illegible]

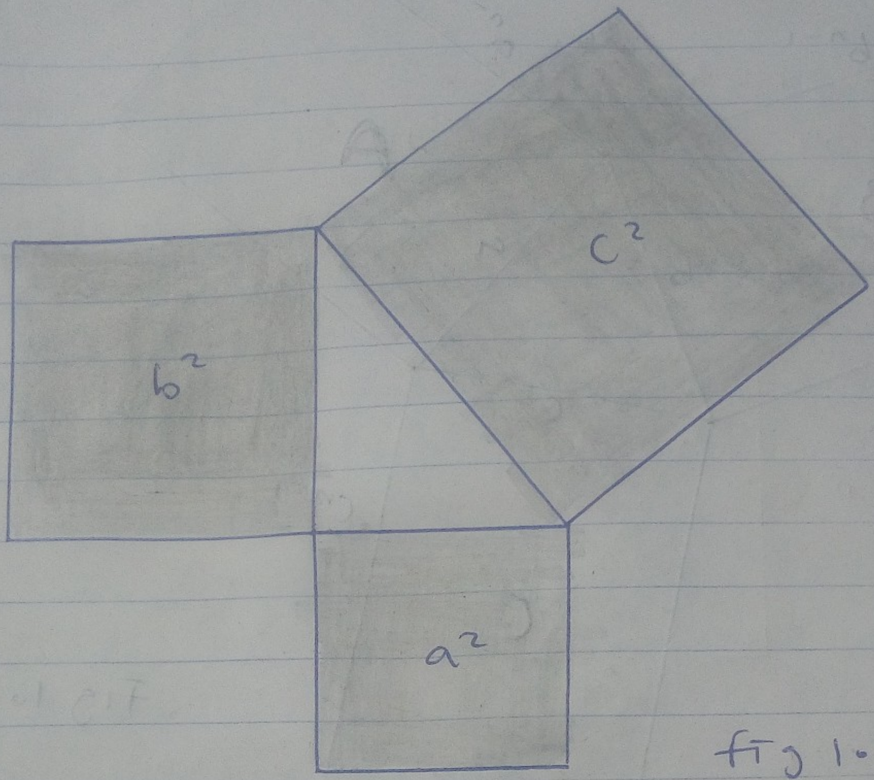


fig 1.2

$n = 2$ is a special case of the pythagoras theorem.

From the generalisation of the pythagorean theorem we have that If similar shapes are drawn on the three sides of the right-angled triangle, then the area of the two smaller shapes shall equal the area of the bigger one. which is a simple incidence of proportionality.

* For case $n > 2$ then we shall find similar diagram to express the equation $a^n + b^n = c^n$

Below is a general triangular model

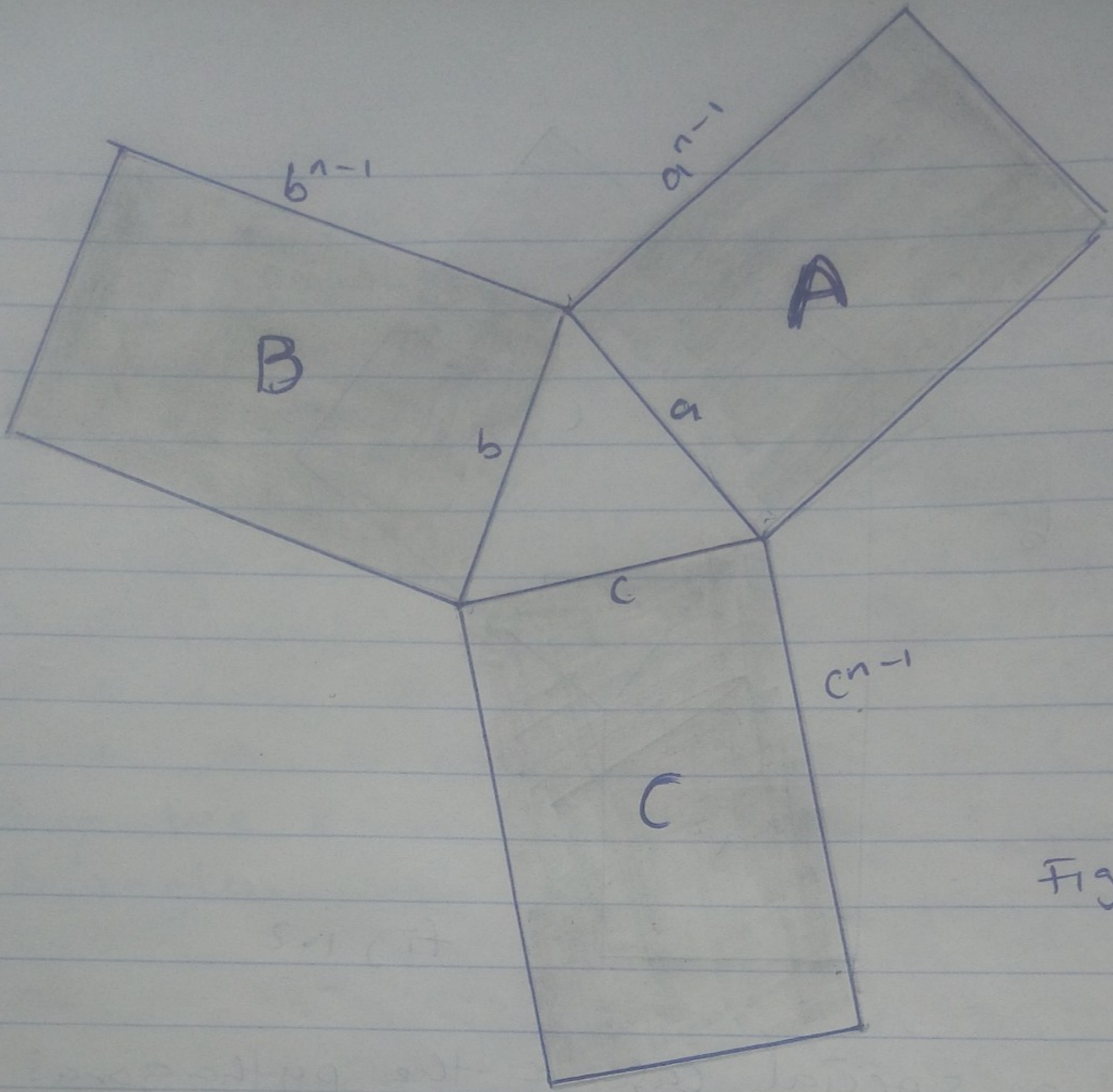


Fig 1.3

Let the general form be assumed to be rectangles with length a, b, c and height a^{n-1}, b^{n-1} and c^{n-1}

* Note: that Squares ~~are~~ is a member of the family of rectangles, which justifies case $n=2$

where $n>3$ we have the rectangles shall get longer

<u>Solution</u>

$$a \times a^{n-1} = a^n = A \quad \text{.......} \quad (1)$$
$$b \times b^{n-1} = b^n = B \quad \text{.......} \quad (2)$$
$$c \times c^{n-1} = c^n = C \quad \text{.......} \quad (3)$$

So that if $a^n + b^n = c^n$ then
$$A + B = C$$

If we have that the area of rectangle A plus rectangle B equals the area of rectangle C.

we then Imply that if $A+B=C$ then Similar shapes drawn on the sides of these rectangles shall also obey the area law

Example

$$\frac{\Delta_1}{A} = \frac{\Delta_2}{B} = \frac{\Delta_3}{C} \quad \text{(For triangles)}$$

$$\frac{\Delta_1}{A} = \frac{\Delta_3}{C} \; ; \quad \Delta_1 = \frac{A\Delta_3}{C} \, , \quad \Delta_2 = \frac{B\Delta_3}{C}$$

$$\Delta_1 + \Delta_2 = \frac{A\Delta_3}{C} + \frac{B\Delta_3}{C}$$

$$\Delta_1 + \Delta_2 = \frac{\Delta_3}{C}(A+B)$$

$$\Delta_1 + \Delta_2 = \frac{\Delta_3}{C}(C)$$

$$\Delta_1 + \Delta_2 = \Delta_3$$

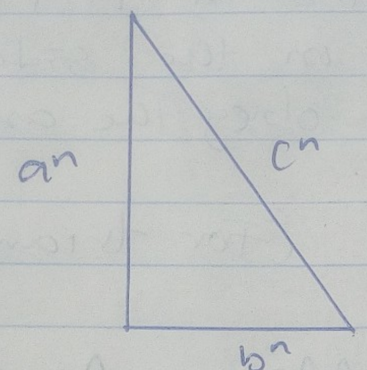
Solution

Let us assume that one of such shapes shall be a square. the we have

$$\square_1 + \square_2 = \square_3 \, , \quad a^2 + b^2 = c^2$$

So we shall have that $a^2+b^2=c^2$ when Squares are drawn on either sides of this ~~to~~ rectangles.

But ~~if~~ $a^2 + b^2 = c^2$
then we have from 'Euclid's Element, Book 1, proposition 48' that the triangle in question must be a right-angled triangle.



Thus we shall go further to prove that where $a^2+b^2=c^2$ then no value of $n>2$ shall satisfy the equation. $a^n+b^n=c^n$

Solution

$a^n + b^n = c^n$ --- (1)

$a^2 + b^2 = c^2$ --- (2)

multiply equation 2 by c^{n-2}

$a^2 c^{n-2} + b^2 c^{n-2} = c^2 c^{n-2}$

$a^2 c^{n-2} + b^2 c^{n-2} = c^n$

where $n > 2$ $a^2 c^{n-2} > a^n$ --- (3)

and $b^2 c^{n-2} > b^n$ --- (4)

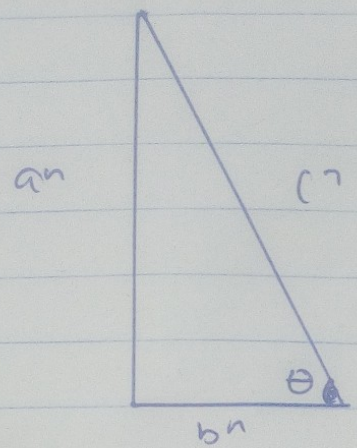
Adding equation --- (3) and (4)

$a^2 c^{n-2} + b^2 c^{n-2} > a^n + b^n$ where $n > 2$

Since $a^2 c^{n-2} + b^2 c^{n-2} = c^n$

we have that $c^n > a^n + b^n$ where $n > 2$

PROOF TWO



$a^n + b^n = c^n$

<u>Solution</u>

$$\frac{a^n}{c^n} + \frac{b^n}{c^n} = \frac{c^n}{c^n}$$

$$\frac{a^n}{c^n} + \frac{b^n}{c^n} = 1$$

$$\left(\frac{a}{c}\right)^n + \left(\frac{b}{c}\right) = 1$$

$$\sin^n\theta + \cos^n\theta = 1$$

which have Solotions only when $n = 2$ not greater than two.

So far our argument has been thus:

IF $a^n + b^n = c^n$ then,

$a^2 + b^2 = c^2$ but if

$a^2 + b^2 = c^2$ then for no value where $n > 2$

does $a^n + b^n = c^n$ holds.

QED.