Elementary integral and some series

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Abstract. We recall a elementary integral and some series.

1. Elementary Integral

$$\pi = 2\int_0^1 \frac{1}{\sqrt[4]{x}\sqrt{1-x}\,(1+x)}\,dx = 2\int_0^1 \frac{1}{\sqrt[4]{1-x}\sqrt{x}\,(2-x)}\,dx \tag{1}$$

Remak 1. $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.14159265...$.

2. Some Series

$$\pi = 2\sum_{n=0}^{\infty}\frac{2^{-n}}{2n+1}F\left(\frac{1}{4},n+\frac{1}{2};n+\frac{3}{2};1\right) \tag{2}$$

$$\pi = \frac{4}{3}\sum_{n=0}^{\infty}2^{-n}F\left(-n+\frac{1}{2},\frac{3}{4};\frac{7}{4};1\right) \tag{3}$$

$$\begin{aligned}\pi = 8\left(\frac{3}{7}\right)^{3/4}\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-2n}(3/7)^n}{4n+3}F\left(n+\frac{3}{4},n+\frac{3}{4};n+\frac{7}{4};\frac{3}{7}\right)+\\ +\frac{4\sqrt{2}}{7\sqrt[4]{3}}\sum_{n=0}^{\infty}\frac{(-3)^{-n}}{n+1}F\left(\frac{1}{2},n+1,n+2;1\right)\sum_{k=0}^{n}\frac{(1/4)_k}{k!}\left(\frac{3}{7}\right)^{n-k}\end{aligned} \tag{4}$$

Remark 2.

- $F(a,b;c;z) \equiv {}_2F_1(a,b;c;z)$ *is the hypergeometric function.*
- $F\left(\frac{1}{2},n+1;n+2;1\right) = \frac{(n+1)2^{2n+1}}{\binom{2n}{n}(2n+1)} \quad ,n\in\mathbb{N}\cup\{0\}$.

References

1. Boros, G., and Moll, V.H.: Irresistible Integrals. Symbolics, Analysis and Experiments in the Evaluation of Integrals. Cambridge University Press, 2004.
2. Gradshteyn, I.S., and Ryzhik, I.M.: Table of Integrals, Series, and Products. 7th ed.edited by Alan Jeffrey and Daniel Zwillinger. Academic Press, 2007.