Dottie number: A simple remark

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abstract
This note presents a simple formula for Pi

1. Introduction

1.1. The unique real root of the equation $x=\cos x$ is (Dottie number) :

$$x=d=0.73908513... \tag{1}$$

1.2. Iteration:

$$x_{n+1}=\cos x_n \quad , x_0=1 \Rightarrow x_n \to d \tag{2}$$

1.3. Notation:

$$d=\cos\cos\cos...\cos 1 \tag{3}$$

2. A simple formula for Pi

2.1. The number Pi is defined by:

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=3.14159265... \tag{4}$$

2.2. A simple formula for Pi:

$$\pi=2d+2\sin^{-1}d \tag{5}$$

$$\pi=2\left(\cos\cos\cos...\cos 1\right)+2\sin^{-1}\left(\cos\cos\cos...\cos 1\right) \tag{6}$$

3. The sequence π_n

3.1. The sequence d_n :

$$d_{n+1}=\cos d_n \quad , d_0=\cos 1 \tag{7}$$

$$d_n \to d \tag{8}$$

3.2. The sequence π_n :

$$\pi_n = 2d_n + 2\sin^{-1} d_n \quad , n = 0,1,2,3,\ldots \tag{9}$$

$$\pi_n \to \pi \tag{10}$$

$$\left|\pi_n - \pi\right| \le 1.18 \cdot \left(\sin\left(\cos\cos 1\right)\right)^n = 1.18 \cdot \left(0.756\ldots\right)^n \quad , n = 0,1,2,3,\ldots \tag{11}$$

3.3. Graphics:

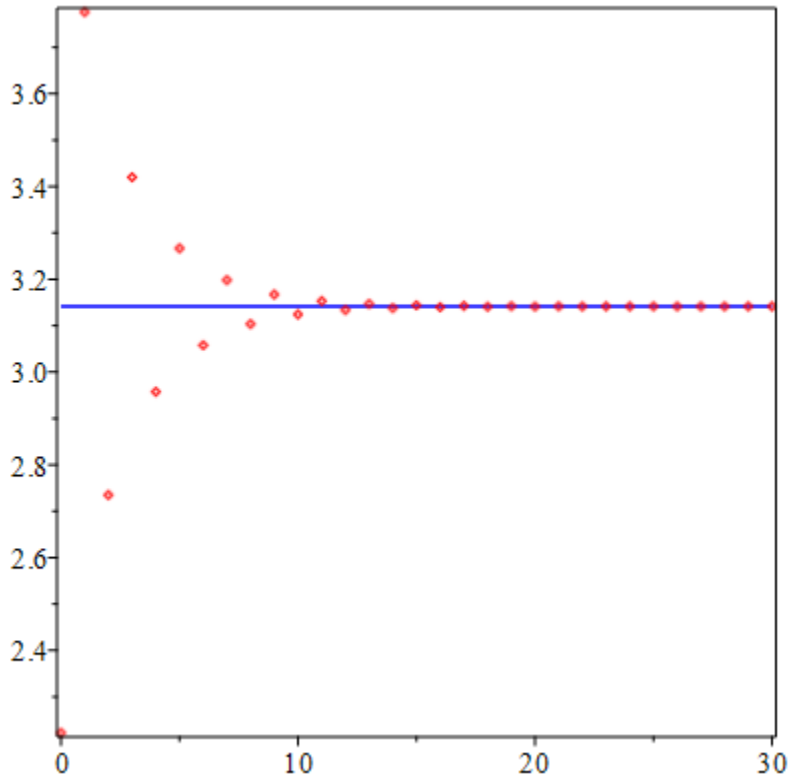

Figure 1. • π_n , • π , $n = 0\ldots30$.

References

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