Three Equations

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Abstract

This note presents Three series for Pi

The number Pi is defined by

$$\pi = 2\int_0^\infty \frac{1}{1+x^2}\,dx = 3.141592...$$ (1)

This note presents three series for π .

Series

Entry 1. If $a = 0.460611208...$ then

$$16a^4 e^{8a} - 8a^2 e^{6a} - 16a^4 e^{4a} + e^{4a} - 8a^2 e^{2a} + 16a^4 = 0$$ (2)

$$\pi = 12 \sum_{n=0}^\infty \binom{2n}{n} \frac{2^{-2n} a^{2n+1} \sinh\left((2n+1)a\right)}{2n+1}$$ (3)

Entry 2. If $a = 0.526737665...$ then

$$4a^4 e^{8a} - 4a^2 e^{6a} + e^{4a} - 4a^2 e^{2a} + 4a^4 = 0$$ (4)

$$\pi = 8 \sum_{n=0}^\infty \binom{2n}{n} \frac{2^{-2n} a^{2n+1} \sinh\left((2n+1)a\right)}{2n+1}$$ (5)

Entry 3. If $a = 0.560031932...$ then

$$16a^4 e^{8a} - 24a^2 e^{6a} + 16a^4 e^{4a} + 9e^{4a} - 24a^2 e^{2a} + 16a^4 = 0$$ (6)

$$\pi = 6 \sum_{n=0}^\infty \binom{2n}{n} \frac{2^{-2n} a^{2n+1} \sinh\left((2n+1)a\right)}{2n+1}$$ (7)

References

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